Department of Economics University of Maryland

Economics 601

Macroeconomic Analysis I

First-Quarter Ph.D. Macro

Problem Set 1

Professor Sanjay Chugh

Fall 2011

Due: Monday, September 19, 2011

Instructions: Written solutions must be submitted no later than 9:30am on the date listed above. Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be thoroughly presented and not leave the reader (i.e., the TA and I) guessing about what you actually meant.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in (small) groups (no larger than three people) to think through issues, ideas, and mechanics; but you must submit your own independently-written solutions, indicating with whom you collaborated. **Under no circumstances will multiple verbatim identical submissions be considered acceptable.**

Solutions should be clearly, logically, and thoroughly presented. Your method of argument(s) and approach to problems is as important as, if not more important than, your "final answer." Throughout, your analysis should be based on the methods and concepts we have developed in class and/or you have studied in related courses.

There are three problems.

Problem 1: Intertemporal Elasticity of Substitution (9 points). Derive the intertemporal elasticity of substitution (IES) for the following utility functions in the deterministic two-period model. Make clear the formal and intuitive definition of the IES. If the IES measure does not exist, explain briefly why.

a. $u(c_1, c_2) = \dfrac{c_1^{1-\sigma} - 1}{1-\sigma} + \dfrac{c_2^{1-\sigma} - 1}{1-\sigma}$, $\sigma > 0$.

b. $u(c_1, c_2) = -\dfrac{1}{a} e^{-ac_1} - \dfrac{1}{a} e^{-ac_2}$

c. $u(c_1, c_2) = \gamma c_1 - \dfrac{\alpha}{2} c_1^2 + \gamma c_2 - \dfrac{\alpha}{2} c_2^2, \gamma > 0, \alpha > 0$

Problem 2: Relative and Absolute Risk Aversion (9 points). Derive measures of relative risk aversion (RRA) and absolute risk aversion (ARA) for the following utility functions in the two-period model (technically, stochastic, but we will soon discuss this further). Make clear the formal and intuitive definition of RRA and ARA. If the RRA and/or ARA measures do not exist, explain briefly why.

a. $u(c_1, c_2) = \dfrac{c_1^{1-\sigma} - 1}{1-\sigma} + \dfrac{c_2^{1-\sigma} - 1}{1-\sigma}$, $\sigma > 0$.

b. $u(c_1, c_2) = -\dfrac{1}{a} e^{-ac_1} - \dfrac{1}{a} e^{-ac_2}$

c. $u(c_1, c_2) = \gamma c_1 - \dfrac{\alpha}{2} c_1^2 + \gamma c_2 - \dfrac{\alpha}{2} c_2^2, \gamma > 0, \alpha > 0$

Problem 3: Arrow-Debreu Securities vs. Non-Arrow-Debreu Securities (15 points). Consider two variations of the stochastic two-period consumption model. Except for what is described below, all other notation and details of the model are exactly as studied in class. In particular, period-2 income has conditional risk characterized by a three-point distribution function *G*(.), with realization y_2^H with probability $1 > q > 0$, realization $\bar{y}_2$ with probability $1 > p > 0$, and realization y_2^L with probability $(1-p-q)$.

One variation is the model studied in class that has a **single** asset a_1 available for purchase at price R in period 1 and that pays a state-contingent return in period 2.

The second variation is that, rather than the single asset a_1, there are **three types of assets** available for purchase. Each unit of asset a_1^H has purchase price R^H in period 1, and pays off one unit of goods in period 2 if state y_2^H is realized and zero in all other realized states; each unit of asset $\bar{a}_1$ has purchase price $\bar{R}$ in period 1, and pays off one unit of goods in period 2 if state $\bar{y}_2$ is realized and zero in all other realized states; and each unit of asset a_1^L has purchase price R^L in period 1, and pays off one unit of goods in period 2 if state y_2^L is realized and zero in all other realized states.

Setting up the budget constraint(s) and Lagrange analysis appropriately, show that the two different asset structures lead to two different allocations.