BASICS OF DYNAMIC STOCHASTIC GENERAL EQUILIBRIUM

SEPTEMBER 6, 2011

HETEROGENEITY

- Implementing representative consumer
  - An infinity of consumers, each indexed by a point on the unit interval [0,1]
  - Each individual is identical in preferences and endowments
  - Implies aggregate consumption demand and asset demand

Aggregate consumption demand = One individual's consumption demand x **1**

Aggregate savings demand = One individual's savings demand x **1**

- Under some particular types of heterogeneity, a representative-consumer foundation of aggregates exists
  - Provided complete set of Arrow-Debreu securities exists…
  - …to allow individuals to diversify away (insure) their idiosyncratic risk

- Consider heterogeneity
  - In income realizations (from Markov process)
  - In initial asset holdings *a*
  - In utility functions (application to CRRA utility)
  - Example: two types of individuals to illustrate

HETEROGENEITY

- **Two types of individuals, $i \in \{1,2\}$, each with population weight 0.5**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u^i(c_t^i) \quad \textbf{subject to} \quad c_t^i + \sum_j R_t^j a_t^{ij} = y_t^i + a_{t-1}^i$$

- **Optimization between period t and state j in period t+1 (conditional on period t outcomes)**

$$\frac{u_c^1(c_t^1)}{\beta u_c^1(c_{t+1}^{1j})} = \frac{R_t^j}{p_{t+1}^j} \qquad \frac{R_t^j}{p_{t+1}^j} = \frac{u_c^2(c_t^2)}{\beta u_c^2(c_{t+1}^{2j})}$$

Given all individuals base choices on same prices and probabilities

Towards A Representative Consumer

RISK SHARING

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In all states at all dates

- **PERFECT RISK SHARING**
  - **IMRS, for each state j, equated across individuals**
  - **Individuals experiencing idiosyncratic shocks can insure them away (provided complete markets)**
- **Risk sharing about equalizing fluctuations of $u'(.)$ across individuals**
  - **Not about equalizing levels of $u'(.)$ or consumption over time**
- **If initial conditions, period-zero outcomes, and $u(.)$ are identical (e.g., due to identical a_0 and realized y_0), then risk sharing → identical outcomes $\forall$ t**

→ A representative consumer

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Risk sharing across individuals ≈ consumption smoothing for a given individual
(= if initial conditions, t=0 outcomes, and $u(.)$ identical)

RISK SHARING

- Example: CRRA utility, but heterogenous RRA/IES
  - $\sigma^1 \neq \sigma^2$

$$\left(\frac{c_t^1}{c_{t+1}^{1j}}\right)^{-\sigma^1} = \left(\frac{c_t^2}{c_{t+1}^{2j}}\right)^{-\sigma^2}$$

Perfect risk sharing

  - IMRS equated across individuals
- Growth rates of consumption not equated unless $\sigma^1 = \sigma^2$

$$\frac{c_{t+1}^{1j}}{c_t^1} = \left(\frac{c_{t+1}^{2j}}{c_t^2}\right)^{\sigma^2/\sigma^1}$$

AGGREGATION

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- **Allocations are Pareto-optimal (implied by First Welfare Theorem)**
  - **All MRS's (across individuals, states, and dates) are equated**
  - **Even though levels of consumption may differ across individuals**
  - **No individual can be made better off without making some agent worse off**
  - **(Pareto welfare concept takes distributions of outcomes as given)**
  - **Due to complete financial markets**

- **Pareto-optimal allocations + heterogeneity of utility functions**
  - **There exists a utility function $u(c)$ in aggregate $c \equiv c^1 + c^2$ that leads to the same aggregates (Constantanides (1982)); if CRRA, $u(.)$ has $\sigma \in (\sigma^1, \sigma^2)$**

AGGREGATION

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- **Pareto-optimal allocations + heterogeneity of utility functions**
  - **There exists a utility function $u(c)$ in aggregate $c \equiv c^1 + c^2$ that leads to the same aggregates; proof relies on general equilibrium theory**

AGGREGATION

- Now consider economy-wide aggregates

$$c_t = 0.5c_t^1 + 0.5c_t^2$$ Aggregate consumption

$$y_t = 0.5y_t^1 + 0.5y_t^2$$ Aggregate income (endowment)

(For each type of asset) $$a_t = 0.5a_t^1 + 0.5a_t^2$$ Aggregate assets?

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- So far have been considering assets as claims (paper!) (partial equilibrium)
- In aggregate, must be some tangible asset(s) backing them (gen. equil.)

- No physical assets in model so far → a_t = ***0 in aggregate*** $\forall$ t !

AGGREGATION

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(For each type of asset) $$0 = a_t = 0.5a_t^1 + 0.5a_t^2$$ **Aggregate assets = 0 if no *physical* assets**

- **So far have been considering assets as claims (paper!) (partial equilibrium)**
- **In aggregate, must be some tangible asset(s) backing them (gen. equil.)**
- **No physical assets in model so far → a_t = *0 in aggregate* ∀ t !**
- **Heterogeneous individuals creating/buying/selling assets vis-à-vis each other**
- **Richer models**
  - **Mediate through "banking" or "insurance" markets, etc.**
  - **But only meaningful if some friction/imperfections in model of financial markets…**
  - **…otherwise identical outcomes (in which case "banking" sector is a "veil")**

AGGREGATION

- **Economy-wide aggregates**

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Asset market clearing condition (for each type of asset) $$0 = a_t = 0.5a_t^1 + 0.5a_t^2$$ **Aggregate assets = 0 if no *physical* assets**

  - **Aggregate savings = $a_t - a_{t-1}$ = 0 ∀ t**
- **Aggregate together two types' budget constraints**

$$c_t^1 + \sum_j R_t^j a_t^{1j} = y_t^1 + a_{t-1}^1 \qquad c_t^2 + \sum_j R_t^j a_t^{2j} = y_t^2 + a_{t-1}^2$$

  - **Weight by share of population**

$$\Rightarrow \quad 0.5(c_t^1 + c_t^2) + \sum_j R_t^j 0.5(a_t^{1j} + a_t^{2j}) = 0.5(y_t^1 + y_t^2) + 0.5(a_{t-1}^1 + a_{t-1}^2)$$

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Towards A Representative Consumer

AGGREGATION

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A general procedure for constructing economy-wide resource constraint

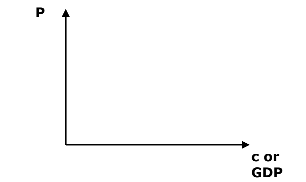
goods available = goods used

$$\Rightarrow \; c_t = y_t$$

Goods market clearing condition – aka resource constraint

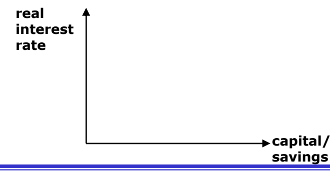
The Three Macro (Aggregate) Markets

- **Goods Markets**



- **Labor Markets**

- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



Towards Dynamic General Equilibrium

- **Lifecycle/permanent income consumption model the most basic building block of all macro models**
- **Dynamic stochastic general equilibrium (DSGE) theory**
  - **(DGE if deterministic)**
  - **GE: simultaneous determination of prices and quantities in all markets (macro markets: goods, labor, capital)**

- **Foundations of baseline DSGE model**
  - **Representative consumer**
  - **Representative firm**
  - **Perfect competition in all markets**
  - **Rational expectations**
  - **Perfect AD financial markets**

THE REAL BUSINESS CYCLE MODEL

Kydland and Prescott (1982), Long and Plosser (1983), King, Plosser, and Rebelo (1988)

- **All modern macro models descend from RBC model – dynamic GE**
  - **No matter how many market imperfections, heterogeneity, etc, etc.**

- **Foundations of the RBC model**
  - **Without optimizing consumers: Solow growth model**
  - **With optimizing consumers: Ramsey/Cass/Koopmans model**

"The Solow growth model"

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

$$y_t + (1+r_t)a_{t-1}$$

- **Model of non-asset income so far: endowment y_t, possibly stochastic**
- **Now suppose y_t is labor income**

$$y_t = w_t n_t$$

- **Normalize "time available" in each time period to one unit**
  - **Individual decides how to divide between "labor" and "leisure"**
    - **(Basic models: leisure is all "non-labor," but empirical and theoretical work recently studying the importance of finer categorizations of "non-labor time" for macro issues – e.g., search and matching theory)**
  - **Labor = n_t ←→ leisure ≡ l_t = 1-n_t**
  - **Time is now the *endowment!***

Macro Fundamentals

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

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- **Assert that individuals care about leisure,** $u(c_t, \ell_t)$
  - $u_{ct} > 0$, $u_{lt} > 0$, $u_{cct} < 0$, $u_{llt} < 0$
  - **Inada conditions on both *c* and *l***
- **Sometimes more convenient to represent as** $u(c_t, n_t)$
  - $u_{ct} > 0$, $u_{nt} < 0$, $u_{cct} < 0$, $u_{nnt} > 0$ **(strictly decreasing and convex in *n*)**

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

- **Intertemporal optimization problem**

  $$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^S) \text{ subject to } c_t + a_t = w_t n_t^S + (1+r_t)a_{t-1}$$

  - **Individual takes as given $\{w_t, r_t\}_{t=0,1,2,\ldots}$ -- price-taker in labor market**
    - **From perspective of individual, (w,r) evolve as Markov**
  - **Notation n^S emphasizes individual's supply of labor**

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- **Recursive representation**
  - **State vector in arbitrary period *t*: $[a_{t-1}; w_t, r_t]$**

Numeraire object: consumption

$$V(a_{t-1}; w_t, r_t) = \max_{\{c_t, n_t^S, a_t\}} \left\{ u(c_t, n_t^S) + \beta E_t V(a_t; w_{t+1}, r_{t+1}) \right\}$$

$$\text{subject to } c_t + a_t = w_t n_t^S + (1+r_t)a_{t-1}$$

- **FOCs**

  ***c_t* :**

  ***n^S_t* :**

LABOR SUPPLY

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- **FOCs**

  c_t :

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  } **CONSUMPTION-LEISURE OPTIMALITY CONDITION**

  A *static* condition

LABOR SUPPLY

$$-\frac{u_n(c_t, n_t^S)}{u_c(c_t, n_t^S)} = w_t \quad \Rightarrow \quad n_t^S = n^S(w_t; c_t)$$

- **Consumption-leisure (aka consumption-labor) optimality condition**
  - **An intratemporal optimality condition**

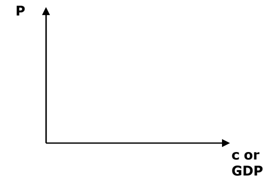
- **Defines period-t labor supply function**
  - **For given individual…**
  - **…but if representative agent, equivalent to aggregate labor supply**
  - **Note: for given c**

- **Example:** $u(c,n) = \ln c - \frac{\theta}{1+1/\psi} n^{1+1/\psi}$
  - **Compute labor supply function?**
  - **Compute elasticity of n^S_t with respect to w_t?**

    Frisch elasticity of labor supply

The Three Macro (Aggregate) Markets

- Goods Markets



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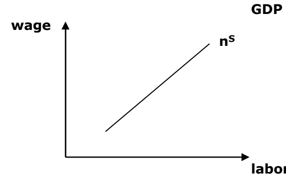

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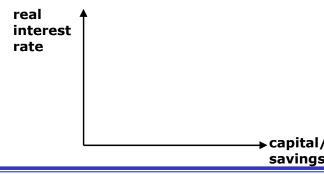

Production of Goods

- **Representative firm produces the numeraire output good of the economy**
- **A homogenous output good**
- **Perfect competition in goods supply**
- **Inputs**
  - **Labor**
  - **Capital**
    - **E.g., machines, factories, computers, intangibles, …**

- **Firm produces using a (aggregate) production technology**

$$y_t = z_t \cdot f(k_t^D, n_t^D)$$

  - **k^D the firm's capital demand**
  - **n^D the firm's labor demand**
  - **$f(.)$ often assumed CRS (Cobb-Douglas, in particular)**
  - **z_t a process that shifts the production function**

- **Empirically identify z_t as Solow residual**
  - **Growth theory: z deterministic**
  - **Business cycle theory: z stochastic (Markov)**

PRODUCTION OF GOODS

- **Representative firm profit maximization**
  - **Price taker in capital market, labor market, and output market**
  - **Baseline model(s)**
    - **Firm hires/rents labor and capital each period**
    - **Firm does not "own" any capital or labor (without loss of generality if no financial market imperfections)**

$$\max_{n_t^D, k_t^D} \left(z_t f(k_t^D, n_t^D) - w_t n_t^D - r_t^k k_t^D \right)$$

- **FOCs**

n^D_t **:** $z_t f_n(k_t^D, n_t^D) - w_t = 0$

k^D_t **:** $z_t f_k(k_t^D, n_t^D) - r_t^k = 0$

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- **FOCs**

n^D_t **:** $z_t f_n(k_t^D, n_t^D) - w_t = 0$ **DEFINES labor demand function $n^D(w_t)$**

k^D_t **:** $z_t f_k(k_t^D, n_t^D) - r_t^k = 0$ **DEFINES capital demand function $k^D(r^k_t)$**

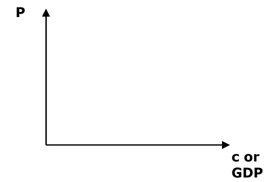
For a given firm

If rep. firm, equivalent to aggregate factor demands

- **Firms entirely static entities in baseline macro model(s)**
  - **Contrast with consumers**
  - **(NK theory and search theory: firms are dynamic entities)**

The Three Macro (Aggregate) Markets

- **Goods Markets**



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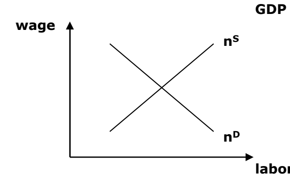

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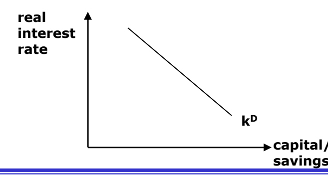

Capital Supply

- **Baseline model(s)**
  - **Physical capital takes "time to build"**
    - **Simplest: one-period lag between building and using capital**
  - **Closed economy**
    - **Aggregate capital demand must be supplied domestically**

- **Consumer intertemporal optimization problem**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^S) \quad \textbf{subject to} \quad c_t + a_t = w_t n_t^S + (1+r_t) a_{t-1}$$

  - **a_{t-1} is a given individual's pre-determined stock of assets**
  - **Representative agent: a_{t-1} is economy's pre-determined stock of assets**

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  - **a_{t-1} is a given individual's pre-determined stock of assets**
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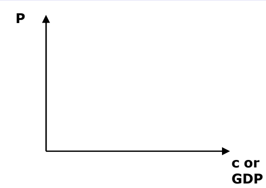
- **Capital-market clearing in each period *t***

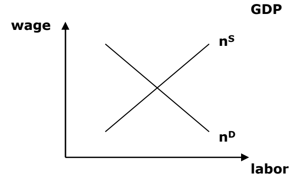
$$k_t^D = a_{t-1} \left(= k_t^S \right)$$

THE THREE MACRO (AGGREGATE) MARKETS

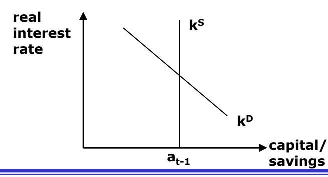
- **Goods Markets**



- **Labor Markets**



- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**



CAPITAL SUPPLY

- **Baseline model(s)**
  - **Physical capital takes "time to build"**
    - **Simplest: one-period lag between building and using capital**
  - **Closed economy**
    - **Aggregate capital demand must be supplied domestically**

- **Consumer intertemporal optimization problem**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^S) \text{ subject to } c_t + a_t = w_t n_t^S + (1+r_t) a_{t-1}$$

  - **a_{t-1} is a given individual's pre-determined stock of assets**
  - **Representative agent: a_{t-1} is economy's pre-determined stock of assets**

- **Capital-market clearing in each period t**

$$k_t^D = a_{t-1} \left(= k_t^S\right)$$

- **Capital depreciates at rate δ each period**
  - **Economic depreciation, due to physical wear and tear of production**
  - **Not accounting depreciation**
  - **Compensation reflected in capital-market-clearing price: $r_t = r^k_t - \delta$**

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- **Euler equation**

$$u'(c_t) = \beta E_t \left\{ u'(c_{t+1})(1 + r_{t+1}^k - \delta) \right\}$$

  - **From perspective of single individual: characterizes optimal savings (flow!) decision between t and $t+1$**
  - **From perspective of entire economy: characterizes optimal investment (flow!) in capital stock between t and $t+1$**
- **Closed economy: domestic savings = domestic investment**
- **Note timing: savings/investment decisions in t alter the available capital stock in period $t+1$ ("time to build")**

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

- **Round out final details**
- **Baseline model(s)**
  - **Consumption goods and capital goods are freely interchangeable**
  - **i.e., capital good in a given period can be "dismantled" and used for consumption in future periods**
  - **No irreversibility of investment process**
  - **Implies relative price (not interest rate…) of capital = …?…**

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- CRS production process *f*(k,n), firms earn profits = …?…
  - Corollary: factors of production are paid …?…

Macro Fundamentals

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- Labor-market clearing

  $n_t \equiv n^D_t = n^S_t, \forall t$ (with clearing price w_t)

- Capital-market clearing

  $k_t \equiv k^D_t = k^S_t, \forall t$ (with clearing price r^k_t)

- Goods market clearing

  $c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t), \forall t$ (with clearing price = …?…)

DYNAMIC GENERAL EQUILIBRIUM

- Economy-wide state vector in period t: $(k_t; z_t)$
- Consider $T \to \infty$

- Definition: a dynamic stochastic general equilibrium is time-invariant state-contingent price functions $w(k_t; z_t)$, $r^k(k_t; z_t)$ and state-contingent consumption, labor, and (one-period-ahead) capital decision rules $c(k_t; z_t)$, $n(k_t; z_t)$, and $k(k_t; z_t)$ that jointly satisfy the following:

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1. (Consumer optimality) Given $w(k_t; z_t)$, $r^k(k_t; z_t)$, the functions $c(k_t; z_t)$, $n(k_t; z_t)$, and $k(k_t; z_t)$ solve the Euler equation (replaced by TVC as $T \to \infty$), labor supply function, and flow budget constraint of the representative consumer

2. (Firm optimality) Given $w(k_t; z_t)$, $r^k(k_t; z_t)$, the function $n(k_t; z_t)$ satisfies the labor demand function and k_t satisfies the capital demand function

3. (Markets clear)
   - Labor-market clearing
     $$n(k_t; z_t) \equiv n^D{}_t = n^S{}_t,\ \forall t$$
   - Capital-market clearing
     $$k_t \equiv k^D{}_t = k^S{}_t,\ \forall t$$
   - Goods market clearing
     $$c(k_t; z_t) + k(k_t; z_t) - (1-\delta)k_t = z_t \cdot f(k_t, n(k_t; z_t)),\ \forall t$$

given the initial capital stock k_0 and (Markov) transition process for $z_t \to z_{t+1}$

DYNAMIC GENERAL EQUILIBRIUM

Applications of DSGE theory

DYNAMIC GENERAL EQUILIBRIUM

Applications of DSGE theory

EVERYWHERE IN MACROECONOMICS

(and related fields)