

ASSET PRICING IN INTERTEMPORAL CONSUMPTION MODELS

OCTOBER 5, 2011

The Border of Macro and Finance

ASSET PRICING APPLICATIONS

- ☐ Lucas-tree model
- ☐ General equilibrium asset pricing
- ☐ Equity premium puzzle
- ☐ Risk-free rate puzzle
- ☐ Alternative preference specifications
 - ☐ Recursive utility (Epstein-Zin)
 - ☐ Habit persistence
 - ☐ Hyperbolic discounting
- ☐ Risk sharing
- ☐ (Constructing the representative consumer)

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3

AN EQUILIBRIUM ASSET PRICING MODEL

- ☐ Modify interpretation of intertemporal model

$$\max_{\{c_t, a_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{subject to} \quad c_t + s_t a_t = d_t a_{t-1} + s_t a_{t-1}, \quad t = 0, 1, 2, \dots$$

a_t : end-of-period- t holdings of shares of an asset

s_t : period- t price of a share

d_t : period- t per-share dividend

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4

AN EQUILIBRIUM ASSET PRICING MODEL

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a_t : end-of-period- t holdings of shares of an asset

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- **Equivalently express budget constraint as**

$$c_t + s_t \underbrace{(a_t - a_{t-1})}_{\text{change in share holdings, i.e., quantity of savings}} = d_t a_{t-1}, \quad t = 0, 1, 2, \dots$$

change in share holdings, i.e., quantity of savings

- **"Supply-side" of financial markets**

- $a_t = 1 \quad \forall t$ – an **equilibrium** outcome

- **Do not/cannot impose when solving consumer problem!**

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5

AN EQUILIBRIUM ASSET PRICING MODEL

- **FOCs yield asset pricing equation**

$$s_t = E_t \left\{ \underbrace{\frac{\beta \lambda_{t+1}}{\lambda_t}}_{\text{Pricing kernel}} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

Pricing kernel
aka stochastic discount factor (SDF)

↓ Can express in terms of marginal utilities...but don't have to

$$s_t = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

Period- t asset price = **SDF** x **Future return**

Two components:

1. Future price of stock
2. Future dividend payment

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6

AN EQUILIBRIUM ASSET PRICING MODEL

- **Basic asset pricing equation**

$$s_t = E_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

↓
Recursively
substitute forward

$$s_t = E_t \left\{ \sum_{\tau=t+1}^{\infty} \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} d_{\tau} \right\} + \lim_{\tau \rightarrow \infty} E_t \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} s_{\tau} = 0$$

- **Impose a TVC**
- **Similar intuition as** $\lim_{t \rightarrow \infty} \beta^t u'(c_t^*) a_t^* = 0$
 - **Value of terminal asset holdings (in present-value utils) must $\rightarrow 0$**
 - **There: had price of asset = 1, so needed terminal condition on quantity**

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7

AN EQUILIBRIUM ASSET PRICING MODEL

- **Basic asset pricing equation**

$$s_t = E_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

↓
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$$s_t = E_t \left\{ \sum_{\tau=t+1}^{\infty} \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} d_{\tau} \right\} + \lim_{\tau \rightarrow \infty} E_t \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} s_{\tau} = 0$$

- **Impose a TVC**
 - **Rules out "asset price bubbles"**
- **The basic form of any asset pricing condition**

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8

EQUILIBRIUM ASSET PRICING

□ Basic asset pricing equation

$$s_t = E_t \left\{ \sum_{\tau=t+1}^{\infty} \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} d_{\tau} \right\} \quad \text{OR} \quad s_t = E_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

with TVC imposed

□ Lucas (1978) model

- $\{d_{\tau}\}_{\tau=t,t+1,\dots}$ follows Markov process
- "Supply-side" of financial markets is $a_{\tau} = 1 \quad \forall \tau$
- Suppose representative consumer

□ Implementing representative consumer

- An infinity of consumers, each indexed by a point on the unit interval $[0,1]$
- Each individual is identical in preferences and endowments
- Implies aggregate consumption demand and asset demand

$$\begin{array}{lcl} \text{Aggregate consumption demand} & = & \text{One individual's consumption demand} \times \text{Total measure of individuals} \\ \text{Aggregate savings demand} & = & \text{One individual's savings demand} \times \text{Total measure of individuals} \end{array}$$

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9

EQUILIBRIUM ASSET PRICING

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$$s_t = E_t \left\{ \sum_{\tau=t+1}^{\infty} \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} d_{\tau} \right\} \quad \text{OR} \quad s_t = E_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

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$$\begin{array}{lcl} \text{Aggregate consumption demand} & = & \text{One individual's consumption demand} \times 1 \\ \text{Aggregate savings demand} & = & \text{One individual's savings demand} \times 1 \end{array}$$

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10

EQUILIBRIUM ASSET PRICING

Basic asset pricing equation

$$s_t = E_t \left\{ \sum_{\tau=t+1}^{\infty} \frac{\beta^{\tau-t} \lambda_{\tau}}{\lambda_t} d_{\tau} \right\} \quad \text{OR} \quad s_t = E_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} (d_{t+1} + s_{t+1}) \right\}, \quad t = 0, 1, 2, \dots$$

with TVC imposed

Lucas (1978) model

$\{d_{\tau}\}_{\tau=t,t+1,\dots}$ follows Markov process

"Supply-side" of financial markets is $a_{\tau} = 1 \quad \forall \tau$

Solution is a consumption and asset price sequence $\{c_{\tau}^*, a_{\tau}^*\}_{\tau=t,t+1,t+2,\dots}$ that satisfies

Sequence of stochastic pricing conditions

$$s_{\tau}^* = E_{\tau} \left\{ \sum_{t=\tau+1}^{\infty} \frac{\beta^{\tau-t} u'(c_{\tau}^*)}{u'(c_t^*)} d_{\tau} \right\}$$

in which TVC ("no-bubbles") is already imposed

Sequence of budget constraints with asset-market clearing condition $a_{\tau} = 1$ imposed

$$c_{\tau}^* = d_{\tau}, \quad \tau = t, t+1, t+2, \dots$$

taking as given exogenous law of motion for d_{τ}

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11

EQUILIBRIUM ASSET PRICING

What if N assets?

$$\max_{\{c_t, a_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{subject to} \quad c_t + \sum_{j=1}^N s_t^j a_t^j = \sum_{j=1}^N d_t^j a_{t-1}^j + \sum_{j=1}^N s_t^j a_{t-1}^j, \quad t = 0, 1, 2, \dots$$

Not necessarily Arrow-Debreu assets

N not necessarily equal to the number of exogenous states of nature

a_t^j : end-of-period- t holdings of shares of asset j

s_t^j : period- t price of a share of asset j

d_t^j : period- t per-share dividend from asset j

FOCs yield N asset pricing equations

$$s_t^j = E_t \left\{ \frac{\beta \lambda_{t+1}}{\lambda_t} (d_{t+1}^j + s_{t+1}^j) \right\}, \quad j = 1, 2, \dots, N, t = 0, 1, 2, \dots$$

Each has identical SDF

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12

EQUILIBRIUM ASSET PRICING

$$E_t \left\{ \underbrace{\frac{\beta \lambda_{t+1}}{\lambda_t}}_{\text{SDF}} \underbrace{\left(\frac{d_{t+1}^j + s_{t+1}^j}{s_t^j} \right)}_{\text{Gross return on one unit of } a^j} \right\} = 1$$

Demand-side pricing condition for asset j

- Finance applications sometimes/often suppose SDF follows exogenous law of motion
 - May be useful for making progress on understanding asset pricing if we have the wrong model of (intertemporal) consumption

EQUILIBRIUM ASSET PRICING

$$E_t \left\{ \underbrace{\frac{\beta \lambda_{t+1}}{\lambda_t}}_{\text{SDF}} \underbrace{\left(\frac{d_{t+1}^j + s_{t+1}^j}{s_t^j} \right)}_{\text{Gross return on one unit of } a^j} \right\} = 1$$

Demand-side pricing condition for asset j

- Finance applications sometimes/often suppose SDF follows exogenous law of motion
 - May be useful for making progress on understanding asset pricing if we have the wrong model of (intertemporal) consumption

- Consumption-based asset pricing

$$E_t \left\{ \underbrace{\frac{\beta u'(c_{t+1})}{u'(c_t)}}_{\text{Growth of marginal utility}} \left(\frac{d_{t+1}^j + s_{t+1}^j}{s_t^j} \right) \right\} = 1 \quad \longleftrightarrow \quad E_t \left\{ \beta u'(c_{t+1}) \left(\frac{d_{t+1}^j + s_{t+1}^j}{s_t^j} \right) \right\} = u'(c_t)$$

Interpretation: on average, in future states in which marginal utility is **low**, asset j 's return must be **high in order to be induced to hold it**

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15

EQUILIBRIUM ASSET PRICING

- ☐ **No-arbitrage conditions**
 - ☐ For any assets a^i, a^k

$$E_t \left\{ \underbrace{\frac{\beta u'(c_{t+1})}{u'(c_t)}}_{\text{Growth of marginal utility}} \underbrace{\left(\frac{d_{t+1}^i + s_{t+1}^i}{s_t^i} - \frac{d_{t+1}^k + s_{t+1}^k}{s_t^k} \right)}_{\substack{\text{Gross return on one unit of } a^i \\ \text{Gross return on one unit of } a^k}} \right\} = 0$$

Return premium of asset i over asset k
 - ☐ Provides foundation for various empirical tests of "market efficiency" ...
 - ☐ ...or simply the underlying structure of the model
- ☐ Mehra and Prescott (1985 JME)
 - ☐ Test how well model does in jointly explaining returns on aggregate equity and risk-free bonds
 - ☐ Representative agent
 - ☐ Only two assets: a Lucas tree (stock) and risk-free bond
 - ☐ Main result: equity premium puzzle

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16

EQUITY PREMIUM PUZZLE

$$\max_{\{c_t, a_t, b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{subject to} \quad c_t + s_t a_t + R_t^f b_t = d_t a_{t-1} + s_t a_{t-1} + b_{t-1}, \quad t = 0, 1, 2, \dots$$

a_t : end-of-period- t holdings of shares of an asset – interpret as stock

s_t : period- t price of a share

d_t : period- t per-share dividend

b_t : end-of-period- t holdings of risk-free bond – interpret as short-term Tbill

R_t^f : period- t price of risk-free bond that matures in period $t+1$

Asset-pricing conditions

$$s_t = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} (d_{t+1} + s_{t+1}) \right\} \quad R_t^f = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

$$\updownarrow \quad \updownarrow$$

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \underbrace{\left(\frac{d_{t+1} + s_{t+1}}{s_t} \right)}_{\equiv 1/R_{t+1}^s} \right\} \quad \text{No arbitrage} \quad = \quad 1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{1}{R_t^f} \right\}$$

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17

EQUITY PREMIUM PUZZLE

Can standard intertemporal consumption model reasonably jointly explain risk-free returns and risky (stock) returns?

$$(1) \quad E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{1}{R_t^f} \right\} = 1 \quad (2) \quad E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \underbrace{\left(\frac{1}{R_{t+1}^s} - \frac{1}{R_t^f} \right)}_{\text{Equity premium}} \right\} = 0$$

Mehra and Prescott (1985 JME)

Originally posed question

Answered in the negative!

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18

EQUITY PREMIUM PUZZLE

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- Mehra and Prescott (1985 JME)
 - Originally posed question
 - Answered in the negative!
- EQUITY PREMIUM PUZZLE
 - An ongoing central issue in finance and macroeconomics
 - Not a qualitative shortcoming of basic model
 - A (spectacular) quantitative shortcoming of basic model
- Good overviews/reviews
 - Kocherlakota (1996 JEL)
 - Mehra and Prescott (2003, *Handbook of Economics and Finance*)
 - Cochrane, Chapter 21

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19

EQUITY PREMIUM PUZZLE

- Can standard intertemporal consumption model reasonably jointly explain risk-free returns and risky (stock) returns?

$$(1) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{1}{R_t^f} \right\} = 1 \quad (2) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \underbrace{\left(\frac{1}{R_{t+1}^s} - \frac{1}{R_t^f} \right)}_{\text{Equity premium}} \right\} = 0$$

- Start with (2)

$$\longrightarrow E_t \left(\frac{1}{R_{t+1}^s} \right) = \frac{1}{R_t^f} - \frac{\text{Cov}_t(u'(c_{t+1})/u'(c_t), 1/R_{t+1}^s)}{E_t[u'(c_{t+1})/u'(c_t)]}$$

- Identify $1/R_{t+1}^s$ as real return on broad stock index (i.e., S&P500)
- Identify $1/R_t^f$ as real return on (nominally) risk-free short T-bills
- Expected return on equity = risk-free return + premium for bearing risk
 - Premium depends on covariance of equity return with marginal utility
- Recall interpretation: on average, in future states in which marginal utility is low, asset j 's return must be high in order to be induced to hold it

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20

EQUITY PREMIUM PUZZLE

$$E_t \left(\frac{1}{R_{t+1}^s} \right) = \frac{1}{R_t^f} - \frac{\text{Cov}_t \left(u'(c_{t+1}) / u'(c_t), 1 / R_{t+1}^s \right)}{E_t [u'(c_{t+1}) / u'(c_t)]}$$

- ❑ Expected return on equity = risk-free return + premium for bearing risk
 - ❑ Premium depends on covariance of equity return with marginal utility
 - ❑ On average, in future states in which marginal utility is low, asset j 's return must be high in order to be induced to hold it
- ↑
- ❑ Assets that co-vary positively with consumption over time (negatively with future MU of consumption) command a high premium because their returns are "destabilizing"
 - ❑ i.e., push individual away from consumption smoothing across future risky states
 - ❑ Test using standard CRRA $u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$

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21

EQUITY PREMIUM PUZZLE

$$E_t \left(\frac{1}{R_{t+1}^s} \right) = \frac{1}{R_t^f} - \frac{\text{Cov}_t \left((c_{t+1} / c_t)^{-\sigma}, 1 / R_{t+1}^s \right)}{E_t (c_{t+1} / c_t)^{-\sigma}}$$

- ❑ Can test with variety of aggregate stock returns (inclusive of dividends)
- ❑ Construct real risk-free rate from nominal short interest rates and inflation
- ❑ Mehra and Prescott (1985) – use data from 1889-1978
- ❑ Can apply various sophisticated econometric tools (GMM, Bayesian, etc)
- ❑ Sample means convey basic problem – approximate with unconditional $E(\cdot)$

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22

EQUITY PREMIUM PUZZLE

$$E\left(\frac{1}{R_{t+1}^s}\right) = \frac{1}{R_t^f} - \frac{\text{Cov}\left((c_{t+1}/c_t)^{-\sigma}, 1/R_{t+1}^s\right)}{E(c_{t+1}/c_t)^{-\sigma}}$$

- ❑ Can test with variety of aggregate stock returns (inclusive of dividends)
- ❑ Construct real risk-free rate from nominal short interest rates and inflation
- ❑ Mehra and Prescott (1985) – use data from 1889-1978
- ❑ Can apply various sophisticated econometric tools (GMM, Bayesian, etc)
- ❑ Sample means convey basic problem – approximate with unconditional $E(\cdot)$
- ❑ Sample statistics, 1889-1978 (annual data)
 - ❑ $\text{mean}(1/R_t^f) = 1.01$
 - ❑ $\text{mean}(1/R_t^s) = 1.07$
 - ❑ $\text{mean}(c_{t+1}/c_t) = 1.018$
 - ❑ $\text{SD}(c_{t+1}/c_t) = 0.036$
 - ❑ $\text{SD}(R_t^s) = 0.0274$
 - ❑ $\text{Cov}(c_{t+1}/c_t, R_t^s) = 0.00219$

Explaining 6 percent equity premium with growth rates of aggregate consumption requires a stand on RRA σ ...
...and then computing $\text{Cov}(\cdot)$ based on it

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23

EQUITY PREMIUM PUZZLE

$$E\left(\frac{1}{R_{t+1}^s}\right) = \frac{1}{R_t^f} - \frac{\text{Cov}\left((c_{t+1}/c_t)^{-\sigma}, 1/R_{t+1}^s\right)}{E(c_{t+1}/c_t)^{-\sigma}}$$

- ❑ What are “reasonable” values of RRA?
- ❑ Business-cycle models require $\sigma \approx 1$ (Econ 701)
- ❑ Microeconomic evidence higher(?) risk aversion
- ❑ Hall (1988) evidence indicates IES ≈ 0.1
 - ❑ If CRRA $\rightarrow \sigma \approx 10$
 - ❑ (Though Hall disavows that he is measuring risk aversion – Kocherlakota says argument is deeply flawed (1990, *JFin* p. 175))
- ❑ Mehra and Prescott assert $\sigma = 10$ as extreme upper bound on risk aversion

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24

EQUITY PREMIUM PUZZLE

$$E\left(\frac{1}{R_{t+1}^s}\right) = \frac{1}{R_t^f} - \frac{\text{Cov}\left((c_{t+1}/c_t)^{-\sigma}, 1/R_{t+1}^s\right)}{E(c_{t+1}/c_t)^{-\sigma}}$$

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- ❑ Mehra and Prescott assert $\sigma = 10$ as extreme upper bound on risk aversion
- ❑ $\sigma = 10$
 - ❑ Standard model explains ≈ 0.3 percent of the 6 percent equity premium
 - ❑ An order of magnitude off! Even though $\sigma = 10$ already “too high”(?)
- ❑ Explaining equity premium with standard model requires $\sigma \approx 50!$

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25

EQUITY PREMIUM PUZZLE

$$(1) \quad \beta E\left\{\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\right\} = \frac{1}{1/R_t^f} \quad (2) \quad E\left(\frac{1}{R_{t+1}^s}\right) = \frac{1}{R_t^f} - \frac{\text{Cov}\left((c_{t+1}/c_t)^{-\sigma}, 1/R_{t+1}^s\right)}{E(c_{t+1}/c_t)^{-\sigma}}$$

- ❑ $\sigma = 10$
 - ❑ Standard model explains ≈ 0.3 percent of the 6 percent equity premium
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- ❑ Explaining equity premium with standard model requires $\sigma \approx 50!$
- ❑ If accept $\sigma \approx 50$
 - ❑ Requires accepting virtually zero IES (not inconsistent with Hall 1988...)...
 - ❑ ...and generates another anomaly
- ❑ If prior is $\beta \approx 0.98-0.99$, (1) implies $1/R^f = \dots?$

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26

EQUITY PREMIUM AND RISK-FREE RATE PUZZLES

$$(1) \quad \beta E \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} = \frac{1}{1/R_t^f} \quad (2) \quad E \left(\frac{1}{R_{t+1}^s} \right) = \frac{1}{R_t^f} - \frac{\text{Cov}((c_{t+1}/c_t)^{-\sigma}, 1/R_{t+1}^s)}{E(c_{t+1}/c_t)^{-\sigma}}$$

- **$\sigma = 10$**
 - Standard model explains ≈ 0.3 percent of the 6 percent equity premium
 - An order of magnitude off!
- **Explaining equity premium with standard model requires $\sigma \approx 50$!**
- **If accept $\sigma \approx 50$**
 - Requires accepting virtually zero IES (not inconsistent with Hall 1988...)...
 - ...and generates another anomaly
- **If prior is $\beta \approx 0.98-0.99$, (1) implies $1/R_t^f \approx 1.1$**
 - **10 percent higher than empirical risk-free rate!**
- **(Extremely) high σ to fix equity premium puzzle causes risk-free rate puzzle!**
 - Which can be offset by $\beta \approx 0.6$...extreme impatience! **"Hyperbolic?"**

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27

EQUITY PREMIUM AND RISK-FREE RATE PUZZLES

$$(1) \quad \beta E \left\{ \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} \right\} = \frac{1}{1/R_t^f} \quad (2) \quad E \left(\frac{1}{R_{t+1}^s} \right) = \frac{1}{R_t^f} - \frac{\text{Cov}((c_{t+1}/c_t)^{-\sigma}, 1/R_{t+1}^s)}{E(c_{t+1}/c_t)^{-\sigma}}$$

- **With CRRA preferences, resolving both puzzles requires adopting view that consumers are**
 - **EXTREMELY risk averse**
 - **EXTREMELY impatient**
- $\left. \begin{array}{l} \text{EXTREMELY risk averse} \\ \text{EXTREMELY impatient} \end{array} \right\}$

CRRA fundamentally entangles risk attitudes and intertemporal attitudes
- **Many assumptions underlying basic model**
 - No transactions costs in financial markets
 - Everyone participates in all financial markets
 - i.e., no "limited participation"
 - A representative agent exists
 - i.e., can ignore heterogeneity in aggregating Euler equations
 - (recalls Attanasio criticism of Hall (1978))
 - Etc...
- **First (most natural?) line of attack – modify $u(\cdot)$ to disentangle IES and RRA**

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28