LABOR MATCHING MODELS: BASIC DSGE IMPLEMENTATION

NOVEMBER 10, 2011

FIRM VACANCY-POSTING PROBLEM

- Dynamic firm profit-maximization problem

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - \underbrace{w_t n_t^f h_t}_{} - \underbrace{g(v_t)}_{} \right) \right]$$

Discount factor between time 0 and *t* because ***dynamic*** firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many "job advertisements")

Desired target ***future*** firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting ***v*** vacancies

  - Subject to (perceived) law of motion for firm's employment stock

- Baseline model
  - Shut down intensive margin: $h_t = 1$
  - Linear posting costs: $g(v) = \gamma v$
  - Firm production function: $y_t = z_t * n_t$
  - Wage-setting (process) taken as given when posting vacancies

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$$\text{s.t. } n_{t+1}^f = \underbrace{(1-\rho^x) n_t^f}_{\text{Number of existing jobs that do not end: } \rho^x \text{ exogenous separation rate, but can also endogenize}} + \underbrace{v_t k^f(\theta_t)}_{\text{Each vacancy has probability } k^f(\theta) \text{ of attracting a prospective employee}}$$

Perceived law of motion for evolution of employment stock

Number of existing jobs that do not end: ρ^x exogenous separation rate, but can also endogenize

Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a ***market*** variable, θ, so taken as given

FOCs with respect to v_t, n_{t+1}

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho^x) \mu_{t+1} \right) \right\} = 0$$

Combine

FIRM VACANCY-POSTING PROBLEM

- Vacancy posting condition (aka job creation condition)

$$\gamma = k^f(\theta_t) E_t \underbrace{\left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}}_{\text{Expected benefit of posting a vacancy}}$$

γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of attracting a worker) x (expected future benefit of an additional worker)

= marginal output – wage payment + expected asset value of an additional worker

Firm Vacancy-Posting Problem

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EMPLOYEES ARE ASSETS

- Vacancy-posting is a type of investment decision
  - Intertemporal dimension makes discount factor potentially important
    - i.e., makes general equilibrium effects potentially important
- Two prices affect posting decision (aside from intertemporal price)
  - (Future) wage
  - Matching probability (loosely interpret probabilities as prices) which depends on the market variable θ

Household Problem

- Dynamic household utility-maximization problem
  - A continuum [0, 1] of households (a standard assumption)
  - A continuum [0, 1] of atomistic individuals live in each household
  - Thus representative household has a continuum of "family members"

$$\max_{c_t, a_t}\left[E_0\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

$$\text{s.t. } c_t + a_t = \underbrace{n_t w_t h_t}_{} + \underbrace{(1-n_t)b}_{} + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

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KEY: Assuming infinite *family* structure delivers *full consumption insurance* – i.e., all employed and unemployed individuals have equal consumption!

Thus individual family members are risk-neutral with respect to their labor-market realization

Analogy with Hansen-Rogerson structure (see Andolfatto 1996 *AER*)

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- Consumption-savings optimality condition: $1 = R_t E_t \underbrace{\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}}_{\text{Stochastic discount factor}}$

- No labor-supply/part. margin in basic model
  - Each family member either works or is looking for work

WAGE BARGAINING

- **(Generalized) Nash Bargaining**

$$\max_{w_t} \underbrace{\left(W(w_t)-U(w_t)\right)^{\eta}}_{\text{Net payoff to an individual/household of agreeing to wage } w \text{ and beginning production}} \underbrace{\left(J(w_t)-V(w_t)\right)^{1-\eta}}_{\text{Net payoff to a firm of agreeing to wage } w \text{ and beginning production}}$$

Bargaining over how to divide the surplus

DSGE Labor Search Model

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Bargaining over how to divide the surplus

- **Asset values**
  - ***W***: value to (representative) household of having one additional member employed
  - ***U***: value to (representative) household of having one additional member unemployed and searching for work
  - ***J***: value to (representative) firm of having one additional employee
  - ***V***: value to (representative) firm of having a job that goes unfilled
    - Free entry in vacancy-posting → ***V*** = 0
- **Define *W* and *U* in terms of household problem**
  - i.e., based on envelope conditions of household value function

WAGE BARGAINING

- (Generalized) Nash Bargaining

$$\max_{w_t} \left(W(w_t)-U(w_t)\right)^{\eta} J(w_t)^{1-\eta}$$

Bargaining over how to divide the surplus

Net payoff to an individual/household of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

- The Nash surplus-sharing rule

$$\eta\left(W'(w_t)-U'(w_t)\right)J(w_t)=(1-\eta)(-J'(w_t))\left(W(w_t)-U(w_t)\right)$$

(FOC with respect to w_t)

  - Present in any model with Nash bargaining
    - (Most) labor matching models
    - (Most) monetary search models
    - Political bargaining games (Albanesi 2007 *JME*)

- Must specify value equations ***W***(.), ***U***(.), ***J***(.)

VALUE EQUATIONS

- Individual/household value equations (constructed from household problem)

Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

$$W(w_t)=\underbrace{w_t}_{\text{Contemporaneous return is wage}}+E_t\left\{\Xi_{t+1|t}\underbrace{\left[(1-\rho^x)W(w_{t+1})+\rho^x U(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual employed

$$U(w_t)=\underbrace{b}_{\text{Contemporaneous return is unemployment benefit/home production}}+E_t\left\{\underbrace{\Xi_{t+1|t}\left[k^h(\theta_t)W(w_{t+1})+(1-k^h(\theta_t))U(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual unemployed and searching

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Value to household of having the marginal individual unemployed and searching

- **Firm value equation**

$$J(w_t) = \underbrace{z_t - w_t}_{\text{Contemporaneous return is marginal output net of wage payment}} + E_t \underbrace{\left\{ \Xi_{t+1|t} (1-\rho^x) J(w_{t+1}) \right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to firm of the marginal employee

WAGE BARGAINING

- **The Nash surplus-sharing rule**

$$\eta\left(W'(w_t) - U'(w_t)\right) J(w_t) = (1-\eta)(-J'(w_t))\left(W(w_t) - U(w_t)\right)$$

(FOC with respect to w_t)

Insert marginal values

$$\eta J(w_t) = (1-\eta)\left(W(w_t) - U(w_t)\right)$$

Firm's surplus J a constant fraction of household's surplus $W - U$

NOTE: NOT a general property of Nash bargaining; here due to the linearity of W, U, and J with respect to wage

Using definitions of W, U, and J, the job-creation condition, and some algebra

$$\boxed{w_t = \eta \underbrace{\left[z_t + \gamma\theta_t \right]}_{} + (1-\eta) b}$$

Contemporaneous marginal output…

…and a term that captures the social savings on future posting costs if match continues

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

NOTE: With CRS matching function, $\theta = k^h(\theta) / k^f(\theta)$

LABOR MARKET MATCHING

- Aggregate matching function displays CRS

$$m(u_t, v_t)$$

$u_t = 1 - n_t$ is measure of individuals searching for work

- For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on ***v / u***

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$$\frac{m(u_t, v_t)}{v_t} = m\left(\frac{u_t}{v_t}, 1\right) = m\left(\theta_t^{-1}, 1\right) \equiv k^f(\theta_t)$$

Probability a given vacancy / job posting attracts a worker

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Probability a given individual finds a job opening

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

DSGE Labor Search Model

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In matching models, θ is the key driving force of efficiency and therefore optimal policy prescriptions (Hosios 1990 *ReStud* the key reference)

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$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

- Market tightness an allocational signal
  - Because matching probabilities depend on it
  - e.g., the higher (lower) is ***v / u,*** the easier (harder) it is for a given individual to find a job opening

LABOR-MARKET EQUILIBRIUM

- **Aggregate law of motion of employment**

$$N_{t+1} = (1-\rho^x)N_t + m(u_t, v_t)$$

- **Flow equilibrium conditions (an accounting identity…)**

$$m(u_t, v_t) = u_t k^h(\theta_t) = v_t k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t)E_t\left\{\Xi_{t+1|t}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

- **Wage determination**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Basic labor-theory literature: impose ss on these and analyze, do comparative statics, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

GENERAL EQUILIBRIUM

- **Aggregate law of motion for employment**
- **Vacancy-posting (aka job-creation) condition**
- **Wage determination**

The labor market equilibrium (*partial* equilibrium from the perspective of the entire environment)

- **Consumption-savings optimality condition (endogenizes real interest rate)**

$$1 = R_t E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\right\}$$

- **Aggregate resource constraint**

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1-N_t)b$$

Vacancy posting costs and "outside option" are real uses of resources

- **Exogenous LOMs for any driving processes (TFP, etc)**

STEADY STATE OF LABOR MARKET

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1-u=(1-\rho^x)(1-u)+m(u,v)$$ (using $N = 1 - u$)

(2) $$\gamma=\beta k^f(\theta)\left(z-w+\frac{(1-\rho^x)\gamma}{k^f(\theta)}\right)$$ w negatively and nonlinearly related to θ (given CRS matching function)

(3) $$w=\eta\left[z+\gamma\theta\right]+(1-\eta)b$$ w positively and linearly related to θ

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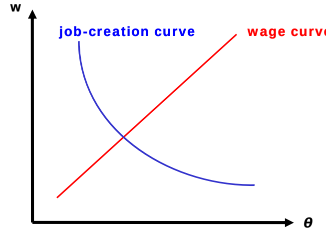
Pissarides Figure 1.1



"Labor supply curve" and "labor demand curve" replaced by "wage curve" and "job-creation curve"

The relevant "quantity" variable θ – but can also loosely think of θ as a "price" because it governs matching probabilities…

Steady State of Labor Market

- Imposing deterministic steady state on labor-market equilibrium conditions

(1) $$u = \frac{m(u,v)+\rho^x}{\rho^x}$$

For a given (w,θ), v and u negatively related (given CRS matching function)

(2) $$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho^x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)

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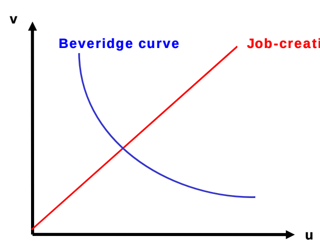
Pissarides Figure 1.2



BEVERIDGE CURVE: Empirical relationship in both long run and short run (i.e., cyclical)

STEADY STATE OF LABOR MARKET

- Labor-market equilibrium is (*w, u, θ*) satisfying (1), (2), (3)

- Comparative statics
  - A rise in *b*…
    - …raises *w*
    - …lowers *θ*
    - …lowers *v* and raises *u*

Higher value (ue benefit) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

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  - A fall in β (or a rise in ρ^x)…
    - …lowers *w*
    - …lowers *θ*
    - …raises *u*
    - …ambiguous effect on *v*

Higher real rate and/or faster job separations (i.e., "faster depreciation of employment stock") makes posting jobs (FOR FIXED *u*) less attractive for firms (both erode firm profits)

- See Pissarides Chapter 1 and RSW (2005 *JEL*) for more

- Next: dynamic stochastic partial equilibrium (Shimer 2005, Hall 2005, and Hagedorn and Manovskii 2008)