Department of Economics University of Maryland

Economics 601

Macroeconomic Analysis I

First-Quarter Ph.D. Macro

Problem Set 3

Professor Sanjay Chugh

Fall 2011

Due: Monday, October 17, 2011

Instructions: Written solutions must be submitted no later than 9:30am on the date listed above. Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be thoroughly presented and not leave the reader (i.e., the TA and I) guessing about what you actually meant.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in (small) groups (no larger than three or four people) to think through issues, ideas, and mechanics; but you must submit your own independently-written solutions, indicating with whom you collaborated. **Under no circumstances will multiple verbatim identical submissions be considered acceptable.**

Solutions should be clearly, logically, and thoroughly presented. Your method of argument(s) and approach to problems is as important as, if not more important than, your "final answer." Throughout, your analysis should be based on the methods and concepts we have developed in class and/or you have studied in related courses.

There are one (lengthy) problem.

"Dynamic" Risk Aversion (34 points). In this problem, you will measure a type of "risk" that can face consumers in an intertemporal setting, even though the underlying model you will use is "deterministic."

Consider the standard **deterministic** infinite-horizon consumer problem, with period-t utility function $u(c_t)$ that satisfies $u'(.) > 0$, $u''(.) < 0$, $u'''(.) \neq 0$, and standard Inada conditions. The consumer's one-period-ahead subjective discount factor is $\beta \in (0,1)$, and lifetime utility starting from period t is

$$\sum_{s=t}^{\infty} \beta^{s-t} u(c_s) .$$

The period-s flow budget constraint, $\forall s \geq t$ is

$$c_s + a_s = y + (1+r)a_{s-1},$$

in which a_s is the consumer's asset holdings at the end of period s, the interest rate $r > 0$ is constant in every period, and endowment income $y > 0$ is constant in every period. Suppose that the consumer's intertemporal optimization occurs at the beginning of period t, at which point the stock of asset holdings is a_{t-1}; suppose also that $\beta(1+r) = 1$.

a. **(3 points). Construct a formal, complete** statement of the solution to the **recursive version** of this problem. In doing so, be sure to include the (appropriately-defined) associated value function $V(.)$ starting from the beginning of period t, with the arguments of $V(.)$ made clear (for parsimony, you may omit any constant parameters of the problem from the list of arguments of $V(.)$). (Note: we are assuming that the "regularity conditions" that ensure an interior solution, the existence of $V(.)$, and that the solution of the recursive problem is also a solution of the sequence problem are all satisfied.)

Starting from this optimal solution, consider a (incremental) change in the consumer's assets a_{t-1}, which are his initial assets at the time of optimization.

For some concreteness, think of this incremental change in initial assets as being a "complete surprise" (in a Knightian uncertainty sense) to the individual, so there is not even a "probability distribution" for this risk that the consumer could have taken into consideration when conducting his optimization. Temporally, think of this incremental change in initial assets as occurring "immediately after" the optimization problem is solved but "before" any of the (optimally-chosen) events of period t unfold. In these senses just described, the incremental change in initial assets (which is ultimately just a perturbation of initial assets) can be considered a form of "risk," one that we can think of as, due to dynamic optimization, potentially having "long-lasting" effects.

For the remainder of the problem, let's view the value function at the start of period t as an argument of only a_{t-1}, suppressing any other arguments you may have identified in part a. That is, from here on, let's write $V(a_{t-1})$, and assume that $V(.)$ is at least twice differentiable.

b. **(9 points).** Define a measure of "generalized absolute risk aversion" as

$$GARA(a_{t-1}) \equiv -\frac{V''(a_{t-1})}{V'(a_{t-1})}.$$

This measure of risk aversion, just like the standard Arrow-Pratt measure of absolute risk aversion, can be motivated from the microeconomics of choice theory under risk, but for our purposes let's take the definition as given.

Derive an expression for $GARA(a_{t-1})$ **which depends on only the optimal choice of consumption in period *t*** (which, given the solution you defined in part a, is simply a number) **and any parameters of the problem, but nothing else. In particular, the final expression for** $GARA(a_{t-1})$ **may not include any endogenous functions.** (Hint(s): the challenge here is of course in constructing $V''(.)$; in doing so, keep in mind, and thus exploit the fact, that the perturbation is occurring around the optimal solution.)

As usual, clearly define any new notation you introduce, and present your arguments/work logically and thoroughly.

c. **(6 points).** Define a measure of "generalized relative risk aversion" as

$$GRRA(a_{t-1}) \equiv -\frac{a_{t-1}V''(a_{t-1})}{V'(a_{t-1})}$$

(which can also be motivated from the microeconomics of choice theory under uncertainty).

Based on your expression for $GARA(a_{t-1})$ above, **derive an expression for** $GRRA(a_{t-1})$ **which depends on only the optimal choice of consumption in period *t*** (which, given the solution you defined in part a, is simply a number) **and any parameters of the problem, but nothing else. In particular, the final expression for** $GRRA(a_{t-1})$ **may not include any endogenous functions.** Again keep in mind the hint(s) from above and, again as usual, clearly define any new notation you introduce, and present your arguments/work logically and thoroughly.

d. **(4 points).** How does the expression you found for $GARA(a_{t-1})$ compare to the standard Arrow-Pratt measure $ARA(c)$ when the latter is evaluated at the optimal choice of consumption in period t? Be as thorough and precise as possible in your comparison, including describing, as fully as you are able, the economic intuition for the way in which the two measures of "absolute risk aversion" compare.

e. **(4 points).** How does the expression you found for $GRRA(a_{t-1})$ compare to the standard Arrow-Pratt measure $RRA(c)$ when the latter is evaluated at the optimal choice of consumption in period t? Be as thorough and precise as possible in your comparison, including describing, as fully as you are able, the economic intuition for the way in which the two measures of "relative risk aversion" compare.

Next, for use below, define A_{t-1} as

$$A_{t-1} \equiv a_{t-1} + \frac{1}{1+r} \cdot \sum_{s=t}^{\infty} \frac{y}{(1+r)^{s-t}}.$$

f. **(6 points)** Consider instead the following proposed definition of "dynamic relative risk aversion"

$$GRRA(A_{t-1}) \equiv -\frac{A_{t-1} \cdot V''(a_{t-1})}{V'(a_{t-1})}.$$

Note that it is A_{t-1} (i.e., CAPITAL A, as defined above) that multiplies the second derivative of the value function here, while (as usual) a_{t-1} is the argument of the value function.

Derive an expression for $GRRA(A_{t-1})$ which depends on only the optimal choice of consumption in period t (which, given the solution you defined in part a, is simply a number) and any parameters of the problem, but nothing else. In particular, the final expression for $GRRA(A_{t-1})$ may not include any endogenous functions. (**Notes:** Show how to construct $V''(.)$; in doing so, keep in mind, and thus exploit the fact, that the perturbation is occurring around the optimal solution. Furthermore, you are free to immediately evaluate all relevant functions at the solution of the problem.)

g. **(2 points)** Comment as fully as you can on the nature of the three different measures of risk aversion derived above.