EMPIRICAL TESTS OF INTERTEMPORAL CONSUMPTION MODELS

OCTOBER 3, 2011

CONSUMPTION AND ASSET PRICING THEORY

- Intertemporal consumption model the backbone of modern macro models
  - Business cycle models
  - Growth models
  - An application of basic consumer theory
- Basic issues/results/questions
  - Intertemporal substitution of consumption
  - Risk aversion
  - Precautionary savings
  - Consumption smoothing (over dates and over states) via financial assets
  - Subjective discount factor
- Asset pricing can be (but need not be…) studied as the flip side of intertemporal consumption theory
- Empirical validity of baseline intertemporal consumption model?
- Empirical validity of baseline asset pricing theory?
- Hall (1978 *JPE*), Hall (1988 *JPE*)
- Lucas (1978 *Econometrica*)

TESTING THE LIFECYCLE MODEL

- **Lifecycle/Permanent Income Model the backbone of modern macro models**
  - **Business cycle models**
  - **Growth models**
  - **A main branch of asset pricing models**
- **Is it a useful positive model?**
  - **For forecastability of consumption? (Hall, 1978 *JPE*)**
  - **For effects of real interest rates on intertemporal substitution? (Hall, 1988 *JPE*)**
- **Test on aggregate data**
  - **Representative-agent assumption in background**
- **Aggregation problems may make estimating Euler equations on aggregate data (seriously) flawed**
  - **Attanasio, Section 4**
- **Appropriate measure of aggregate consumption?**
  - **Exclude durables consumption**
  - **Measure as nondurables + services**
  - **(Alternative: add back in imputed flows of stock of durable goods)**

TESTING THE LIFECYCLE MODEL

- **Standard infinite-horizon setup**

$$\max_{\{c_t,a_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ subject to } c_t + a_t = y_t + (1+r)a_{t-1}, \quad t = 0,1,2,\ldots$$

- **Assumptions**
  - **Time-invariant interest rate**
  - ***u*'(.) > 0, *u*''(.) < 0**
  - **y_t the only source of exogenous risk**
  - **$\{y_t\}_{t=0,1,2,}$ evolves as Markov**
- **Solution is a consumption sequence $\{c_t^*\}_{t=0,1,2,\ldots}$ that satisfies**
  - **Sequence of stochastic Euler equations**

$$u'(c_t^*) = \beta(1+r)E_t u'(c_{t+1}^*), \quad t = 0,1,2,\ldots$$

  - **(which is the TVC in the limit)** $\lim_{t\to\infty} E_0 \beta^t u'(c_t^*) a_t^* = 0$

taking as given (y_0, a_{-1}) and exogenous law of motion for y_t

STOCHASTIC EULER EQUATION

- **Centerpiece of analysis**

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1}) \quad \Leftrightarrow \quad E_t u'(c_{t+1}) = u'(c_t)\left[\beta(1+r)\right]^{-1}$$

- **Hall's key observation**
  - **Conditional expectation of future (one-period-ahead) consumption depends only on current consumption**
  - **Use as basis for testing**
  - **Don't need to use income data as in tests of "Keynesian consumption function"**
- **First study to seriously treat uncertainty in testing lifecycle model**

Hall (1978)

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- **Regression relation**

$$u'(c_{t+1}) = u'(c_t)\left[\beta(1+r)\right]^{-1} + \varepsilon_{t+1}, \quad E_t\varepsilon_{t+1} = 0$$

  - **ε_{t+1} the unforecastable component of marginal utility**
- **All tests have the form of estimating a (one-period-ahead) conditional expectation**

STOCHASTIC EULER EQUATION

- **Consider two forms for *u*(.)**
- **Quadratic**

$$u(c_t) = -\frac{1}{2}(\bar{c} - c_t)^2 \quad \Leftrightarrow \quad u'(c_t) = \bar{c} - c_t$$

  - $\bar{c}$ **the satiation point**
  - **But does satisfy local nonsatiation**
- **CRRA**

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} \quad \Leftrightarrow \quad u'(c_t) = c_t^{-\sigma}$$

- **Estimating equations**
  - **Quadratic** $c_{t+1} = \bar{c}\left[\frac{\beta(1+r)-1}{\beta(1+r)}\right] + \frac{c_t}{\beta(1+r)} + \varepsilon_{t+1}$ — **Tells us something about dynamics of consumption**
  - **CRRA** $c_{t+1}^{-\sigma} = \frac{c_t^{-\sigma}}{\beta(1+r)} + \varepsilon_{t+1}$ — **Tells us something about dynamics of marginal utility of consumption**

STOCHASTIC EULER EQUATION

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  - **CRRA** $c_{t+1}^{-\sigma} = \frac{c_t^{-\sigma}}{\beta(1+r)} + \varepsilon_{t+1}$
- **Suppose *β*(1+r) = 1 for intuitive clarity**
  - **Quadratic** $c_{t+1} = c_t + \varepsilon_{t+1}$ — **Consumption is a random walk!**
  - **CRRA** $c_{t+1}^{-\sigma} = c_t^{-\sigma} + \varepsilon_{t+1}$ — **Marginal utility of consumption is a random walk…**
- **Nonlinearity of marginal utility implies**
  - **Estimates/inference depend on assumption about *σ***
  - **Aggregate consumption data may be misleading for empirical tests, even conditional on a *σ* (Attanasio, Section 4)**

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- **Suppose $\beta(1+r) = 1$ for intuitive clarity**
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    Consumption is a random walk!
  - **CRRA**

    $$c_{t+1}^{-\sigma} = c_t^{-\sigma} + \varepsilon_{t+1}$$

    Marginal utility of consumption is a random walk…
- **What if $c_{t+1} \approx c_t \ \forall$ t?**
  - **Equivalently, what if ε_t is "small" $\forall$ t?**
  - **Construct first-order approximation around c_t**
    - **Start with arbitrary $\beta(1+r)$**

STOCHASTIC EULER EQUATION

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First-order Taylor expansion

$$c_{t+1} = c_t + \varepsilon_{t+1}$$

Consumption is a random walk…to first order!

Stochastic Euler Equation

- Suppose $\beta(1+r) = 1$ for intuitive clarity
  - Quadratic

$$c_{t+1} = c_t + \varepsilon_{t+1}$$

Consumption is a random walk!

  - CRRA

$$c_{t+1}^{-\sigma} = c_t^{-\sigma} + \varepsilon_{t+1}$$

Marginal utility of consumption is a random walk…

First-order Taylor expansion

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Consumption is a random walk…to first order!

- **Consumption is a random walk**
  - Benchmark empirical/theoretical idea
  - Exactly for quadratic utility
  - Approximately for arbitrary utility
  - (Strictly for $\beta(1+r) = 1$ – what if not?)
- Implies expected growth rate of consumption is zero

Results

- Run using OLS on aggregate (nondurables + services)
- Estimating equations
  - Quadratic $c_t = \bar{c}\left[\frac{\beta(1+r)-1}{\beta(1+r)}\right] + \frac{c_{t-1}}{\beta(1+r)} + \varepsilon_t$
  - CRRA $c_t^{-\sigma} = \frac{c_{t-1}^{-\sigma}}{\beta(1+r)} + \varepsilon_t$

Hall, Table 1

Model	σ	Constant	$[\beta(1+r)]^{-1}$	S.E.	R^2
CRRA	0.2	---	0.983 (0.003)	0.00073	0.996
CRRA	1.0	---	0.996 (0.001)	0.0027	0.998
Quadratic	-1.0	-0.014	1.011 (0.003)	0.0146	0.999

RESULTS

- $\beta(1+r) \approx 1$ no matter the utility form
- (Very!) high R^2 values indicate/reflect very small residuals ε_t...
- ...which in turn suggests random-walk (of consumption) interpretation may be fair
  - "...the effective content of the life cycle-permanent income theory is [that] consumption itself evolves as a random walk with trend." (Hall, 1978, p. 980)
- Simply a reflection of high degree of serial correlation of aggregate consumption
  - "Consumption smoothing" – AR(1) estimate ≈ 0.9
  - So high R^2 in and of itself not a confirmation of lifecycle model
- Residuals ε_t should be unforecastable based on data t-1 or earlier
  - Main testable implication of stochastic lifecycle model
  - (Ad-hoc) addition of lagged income, lagged consumption, etc, should not affect inference – using the theory to guide the testing

RESULTS

- Include further lags of consumption as regressors
  - Statistically insignificantly different from zero
- Include lags of disposable income as regressors
  - On the verge of significance, but essentially zero
- Include lags of wealth as regressors
  - S&P (real) stock index as proxy
  - Is statistically different from zero
- Formally rejects pure lifecycle hypothesis
- Does this invalidate the lifecycle model?
  - Hall (p. 985): Can modify the model to account for this...
  - ...relation between random-walk behavior of stock prices and random-walk behavior of consumption?

INTERTEMPORAL SUBSTITUTION

- **Intertemporal elasticity of substitution (IES)**
  - **Definition: elasticity of optimal choice of (c_{t+1}/c_t) with respect to intertemporal price (1+r) between t and t+1**

$$\text{IES} \equiv \frac{\%\text{ change in } (c_2^*/c_1^*)}{\%\text{ change in } (1+r_1)} = \frac{\partial \ln(c_2^*/c_1^*)}{\partial \ln(1+r_1)}$$

- **CRRA utility**

$$\text{IES} = \frac{1}{\sigma}$$

- **Empirical value of IES important for**
  - **Policy evaluation (e.g., distortionary effects of taxes that hit the intertemporal margin?)**
  - **Policy reform (e.g., consumption effects of changes in fiscal and/or monetary policy?)**
  - **Magnitude of business-cycle fluctuations**
  - **Speed of long-run economic growth**

INTERTEMPORAL SUBSTITUTION

- **Hall (1978) – test**

$$u'(c_t) = \beta(1+r)E_t u'(c_{t+1})$$

- **Hall (1988) – test**

$$u'(c_t) = \beta E_t\left[u'(c_{t+1})(1+r_{t+1})\right]$$

INTERTEMPORAL SUBSTITUTION

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- **Effectively a test using time-averaged data of**

$$\frac{c_{t+1}-c_t}{c_t} \equiv \gamma^c_{t+1} = \frac{r_{t+1}-\rho}{\sigma(1+r_{t+1})} = -\frac{\rho}{\sigma} + \frac{1}{\sigma}\frac{r_{t+1}}{1+r_{t+1}}$$

  - **Though econometric issues regarding appropriate time aggregation (p. 346-349)**

- **Main data**
  - **Nondurables consumption**
  - **Real S&P stock returns; T-bill returns; savings accounts returns**

INTERTEMPORAL SUBSTITUTION

- **Back-of-the-envelope "estimation" based on five-year averages**

- **Clearly not a strong positive association between *r* and γ^c, which meaningful intertemporal substitution would predict**

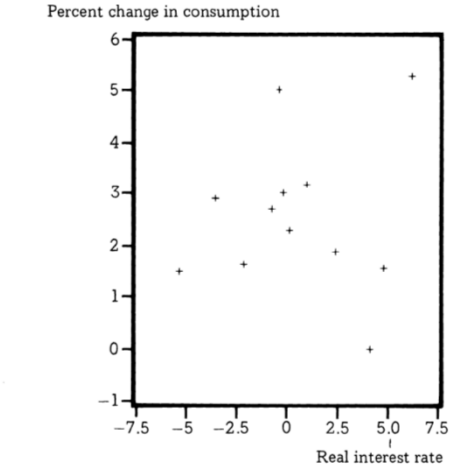

FIG. 1.—Five-year averages of the real return on Treasury bills (horizontal axis) and the rate of change of consumption (vertical axis), 1921–40 and 1946–83.

RESULTS

- Run using OLS

Asset	Estimate of IES	Implied point estimate for σ
T-bills	0.346 (0.337)	2.9
Savings accounts	0.271 (0.330)	3.7
Stocks	0.066 (0.050)	15.1

- Only estimate using S&P returns is statistically significantly different from zero…
- …and indicates very low IES!
- Robustness: different sample periods, different frequencies of data
  - Point estimates of IES all < ~ 0.4

INTERTEMPORAL SUBSTITUTION

- Implications of small point estimates of IES
  - High real rates do not provide much incentive to postpone consumption to the future…
  - …hence do not incentivize saving in the present
- If a CRRA view of the world
  - Small IES → High relative risk aversion (RRA)

$$RRA(c) \equiv -\frac{cu''(c)}{u'(c)} = \sigma \qquad \text{IES} \equiv \frac{\partial \ln(c_{t+1}/c_t)}{\partial \ln(1+r)} = \frac{1}{\sigma}$$

- Must/should IES and RRA be so directly related in reality?
  - Not at al
  - Epstein-Zin (EZ) utility function disentangles the two concepts
- Disentangling IES and RRA a central issue in empirical asset-pricing models

An Equilibrium Asset Pricing Model

- **Modify interpretation of intertemporal model**

$$\max_{\{c_t, a_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ subject to } c_t + s_t a_t = d_t a_{t-1} + s_t a_{t-1}, \quad t = 0, 1, 2, \ldots$$

  - a_t : end-of-period-t holdings of shares of an asset
  - s_t : period-t price of a share
  - d_t : period-t per-share dividend

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- **Equivalently express budget constraint as**

$$c_t + s_t \underbrace{(a_t - a_{t-1})}_{\text{change in share holdings}} = d_t a_{t-1}, \quad t = 0, 1, 2, \ldots$$

- **"Supply-side" of financial markets**
  - $a_t = 1 \; \forall \; t$ – an equilibrium outcome
  - Do not/cannot impose when solving consumer problem!

An Equilibrium Asset Pricing Model

- **FOCs yield asset pricing equation**

$$s_t = E_t\left\{\frac{\beta\lambda_{t+1}}{\lambda_t}\left(d_{t+1}+s_{t+1}\right)\right\}, \quad t=0,1,2,\ldots$$

Pricing kernel

Aka stochastic discount factor (SDF)

↓ In terms of marginal utilities

$$s_t = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(d_{t+1}+s_{t+1}\right)\right\}, \quad t=0,1,2,\ldots$$

Period-*t* asset price = SDF x Future return

Two components:
1. Future price of stock
2. Future dividend payment

An Equilibrium Asset Pricing Model

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$$s_t = E_t\left\{\frac{\beta\lambda_{t+1}}{\lambda_t}\left(d_{t+1}+s_{t+1}\right)\right\}, \quad t=0,1,2,\ldots$$

↓ Recursively substitute forward

$$s_t = E_t\left\{\sum_{\tau=t+1}^{\infty}\frac{\beta^{\tau-t}\lambda_\tau}{\lambda_t}d_\tau\right\}+\lim_{\tau\to\infty}E_t\frac{\beta^\tau\lambda_\tau}{\lambda_t}s_\tau$$

AN EQUILIBRIUM ASSET PRICING MODEL

- **Basic asset pricing equation**

$$s_t = E_t\left\{\frac{\beta\lambda_{t+1}}{\lambda_t}\left(d_{t+1}+s_{t+1}\right)\right\}, \quad t=0,1,2,\ldots$$

Recursively substitute forward

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= 0

- **Impose a TVC**
  - **Rules out asset price bubbles!**
- **The basic form of any asset pricing condition**