CONSUMPTION-LEISURE MODEL

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THE THREE MACRO (AGGREGATE) MARKETS

- **Goods Markets**
- **Labor Markets**
- **Capital/Savings/Funds/Asset Markets**

Will put micro-foundations under all three...
BASICS

- **Consumption-Leisure Model** – provides foundation for
  - Labor-market supply function
  - Goods-market demand function
  - An application of the basic consumer theory model...
  - ...we will put a macro interpretation on it
  - Only one time period – no “future” for which to save

- **Notation**
  - **c**: consumption ("all stuff")
  - **n**: number of hours spent working per week
  - **l**: number of hours leisure per week (time spent not working)
  - **P**: dollar price of one unit of consumption (a nominal variable)
  - **W**: hourly wage rate in terms of dollars (a nominal variable)
  - **t**: tax rate on labor income

- **“Weekly” model a detail**
  - Could have called it a daily model, a monthly model, a yearly model, ...
  - Just need to take SOME stand on the length of a “period”

UTILITY

- **Preferences u(c, l)** with all the “usual properties”
  - Strictly increasing in c
  - Strictly increasing in l
  - Diminishing marginal utility in c
  - Diminishing marginal utility in l
  - Plotted in good-by-good spaces:

- Plotted as indifference curves

- Utility side of consumption-leisure model no different than Chapter 1 model
BUDGET CONSTRAINT

- Consumer must **work** for his income
- Y no longer “falls from the sky”

\[ P_c = Y \]
\[ P_c = (1-t) W_n \]
\[ P_c = (1-t) W (168 - l) \]
Rearrangement
\[ P_c (1-t) W_l = 168 (1-t) W \]

Simply an application/reinterpretation of our basic consumer theory model

Chapter 1 budget constraint

- Spending on consumption
- "Spending" on leisure
- A constant from the point of view of the individual (price-taker)

\[ P_{c1} + P_{c2} = Y \]

- Spending on \( c_1 \)
- Spending on \( c_2 \)
- A constant from the point of view of the individual

(After-tax) wage is opportunity cost of leisure, hence the “price” of leisure - opportunity costs are real economic costs/prices

Simply an application/reinterpretation of our basic consumer theory model

Chapter 1 budget constraint
CONSUMER OPTIMIZATION

- Consumer’s decision problem: maximize utility subject to budget constraint – bring together both cost side and benefit side
  - Choose c and l subject to \( Pc + (1-t)Wl = 168(1-t)W \)
  - Plot budget line
  - Superimpose indifference map

- At the optimal choice
  \[
  u(c^*, l^*) = \frac{(1-t)W}{P} \]

CONSUMPTION-LEISURE OPTIMALITY CONDITION
- A key building block of modern macro models

\[ \text{MRS (between consumption and leisure)} \frac{u(c, l)}{u(c^*, l^*)} = \frac{1-t}{t} \]

IMPORTANT: the larger is \( \frac{1-t}{t} \), the steeper is the budget line

REAL WAGE

- \( W/P \) a key variable for macroeconomic analysis
- Unit Analysis (i.e., analyze algebraic units of variables)
  - Units(\( W \)) = $/hour of work
  - Units(\( P \)) = $/unit of consumption
  - Units(\( W/P \)) = $/hour of work \times \text{unit of consumption}

- Economic decisions depend on real wages (\( W/P \)), not nominal wages (\( W \))
  - Measures the purchasing power of (nominal) wage earnings...
  - ...which is presumably what people most care about

- 2008: nominal \( P \) (CPI) rose by 0.1%, nominal \( W \) rose by 2.9%
  - Real wages rose in 2008...but doesn’t mean real income rose
CONSUMER OPTIMIZATION

- **Consumer’s decision problem**: maximize utility subject to budget constraint – bring together both cost side and benefit side
  - Choose \( c \) and \( l \) subject to \( Pc + (1-t)Wl = 168(1-t)W \)
  - Plot budget line
  - Superimpose indifference map

- **At the optimal choice**

  **CONSUMPTION-LEISURE OPTIMALITY CONDITION**
  - Key building block of modern macro models
  - MRS (between consumption and leisure)
  - After-tax real wage

  \[ \frac{u_c(c^*, l^*)}{u_l(c^*, l^*)} = \frac{(1-t)W}{P} \]

  Derive consumption-leisure optimality condition using Lagrange analysis

LAGRANGE ANALYSIS

- **Apply Lagrange tools to consumption-leisure optimization**
- **Objective function**: \( u(c, l) \)
- **Constraint**: \( g(c, l) = 168(1-t)W - Pc - (1-t)Wl = 0 \)

- **Step 1**: Construct Lagrange function
  \[ L(c, l, \lambda) = u(c, l) + \lambda [168(1-t)W - Pc - (1-t)Wl] \]

- **Step 2**: Compute first-order conditions with respect to \( c, l, \lambda \)
LAGRANGE ANALYSIS

- Apply Lagrange tools to consumption-leisure optimization
- Objective function: $u(c,l)$
- Constraint: $g(c,l) = 168(1-t)W - Pc - (1-t)WL = 0$

- **Step 1:** Construct Lagrange function
  $$L(c,l,\lambda) = u(c,l) + \lambda [168(1-t)W - Pc - (1-t)WL]$$

- **Step 2:** Compute first-order conditions with respect to $c$, $l$, $\lambda$

- **Step 3:** Solve (with focus on eliminating multiplier)
  $$\frac{u(c',l') - u(c,l)}{P} = \frac{(1-t)W}{P}$$

CONSUMPTION-LEISURE OPTIMALITY CONDITION

MRS (between consumption and leisure) = After-tax real wage

MICRO-LEVEL LABOR SUPPLY

An experiment: how do micro-level consumption/leisure choices change as the real wage changes (assume $t = 0$ here for simplicity)
**Micro-Level Labor Supply**

**An experiment:** how do micro-level consumption/leisure choices change as the real wage changes (assume $t = 0$ here for simplicity)

**Real Wages:** $(W/P)_1 < (W/P)_2 < (W/P)_3 < (W/P)_4 < (W/P)_5

**Summary**
1. For low levels of real wages, a rise in the real wage causes optimal leisure to decrease
2. For intermediate levels of real wages, a rise in the real wage causes optimal leisure to remain unchanged
3. For high levels of real wages, a rise in the real wage causes optimal leisure to increase
**Labor Supply**

Using the relation $n = 168 - l$

- Backward-bending labor supply curve at the micro level...
- ..., but not at the macro level

Income effect dominates the substitution effect
- Income effect and substitution effect roughly cancel
- Sum over all individuals
- Substitution effect dominates the income effect

<table>
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<tr>
<th>Individual-level labor supply function</th>
<th>Aggregate-level labor supply function</th>
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**Consumption Demand**

- Optimal choice of consumption was always rising as real wage was rising
- Could have conducted the entire analysis assuming nominal $W$ was held fixed and nominal $P$ was falling
  - Which means real wage $W/P$ is rising

- Result: Fall in $P \rightarrow$ rise in optimal $c$ always
  - Implies downward-sloping consumption demand function at the micro level...
  - ...and at the aggregate level

- Consumption demand over two-thirds of aggregate demand in developed countries
THE THREE MACRO (AGGREGATE) MARKETS

- **Goods Markets**
  - Demand derived from C-L model

- **Labor Markets**
  - Supply derived from C-L model

- **Capital/Savings/Funds/Asset Markets**
  (aka Financial Markets)

THE MACROECONOMICS OF TIME

- **Consumption-leisure model** a static (i.e., one time period) model

- **Dynamic** models the core of modern macroeconomic theory

- Explicit consideration of how economic decisions/behaviors/outcomes unfold over multiple time periods

- **Two-period model** (Chapters 3 and 4) the simplest possible multi-period framework
  - Will allow us to begin analyzing issues regarding interest rates and inflation (phenomena that occur across time)
  - Will allow us to think about credit restrictions and the "credit crunch"

- **Infinite-period model** (Chapter 8)
  - Allows a richer quantitative description of the macroeconomy
  - Highlights the role of assets and the intersection between finance and macroeconomics