NOMINAL RIGIDITIES IN A DSGE MODEL: BASIC CALVO-YUN MODEL

OCTOBER 13, 2011

DIFFERENTIATED-GOODS FIRMS

- ❒ Optimal-pricing condition

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\underbrace{\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]}_{\text{Real marginal revenue}}\right\}=0$$

- ❒ With sticky prices, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization

- ❒ Differentiated firm *i*'s (and hence the aggregate) markup will be time-varying

  As inflation erodes the *relative* price of firm *i*

  - ❒ As "initial marginal revenues" > "initial marginal costs" to balance against "later marginal revenues" < "later marginal costs"
  - ❒ See King and Wolman (1999)

- ❒ Conduct full non-linear analysis (around distorted steady state)
  - ❒ New Keynesian analysis often conducted around efficient steady state

OPTIMAL-PRICING CONDITION

- **Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

- **Define**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

PDV of nominal marginal revenues until next price change

$$P_t x_t^2 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{t+s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

PDV of nominal marginal costs until next price change

- **Optimal-pricing condition:** $x_t^1 = x_t^2$
  - **Emphasizes that optimal *P_i* balances current and future *mr* against current and future *mc***

- **Write x_t^1, x_t^2 recursively (following SGU (2005 *NBER Macro Annual*))**

OPTIMAL-PRICING CONDITION

- **Some notation and definitions**

$$P_{it}$$

Nominal price of good *i* in period *t*

$$p_{it} \equiv \frac{P_{it}}{P_t}$$

Relative price of good *i* in period *t*

$$P_{it+1} = P_{it}$$

Evolution of nominal price if no price change

$$\downarrow$$

$$p_{it+1} = \frac{P_{it+1}}{P_{t+1}} = \frac{P_{it}}{P_{t+1}} = \frac{P_{it}}{P_t}\frac{P_t}{P_{t+1}} = \frac{p_{it}}{\pi_{t+1}}$$

As long as no nominal price change, a firm's relative price erodes at the rate of inflation

($\pi_{t+1} = P_{t+1}/P_t$)

OPTIMAL-PRICING CONDITION

- **Write x_t^1 recursively**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

↓ **Divide by P_t; write out first two terms**

$$x_t^1 = \frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

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↓ **P_{it+1} = P_{it} while no price change opportunity**

$$x_t^1 = \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it+1}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

↓ **Use definitions**

$$x_t^1 = p_{it}^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}p_{it+1}^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

↓ **Use** $p_{it+1} = p_{it}/\pi_{t+1}$

OPTIMAL-PRICING CONDITION

❐ **Write x_t^1 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}\left(\frac{p_{it}}{\pi_{t+1}}\right)^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

↓ **Rearrange**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon}p_{it}^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

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↓ **Rearrange**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon}p_{it}^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

↓ **Multiply and divide by $p_{it+1}^{1-\varepsilon}$**

$$\underbrace{x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\Big\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon}\left(\frac{p_{it}}{p_{it+1}}\right)^{1-\varepsilon}}_{\text{Have generated a recursive term}} \underbrace{p_{it+1}^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\Big\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\frac{\varepsilon-1}{\varepsilon}\right\}}$$

↓ **Express recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}^{\varepsilon}\left(\frac{p_{it}}{p_{it+1}}\right)^{1-\varepsilon} x_{t+1}^1\right\}$$

x^1 expressed recursively

OPTIMAL-PRICING CONDITION

- **Both x_t^1, x_t^2 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} x_{t+1}^1 \right\}$$

x¹ expressed recursively

$$x_t^2 = p_{it}^{-\varepsilon} y_t mc_t + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{1+\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{-\varepsilon} x_{t+1}^2 \right\}$$

x² expressed recursively

- **Optimal-pricing condition expressed compactly**

$$x_t^1 = x_t^2$$

- **Now can use usual numerical solution methods**

AGGREGATE PRICE LEVEL

- **Aggregate price index follows from Dixit-Stiglitz aggregation**

non-adjusters

adjusters

Emphasize *optimal* new price

TRACTABILITY DUE TO THE CALVO *RANDOM-ADJUSTMENT* ASSUMPTION

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di = \int_0^{\alpha} P_{it-1}^{1-\varepsilon} di + \int_{\alpha}^1 P_{it}^{*1-\varepsilon} di$$

$$= \alpha P_{t-1}^{1-\varepsilon} + (1-\alpha) P_t^{*1-\varepsilon}$$

Obtained by substituting demand functions into D-S aggregator

KEY: Because adjusters were *randomly* selected, average (aggregate) price of non-adjusters is identical to previous period's average (aggregate) price

Fraction 1 - α re-set price optimally (and symmetrically)

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KEY: Because adjusters were ***randomly*** selected, average (aggregate) price of non-adjusters is identical to previous period's average (aggregate) price

Fraction 1 - α re-set price optimally (and symmetrically)

Together form the "aggregate supply" block of New Keynesian sticky-price model

$$1 = \alpha \pi_t^{\varepsilon-1} + (1-\alpha) p_t^{*1-\varepsilon}$$

EQUILIBRIUM EVOLUTION OF AGGREGATE INFLATION – depends on relative price set by firms currently adjusting nominal price

$$x_t^1 = x_t^2$$

Optimal pricing condition

PRICE DISPERSION

- ❐ **Calvo model implies dispersion of relative prices**
  - ❐ **As does Taylor model (see Chari, Kehoe, McGrattan (2000 *Econometrica* for an example))…**
  - ❐ **…but not Rotemberg model (quadratic cost of nominal price adjustment)**

- ❐ **Dispersion often ignored until recently…**
  - ❐ **…due to linearization around a zero-inflation steady state (typical simple New Keynesian model soon…)**
  - ❐ **With better numerical tools, easier to take account of dispersion**

- ❐ **Price dispersion the basic source of welfare losses of non-zero inflation**
  - ❐ **Because it implies quantity dispersion across intermediate producers…**
  - ❐ **…which is inefficient because Dixit-Stiglitz aggregator is symmetric and concave in every good *i***

The basic driving force of optimal policy in any NK model

PRICE DISPERSION

- ❒ **For firm *i*,** $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t = z_t f(k_{it}, n_{it})$
- ❒ **Integrating over *i***

$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t \int_0^1 f(k_{it}, n_{it}) di$$

↓ Symmetric choices of ***k* / *n* ratio** across all firms ***i***…

$$y_t \underbrace{\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon}}_{\equiv s_t} di = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$

A measure of dispersion: relative price dispersion leads to dispersion of factor usage across differentiated firms, hence dispersion of quantity across differentiated firms

↓ Express s_t recursively

PRICE DISPERSION

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = \int_\alpha^1 \left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} di + \int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$$

↓ Re-setters all choose same price

$$= (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$$

↓ $P_{it} = P_{it-1}$ for firms that cannot re-set price

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$$\boxed{s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = \frac{1}{\alpha}\int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di}$$

Because of Calvo *random* adjustment and *all* adjusters choose *same* price

↓ **Multiply by $(P_{t-1}/P_{t-1})^{-\varepsilon}$**

$$= (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \underbrace{\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon}}_{\equiv \pi_t^{\varepsilon}} \underbrace{\int_0^\alpha \left(\frac{P_{it-1}}{P_{t-1}}\right)^{-\varepsilon} di}_{\equiv \alpha s_{t-1}}$$

NOTE:

$\alpha = 0$: $s_t = 1$ (no dispersion)

$\alpha > 0$: $s_t > 1$ (dispersion)

RESOURCE CONSTRAINT

- **Summarized by *three* conditions**

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + g_t$$

"Usual" resource constraint

And using factor market clearing conditions here

$$k_t = \int_0^1 k_{it} di,\ n_t = \int_0^1 n_{it} di$$

$$y_t = \frac{z_t f(k_t, n_t)}{s_t}$$

Some output is a pure deadweight loss

(note s_t < 1 cannot occur)

$$s_t = (1-\alpha)\, p_t^{*-\varepsilon} + \alpha \pi_t^{\varepsilon} s_{t-1}$$

Law of motion for deadweight loss

RESOURCE CONSTRAINT

- Summarized by *three* conditions

= 0 in Yun model

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + \cancel{g_t}$$

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Law of motion for deadweight loss

- Law of motion for s_t represented using laws of motion for both P_{t-1} and P^*_{t-1}
  - See equations (25) and (26)

OTHER MODEL DETAILS

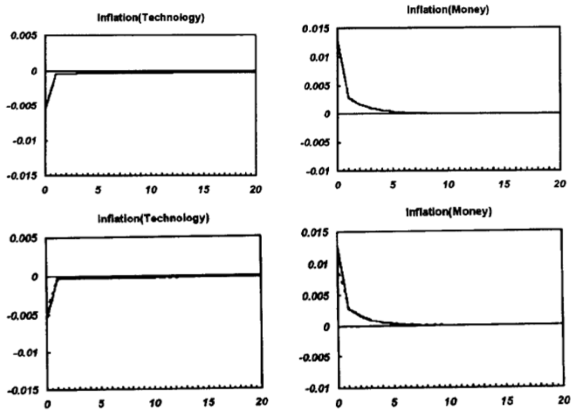
- Cash/credit to motivate money demand
  - i.e., Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991)

Could have been cashless…

- (Habit persistence (i.e., time-non-separability) in leisure)
  - As in Kydland and Prescott (1982); unimportant for main results…
- Exogenous AR(1) money growth process
  - Also "endogenous" money supply process, but not as interesting
- Exogenous AR(1) TFP process
- Indexation of prices to average (i.e., steady-state) inflation
  - For firms not re-setting price, $P_{it} = \pi P_{it-1}$ (will see again in Christiano, Eichenbaum, and Evans (2005 *JPE*))
- Approximated and simulated using linear methods
  - Using King, Plosser, Rebelo (1988) linear approximation method

NOMINAL EFFECTS OF STICKY PRICES

- ❐ **Effects on inflation not very different compared to flex-price case**



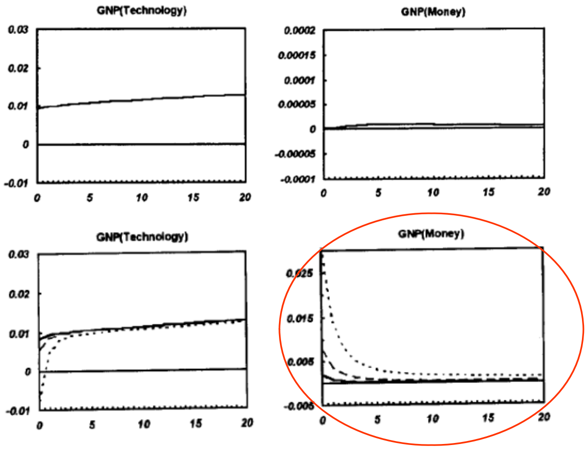
Flexible Prices

Sticky Prices

Lack of inflation persistence in basic Calvo-Yun model (now) well-known – see Steinsson (2001 *JME*)

REAL EFFECTS OF STICKY PRICES

- ❐ **Effects on GDP much bigger the stickier are prices**



Flexible Prices

Sticky Prices

MARKUP DYNAMICS

- **Money (i.e., nontechnology) shock → output expands**
- **With z_t, k_t fixed, output expansion due to increased (equilibrium) employment**
  - **Downward-sloping product demand curves → individual (differentiated) firms must expand their output (partial equilibrium)**
    - **Recall aggregate output a shifter of firm *i* demand function**
  - **Can only be achieved in the short-run if a given firm *i* hires more labor at any real wage**

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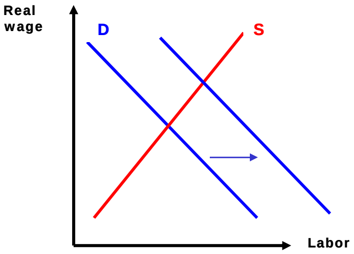

$$\frac{w_t}{z_t f_n(k_t, n_t)} = \overbrace{\frac{1}{\mu_t}}^{= mc_t}$$

Sticky-price model delivers endogenously-countercyclical markup

DSGE STICKY-PRICE MODELS

- **Nominal rigidities embedded in DSGE model**
  - **Monetary shifts → quantitatively "big" effects on output**
  - **(Re-)articulates "old" Keynesian ideas**
  - **Goodfriend and King (1997 *NBER Macroeconomics Annual*): the New-Neoclassical Synthesis**
- **Output effect not very long-lasting (peak response occurs in period of monetary shock, inconsistent with data)**
  - **The "Persistence Puzzle"**
  - **Examined in Chari, Kehoe, and McGrattan (2000 *Econometrica*) and Christiano, Eichenbaum, and Evans (2005 *JPE*)**
- **Inflation not very persistent**
  - **Steinsson (2001 *JME*): "backward-looking price-setting"**
- **A Phillips Curve?**
- **Optimal policy?**