Department of Economics University of Maryland

Economics 601
Macroeconomic Analysis I
First-Quarter Ph.D. Macro
Problem Set 2
Professor Sanjay Chugh
Fall 2011
Due: Monday, October 3, 2011

Instructions: Written solutions must be submitted no later than 9:30am on the date listed above. Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be thoroughly presented and not leave the reader (i.e., the TA and I) guessing about what you actually meant.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in (small) groups (no larger than three people) to think through issues, ideas, and mechanics; but you must submit your own independently-written solutions, indicating with whom you collaborated. **Under no circumstances will multiple verbatim identical submissions be considered acceptable.**

Solutions should be clearly, logically, and thoroughly presented. Your method of argument(s) and approach to problems is as important as, if not more important than, your "final answer." Throughout, your analysis should be based on the methods and concepts we have developed in class and/or you have studied in related courses.

There are two problems.

Problem 1: Infinite-Horizon Consumption Model (16 points). Consider the infinite-horizon consumption model. Suppose the consumer has lifetime utility function given by

$$\lim_{T\to\infty} E_0 \sum_{t=0}^{T} \beta^t u(c_t)$$

with budget constraints given by $c_t + \sum_{i=1}^{J} R_{it} a_{it} = y_t + a_{t-1}$ for each *t*, and state-contingent (i.e., Arrow-Debreu) assets available for purchase in each period, $i \in \{1, 2, ..., J\}$. The function *u*(.) is strictly increasing and strictly concave. (And we are of course considering rational expectations in everything below.)

a. **(2 points)** Construct a recursive problem (i.e., dynamic programming problem) based on the above. State explicitly what other (if any other) assumptions are required in order to make the problem have a unique solution.

b. **(2 points)** Based on the recursive problem above, construct the Euler equation(s) for period *t*. State any further (if any other) assumptions required to make the Euler equation(s) well-behaved.

c. **(2 points)** State the complete/proper definition of the equilibrium solution of the recursive problem above. Be sure to include every detail.

d. **(2 points)** Suppose instead of the recursive problem above, you want to study the sequential problem (i.e., sequential Lagrangian). Construct a sequential problem based on the above. In particular, do **not** make any other assumptions required in order to make the problem have a unique solution.

e. **(2 points)** Based on the sequential problem above, construct the Euler equation(s) for period *t*.

f. **(2 points)** State the complete/proper definition of the equilibrium solution of the sequential problem above. Be sure to include every detail.

g. **(2 points)** Construct the value function, carefully labeling variables with their optimum values, of either the recursive problem, the sequential problem, or both.

h. **(2 points)** Suppose we are looking to differentiate the value function with respect to parametric assets. Which variable(s), if any variable(s), can we differentiate the value function with respect to? If it exists, differentiate the value function with respect to those variable(s). Provide brief interpretation.

Problem 2: Finite-Horizon Dynamic Programming (17 points). Consider a three-period (period 0, period 1, period 2) **deterministic** consumer problem. The real endowment incomes in periods 0, 1, and 2, are constant at y, and the real interest rate on assets brought into each of the three periods is also constant at r. All of this is known from the beginning of period zero (because the problem is deterministic).

Denote the value functions for periods 0, 1, and 2, respectively, by $V^0(a_{-1})$, $V^1(a_0)$, and $V^2(a_1)$; the state variables that are arguments to the value functions are already provided to you, and there are no other arguments. The value functions for "period 3" and beyond are all zero.

The lifetime utility function of the consumer, starting from the beginning of period zero, is

$$\frac{c_0^{1-\sigma}-1}{1-\sigma}+\frac{\beta(c_1^{1-\sigma}-1)}{1-\sigma}+\frac{\beta^2(c_2^{1-\sigma}-1)}{1-\sigma},$$

in which the scalar $\beta \in (0,1)$ is a standard one-period-ahead subjective discount factor, $\sigma > 0$ is a parameter of the utility function, and, as always, c_t, denotes consumption in periods t = 0, 1, 2. The sequence of budget constraints faced by the consumer are also as usual,

$$\begin{aligned}
c_0 + a_0 &= y + (1+r)a_{-1} \\
c_1 + a_1 &= y + (1+r)a_0 \\
c_2 + a_2 &= y + (1+r)a_1
\end{aligned}$$

in which a_t denotes asset holdings at the end of period t. The **terminal condition of this problem is a_2 = 0**, and the parameters of the entire lifetime utility maximization problem are (y, r, a_{-1}).

You have the following three (related) tasks:

1. Conduct a **value function iteration to develop expressions for the value functions** $V^0(a_{-1})$**,** $V^1(a_0)$ **, and** $V^2(a_1)$ (where the superscript denotes the period from which $V(.)$ begins).

2. Develop **closed-form expressions for the optimal decision rules (aka policy functions)** for consumption in each of the three periods and (end-of-period) asset holdings for period zero and period one. These policy functions should be functions of only the state variables for any given period. That is, develop closed-form expressions for the optimal choices $c_0^* = c^0(a_{-1})$, $a_0^* = a^0(a_{-1})$, $c_1^* = c^1(a_0)$, $a_1^* = a^1(a_0)$, and $c_2^* = c^2(a_1)$ that depend on only the given state variables and fixed

parameters of the problem. (You do **not** have to obtain closed-form solutions for these optimal choices in terms of only parameters of the problem.)

3. Answer the following: are there any restriction(s) on parameters that make the value functions $V^0(a_{-1}) = V^1(a_0) = V^2(a_1)$? If so, develop the restriction(s) and describe the economic intuition; if not, describe intuitively why there is no such restriction(s). (This question is likely best answered after completing the above.)

Note that while there are several ways one can solve the underlying optimization problem, you are being asked to do so via a value function iteration (i.e., you are asked to demonstrate conducting a value function iteration).