Economics 325
Intermediate Macroeconomic Analysis
Midterm Exam – Suggested Solutions
Professor Sanjay Chugh
Fall 2009

NAME: _____

Each problem's total number of points is shown below. Your solutions should consist of some appropriate combination of mathematical analysis, graphical analysis, logical analysis, and economic intuition, but in no case do solutions need to be exceptionally long. Your solutions should get straight to the point – **solutions with irrelevant discussions and derivations will be penalized.**   You are to answer all questions in the spaces provided.

You may use one page (double-sided) of notes. You may **not** use a calculator.

TOTAL PART 1	**/ 50**
TOTAL PART 2	**/ 50**
TOTAL	**/ 100**

Problem 1: Core Inflation and Non-Core Inflation in the Two-Period Economy (30 points).
Two distinct measures of inflation – called core inflation and non-core inflation – generally
attract attention by policy-makers and the media. The core inflation rate is the rate of growth of
prices of so-called "core goods" (such as food, clothing, and shelter), while the non-core
inflation rate is the rate of growth of prices of so-called "non-core goods" (generally energy
items).

Consider our usual two-period economy (with no government), in which the representative
consumer has no control over his **nominal** income. Rather than there being only one "type" of
good the consumer purchases each period, however, suppose that each period there are two
"types" of goods: core good and non-core goods. The lifetime utility function of the
representative consumer is

$$u\left(c_1^{CORE}, c_1^{NONCORE}, c_2^{CORE}, c_2^{NONCORE}\right) = \ln\left(c_1^{CORE}\right) + \ln\left(c_1^{NONCORE}\right) + \ln\left(c_2^{CORE}\right) + \ln\left(c_2^{NONCORE}\right),$$

where $\ln$ stands for the natural logarithm, c_1^{CORE} stands for consumption of core goods in period
1, $c_1^{NONCORE}$ stands for consumption of non-core goods in period 1, and similarly for c_2^{CORE} and
$c_2^{NONCORE}$.

The representative consumer begins period one with zero assets (i.e., $A_0 = 0$). The period-by-
period budget constraints of the representative consumer are thus

$$P_1^{CORE} c_1^{CORE} + P_1^{NONCORE} c_1^{NONCORE} + A_1 = Y_1$$
$$P_2^{CORE} c_2^{CORE} + P_2^{NONCORE} c_2^{NONCORE} + A_2 = Y_2 + (1+i) A_1$$

where P_1^{CORE} denotes the **nominal** price of core goods in period 1, $P_1^{NONCORE}$ denotes the **nominal**
price of non-core goods in period 1, and similarly for P_2^{CORE} and $P_2^{NONCORE}$. As usual, Y_1 and Y_2
denote **nominal** income in periods 1 and 2, respectively, and i is the nominal interest rate.

Finally, we can construct as usual the representative consumer's nominal lifetime budget
constraint, which here is:

$$P_1^{CORE} c_1^{CORE} + P_1^{NONCORE} c_1^{NONCORE} + \frac{P_2^{CORE} c_2^{CORE}}{1+i} + \frac{P_2^{NONCORE} c_2^{NONCORE}}{1+i} = Y_1 + \frac{Y_2}{1+i}$$

This nominal LBC has the same interpretation as always: the PDV of all lifetime nominal
consumption (which here takes into account both consumption of core and non-core goods) is
equal to the PDV of all lifetime nominal income. That is, in a lifetime sense, all consumption
spending equals all income regardless of "how many" goods there are to purchase.
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Problem 1 continued

a. **(18 points)** Using an appropriate **nominal** Lagrange formulation (you are free to choose either a lifetime formulation or a sequential formulation), derive **four** optimality conditions: one between period-1 core consumption and period-1 non-core consumption; one between period-2 core consumption and period-2 non-core consumption; one between period-1 core consumption and period-2 core consumption; and one between period-1 non-core consumption and period-2 non-core consumption. These optimality conditions should be based on the utility function given above. Show all important steps.

Solution: Taking the lifetime approach, the Lagangian is

$$\ln\left(c_1^{CORE}\right) + \ln\left(c_1^{NONCORE}\right) + \ln\left(c_2^{CORE}\right) + \ln\left(c_2^{NONCORE}\right)$$
$$+ \lambda\left[Y_1 + \frac{Y_2}{1+i} - P_1^{CORE}c_1^{CORE} - P_1^{NONCORE}c_1^{NONCORE} - \frac{P_2^{CORE}c_2^{CORE}}{1+i} - \frac{P_2^{NONCORE}c_2^{NONCORE}}{1+i}\right]$$

The FOCs are

$$\frac{1}{c_1^{CORE}} - \lambda P_1^{CORE} = 0$$

$$\frac{1}{c_1^{NONCORE}} - \lambda P_1^{NONCORE} = 0$$

$$\frac{1}{c_2^{CORE}} - \frac{\lambda P_2^{CORE}}{1+i} = 0$$

$$\frac{1}{c_2^{NONCORE}} - \frac{\lambda P_2^{NONCORE}}{1+i} = 0$$

Solving out for λ appropriately and then rearranging, we have that the four optimality conditions are, in order,

**Problem 1a continued (if you need more space)**

$$\frac{c_1^{NONCORE}}{c_1^{CORE}} = \frac{P_1^{CORE}}{P_1^{NONCORE}}$$

$$\frac{c_2^{NONCORE}}{c_2^{CORE}} = \frac{P_2^{CORE}}{P_2^{NONCORE}}$$

$$\frac{c_2^{CORE}}{c_1^{CORE}} = \frac{P_1^{CORE}(1+i)}{P_2^{CORE}}$$

$$\frac{c_2^{NONCORE}}{c_1^{NONCORE}} = \frac{P_1^{NONCORE}(1+i)}{P_2^{NONCORE}}$$

b. **(6 points)** In each of the following three diagrams, appropriately label the slope of the budget line in terms of variables defined above. **Briefly** describe how you determined the relevant slopes (you may refer to your work in part a if needed).

$c1^{NONCOR}$ $c2^{NONCOR}$ $c2^{COR}$

$c1^{COR}$ $c2^{COR}$ $c1^{COR}$
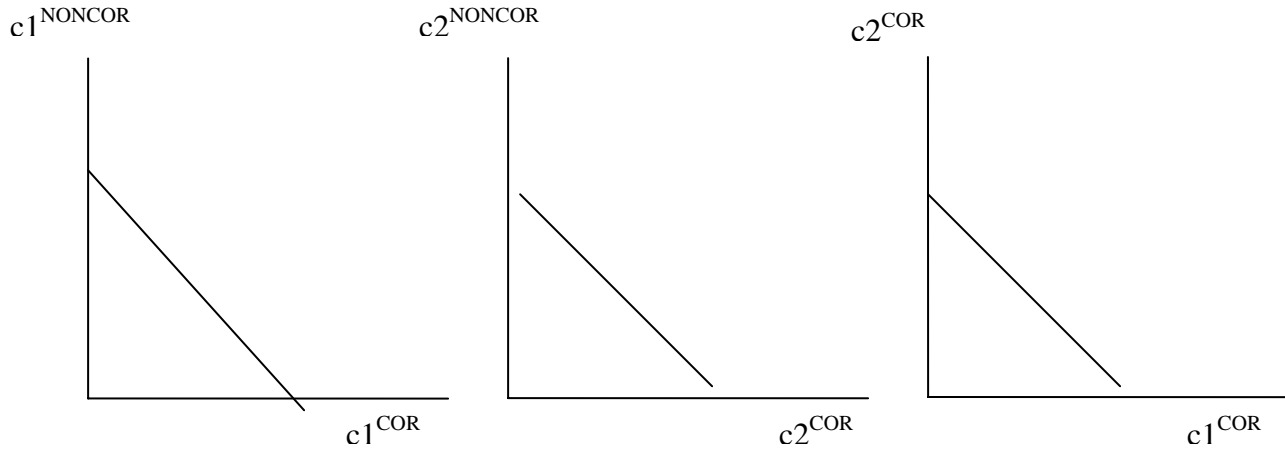
Solution: Based on the computed optimality conditions in part b, it is easy to conclude that the slope in the left-most diagram is $-P_1^{CORE}/P_1^{NONCORE}$; the slope in the middle diagram is $-P_2^{CORE}/P_2^{NONCORE}$; and the slope in the right-most diagram is $-P_1^{CORE}(1+i)/P_2^{CORE}$. This follows from the fact that in each of the optimality conditions in part b, the right-hand-side represents the (absolute value of the) slope of the relevant budget line. Alternatively, you could have take the LBC constructed in part a and successively rearranged it to isolate $c_1^{NONCORE}$, $c_2^{NONCORE}$, and c_2^{CORE}, which would give you (in each line, the focus is just on the relevant slope term):

$$c_1^{NONCORE} = -\frac{P_1^{CORE}}{P_1^{NONCORE}} c_1^{CORE} + (\text{other stuff})$$

$$c_2^{NONCORE} = -\frac{P_2^{CORE}}{P_2^{NONCORE}} c_2^{CORE} + (\text{other stuff})$$

$$c_2^{CORE} = -\frac{P_1^{CORE}(1+i)}{P_2^{CORE}} c_1^{CORE} + (\text{other stuff})$$

Problem 1 continued

Suppose that the Fed changes the nominal interest rate, and suppose that this change in policy does not at all affect P_1^{CORE}, $P_1^{NONCORE}$, P_2^{CORE}, or $P_2^{NONCORE}$, nor does it affect Y_1 or Y_2.

c. **(3 points)** At the resulting new optimal choice, will this change in monetary policy affect consumers' MRS between period-1 core and period-1 non-core consumption? If so, briefly explain why/how; if not, briefly explain why not. (You may refer to the diagrams in part b if needed.)

Solution: Simply examining the analysis in parts a and b (and with the stated assumption that the monetary policy action has no effect on prices), it is clear that the slopes of the budget lines in the left-most and middle diagrams above will be completely unaffected. Thus, at consumers' optimal choices, their MRS along the period-1 core/period-1 non-core margin (i.e., the left-most diagram above) will be **un**affected.

d. **(3 points)** At the resulting new optimal choice, will this change in monetary policy affect consumers' MRS between period-1 core and period-2 core consumption? If so, briefly explain why/how; if not, briefly explain why not. (You may refer to the diagrams in part b if needed.)

Solution: In contrast to the analysis above, now the focus is on the right-most diagram above. The right-most diagram describes consumption decisions that **span time periods.** Hence, the relevant price across time periods involves interest rates (recall our general interpretation of interest rates as the "price of time"). Thus, when the Fed changes interest rates (note that in this problem, changes in nominal interest rates and changes in real interest rates are one and the same because you are told to assume that **inflation** never changes), the slope of the budget line in the right-most diagram will indeed be affected. Hence, at consumers' optimal choices, their MRS along the period-1 core/period-2 core margin **will** be affected.

Problem 2: Quasi-Linear Utility (20 points). In the static consumption-leisure model, suppose the representative consumer has the following utility function over consumption and leisure,

$$u(c,l) = \ln(c) + A \cdot l,$$

where, as usual, c denotes consumption and l denotes leisure. In this utility function, $\ln(\cdot)$ is the natural log function, and A is a number (a constant) smaller than one that governs how much utility the individual obtains from a given amount of leisure. Suppose the budget constraint the individual faces is simply $c = (1-t) \cdot w \cdot n$, where t is the labor tax rate, w is the **real** hourly wage rate, and n is the number of hours the individual works. (Notice that this budget constraint is expressed in real terms, rather than in nominal terms.)

a. **(3 points)** Does this utility function display diminishing marginal utility in consumption? Briefly explain.

Solution: Marginal utility with respect to consumption is the slope of the utility function when viewed as a function of consumption alone. The given function is logarithmic in consumption, and the natural log function, is strictly increasing and strictly concave, meaning the slope with respect to consumption is always decreasing and asymptotes to zero. Hence, this function does display diminishing marginal utility in consumption.

b. **(3 points)** Does this utility function display diminishing marginal utility in leisure? Briefly explain.

Solution: Just as above, marginal utility with respect to leisure is the slope of the utility function when viewed as a function of leisure alone. The given function is **linear** in leisure, hence its slope with respect to leisure is constant. Thus, this function does **not** display diminishing marginal utility in leisure.

c. **(14 points)** Assume (as usual) the representative consumer maximizes utility. For the given utility function, **plot this representative consumer's labor supply function**, explaining the logic behind your plotted function. Also, how would a decrease in the tax rate t affect the optimal amount of labor supply (i.e., increase it, decrease it, or leave it unchanged)? **Carefully explain your logic/derivation. (Note: Be sure to base your analysis here on the utility function that is _given_ above.)**

Solution: The consumption-leisure optimality condition (which can be derived using a Lagrangian, which is omitted here because the general derivation proceeds exactly as we've seen several times) is

$$\frac{u_l}{u_c} = \frac{A}{1/c} = (1-t) \cdot w,$$

from which we get that $Ac = (1-t)w$ at the consumer's optimal choice. Substituting the given budget constraint into this (i.e., substituting for c) we have $A \cdot (1-t) \cdot w \cdot n = (1-t) \cdot w$. Canceling terms and solving for n, we find

$$n = \frac{1}{A},$$

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Problem 2c continued (if you need more space)

which shows that labor supply here is independent of taxes, hence changes in the tax rate cannot affect the quantity of labor. The labor supply function, plotted with the wage (pre- or after-tax, it doesn't make a difference) on the vertical axis and n on the horizontal axis, is a vertical line at the numerical value $1/A$. This (perfectly inelastic) labor supply is clearly unaffected by changes in taxes; indeed, it is completely unaffected by changes in changes in the pre-tax real wage w as well.

Further discussion (which was not required): The reason why labor (equivalently, leisure) here doesn't depend at all on the (pre- or after-tax) wage is that there is no diminishing marginal utility in leisure (i.e., utility is linear with respect to leisure, as we saw above). When a multi-dimensional utility function is linear in one argument and has diminishing marginal utility in its other argument(s), it is said to be "quasi-linear." Quasi-linear utility functions give rise to demand functions for the linear object that are completely insensitive (inelastic) to price – here, the demand for leisure (the flip side of which is the supply of labor) is completely insensitive (inelastic) to the wage.

Problem 3: "Hyperbolic Impatience" and Stock Prices (28 points). In this problem you will study a slight extension of the infinite-period economy from Chapter 8. Specifically, suppose the representative consumer has a lifetime utility function given by

$$u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + ...,$$

in which, as usual, $u(.)$ is the consumer's utility function in any period and β is a number between zero and one that measures the "normal" degree of consumer impatience. **The number γ (the Greek letter "gamma," which is the new feature of the analysis here) is also a number between zero and one, and it measures an "additional" degree of consumer impatience, but one that ONLY applies between period t and period t+1.[1] This latter aspect is reflected in the fact that the factor γ is NOT successively raised to higher and higher powers as the summation grows.**

The rest of the framework is exactly as studied in Chapter 8: a_{t-1} is the representative consumer's holdings of stock at the beginning of period t, the nominal price of each unit of stock during period t is S_t, and the nominal dividend payment (per unit of stock) during period t is D_t. Finally, the representative consumer's consumption during period t is c_t and the nominal price of consumption during period t is P_t. As usual, analogous notation describes all these variables in periods $t+1$, $t+2$, etc.

The Lagrangian for the representative consumer's utility-maximization problem (starting from the perspective of the beginning of period t) is

$$u(c_t) + \gamma\beta u(c_{t+1}) + \gamma\beta^2 u(c_{t+2}) + \gamma\beta^3 u(c_{t+3}) + ...$$
$$+\lambda_t\left[Y_t + (S_t + D_t)a_{t-1} - P_t c_t - S_t a_t\right]$$
$$+\gamma\beta\lambda_{t+1}\left[Y_{t+1} + (S_{t+1} + D_{t+1})a_t - P_{t+1}c_{t+1} - S_{t+1}a_{t+1}\right]$$
$$+\gamma\beta^2\lambda_{t+2}\left[Y_{t+2} + (S_{t+2} + D_{t+2})a_{t+1} - P_{t+2}c_{t+2} - S_{t+2}a_{t+2}\right]$$
$$+\gamma\beta^3\lambda_{t+3}\left[Y_{t+3} + (S_{t+3} + D_{t+3})a_{t+2} - P_{t+3}c_{t+3} - S_{t+3}a_{t+3}\right]$$
$$+...$$

NOTE CAREFULLY WHERE THE "ADDITIONAL" IMPATIENCE FACTOR γ APPEARS IN THE LAGRANGIAN.

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[1] The idea here, which goes under the name "hyperbolic impatience," is that in the "very short run" (i.e., between period t and period t+1), individuals' degree of impatience may be different from their degree of impatience in the "slightly longer short run" (i.e., between period t+1 and period t+2, say). "Hyperbolic impatience" is a phenomenon that routinely recurs in laboratory experiments in experimental economics and psychology, and has many far-reaching economic, financial, policy, and societal implications.

Problem 3 continued

a. **(4 points)** Compute the first-order conditions of the Lagrangian above with respect to **both** a_t **and** a_{t+1}. (**Note:** There is no need to compute first-order conditions with respect to any other variables.)

Solution: The two FOCs are

$$-\lambda_t S_t + \gamma\beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0$$
$$-\gamma\beta\lambda_{t+1}S_{t+1} + \gamma\beta^2\lambda_{t+2}(S_{t+2} + D_{t+2}) = 0$$

b. **(4 points)** Using the first-order conditions you computed in part a, construct two distinct stock-pricing equations, one for the price of stock in period t, and one for the price of stock in period $t+1$. Your final expressions should be of the form $S_t = ...$ and $S_{t+1} = ...$ (**Note:** It's fine if your expressions here contain Lagrange multipliers in them.)

Solution: Simply rearranging the two FOCs above and canceling the γ term (along with one β term) in the second FOC, we have

$$S_t = \frac{\gamma\beta\lambda_{t+1}}{\lambda_t}(S_{t+1} + D_{t+1})$$

$$S_{t+1} = \frac{\beta\lambda_{t+2}}{\lambda_{t+1}}(S_{t+2} + D_{t+2})$$

For the questions next, observe that the S_t expression and the S_{t+1} expression are subtly, but importantly, different here. They would be identical to each other (other than the fact that the time subscripts are different, but that is as usual) if and only if $\gamma = 1$. If $\gamma < 1$, which is the case of "hyperbolic impatience," then stock prices are determined in a somewhat "different way" in the "very short run" compared to the "longer short run" or "medium run."

Problem 3 continued

For the remainder of this problem, suppose that it is known that $D_{t+1} = D_{t+2}$, and that $S_{t+1}=S_{t+2}$, and that $\lambda_t = \lambda_{t+1} = \lambda_{t+2}$.
c. **(5 points).** Does the above information necessarily imply that the economy is in a steady-state? **Briefly and carefully explain why or why not; your response should make clear what the definition of a "steady state" is.** (**Note:** To address this question, it's possible, though not necessary, that you may need to compute other first-order conditions besides the ones you have already computed above.)

Solution: No, none of these statements necessarily imply that the economy is in a steady state, which, recall, means that all real variables become constant and never again change. There are two ways of observing that the above information does not imply the economy is in steady state. First, the above statements are all about **nominal** variables, and in a steady state it can be the case that nominal variables continue fluctuating over time, even though all real variables do not. Another way of arriving at the correct conclusion here is that the statements above only refer to periods t, t+1, and t+2. In a steady-state, (real) variables settle down to constant values **forever,** not just for a few time periods.

d. **(5 points)** Based on the above information and your stock-price expressions from part b, can you conclude that the period-t stock price (S_t) is higher than S_{t+1}, lower than S_{t+1}, equal to S_{t+1}, or is it impossible to determine? **Briefly and carefully explain the economics (i.e., the economic reasoning, not simply the mathematics) of your finding.**

Solution: You are given that nominal stock prices, nominal dividends, and the Lagrange multiplier in period t+1 and t+2 are equal to each other. Let's call these common values $\overline{S}$, $\overline{D}$, and $\overline{\lambda}$ (that is, $\overline{S} = S_{t+1} = S_{t+2}$ $\overline{D} = D_{t+1} = D_{t+2}$, and $\overline{\lambda} = \lambda_{t+1} = \lambda_{t+2}$). Inserting these common values in the period-t+1 stock price equation, we have $\overline{S} = \dfrac{\beta\overline{\lambda}}{\overline{\lambda}}(\overline{S} + \overline{D})$. Canceling terms, we have that the nominal stock price in period t+1 (and t+2) is $\overline{S} = \beta\overline{S} + \beta\overline{D}$ (which we could of course solve for the stock price as $\overline{S} = \dfrac{\beta}{1-\beta}\overline{D}$ if we needed to).

Now, using the common values of *S, D*, and the multiplier in the period-t stock price equation gives us $S_t = \gamma\beta(S_{t+1} + D_{t+1}) = \gamma\beta(\overline{S} + \overline{D}) = \gamma(\beta\overline{S} + \beta\overline{D})$. Note that the final term in parentheses is nothing more than $\overline{S}$, hence we have

$$S_t = \gamma\overline{S}.$$

If $\gamma < 1$, then clearly the stock-price in period t is smaller than it is in period t+1 (and period t+2). The economics of this is due to the "hyperbolic impatience" which makes consumers more impatient to purchase consumption in the "very short run" (period t) compared to the "longer short run." All else equal, this means that in the very short run, consumers' do not care to save as much (due to the their extreme impatience in the very short run), which means their demand for saving --- i.e., their demand for stock – is lower. Lower demand for stock means a lower price of stock, all else equal.

Problem 3 continued

Now also suppose that the utility function in every period is $u(c) = \ln c$, and also that the **real interest rate is zero in every period.**

e. **(5 points) Based on the utility function given, the fact that $r = 0$, and the basic setup of the problem described above, construct two marginal rates of substitution (MRS): the MRS between period-t consumption and period-t+1 consumption, and the MRS between period-t+1 consumption and period-t+2 consumption.**

Solution: This only requires examining the lifetime utility function (the first line of the Lagrangian above). By definition, the MRS between period t consumption and t+1 consumption is $\dfrac{u'(c_t)}{\gamma\beta u'(c_{t+1})} = \dfrac{c_{t+1}}{\gamma\beta c_t}$, and the MRS between period t+1 consumption and t+2 consumption is $\dfrac{\gamma\beta u'(c_{t+1})}{\gamma\beta^2 u'(c_{t+2})} = \dfrac{u'(c_{t+1})}{\beta u'(c_{t+2})} = \dfrac{c_{t+2}}{\beta c_{t+1}}$. Note that the form of the two MRS functions is different: the hyperbolic impatience affects the former MRS, but not the latter MRS.

f. **(5 points – Harder)** Based on the two MRS functions you computed in part e and on the fact that $r = 0$ in every period, determine which of the following two consumption growth rates

$$\frac{c_{t+1}}{c_t} \quad \text{OR} \quad \frac{c_{t+2}}{c_{t+1}}$$

is larger. That is, is the consumption growth rate between period t and period t+1 (the fraction on the left) expected to be larger than, smaller than, or equal to the consumption growth rate between period t+1 and period t+2 (the fraction on the right), or is it impossible to determine? **Carefully explain your logic, and briefly explain the economics (i.e., the economic reasoning, not simply the mathematics) of your finding.**

Solution: The basic consumption-savings optimality condition states that the MRS between two consecutive time periods is equated to (1+r). You are told here that r = 0 always. Based on the two MRS functions constructed above, then, it follows immediately that the consumption growth rate between period t and t+1 is **smaller than** the consumption growth rate between period t+1 and period t+2. This follows because $\gamma < 1$. The economics is similar to above: hyperbolic impatience makes consumers consume "much more" in the very short run (i.e., period t), which means that the growth rate of consumption between period t (already a very high consumption period) and t+1 will be low, compared to the similar comparison one period later.

Problem 4: Government Debt Ceilings (22 points). Just like we extended our two-period analysis of consumer behavior to an infinite number of periods, we can extend our two-period analysis of fiscal policy to an infinite number of periods.

The government's budget constraints (expressed in real terms) for the years 2009 and 2010 are

$$g_{2009} + b_{2009} = t_{2009} + (1+r)b_{2008}$$
$$g_{2010} + b_{2010} = t_{2010} + (1+r)b_{2009}$$

and analogous conditions describe the government's budget constraints in the years 2011, 2012, 2013, etc. The notation is as in Chapter 7: g denotes real government spending during a given time period, t denotes real tax revenue during a given time period (all taxes are assumed to be lump-sum here), r denotes the real interest rate, and b denotes the government's asset position (b_{2008} is the government's asset position at the end of the year 2008, b_{2009} is the government's asset position at the end of 2009, and so on).

At the end of 2008, the government's asset position was roughly a **debt** of $10 trillion (that is, b_{2008} = -$10 trillion).

The current fiscal policy plans/projections call for: g_{2009} = $4 trillion, t_{2009} = $2 trillion, g_{2010} = $3 trillion, and t_{2010} = $2 trillion.

Finally, given how low interest rates are right now and how low they are projected to remain for the next few years, suppose that the real interest rate is always zero (i.e., $r = 0$ always).

a. **(3 points)** Assuming the projections above prove correct, what will be the numerical value of the federal government's asset position at the end of 2009? Briefly explain/justify.

Solution: Using the given numerical values and using the 2009 government budget constraint given above, it is straightforward to calculate b_{2009} = -$12 trillion.

b. **(3 points)** Assuming the projections above prove correct, what will be the numerical value of the federal government's asset position at the end of 2010? Briefly explain/justify.

Solution: Using the given numerical values, the value for b_{2009} found in part a, and using the 2010 government budget constraint given above, it is straightforward to calculate b_{2010} = -$13 trillion.

Problem 4 continued

Under current federal law, the U.S. government's debt cannot be larger than $12 trillion at any point in time. This limit is known as the "debt ceiling."

c. **(3 points) Based on your answer in part a above,** does the debt ceiling pose a problem for the government's fiscal policy plans during the course of the year 2009? If it poses a problem, briefly describe the problem; if it poses no problem, briefly describe why it poses no problem.

Solution: No, the debt ceiling poses no problem for the fiscal policy plans for the year 2009. This is because the t and g plans call for a debt at the end of 2009 of $12 trillion, which does not exceed the ceiling.

d. **(3 points) Based on your answer in part b above,** does the debt ceiling pose a problem for the government's fiscal policy plans during the course of the year 2010? If it poses a problem, briefly describe the problem; if it poses no problem, briefly describe why it poses no problem.

Solution: Yes, the debt ceiling poses a problem for the fiscal policy plans for the year 2010. This is because the t and g plans call for a debt at the end of 2010 of $13 trillion, which violates the ceiling.

Suppose that the Obama administration only becomes aware of the $12 trillion debt ceiling at the very end of 2009 – to be precise, suppose the administration only becomes aware of it on December 31, 2009, when all of the year's spending and tax collections have ended. Furthermore, suppose Congress does not alter the debt ceiling at all.

e. **(3 points)** Will the government be **forced** (note the emphasis here) to change t_{2010} compared to the projection of $t_{2010} = 2 trillion? If not, explain why not. If so, explain in which direction (up or down)?

Solution: Because all that matters for the end-of-year debt position is the fiscal **flow** during the year 2010 (i.e., what matters is the difference $g_{2010} - t_{2010}$), **no,** the government does not **have** to increase taxes in 2010 in order to stay within the debt ceiling. The government could instead achieve the entire adjustment required to stay within the debt ceiling by cutting government spending in 2010, and leave taxes in 2010 unchanged.

Problem 4 continued
While we did not formally study the idea of a "government utility function," in complete analogy with consumer theory, we can imagine that the government has a "utility function" for its own spending.[2] Suppose the government's lifetime utility function, starting from the perspective of the very beginning of the year 2009, can be described by the function $u(g_{2009}, g_{2010}, g_{2011}, g_{2012}, ...)$, and this utility function satisfies all the "usual properties" we have been studying (i.e., it is strictly increasing in each argument, with diminishing marginal utility in each argument).
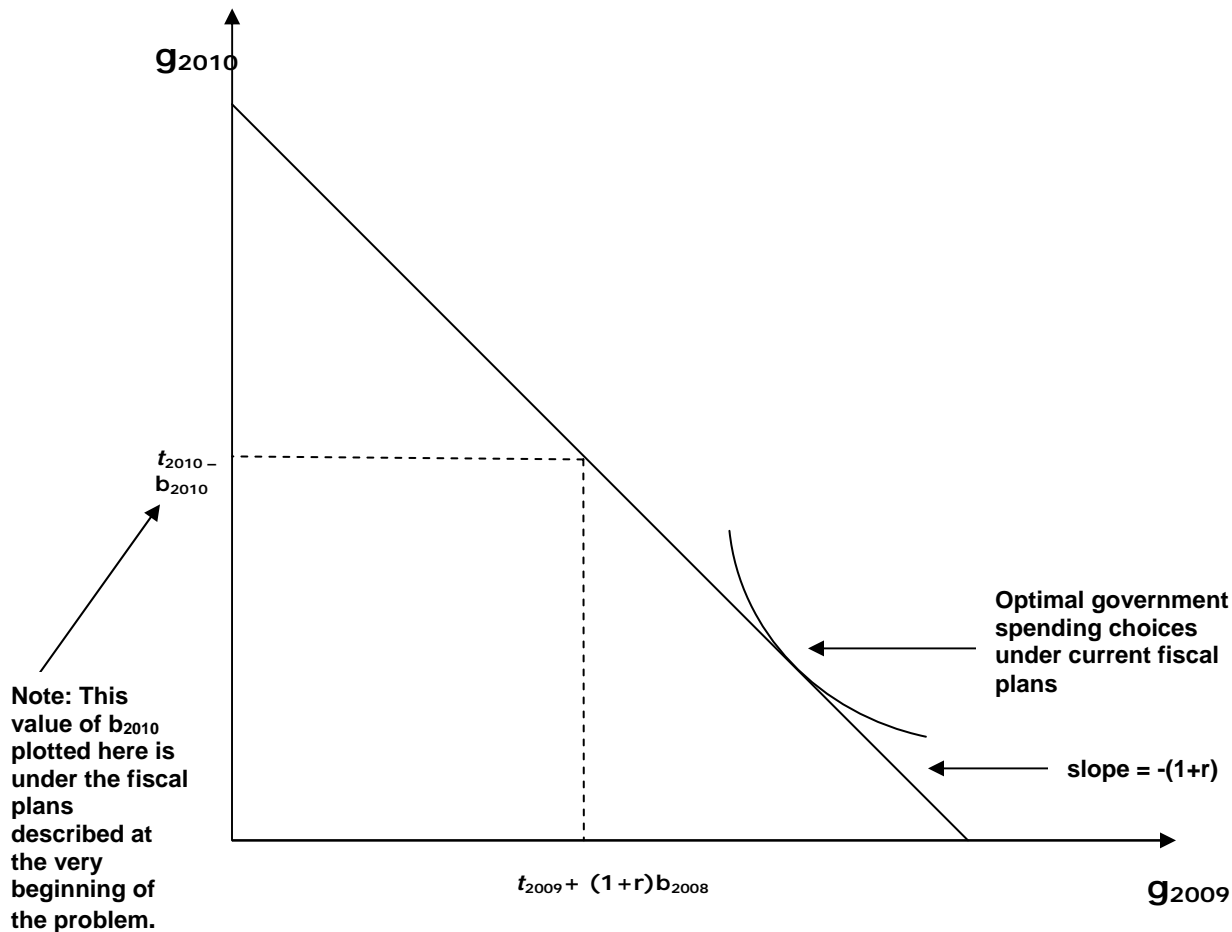
f. **(7 points – Harder)** The following diagram (on the next page) focuses on the two-year time span 2009-2010 and plots the government's budget constraint over the two-year time span along with the government's choices of g_{2009} and g_{2010} as described in parts a and b. The diagram below depicts these choices of g as optimal choices. **Note that this budget line is NOT a LIFETIME budget constraint because the government is NOT assumed to cease operations at the end of 2010.**

Suppose that the debt ceiling law never changes. **Furthermore, because of political reasons, it sometimes seems much easier to change government spending than to change taxes. Let's make this idea black-and-white by now supposing that taxes can never change.**

Once the Obama administration becomes aware of the debt ceiling on December 31, 2009, **illustrate in the diagram below any and all effects that must happen to comply with the debt ceiling. If there are no effects to illustrate, explain why there are none. If there are effects to illustrate, be clear to illustrate _all_ of them (it is up to you determine what and how many effects there are), and briefly explain your illustration. (Note:** examples of effects to illustrate may be things such as "the budget line pivots outward," etc.)

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[2] Ideally, the government's utility function is "benevolent" in the sense that it "should" reflect the needs and desires of its citizens, but corruption etc. can sometimes distort government utility functions. But let's leave aside such issues here and think of the government as "benevolent."

Problem 4g continued

g_{2010}

$t_{2010} - b_{2010}$

Note: This value of b_{2010} plotted here is under the fiscal plans described at the very beginning of the problem.

Optimal government spending choices under current fiscal plans

slope = -(1+r)

$t_{2009} + (1+r)b_{2008}$

g_{2009}

Solution: You are told that the administration only becomes aware of the debt ceiling at the end of 2009, at which point it is too late to change either g_{2009} or t_{2009}. Furthermore, you are told that t_{2010} cannot change (due to political reasons, say). The only way for the debt ceiling law to not be violated in 2010, then, is for the government to lower g_{2010}. Thus, the "restricted" choice of government spending must then lie directly vertically below the original optimal choice shown in the diagram (directly vertically below because, again, g_{2009} is something that can no longer be changed once December 31, 2009 arrives).

Passing through this new point will be a budget line that is parallel to the original one, but on which the vertical intercept is lower, by exactly the amount by which g_{2010} falls. (Indeed, the fall in g_{2010} is exactly mirrored by an equal rise in b_{2010} – see the 2010 budget constraint above.)