BASIC CONSUMPTION-SAVINGS FRAMEWORK: THE STOCHASTIC CASE

SEPTEMBER 12, 2011

BASICS

- Consumption-Savings Framework
  - So far only a deterministic analysis…
  - …now introduce risk
  - Still an application of basic consumer theory
  - The cornerstone of modern macro theory
- Starting point: two time periods
  - Important: all analysis conducted from the perspective of the very beginning of period 1…
  - …so a "future" (period 2) for which to save
  - But risk exists about (some) period-2 primitives
  - Soon will extend to infinite number of periods
- Dynamic stochastic analysis the foundation of modern macroeconomic theory
  - An explicit accounting of time
  - An explicit accounting of risk
  - Two-period stochastic model illustrates many central ideas, results, and methods

REPRESENTATIVE-AGENT MACROECONOMICS

- Consumer A: Consumed \$50 in Year X
- Consumer B: Consumed \$75 in Year X
- Consumer C: Consumed \$100 in Year X
- Consumer D: Consumed \$125 in Year X
- Consumer E: Consumed \$150 in Year X

No other consumers in the economy

THE REPRESENTATIVE CONSUMER

- Aggregate (i.e., economy-wide) consumption = \$500
- Average consumption = \$100

- If we are interested only in aggregate outcomes
- …and if we want to take a micro-based approach to explaining aggregate outcomes…
- …model Consumer C's behavior/decision-making

- A simplistic approach – turns out to yield surprisingly rich results, insights, and predictions

RISK

- Where to introduce risk?
  - In simple model so far, only two possibilities (i.e., only two primitives of model)

- Period-2 income (endowment) risk
  - Endowment y_2 drawn from probability distribution $G(.)$
  - Support of $G(.)$ could be continuous or discrete
  - Begin with simple example of discrete period-2 endowments

$$y_2 = \begin{cases} y_2^H & \text{probability q} \\ \bar{y}_2 & \text{probability p} \\ y_2^L & \text{probability 1-p-q} \end{cases}$$

Three possible period-2 states

- Period-2 preference risk
  - Don't know the subjective way in which you'll enjoy consumption in the future
  - If assume additive separability, $v(c_1, c_2) = u(c_1) + \beta u(c_2)$, with risk about β

RISK

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Study this stochastic version of two-period model

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Three possible period-2 states

  - Can measure income, so risk about income can be incorporated into the theory in an empirically-disciplined way

- Period-2 preference risk
  - Don't know the subjective way in which you'll enjoy consumption in the future
  - If assume additive separability, $v(c_1,c_2) = u(c_1) + \beta u(c_2)$, with risk about β
  - Cannot measure preferences, so how can risk about preferences be incorporated into the theory in empirically-disciplined way? (though some models do allow preference risk…)

But utility written in suggestive way… soon consider $v(.)$ that "looks like" this, but different interpretation of β

ASSET MARKETS

- Risk about the future (period 2) requires adopting a view about the nature of future (period-2) returns on assets

- State-contingent asset returns
  - Suppose period-2 realized return on asset depends on realized period-2 endowment

Asset return $r_1 = \begin{cases} r_1^H & \text{probability q} \\ \bar{r}_1 & \text{probability p} \\ r_1^L & \text{probability 1-p-q} \end{cases}$ Corresponding to $y_2 = \begin{cases} y_2^H & \text{probability q} \\ \bar{y}_2 & \text{probability p} \\ y_2^L & \text{probability 1-p-q} \end{cases}$

  - Schedule of returns known in period 1

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  - Schedule of returns known in period 1

- NOT equivalent to complete set of Arrow-Debreu assets
  - A-D security: asset that pays one unit of numeraire in a particular realized state, zero otherwise
  - Complete markets: A-D security exists for each of the possible realizations of uncertainty
  - Complete asset markets span the risk space – basics of macro-finance connections next week

Budget Constraint(s)

- Period 1: budget constraint identical to deterministic case

$$c_1 + a_1 = y_1 + (1+r_0)a_0$$

- Period 2: state-contingent budget constraints, one for each of the possible realized states

$$c_2^H + \cancel{a_2^H} = y_2^H + (1+r_1^H)a_1 \quad \text{probability q}$$
$$c_2^M + \cancel{a_2^M} = \bar{y}_2 + (1+\bar{r}_1)a_1 \quad \text{probability p}$$
$$c_2^L + \cancel{a_2^L} = y_2^L + (1+r_1^L)a_1 \quad \text{probability 1-p-q}$$

= 0 (impose terminal asset holdings state by state)

Period-2 consumption (endogenous) is state-contingent

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$$c_2^L + \cancel{a_2^L} = y_2^L + (1+r_1^L)a_1 \qquad \text{probability 1-p-q}$$

= 0 (impose terminal asset holdings state by state)

Period-2 consumption (endogenous) is state-contingent

- P2BC satisfied for each state $\Rightarrow$ P2BC satisfied in expectation
  - But reverse is not true!

$$0 = q\left[y_2^H + (1+r_1^H)a_1 - c_2^H\right] + p\left[\bar{y}_2 + (1+\bar{r}_1)a_1 - c_2^M\right] + (1-p-q)\left[y_2^L + (1+r_1^L)a_1 - c_2^L\right]$$
$$= E_1\left\{y_2 + (1+r_1)a_1 - c_2\right\}$$

  - Notation: $E_1\{.\}$ is expectation conditional on period-1 information set
  - Expectation computed with respect to objective distribution of period-2 risk, $G(.)$
    - An aspect of rational expectations: economic agents know and use the true (objective) distribution of risk

Important building block of (most) modern macro models

Budget Constraint(s)

- Terms arising from budget constraints in sequential Lagrangian formulation

$$+\lambda_1\left[y_1 + (1+r_0)a_0 - c_1 - a_1\right] + q\lambda_2^H\left[y_2^H + (1+r_1^H)a_1 - c_2^H\right] + p\lambda_2^M\left[\bar{y}_2 + (1+\bar{r}_1)a_1 - c_2^M\right]$$
$$+(1-p-q)\lambda_2^L\left[y_2^L + (1+r_1^L)a_1 - c_2^L\right]$$

  - Period-specific multipliers, with state-contingent $\lambda_2 \in \left\{\lambda_2^H, \lambda_2^M, \lambda_2^L\right\}$

BUDGET CONSTRAINT(S)

- Terms arising from budget constraints in sequential Lagrangian formulation

$$+\lambda_1\left[y_1+(1+r_0)a_0-c_1-a_1\right]+q\lambda_2^H\left[y_2^H+(1+r_1^H)a_1-c_2^H\right]+p\lambda_2^M\left[\bar{y}_2+(1+\bar{r}_1)a_1-c_2^M\right]$$
$$+(1-p-q)\lambda_2^L\left[y_2^L+(1+r_1^L)a_1-c_2^L\right]$$

expressed more compactly in expectations

  - Period-specific multipliers, with state-contingent $\lambda_2\in\left\{\lambda_2^H,\lambda_2^M,\lambda_2^L\right\}$

- Or terms arising from LBCs in lifetime Lagrangian formulation
  - Three possible lifetime paths, so three possible LBCs

$$+q\lambda^H\left[y_1+\frac{y_2^H}{1+r_1^H}+(1+r_0)a_0-c_1-\frac{c_2^H}{1+r_1^H}\right]+p\lambda^M\left[y_1+\frac{\bar{y}_2}{1+\bar{r}_1}+(1+r_0)a_0-c_1-\frac{c_2^M}{1+\bar{r}_1}\right]$$
$$+(1-p-q)\lambda^L\left[y_1+\frac{y_2^L}{1+r_1^L}+(1+r_0)a_0-c_1-\frac{c_2^L}{1+r_1^L}\right]$$

a_1 eliminated using the state-contingent period-2 budget constraints

expressed more compactly in expectations

  - Three present-value state-contingent multipliers $\left\{\lambda^H,\lambda^M,\lambda^L\right\}$
  - Same interpretation as deterministic case: expected PDV of lifetime expenditure equals expected PDV of lifetime wealth, except now for each realized path of the economy

BUDGET CONSTRAINT(S)

- Using probability distribution ***G*(.)**
- Terms arising from budget constraints in sequential Lagrangian formulation

expressed more compactly in expectations

$$+\lambda_1\left[y_1+(1+r_0)a_0-c_1-a_1\right]+E_1\left\{\lambda_2\left[y_2+(1+r_1)a_1-c_2\right]\right\}$$

  - Period-specific multipliers, with state-contingent $\lambda_2\in\left\{\lambda_2^H,\lambda_2^M,\lambda_2^L\right\}$

- Or terms arising from LBCs in lifetime Lagrangian formulation

expressed more compactly in expectations

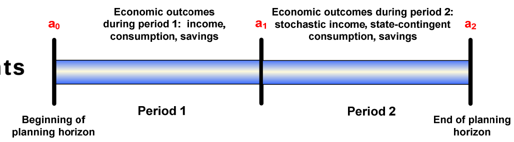
$$+E_1\left\{\lambda\left[y_1+\frac{y_2}{1+r_1}+(1+r_0)a_0-c_1-\frac{c_2}{1+r_1}\right]\right\}=0$$

a_1 eliminated using the state-contingent period-2 budget constraints

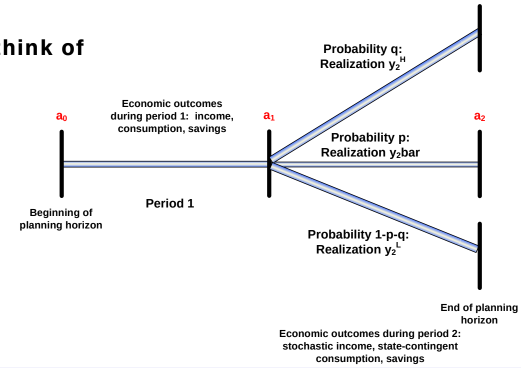
- Compact representation useful for writing/analyzing the problem
- But in constructing solution, each of the possible budget paths must be satisfied, not just the "average budget path"
- Will pursue sequential analysis; try LBC formulation yourself

Event Tree

- ❑ **Timeline of events**

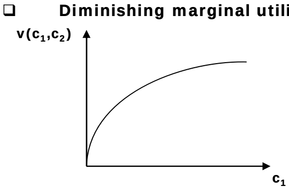


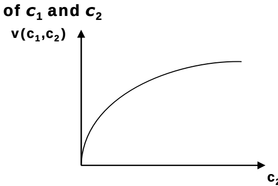
- ❑ **More useful to think of as event tree**



Utility

- ❑ **Preferences $v(c_1, c_2)$ with all the "usual properties"**
  - ❑ **Lifetime utility function**
  - ❑ **Assume separable across time periods: $v(c_1,c_2) = u(c_1) + u(c_2)$ (deterministic case)**
  - ❑ **Strictly increasing in each of c_1 and c_2**
  - ❑ **Diminishing marginal utility in each of c_1 and c_2**



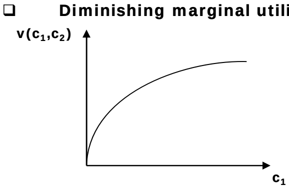


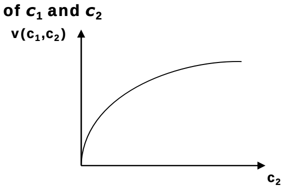
  - ❑ **But realized c_2 cannot be known at time decisions are made in period 1, due to period-2 income risk**

- ❑ **How to incorporate risk into utility metric?**

Expected Utility

- Preferences $v(c_1, c_2)$ with all the "usual properties"
  - Lifetime ***expected*** utility function
  - Assume separable across time periods: $v(c_1,c_2) = u(c_1) + u(c_2)$ (deterministic case)
  - Strictly increasing in each of c_1 and c_2
  - Diminishing marginal utility in each of c_1 and c_2





  - But realized c_2 cannot be known at time decisions are made in period 1, due to period-2 income risk
- How to incorporate risk into utility metric?
- Expected lifetime utility
  - Assume consumers maximize

$$E_1 v(c_1,c_2) = u(c_1) + E_1 u(c_2) \left(= u(c_1) + qu(c_2^H) + pu(c_2^M) + (1-p-q)u(c_2^L) \right)$$

  - A decision-theoretic (not experiential) utility metric
  - von-Neumann-Morgenstern (1944) foundations / Econ 603 (Mas-Colell, Chapter 6)

State-Contingent Choices

- Sequential Lagrange formulation

$$\max u(c_1) + E_1 u(c_2) + \lambda_1 \left[y_1 + (1+r_0)a_0 - c_1 - a_1 \right] + E_1 \left\{ \lambda_2 \left[y_2 + (1+r_1)a_1 - c_2 \right] \right\}$$

- FOCs

(1) c_1: $u'(c_1) - \lambda_1 = 0$ Marginal value of period-1 resources = marginal utility of c_1

(2) a_1: $-\lambda_1 + E_1 \left\{ \lambda_2 (1+r_1) \right\} = 0$

(3) c_2: $E_1 u'(c_2) - E_1 \lambda_2 = 0$

  - Note that (3) is not the period-2 statement of (1)

STATE-CONTINGENT CHOICES

- **Sequential Lagrange formulation**

$$\max u(c_1)+E_1u(c_2)+\lambda_1\left[y_1+(1+r_0)a_0-c_1-a_1\right]+E_1\left\{\lambda_2\left[y_2+(1+r_1)a_1-c_2\right]\right\}$$

- **FOCs**

(1) c_1: $u'(c_1)-\lambda_1=0$ — Marginal value of period-1 resources = marginal utility of c_1

(2) a_1: $-\lambda_1+E_1\left\{\lambda_2(1+r_1)\right\}=0$

(3) c_2: $E_1u'(c_2)-E_1\lambda_2=0$

  - **Note that (3) is not the period-2 statement of (1)**

- **But with state-contingent asset returns, condition (1) does hold in period 2**
  - **Write out expectations in optimization explicitly**

$$\max u(c_1)+\lambda_1\left[y_1+(1+r_0)a_0-c_1-a_1\right]+qu(c_2^{\,H})+q\lambda_2^{\,H}\left[y_2^{\,H}+(1+r_1^{\,H})a_1-c_2^{\,H}\right]$$
$$+pu(c_2^M)+p\lambda_2^M\left[\bar{y}_2+(1+\bar{r}_1)a_1-c_2^M\right]+(1-p-q)u(c_2^L)+(1-p-q)\lambda_2^L\left[y_2^{\,L}+(1+r_1^{\,L})a_1-c_2^{\,L}\right]$$

$$\Big\downarrow \text{FOCs}$$

STATE-CONTINGENT CHOICES

- **FOCs**

(1) c_1: $u'(c_1)-\lambda_1=0$ — Marginal value of period-1 resources = marginal utility of c_1

(2) a_1: $-\lambda_1+E_1\left\{\lambda_2(1+r_1)\right\}=0$ — Euler equation

(3) $\left\{\begin{array}{ll} c_2^H: & u'(c_2^{\,H})-\lambda_2^{\,H}=0 \\ c_2^M: & u'(c_2^M)-\lambda_2^M=0 \\ c_2^L: & u'(c_2^L)-\lambda_2^L=0 \end{array}\right\}$ — Marginal value of period-2 resources = marginal utility of c_2

IMPORTANT: Holds state-by-state

(and thus also holds in expectation)

- **Analyze (2)**
  - **Express as an asset-pricing condition** $\underbrace{1}_{\text{Price in period 1 of purchasing one unit of } a_1}=\underbrace{E_1\left\{\frac{\lambda_2}{\lambda_1}(1+r_1)\right\}}_{\text{Expected return in period 2}}$

Expected return in period 2

Note covariance between (λ_2/λ_1) and ($1+r_1$)

STATE-CONTINGENT CHOICES

- **FOCs**
  - (1) c_1: $u'(c_1) - \lambda_1 = 0$ — Marginal value of period-1 resources = marginal utility of c_1
  - (2) a_1: $-\lambda_1 + E_1\{\lambda_2(1+r_1)\} = 0$ — Euler equation
  - (3) $\left\{\begin{array}{ll} c_2^H: & u'(c_2^H) - \lambda_2^H = 0 \\ c_2^M: & u'(c_2^M) - \lambda_2^M = 0 \\ c_2^L: & u'(c_2^L) - \lambda_2^L = 0 \end{array}\right\}$ — Marginal value of period-2 resources = marginal utility of c_2. IMPORTANT: Holds state-by-state (and thus also holds in expectation)

- **Analyze (2)**
  - Express as an asset-pricing condition $\underbrace{1}_{\text{Price in period 1 of purchasing one unit of } a_1} = E_1\underbrace{\left\{\frac{\lambda_2}{\lambda_1}(1+r_1)\right\}}_{\text{Expected return in period 2}}$

    Note covariance between (λ_2/λ_1) and $(1+r_1)$
  - Or write out expectations in (2) explicitly

$$-\lambda_1 + q\lambda_2^H(1+r_1^H) + p\lambda_2^M(1+\bar{r}_1) + (1-p-q)\lambda_2^L(1+r_1^L) = 0 \longleftrightarrow 1 = \underbrace{\frac{q\lambda_2^H}{\lambda_1}}(1+r_1^H) + \underbrace{\frac{p\lambda_2^M}{\lambda_1}}(1+\bar{r}_1) + \underbrace{\frac{(1-p-q)\lambda_2^L}{\lambda_1}}(1+r_1^L)$$

The prices of Arrow-Debreu securities.

Terminology: "state prices"

CONSUMPTION DYNAMICS

- **Solution to consumer problem is an asset position and state-contingent consumption profile** $\left(c_1, c_2^H, c_2^M, c_2^L; a_1\right)$ **that satisfies**
  - State-by-state period-2 budget constraint

    5 equations, 5 unknowns

    In principle, can solve

    $c_2^H + \underbrace{a_2^H}_{=0} = y_2^H + (1+r_1^H)a_1$ $\quad c_2^M + \underbrace{a_2^M}_{=0} = \bar{y}_2 + (1+\bar{r}_1)a_1$ $\quad c_2^L + \underbrace{a_2^L}_{=0} = y_2^L + (1+r_1^L)a_1$
  - Euler equation

$$1 = \frac{qu'(c_2^H)}{u'(c_1)}(1+r_1^H) + \frac{pu'(c_2^M)}{u'(c_1)}(1+\bar{r}_1) + \frac{(1-p-q)u'(c_2^L)}{u'(c_1)}(1+r_1^L)\left(= E_1\left[\frac{u'(c_2)}{u'(c_1)}(1+r_1)\right]\right)$$

  - Period-1 budget constraint $c_1 + a_1 = y_1 + (1+r_0)a_0$

  taking as given $\left(r_1^H, \bar{r}_1, r_1^L; y_1, a_0, r_0\right)$ **and the stochastic distribution $G(.)$ of y_2**

- **Could express solution in alternative ways**
  - e.g., using lifetime budget constraints

APPLICATIONS

- Use (solution to) stochastic two-period model to illustrate some basic results and ideas in
  - Consumption research
  - Asset pricing research
- Certainty-equivalent consumption
  - Assuming
    - Quadratic period-utility $u(c) = \gamma c - \dfrac{\alpha c^2}{2}$
    - Risk-free asset returns
    - Risky period-2 income (with arbitrary distribution)
- Risk aversion
- Precautionary savings
- Introduction to asset pricing

CERTAINTY EQUIVALENCE

- Assume quadratic utility

$$v(c_1, c_2) = u(c_1) + u(c_2) = \gamma c_1 - \frac{\alpha c_1^2}{2} + \gamma c_2 - \frac{\alpha c_2^2}{2}$$

- Assume interest rate is not state contingent

$$r_1^H = \bar{r}_1 = r_1^L = r_1$$ risk-free interest rate

CERTAINTY EQUIVALENCE

- **Assume quadratic utility**

$$v(c_1,c_2)=u(c_1)+u(c_2)=\gamma c_1-\frac{\alpha c_1^2}{2}+\gamma c_2-\frac{\alpha c_2^2}{2}$$

- **Assume interest rate is not state contingent**

$$r_1^H=\bar{r}_1=r_1^L=r_1 \qquad \text{risk-free interest rate}$$

- **Insert in definition of solution to intertemporal problem**

$$c_2^H+\underbrace{\cancel{a_2}^H}_{=0}=y_2^H+(1+r_1)a_1 \qquad c_2^M+\underbrace{\cancel{a_2}^M}_{=0}=\bar{y}_2+(1+r_1)a_1 \qquad c_2^L+\underbrace{\cancel{a_2}^L}_{=0}=y_2^L+(1+r_1)a_1 \qquad c_1+a_1=y_1+(1+r_0)a_0$$

Euler eqn often the key $u'(c_1)=E_1\left[u'(c_2)(1+r_1)\right] \longrightarrow \gamma-\alpha c_1=E_1\left[(\gamma-\alpha c_2)(1+r_1)\right]$

$$\longrightarrow \gamma-\alpha c_1=q(\gamma-\alpha c_2^H)(1+r_1)+p(\gamma-\alpha c_2^M)(1+r_1)+(1-p-q)(\gamma-\alpha c_2^L)(1+r_1)$$

$$\longrightarrow \gamma-\alpha c_1=(1+r_1)\left[q(\gamma-\alpha c_2^H)+p(\gamma-\alpha c_2^M)+(1-p-q)(\gamma-\alpha c_2^L)\right]$$
$$=(1+r_1)\left[\gamma-\alpha\underbrace{\left(qc_2^H+pc_2^M+(1-p-q)c_2^L\right)}_{=?}\right]$$

CERTAINTY EQUIVALENCE

- **Assume quadratic utility**

$$v(c_1,c_2)=u(c_1)+u(c_2)=\gamma c_1-\frac{\alpha c_1^2}{2}+\gamma c_2-\frac{\alpha c_2^2}{2}$$

- **Assume interest rate is not state contingent**

$$r_1^H=\bar{r}_1=r_1^L=r_1 \qquad \text{risk-free interest rate}$$

- **Insert in definition of solution to intertemporal problem**

$$c_2^H+\underbrace{\cancel{a_2}^H}_{=0}=y_2^H+(1+r_1)a_1 \qquad c_2^M+\underbrace{\cancel{a_2}^M}_{=0}=\bar{y}_2+(1+r_1)a_1 \qquad c_2^L+\underbrace{\cancel{a_2}^L}_{=0}=y_2^L+(1+r_1)a_1 \qquad c_1+a_1=y_1+(1+r_0)a_0$$

Euler eqn often the key $u'(c_1)=E_1\left[u'(c_2)(1+r_1)\right] \longrightarrow \gamma-\alpha c_1=E_1\left[(\gamma-\alpha c_2)(1+r_1)\right]$

$$\longrightarrow \gamma-\alpha c_1=q(\gamma-\alpha c_2^H)(1+r_1)+p(\gamma-\alpha c_2^M)(1+r_1)+(1-p-q)(\gamma-\alpha c_2^L)(1+r_1)$$

$$\longrightarrow \gamma-\alpha c_1=(1+r_1)\left[q(\gamma-\alpha c_2^H)+p(\gamma-\alpha c_2^M)+(1-p-q)(\gamma-\alpha c_2^L)\right]$$
$$=(1+r_1)\left[\gamma-\alpha\underbrace{\left(qc_2^H+pc_2^M+(1-p-q)c_2^L\right)}\right]$$

$$\longrightarrow \gamma-\alpha c_1=(1+r_1)\left[\gamma-\alpha E_1c_2\right] \longrightarrow c_1=-\frac{\gamma}{\alpha}r_1+(1+r_1)E_1c_2$$

CERTAINTY EQUIVALENCE

- If not concerned with state-contingent solutions for c_2…
- …solution to consumer problem is an asset position and expected consumption profile $(c_1, E_1c_2; a_1)$ that satisfies
  - Period-2 budget constraint in expectation
    $$E_1c_2 = (1+r_1)a_1 + E_1y_2$$
  - Euler equation
    $$c_1 = -\frac{\gamma}{\alpha}r_1 + (1+r_1)E_1c_2$$
  - Period-1 budget constraint $c_1 + a_1 = y_1 + (1+r_0)a_0$

  taking as given $(r_1; y_1, a_0, r_0)$ and the stochastic distribution ***G*(.)** of y_2
- Optimal period-1 consumption

$$c_1 = \underbrace{-\frac{\gamma}{\alpha}\left(\frac{r_1}{1+(1+r_1)^2}\right)}_{\equiv A} + \underbrace{\left(\frac{(1+r_1)^2}{1+(1+r_1)^2}\right)}_{\equiv B}\left(y_1+(1+r_0)a_0\right) + \underbrace{\left(\frac{1+r_1}{1+(1+r_1)^2}\right)}_{\equiv C}E_1y_2$$

Stochastic Consumption-Savings Model: Application

CERTAINTY EQUIVALENCE

- If not concerned with state-contingent solutions for c_2…
- …solution to consumer problem is an asset position and expected consumption profile $(c_1, E_1c_2; a_1)$ that satisfies
  - Period-2 budget constraint in expectation
    $$E_1c_2 = (1+r_1)a_1 + E_1y_2$$
  - Euler equation
    $$c_1 = -\frac{\gamma}{\alpha}r_1 + (1+r_1)E_1c_2$$
  - Period-1 budget constraint $c_1 + a_1 = y_1 + (1+r_0)a_0$

  taking as given $(r_1; y_1, a_0, r_0)$ and the stochastic distribution ***G*(.)** of y_2
- Optimal period-1 consumption

$$c_1 = A + B\cdot\underbrace{\left(y_1+(1+r_0)a_0\right)}_{\text{current resources}} + C\cdot\underbrace{E_1y_2}_{\text{expected future income}}$$

  - Depends only on the mean of risky future income, E_1y_2
  - Independent of second- and higher-moments of risky future income

Certainty Equivalence

- Optimal period-1 (current) consumption

$$c_1 = A + B\cdot\left(y_1 + (1+r_0)a_0\right) + C\cdot E_1 y_2$$

  - Depends only on the mean of risky future income, $E_1 y_2$
  - Independent of second- and higher-moments of risky future income

- Distribution function $G(.)$ of period-2 income

$$y_2 = \begin{cases} y_2^H & \text{probability q} \\ \bar{y}_2 & \text{probability p} \\ y_2^L & \text{probability 1-p-q} \end{cases} \quad \left. \begin{array}{l} E_1 y_2 = \bar{y}_2 \\ \operatorname{Var} y_2 = q\left(y_2^H - \bar{y}_2\right)^2 + (1-p-q)\left(y_2^L - \bar{y}_2\right)^2 \end{array} \right.$$

- Certainty Equivalence
  - Mean-preserving spreads of $G(.)$ do not affect optimal choice of c_1
  - E.g., (p = 1, q = 0)
    - Period-2 income has no risk
    - But c_1 is identical
    - s_1 (period-1 savings) is identical

Certainty Equivalence

- A benchmark result in intertemporal consumption theory

- Result depends on
  - Quadratic utility
  - Riskless (aka non-state-contingent) asset returns
  - Only source of risk is income risk

- Only version of the intertemporal consumption model with analytical solution

- Strong implication: risk about future (income) does not affect current consumption and savings decisions
  - Intuitively plausible?
  - Empirically relevant?
  - Probably not…but why not?

- After all, model does feature both
  - Income risk (Var y_2 > 0)
  - Risk averse utility with respect to consumption – need to define formally