Department of Economics University of Maryland

Economics 325

Intermediate Macroeconomic Analysis

Final Exam Suggested Solutions

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NAME: ______________________________

The Exam has a total of four (4) problems and pages numbered one (1) through nine (9). Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.** You are to answer all questions in the spaces provided.

You may use two pages (double-sided) of notes. You may **not** use a calculator.

Problem 1	**/ 25**
Problem 2	**/ 25**
Problem 3	**/ 25**
Problem 4	**/ 25**
TOTAL	**/ 100**

Problem 1: Consumption and Savings in the Two-Period Economy (25 points). Consider a two-period economy (with no government), in which the representative consumer has no control over his income. The lifetime utility function of the representative consumer is $u\left(c_1, c_2\right) = \ln c_1 + \ln c_2$, where $\ln$ stands for the natural logarithm. We will work here in purely real terms: suppose the consumer's **present discounted value of ALL lifetime REAL income is 26.** Suppose that the real interest rate between period 1 and period 2 is zero (i.e., r = 0), and also suppose the consumer begins period 1 with zero net assets.

a. **(17 points)** Set up the lifetime Lagrangian formulation of the consumer's problem, in order to answer the following: i) is it possible to numerically compute the consumer's optimal choice of consumption in period 1? If so, compute it; if not, explain why not. ii) is it possible to numerically compute the consumer's optimal choice of consumption in period 2? If so, compute it; if not, explain why not. iii) is it possible to numerically compute the consumer's real asset position at the end of period 1? If so, compute it; if not, explain why not.

Solution: We know that with zero initial assets, the LBC of the consumer is

$$c_1 + \frac{c_2}{1+r_1} = y_1 + \frac{y_2}{1+r_1},$$

where the notation is standard from class. The Lagrangian is thus

$$u(c_1, c_2) + \lambda\left[y_1 + \frac{y_2}{1+r_1} - c_1 - \frac{c_2}{1+r_1}\right],$$

where λ of course is the Lagrange multiplier (note there's only one multiplier since this is the lifetime formulation of the problem not the sequential formulation of the problem). The first-order conditions with respect to c_1 and c_2 (which are the objects of choice) are, as usual:

$$u_1(c_1, c_2) - \lambda = 0$$

$$u_2(c_1, c_2) - \frac{\lambda}{1+r_1} = 0$$

(And of course the FOC with respect to the multiplier just gives back the LBC.) Also as usual, these FOCs can be combined to give the consumption-savings optimality condition, $\frac{u_1(c_1, c_2)}{u_2(c_1, c_2)} = 1 + r_1$. With the given utility function, the marginal utility functions are $u_1 = 1/c_1$ and $u_2 = 1/c_2$, so the consumption-savings optimality condition in this case becomes $c_2 / c_1 = 1 + r_1$. This can be rearranged to give $c_2 = (1+r_1)c_1$, which we can then insert in the LBC to give $c_1 + c_1 = y_1 + \frac{y_2}{1+r_1}$ (no, that's not a typo, it's $c_1 + c_1$ after the substitution…).

In this problem, you are given neither y_1 nor y_2. Instead, what you are given is $y_1 + \frac{y_2}{1+r_1} = 26$.

Thus, we have that the optimal quantity of period-1 consumption is $c_1^* = 13$ (which solves part i). We can **not** compute c_2^*, however, because we are not given the interest rate r_1 (which you would need in order to use the expression $c_2 = (1+r_1)c_1$ computed above. (This solves part ii). To compute the asset position at the end of period 1, we would need to compute $y_1 - c_1^*$, but since we don't know y_1, we cannot compute this either (which solves part iii).

Problem 1 continued

b. **(8 points)** To demonstrate how important the concept of the real interest rate is in macroeconomics, an interpretation of it (in addition to the several different interpretations we have already discussed in class) is that it reflects the rate of consumption growth between two consecutive periods. Using the consumption-savings optimality condition for the given utility function, **briefly** describe/discuss **(rambling essays will not be rewarded)** whether the real interest rate is **positively related to, negatively related to, or not at all related to the rate of consumption growth between period one and period two.** For your reference, the definition of the rate of consumption growth rate between period 1 and period 2 is $\frac{c_2}{c_1} - 1$ (completely analogous to how we defined in class the rate of growth of prices between period 1 and period 2). (**Note:** No mathematics are especially required for this problem; also note this part can be fully completed even if you were unable to get all the way through part a).

Solution:

The familiar consumption-savings optimality condition is $\frac{u_1}{u_2} = 1 + r$. As we just saw above, for the given utility function, this becomes $\frac{1/c_1}{1/c_2} = 1 + r$, or, rewriting,

$$\frac{c_2}{c_1} = 1 + r.$$

The left-hand-side of this expression obviously measures the consumption growth rate between period 1 and period 2. That is, if $c_1 = 100$ and $c_2 = 103$, clearly the consumption growth rate is 3 percent between period 1 and period 2. Which would mean that $r = 0.03$. If the real interest rate were instead larger, clearly the left-hand-side, c_2/c_1, would be larger as well. Thus, **the higher is the real interest rate, the higher is the consumption growth rate between periods – the real interest rate and the consumption growth rate are positively related to each other.**

This is thus yet another way to think about the real interest rate. The two other ways we discussed in class of thinking intuitively about the real interest rate is that it measures the price of current (period-1) consumption in terms of future (period-2) consumption; and as reflecting the fundamental degree of (human) impatience of individuals in the economy. All of these various (and ultimately inter-related) ways of thinking about the real interest underline its fundamental importance in macroeconomic theory.

Note that simply arguing/explaining here that a rise in the real interest rate leads to a fall in period-1 consumption does not address the question – the question is about the **rate of change of consumption between period 1 and period 2,** not about the **level** of consumption in period 1 by itself.

Problem 2: The Keynesian-RBC-New Keynesian Evolution (25 points). Here you will briefly analyze aspects of the evolution in macroeconomic theory over the past 25 years. Address each of the following **in no more than three brief phrases/sentences each.**

a. **(8 points)** Describe **briefly** what the Lucas critique is and how/why it led to the demise of (old) Keynesian models.

Solution: The old Keynesian models were large estimated systems of equations, and the estimated coefficients could not (because they were just based on historical observations) take into account how behavior might change if policy changed. In the 1970's, this led to the downfall of such models as policy-makers tried more and more to exploit these relationships, but the "coefficients" began to vary a lot (for some reason…) with policy, eventually causing the profession (through the Lucas critique) to understand that such models really were not all that useful for policy advice after all.

b. **(9 points)** In writing down utility functions and production functions for use in "RBC-style" macro models, the assumed functions are typically "estimated" using data (i.e., a common assumption is the logarithmic utility function we have often used, based on some statistical evidence that it is consistent with observed microeconomic and macroeconomic evidence). Is this practice subject to a "Lucas-type critique?" **Briefly** explain why or why not?

Solution: Yes, it seems that this practice is also subject to a Lucas-type critique – the parameters/coefficients in the utility and production functions, for example, **could** in principle be dependent on policy. If they are, and policy changes in particular way that, say, changes consumers' utility functions, then the same pitfalls facing the old Keynesian models could arise. So far, it seems we have not witnessed this aspect of the Lucas critique.

c. **(8 points)** **Briefly** define and describe the neutrality vs. nonneutrality debate surrounding monetary policy today. Which type of shock does this debate concern?

Solution: The RBC view holds that money shocks do not affect real variables (i.e., consumption or GDP) in the economy (neutrality), while the New Keynesian view holds that they do (nonneutrality) because prices take time to adjust (are "sticky").

Problem 3: Optimal Tax Policy (25 points). Consider our static (i.e., one period) consumption-leisure framework from Chapter 2. In this problem, you will use this framework as a basis for offering guidance regarding optimal (i.e., the "best") labor income tax policy.

Recall the basic consumption-leisure optimality condition

$$\frac{u_l(c,l)}{u_c(c,l)} = (1-t)w,$$

in which all of the notation is as in Chapter 2: t denotes the labor income tax rate, w denotes the real wage, c denotes consumption, l denotes leisure, u_c denotes the marginal utility of consumption, and u_l denotes the marginal utility of leisure.

Suppose that **firms are monopolistically competitive** (rather perfectly competitive). It can be shown in this case that when firms are making their profit-maximizing choice regarding labor hiring, the following condition is true:

$$mpn = w(1-monpol).$$

Here, *mpn* denotes the marginal product of labor and *monpol* is a measure of the degree of monopoly power that firms wield. For example, if *monpol* = 0, then firms wield no monopoly power whatsoever, in which case we are back to our perfectly-competitive framework of firm profit maximization from Chapter 6. If instead *monpol* > 0, then firms do wield some monopoly power. (**Notes:** The variable *monpol* can never be less than zero. You also do not need to be concerned here with how the above expression is derived – just take it as given. **Further,** note that there are **no** financing constraint issues here whatsoever.)

Suppose the following:

1. The only goal policy makers have in choosing a labor tax rate t is to ensure that the **perfectly-competitive outcome in labor markets is attained.**
2. Any monopoly power that firms have cannot be **directly** eliminated by policy makers. That is, if *monpol* > 0, the government cannot do anything about that; all the government can do is choose a tax rate t.

Based on all of the above, derive a relationship between **the optimal (i.e., in the sense that it attains the goal of policymakers described in point #1 above) labor income tax rate and the degree of firms' monopoly power. Very carefully explain all your logic and arguments**, including any mathematical derivations involved. (**Note:** There are a number of logical steps to the argument, which are left to you to determine.)

(OVER)

Problem 3 continued

Solution:

Note for use below that we can express the second equation above as $w = \frac{mpn}{1-monpol}$.

The first logical step in the argument is the observation that there is *no* monopoly power if markets are perfectly competitive (by definition, obviously), in which case *monpol* = 0. In this case, we have that *mpn* = *w*. Putting this conclusion together with the consumption-leisure optimality condition gives us

$$\frac{u_l(c,l)}{u_c(c,l)} = (1-t)mpn.$$

By the basic theory of perfect competition, you had to then recognize (implicitly or explicitly) that any tax rate different from zero creates a "wedge" (i.e., a deadweight loss) in the labor market. Hence since perfect competition means *zero* deadweight losses, the optimal tax policy in the case of *monpol* = 0 is *t* = 0. This implies that in perfect competition, it must be that $\frac{u_l(c,l)}{u_c(c,l)} = mpn$.

Now let's generalize the argument for the case of *monpol* > 0. Inserting the expression $w = \frac{mpn}{1-monpol}$ into the consumption-leisure optimality condition, we have

$$\frac{u_l(c,l)}{u_c(c,l)} = \frac{(1-t)mpn}{1-monpol}.$$

The goal of tax policy now is to pick a *t* so that the perfect-competition outcome $\frac{u_l(c,l)}{u_c(c,l)} = mpn$ is achieved despite the fact that *monpol* > 0. Examining the previous condition, it is clear that setting *t* = *monpol* achieves the perfect-competition outcome.

Thus, the optimal labor-income tax rate is *t* = *monpol*.

Problem 4: Financing Constraints and Labor Demand (25 points). In our class discussion about the way in which financing constraints affect firms' profit maximization decisions, we focused on the effects on firms' physical capital investment. In reality, most firms spend twice as much on their wage costs (i.e., their labor costs) than on their physical investment costs. (That is, for most firms, roughly two-thirds of their total costs are wages and salaries, while roughly one-third of their total costs are devoted to improving or expanding their physical capital.)

For many firms, payment of wages must be made **before** the receipt of revenues within any given period. (For example, imagine a firm that has to pay its employees to build a computer; the revenues from the sale of this computer typically don't arrive for many weeks or months later because of delays in the shipping process, the retail process, etc.) For this reason, firms typically need to borrow to pay for their ongoing wage costs.[1] But, because of asymmetric information problems, lenders typically require that the firm put up some financial collateral to secure loans for this purpose.

Here, you will analyze the consequences of financing constraints on firms' wage payments using a variation of the accelerator framework we studied in class.

For simplicity, suppose that the representative firm, which operates in a two-period economy, must borrow in order to finance only period-2 wage costs; for some unspecified reason, suppose that period-1 wage costs are not subject to a financing constraint.

As in our study of the accelerator framework in class, the representative firm's two-period discounted profit function is

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1$$
$$+ \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i}$$

and suppose now the financing constraint that is relevant for firm profit-maximization is

$$\frac{P_2 w_2 n_2}{1+i} = S_1 a_1 .$$

(The present-discounted-value appears on the left-hand-side because we are conducting the analysis, as always, from the perspective of the beginning of period 1.) The notation is as always: P denotes the nominal price of the output the firm produces and sells; S denotes the nominal price of stock; D denotes the nominal dividend paid by each unit of stock; n denotes the quantity of labor the firm hires; w is the **real** wage; a_0, a_1, and a_2 are, respectively, the firm's holdings of stock at the end of period 0, period 1, and period 2; k_1, k_2, and k_3 are, respectively, the firm's ownership of physical capital at the end of period 0, period 1, and period 2; i denotes the nominal interest rate between period 1 and period 2; and the production function is denoted by $f(.)$. Also as usual, subscripts on variables denote the time period of reference for that variable. Finally, because this is a two-period framework, we know $a_2 = 0$ and $k_3 = 0$. **(OVER)**

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[1] The commercial paper market, about which much has been discussed in the news media in the past year, is one type of channel for such firm financing needs.

Problem 4 continued

The Lagrangian for the firm's profit maximization problem is thus

$$
\begin{aligned}
&P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1)a_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 \\
&+ \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2)a_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i} \\
&+ \lambda \left[S_1 a_1 - \frac{P_2 w_2 n_2}{1+i} \right]
\end{aligned}
$$

in which λ denotes the Lagrange multiplier on the financing constraint.

a. **(5 points)** Based on the Lagrangian above, compute the first-order conditions with respect to k_2 and a_1.

Solution: The first-order conditions are simply:

$$
\begin{aligned}
&-P_1 + \frac{P_2 f_k(k_2, n_2)}{1+i} + \frac{P_2}{1+i} = 0 \\
&-S_1 + \frac{S_2 + D_2}{1+i} + \lambda S_1 = 0
\end{aligned}
$$

b. **(5 points)** Based on the Lagrangian above, compute the first-order conditions with respect to n_1 and n_2.

Solution: The first-order conditions are simply:

$$
\begin{aligned}
&P_1 f_n(k_1, n_1) - P_1 w_1 = 0 \\
&\frac{P_2 f_n(k_2, n_2)}{1+i} - \frac{P_2 w_2}{1+i} - \frac{\lambda P_2 w_2}{1+i} = 0
\end{aligned}
$$

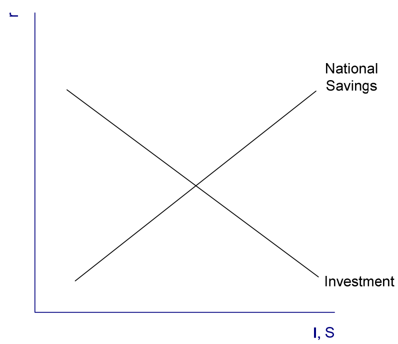
Note for reference below that the second equation here can be expressed as $f_n(k_2, n_2) = w_2(1+\lambda)$.

Problem 4 continued

Suppose that at the beginning of period 1, the real return on STOCK, r^{STOCK}, all of a sudden falls below r, the real return on riskless ("safe") assets. Suppose that before this shock occurred (i.e., in "period zero"), it was the case that $r = r^{STOCK}$.

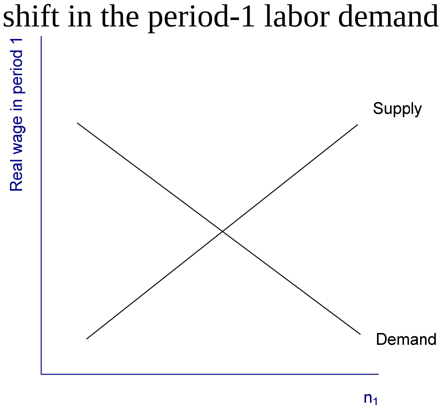
c. **(5 points)** Below is a graph of the investment (capital) market in period 1. Does the adverse shock to r^{STOCK} shift either the investment demand and/or the savings supply function? If so, explain how, in what direction, and why.

Solution: The investment demand function is unaffected by the financing constraint (see the FOC on $k1$ above), hence exogenous changes in $rSTOCK$ have no effect on capital demand/investment demand. Furthermore, because nothing is said about whether financing frictions impinge on the savings supply side of the economy (i.e., on consumers' consumption-savings decisions), there is no basis for asserting any shift of the savings function. Hence, there is no direct effect on the market for physical capital.



d. (5 points) Below is a graph of the labor market in period 1. Does the adverse shock to r^{STOCK} shift either the labor demand and/or the labor supply function? If so, explain how, in what direction, and why.

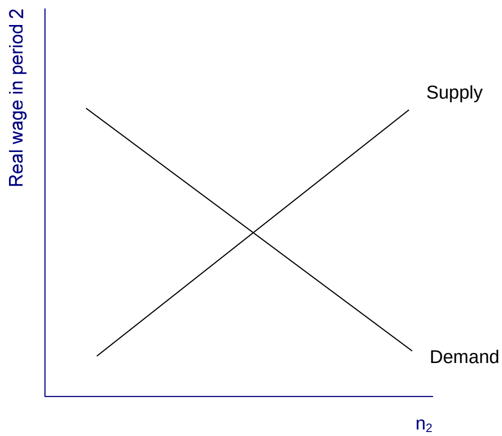
Solution: No shift in labor supply because, as above, no statements are made about whether financing frictions affect consumers' behavior (which is what would be required for a shift of the labor supply function). The financing constraint does NOT affect period-1 wage payments, hence a fall in $rSTOCK$ has no direct effect on the period-1 labor demand function: there is no shift in the period-1 labor demand function.



Problem 4 continued

e. **(5 points)** Below is a graph of the labor market in period 2. Does the adverse shock to r^{STOCK} shift either the labor demand and/or the labor supply function? If so, explain how, in what direction, and why.

Solution: No shift in labor supply because, once again, no statements are made about whether financing frictions affect consumers' behavior (which is what would be required for a shift of the labor supply function. A fall in *rSTOCK* will cause a RISE in λ, hence for a given level of $w2$, the "effective" marginal product of period-2 labor FALLS, hence the labor demand curve shifts **inwards**. This effect arises because λ directly appears in the period-2 FOC on labor above.



Period 2 Labor Market

END OF EXAM