Department of Economics University of Maryland

Economics 325
Intermediate Macroeconomic Analysis
Problem Set 1
Professor Sanjay Chugh
Fall 2011
Due: Wednesday, October 5, 2011

Instructions: Written (typed is strongly preferred, but not required) solutions must be submitted no later than 2:00pm on the date listed above.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in groups to think through issues and ideas, but you must submit your own independently-written solutions. **Under no circumstances will multiple verbatim identical solutions be considered acceptable.**

Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be **clearly, logically, and thoroughly presented;** they should not leave the reader (i.e., your TAs and I) guessing about what you actually meant. Your method of argument(s) and approach to problems is as important as, if not more important than, your "final answer." Throughout, your analysis should be based on the frameworks, concepts, and methods developed in class.

There are a total of three problems, each with multiple subparts.

Problem 1: Consumption, Savings, and Borrowing Constraints (40 points). In this problem, you will **formally** study how borrowing constraints might affect the representative consumer's optimization problem. To keep things tractable, you will numerically study optimal choices when there are no borrowing constraints at all; and then you will study optimal choices when there are borrowing constraints that affect the consumer's optimal decisions (i.e., all of the analysis is Lagrangian analysis).

The representative consumer's lifetime utility function is $\ln c_1 + \ln c_2$, in which there is no discounting (of future utility) at all. There is also no government at all, hence taxes and government spending are always zero.

Numerical values for required items are: $y_1 = 5$, $y_2 = 15$, $a_0 = 5$, $r = 0.05$, $a_2 = 0$ (this last is as usual); furthermore, suppose this is a purely real economy (i.e., there is never any inflation).

For the first parts of the question, suppose there is no borrowing constraint at all.

a. **(3 points)** Set up a **sequential** Lagrange optimization problem consistent with the above facts.

b. **(8 points)** Based on the Lagrange optimization problem you constructed in part a, solve for the numerical values of the optimal choices of period-1 consumption and period-2 consumption.

c. **(4 points)** What is the numerical value of the consumer's asset position at the end of period 1? And, related, is period-1 savings of the consumer positive, negative, or zero? Briefly explain the economics.

For the remainder of the question, suppose there is a borrowing constraint. In particular, suppose the consumer can borrow zero during period one. For possible use in the Lagrangian below, write this term as

$$... + \mu \cdot (\text{zero borrowing})$$

where the ellipsis indicate things that come before the borrowing constraint, and $\mu > 0$ (the Greek letter "mu") is the Lagrange multiplier on the borrowing constraint**.**

d. **(4 points)** Starting from the sequential Lagrange you constructed in part a, what is now the Lagrange optimization problem? If there are no other terms in the problem, briefly explain why not. If there are other terms in the problem, briefly explain their economic content.

e. **(9 points)** Starting from the Lagrange optimization problem you constructed in part d, solve for the numerical values of the optimal choices of period-1 consumption and period-2 consumption.

f. **(4 points)** At the optimal choice computed in part e, what is the numerical value of the Lagrange multiplier on the borrowing constraint (i.e., of μ)?

g. **(4 points)** What is the numerical value of the consumer's asset position at the end of period 1? And, related, is period-1 savings of the consumer positive, negative, or zero? Are these answers different from, or identical to, your answers in part c? Briefly explain the economics.

h. **(4 points)** Under which scenario (no borrowing constraint, or a borrowing constraint that exists) is the individual's lifetime utility maximized? Briefly explain the economics.

Problem 2: Taxation Dynamics in the Two-Period Model (24 points). Suppose the government is considering how to balance its two-period (i.e., its lifetime) budget constraint. No matter what, it must be the case that $b_2 = 0$ (i.e., just like the representative consumer, the government cannot end its existence in debt, nor will it, due to some unnamed lifetime utility function, end with strictly positive assets).

For the analysis of this problem, consider four successively simplifying assumptions:

1. Consider **ONLY** the optimality conditions of the consumer sector during the two periods (i.e., do not consider any of the budget constraints at all).

2. More precisely, take the results of other models (in particular, consumption-labor and consumption-savings) as given. In particular, the consumption-labor optimality conditions are $\frac{u_l(c_1,l_1)}{u_c(c_1,l_1)} = (1-t_1)w_1$ and $\frac{u_l(c_2,l_2)}{u_c(c_2,l_2)} = (1-t_2)w_2$. And the consumption-savings optimality condition is $\frac{u_{c_1}(c_1,c_2)}{u_{c_2}(c_1,c_2)} = 1+r$.

3. All of the taxes that appear in the three equations above are at play, but there are **NO** other types of non-lump-sum taxes that can be implemented.

4. Prices in both labor markets (i.e., $w_1 > 0$ and $w_2 > 0$) and in the financial market (i.e., $r > 0$) are unchanging as various fiscal policy choices are considered.

Suppose that government spending is constant (and strictly positive) in each of periods one and two (of course, the practical policy discussions are also about government spending). Thus, you can think of the government debating only how to change its collection of **BOTH** lump-sum taxes $T_1 > 0$ and $T_2 > 0$, **AND** of non-lump-sum labor income taxes $t_1 > 0$ and $t_2 > 0$, **AND** (by implication) bond holdings b_1 between period one and period two.

a. **(4 points)** Construct the single two-period (i.e., lifetime) government budget constraint starting from the two period-by-period (i.e., period one and period two) budget constraints. Show any important steps, and briefly explain the economics.

b. **(4 points)** Suppose the government proposes to collect very low labor income taxes in period one, and much higher labor income taxes in period two. From the perspective of the very beginning of period one, **briefly (in no more than three sentences)** show/discuss whether this proposal is optimal (i.e., enhances **consumers' lifetime utility**) or not? **Briefly discuss (among your three sentences) the economic intuition.**

c. **(4 points)** Suppose the government proposes to collect very high labor income taxes in period one, and much lower labor income taxes in period two. From the

perspective of the very beginning of period one, **briefly (in no more than three sentences)** show/discuss whether this proposal is optimal (i.e., enhances **consumers' lifetime utility**) or not? **Briefly discuss (among your three sentences) the economic intuition.**

d. **(4 points)** Suppose the government proposes to bring the two labor income tax rates into exact equality. In terms of consumer lifetime utility, is this solution a better solution, a worse solution, or is it impossible to determine? Show any key steps. Also briefly explain the economics of why it is better or worse, or, if it is impossible to determine, explain the economics of why.

e. **(4 points)** Given your assessment of the tax system in part d, consider the following: suppose the government collected more of its total tax revenue via lump-sum taxes, T_1 and T_2, which leaves less total taxation to collect via labor income taxes. If the two labor income tax rates are still left exactly equal to each other (but at a lower rate), is consumer lifetime utility **even** better off, **even** worse off, or is it impossible to determine? As above, show any key steps. Also briefly explain the economics of why it is better or worse, or, if it is impossible to determine, explain the economics of why.

f. **(4 points)** For this part only, suppose labor income tax rates can be set (either one of them, or both of them simultaneously) to **negative** values (i.e., $t_1 < 0$ and $t_2 < 0$). Noting the results of parts d and e, what if lump-sum taxes are set so high that the government can set both t_1 and t_2 each to strictly negative (and still equal) values. Is consumer lifetime utility **EVEN** better off, **EVEN** worse off, or is it impossible to determine? As above, show any key steps. Also briefly explain the economics of why it is better or worse, or, if it is impossible to determine, explain the economics of why.

Problem 3: Unemployment, Labor Markets, and Inequality (36 points). Consider the static (i.e., Chapter 2) model of consumption and labor hours (aka, leisure), modified a bit by taking account of, qualitatively, a matching process between unemployed individuals (i.e., those that would like to be working, but for some reason are not) with firms. As in Chapter 2, suppose there are 168 hours total for each individual.

Formally, one can **only** suppose that an individual makes a "consumption-labor hours decision" **if** he has a job. If an individual does not have a job, our analysis simply leaves his labor-income life outside the model (note well this statement). Doing so makes it easier to keep all analysis, loosely speaking, within the realm of the "representative agent" view. **However,** in this problem, you will look somewhat outside the strict representative-agent view, as noted above.

Suppose all of the people (which we will leave unnumbered) in the economy would like a job. Having a job means an individual **must** choose to work 40 hours, and there is no other choice possible. However, not everyone has a job – those that do not have a job can only work zero hours.

There are no numerical values in this problem; all analysis is qualitative, but can be (as you will read below) somewhat mathematical.

a. **(4 points)** If we are considering only those individuals that are employed, how many **hours** does the average individual work? Briefly, and clearly, explain.

b. **(10 points)** Consider two different scenarios, labeled scenario #1 and scenario #2. In the different scenarios, the number of individuals who work are different. Thus, the number of individuals who are **not** working are also different under the two scenarios. **Suppose we do not want to consider everyone (employed and unemployed) in both scenarios as consuming the consumption goods of the consumption-labor model**. That is, we want to consider **only** the employed individuals as consuming equal quantities of those goods, and **hence** ignore the other individuals. Is this possible to do? If so, how, both mathematically and conceptually? If not, why not, both mathematically and conceptually? Briefly, and clearly, explain.

c. **(10 points)** Consider two different scenarios, labeled scenario #1 and scenario #2. In the different scenarios, the number of individuals who work are different. Thus, the number of individuals who are **not** working are also different under the two scenarios. **Suppose we do want to consider everyone (employed and unemployed) in both scenarios as consuming the consumption goods of the consumption-labor model.** That is, we want to consider **both** the employed and unemployed individuals as consuming an equal quantity of those goods, and **hence not** ignore the other individuals. Is this possible to do? If so, how, mathematically and conceptually? If not, why not, both mathematically and conceptually? Briefly, and clearly, explain.

d. **(6 points)** Start from the scenario outlined in part c. Politicians notice if the level of average consumption per person (**regardless** of employment status) varies. Name three distinct things that would encourage politicians to try to boost programs that would enhance employment. **(Note: we will ONLY read the first two sentences of each of your three responses (which must be of reasonable length) of any response you write. So be BRIEF.)**

e. **(6 points)** Start from the scenario outlined in part c. Politicians notice if the level of consumption per person (**regardless** of employment status) varies. Name three distinct things that would encourage politicians to try to boost programs that would enhance firms' rights to demand labor as they see fit. **(Note: we will ONLY read the first two sentences of each of your three responses (which must be of reasonable length) of any response you write. So be BRIEF.)**