# LABOR MATCHING MODELS: EFFICIENCY PROPERTIES

## NOVEMBER 22, 2011



## LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching "technology"**

$$\max_{c_t, v_t, N_{t+1}} \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b \qquad \text{Fix } h = 1$$

$$N_{t+1} = (1 - \rho^x) N_t + m(u_t, v_t) \qquad \text{And } N = 1 - u$$

## LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching "technology"**

$$\max_{c_t, v_t, N_{t+1}} \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b \qquad \text{Fix } h = 1$$

$$N_{t+1} = (1 - \rho^x) N_t + m(1 - N_t, v_t) \qquad \text{And } N = 1 - u$$





## LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching "technology"**

$$\max_{c_t, v_t, N_{t+1}} \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

Multipliers

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b \qquad \text{Fix } h = 1 \qquad \lambda_t$$

$$N_{t+1} = (1 - \rho^x) N_t + m(1 - N_t, v_t) \qquad \text{And } N = 1 - u \qquad \mu_t$$

  - **FOCs**

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2(1 - N_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} \left[ z_{t+1} - b \right] \right\} + \beta E_t \left\{ \mu_{t+1} \left[ (1 - \rho^x) - m_1(1 - N_t, v_t) \right] \right\} = 0$$

↓ Eliminate multipliers

## LABOR-MATCHING EFFICIENCY

- **Social Planning problem**

$$\frac{\gamma}{m_2(1-N_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-N_t,v_t)}{m_2(1-N_t,v_t)} + \frac{(1-\rho^x)\gamma}{m_2(1-N_t,v_t)}\right]\right\}$$





## LABOR-MATCHING EFFICIENCY

- **Social Planning problem**

$$\frac{\gamma}{m_2(1-N_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-N_t,v_t)}{m_2(1-N_t,v_t)} + \frac{(1-\rho^x)\gamma}{m_2(1-N_t,v_t)}\right]\right\}$$

Cobb-Douglas matching

$m(u,v) = u^{\alpha}v^{1-\alpha}$

$m_1(u,v) = \alpha u^{\alpha-1}v^{1-\alpha} = \alpha\theta^{1-\alpha}$

$m_2(u,v) = (1-\alpha)u^{\alpha}v^{-\alpha} = (1-\alpha)\theta^{-\alpha}$

AND

$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$

$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$

Combine and rearrange

AND $m_1(u,v) = \alpha k^h(\theta)$ $m_2(u,v) = (1-\alpha)k^f(\theta)$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

**KEY IDEAS**

**Taking the pricing kernel as given, the only unknown process here is $\theta_t$!**

**Efficiency in job-postings is governed by "getting market tightness right!"**

## LABOR-MATCHING EFFICIENCY

- Socially-efficient vacancy posting characterized by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

- Recall decentralized vacancy posting characterized by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\} \quad \text{AND} \quad w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

$$\downarrow$$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - \left(\eta\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\eta)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$





## LABOR-MATCHING EFFICIENCY

- Socially-efficient vacancy posting characterized by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

- Recall decentralized vacancy posting characterized by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\} \quad \text{AND} \quad w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

$$\downarrow$$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - \left(\eta\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\eta)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

- **Efficiency in vacancy posting requires $\eta = \alpha$!**

## HOSIOS CONDITION

- Cobb-Douglas matching technology + Nash bargaining
  - Efficient level of job-creation requires $\eta = \alpha$
  - Hosios (1990 *ReStud*)
- Intuition: search activity generates externalities
  - One extra individual (firm) searching for a job (worker) lowers the probability that *all other* individuals (firms) will find a match…
  - …but raises the probability that *all other* firms (individuals) will find a match
  - Congestion externality – search imposes both positive and negative externalities (on opposite sides of the market)



Efficiency Considerations

## HOSIOS CONDITION

- Cobb-Douglas matching technology + Nash bargaining
  - Efficient level of job-creation requires $\eta = \alpha$
  - Hosios (1990 *ReStud*)
- Intuition: search activity generates externalities
  - One extra individual (firm) searching for a job (worker) lowers the probability that *all other* individuals (firms) will find a match…
  - …but raises the probability that *all other* firms (individuals) will find a match
  - Congestion externality – search imposes both positive and negative externalities (on opposite sides of the market)
- Nash bargaining: $\eta$ governs the private returns to search
  - Share of total match surplus kept by individual
- Cobb-Douglas matching: $\alpha$ governs the social returns to search
  - Elasticity of aggregate number of matches with respect to $u$
- Efficiency requires equating private and social returns: $\eta = \alpha$

## HOSIOS CONDITION

- Also holds under some more general conditions
  - Endogenous search intensity
  - Endogenous "vacancy posting intensity" (Pissarides Chapter 5)
- Pissarides (2000, p. 198): "..we are not likely to find intuition for it..."
- RSW (2005 *JEL* p. 982): "...genuinely surprising result..."
- Is the Hosios condition empirically relevant?
  - Almost surely not...it's a nongeneric parameterization...
  - Nonetheless has become a focal point for calibrated models
- Hosios efficiency emerges endogenously in competitive search equilibrium concept
  - Moen (1997 *JPE*): basic static partial labor search model
  - A well-understood concept in labor theory, but little incorporation into DSGE models





## COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?
  - Would get away from the non-genericity of the Hosios bargaining parameterization
  - May be apriori an appealing way of describing labor markets
    - Locating a firm or a worker is costly and time-consuming...
    - ...but once matched, wages are more or less determined by "market forces," perhaps with little/no room for "bargaining"

## COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?
  - Would get away from the non-genericity of the Hosios bargaining parameterization
  - May be apriori an appealing way of describing labor markets
    - Locating a firm or a worker is costly and time-consuming…
    - …but once matched, wages are more or less determined by "market forces," perhaps with little/no room for "bargaining"
- Moen (1997 ***JPE***) and Shimer (1996) the original implementations of CSE
  - Static partial-equilibrium labor search models
- Will implement in the context of our full DSGE labor-search model
  - Only recently have started to become incorporated into DSGE search models….
  - …but goods-search models, not labor-search (Arseneau and Chugh (2009), Gourio and Rudanko (2009) (Menzio and Shi (2010 ***JET***) a labor-search application)





## CSE – BASICS OF ENVIRONMENT

- Need "many markets" and "many firms"
  - To rationalize "competition," so can operationalize decentralized wage-formation process
- Index continuum of labor "submarkets" by $j$ – e.g., local labor markets
- Within a submarket $j$, many firms looking to hire workers
  - Even within a "local" labor market, coordination frictions in finding workers may exist
  - Index by $i$

## CSE – BASICS OF ENVIRONMENT

- Need "many markets" and "many firms"
  - To rationalize "competition," so can operationalize decentralized wage-formation process
- Index continuum of labor "submarkets" by *j* – e.g., local labor markets
- Within a submarket *j*, many firms looking to hire workers
  - Even within a "local" labor market, coordination frictions in finding workers may exist
  - Index by *i*
- Unemployed individuals *direct* their job search ("send an application") to a particular submarket
  - Based on wages announced by firms in that submarket, and on likelihood of getting a job in that submarket
  - Not random search – directed search is key for concept of CSE
  - Once search is directed, random matching process governs whether an individual gets a job – match formation is still subject to frictions
- Wages determined before matching, not after matching (ex-ante vs. ex-post)
  - All parties direct search according to "posted" wages





## CSE – BASICS OF ENVIRONMENT

- Wages determined before matching, not after matching
  - All parties direct search according to "posted" wages
- Several equivalent ways to implement
  - Perfectly-competitive "market-maker" sector
  - Individuals announce wages before firms search for workers
  - Firms announce wages before individuals search for jobs
    - The implementation we will pursue
  - See RSW 2005 *JEL* survey for alternative implementations

# CSE – BASICS OF ENVIRONMENT

- Wages determined before matching, not after matching
  - All parties direct search according to "posted" wages
- Several equivalent ways to implement
  - Perfectly-competitive "market-maker" sector
  - Individuals announce wages before firms search for workers
  - Firms announce wages before individuals search for jobs
    - The implementation we will pursue
  - See RSW 2005 *JEL* survey for alternative implementations
- Idea of firm wage-posting/wage-announcement implementation
  - Define (expected) payoff function to firm *ij* of finding an additional worker
  - Define (expected) payoff function to individual searching for/applying to a job at firm *ij*
  - Firm *ij* maximizes its payoff subject to the reaction function defined by the individual's payoff function
    - i.e., firm *internalizes* the effect of wages on the other side of the market…
    - …can already see how congestion *externality* issues will be taken care of…
- Internalizing congestion externalities would also be achieved by…
  - Individuals announcing wages taking into account reactions by firms
  - "Market maker" calling out wages taking account reactions by both sides of market





# CSE – IMPLEMENTATION

- Firm *ij* payoff function described by vacancy-posting decision!

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]$$

Cost of posting a vacancy

Expected benefit of posting a vacancy
= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*…

…but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

# CSE – IMPLEMENTATION

- **Firm *ij* payoff function described by vacancy-posting decision!**

$$\gamma = k^f(\theta_{ijt})\left[ z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]$$

Cost of posting a vacancy

Expected benefit of posting a vacancy
= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*...

...but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

- **Value equations for an individual searching for a match at firm *ij***

$$W(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho^x)W(w_{ijt+1}) + \rho^x U_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$U_t = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)W(w_{t+1}) + (1-k^h(\theta_t))U_{t+1}\right]\right\}$$

With probability $1-k^h(\theta_{ijt})$, individual gets this payoff

- **With individuals (households) optimally directing their search, the expected payoff of searching for / applying to a job at firm *ij* is**

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

Payoff of searching at another firm or another submarket independent of *ij*





# CSE – IMPLEMENTATION

- **Firm *ij* maximizes**

$$\gamma = k^f(\theta_{ijt})\left[ z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]$$

**taking as constraint**

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

- **Choice variables: $w_{ijt}$ and $\theta_{ijt}$ (isomorphic to choosing $v_{ijt}$ for a given number of searchers $u_{ijt}$)**

# CSE – IMPLEMENTATION

- **Firm *ij* maximizes**

$$\gamma = k^f(\theta_{ijt})\left[ z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left( \frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]$$

**taking as constraint**

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

- **Choice variables: $w_{ijt}$ and $\theta_{ijt}$ (isomorphic to choosing $v_{ijt}$ for a given number of searchers $u_{ijt}$)**

- **First-order conditions**

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})W'(w_{ijt}) = 0$$

$$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}}\left[ z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left( \frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}}\left[ W(w_{ijt}) - U_t \right] = 0$$

The central idea of CSE: taking into account how matching probabilities are affected by tightness





# CSE – IMPLEMENTATION

- **First-order conditions**

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})W'(w_{ijt}) = 0 \xrightarrow{W'(.)\,=\,1} \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

$$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}}\left[ z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left( \frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}}\left[ W(w_{ijt}) - U_t \right] = 0$$

$= J_{ijt}$ if firms are optimizing

# CSE – IMPLEMENTATION

- **First-order conditions**

$$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) W'(w_{ijt}) = 0 \xrightarrow{W'(.)=1} \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

$$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}} \underbrace{\left[ z_t - w_{ijt} + (1-\rho^x) E_t \left\{ \Xi_{t+1|t} \left( \frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt} \text{ if firms are optimizing}} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}} \left[ W(w_{ijt}) - U_t \right] = 0$$

Cobb-Douglas matching

$$m(u,v) = u^{\alpha} v^{1-\alpha}$$

Combine and rearrange

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

AND

$$\frac{\partial k^h(\theta)}{\partial \theta} = (1-\alpha)\theta^{-\alpha}$$

$$\frac{\partial k^f(\theta)}{\partial \theta} = -\alpha\theta^{-\alpha-1}$$

(Competition within submarket *j* and symmetry across submarkets: drop *ij* indices)

$$\boxed{\alpha\left(W(w_t) - U_t\right) = (1-\alpha) J(w_t)}$$

Exactly the Nash-bargaining sharing rule with endogenous emergence of Hosios condition ($\eta = \alpha$)!!!

Inserting value equations and solving explicitly for wage obviously gives same outcome as Nash-bargained wage with $\eta = \alpha$...





# CSE – INTEREPRETATIONS

- **Mortensen and Pissarides (1999 *Handbook Chapter* p. 2589-2592)**
  - "Price of time" priced efficiently by markets in CSE
  - "Price of time" generically mispriced in bargaining equilibrium
  - ("Price of time" = matching probabilities, which reflect congestion externalities)
  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting
  - CSE effectively fills in this missing market...
  - ...provided we're willing to assume/believe the strong degree of commitment built into CSE model
    - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
    - An open question in search theory

## CSE – INTEREPRETATIONS

- Mortensen and Pissarides (1999 *Handbook Chapter* p. 2589-2592)
  - "Price of time" priced efficiently by markets in CSE
  - "Price of time" generically mispriced in bargaining equilibrium
  - ("Price of time" = matching probabilities, which reflect congestion externalities)
  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting
  - CSE effectively fills in this missing market…
  - …provided we're willing to assume/believe the strong degree of commitment built into CSE model
    - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
    - An open question in search theory
- CSE in principle an alternative equilibrium concept in search models
  - But turns out to be equivalent to bargaining equilibrium with Hosios condition
  - (At least in simple environments….will equivalence hold in richer environments?…)
- Little explored in DSGE contexts
  - Question: Would some types of market frictions, tax issues, etc break the equivalence between CSE and Nash-Hosios bargaining?…





## RELEVANCE OF HOSIOS CONDITION IN DSGE

- Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$
  - Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to
  - Deviation from Friedman Rule can be used to correct search externalities (Cooley and Quadrini (2004 *JET*), Arseneau and Chugh (2008 *JME*, 2010), Faia (2008 *JEDC*))
- Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$
  - Walsh (2005 *RED*)

## RELEVANCE OF HOSIOS CONDITION IN DSGE

- Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$
  - Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to
  - Deviation from Friedman Rule can be used to correct search externalities (Cooley and Quadrini (2004 ***JET***), Arseneau and Chugh (2008 ***JME***, 2010), Faia (2008 ***JEDC***))
- Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$
  - Walsh (2005 ***RED***)
- Hosios issues arise in any DGE model with any type of search/matching market
  - Money search models
    - Rocheteau and Wright (2005 ***Econometrica***)
    - Aruoba and Chugh (2010 ***JET***)
  - Product search models
    - Hall (2007)
    - Arseneau and Chugh (2009)





## DSGE (LABOR) MATCHING MODELS

- Matching models articulate trading frictions – cannot instantaneously/costlessly find trading partners
  - An appealing description of labor markets
  - Maybe of other markets also
- Tractable to incorporate in DSGE models because of assumption of aggregate matching function
- Too ad-hoc or "reduced-form" because of assumption of (black box) aggregate matching friction?

# DSGE (LABOR) MATCHING MODELS

- Matching models articulate trading frictions – cannot instantaneously/costlessly find trading partners
  - An appealing description of labor markets
  - Maybe of other markets also
- Tractable to incorporate in DSGE models because of assumption of aggregate matching function
- Too ad-hoc or "reduced-form" because of assumption of (black box) aggregate matching friction?
- The Shimer Puzzle and attempted answers continue…
- …as do New Keynesian modelers' incorporation of labor matching structure
  - Enables talking meaningfully about the tradeoffs between inflation and unemployment…
  - …i.e., seemingly resuscitates the original Phillips Curve, not the NK Phillips Curve (which links inflation to marginal costs…)