THE BASELINE RBC MODEL: THEORY AND COMPUTATION

SEPTEMBER 8, 2011

STYLIZED MACRO FACTS

- **Foundation of (virtually) all DSGE models (e.g., RBC model) is Solow growth model**
- **So want/need/desire business-cycle models to be consistent with basic growth facts**
- **Kaldor's Stylized Growth Facts (time averages)**
  1. **Output per worker exhibits ~constant growth**
  2. **Capital per worker exhibits ~constant growth**
  3. **Rate of return on capital is ~constant**
  4. **Capital-output ratio is ~constant**
  5. **Factor shares (i.e., payments to capital and payments to labor as fraction of GDP) are ~constant**
- **Business-cycle model**
  - **Interested in fluctuations around long-run growth path**
- **How to construct/extract long-run trend?**
  - **Most common procedure: HP filter**
  - **Eliminates (if data were stationary) fluctuations at frequencies lower than eight years**

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- How to construct/extract long-run trend?
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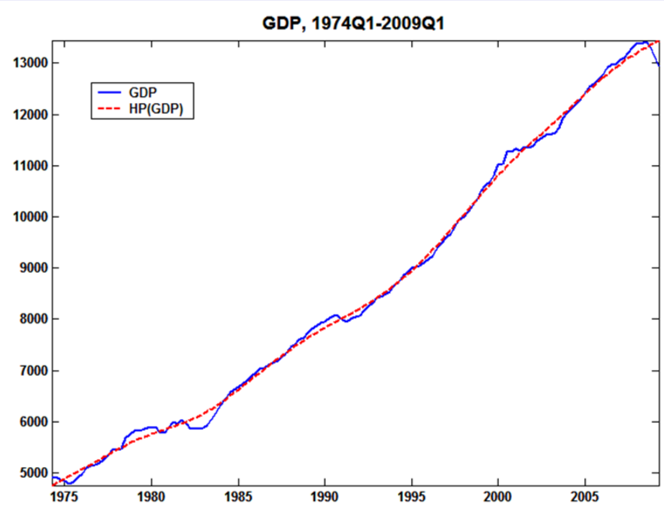
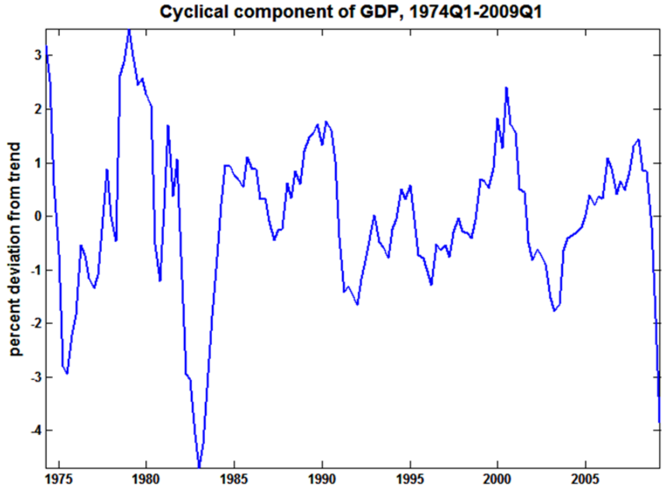
  - Eliminates (if data were stationary) fluctuations at frequencies lower than eight years
  - An alternative: band pass (BP) filter – allows specifying upper and lower frequencies to be eliminated

Cooley volume, Chapter 1

STYLIZED MACRO FACTS



STYLIZED MACRO FACTS



STYLIZED MACRO FACTS

- **Kaldor's Stylized Facts:**
  1. **Output per worker exhibits ~constant growth**
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- **Some basic cyclical volatilities – SD% (i.e., time-series SD of HP-filtered component)**

GDP: 1.47% (1974Q1-2009Q4)	**C: 1.16% (1974Q1-2009Q4)** **CNDUR: 1.05%** **CSERV: 0.74%** **CDUR: 3.82%**
I: 7.03% (1974Q1-2009Q4)	**TOTAL HOURS: 1.59% (K&R)** **AVG HOURS: 0.63% (K&R)**
WAGE: 0.76% (K&R)	

WHAT DO WE WANT TO MODEL?

- **Relative volatilities**
- **Persistence (i.e., first-order serial correlation) of various series**
- **Business cycle comovements**
  - **Corr(C, Y) = 0.83**
  - **Corr(I, Y) = 0.91**
  - **Corr(HOURS, Y) = 0.86**
  - **Corr(WAGE, Y) = 0.68**
- **Labor markets?**
  - **Extensive margin – movements of individuals in and out of employment (i.e., work H = 0 hours or H > 0 hours)**
  - **Intensive margin – how many hours to work given an individual already works (i.e., work H = 39 hours or H = 40 hours or H = 41 etc…)**
  - **The basic RBC model blurs the difference**
    - **Prescott: "LS elasticity of 3 is right…"**

THE THREE MACRO MARKETS

- **Goods Market(s)**
- **Labor Market(s)**
- **Asset/Savings Market(s)**
- **Consumers**
  - **Demand goods**
  - **Supply labor**
  - **Supply assets/savings**
- **Firms**
  - **Produce goods**
  - **Demand labor**
  - **Demand assets/savings (capital)**
- **Government: auxiliary in the basic model**

BUILDING BLOCKS

- **Consumers**
  - **Maximize lifetime utility (i.e., a dynamic problem)**
- **Firms**
  - **Maximize profits**
- **Prices adjust to clear all markets**
  - **Hence a general equilibrium model**
- **Unpredictable fluctuations in total factor productivity (TFP) are the driving source of business cycles in baseline RBC model**
  - **Identify TFP as the Solow residual**
    - **y(t) = z(t)*f(k(t), n(t))**

HOUSEHOLDS

- **Maximize lifetime utility (i.e., a dynamic problem) subject to sequence of budget constraints:**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \quad \textbf{s.t.} \quad c_t + k_{t+1} - (1-\delta)k_t = w_t n_t + r_t k_t$$

- **Set up Lagrangian; optimality conditions**
  - **Consumption-Leisure Optimality Condition: MRS between consumption and labor equals real wage**

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t$$

  - **Consumption-Savings Optimality Condition (Euler equation): MRS between present and future consumption equals real return on savings (a difference equation)**

$$u_c(c_t, n_t) = \beta E_t \left\{ u_c(c_{t+1}, n_{t+1})(1 + r_{t+1} - \delta) \right\}$$

THE REST OF THE MODEL

- **Firms: maximize profits period-by-period** $\max_{n_t,k_t} z_t f(k_t,n_t) - w_t n_t - r_t k_t$
  - **FOCs yield factor-pricing conditions:** $w_t = z_t f_n(k_t,n_t), r_t = z_t f_k(k_t,n_t)$
- **Government: omit from baseline RBC model**
- **Resource constraint:** $c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t,n_t)$
- **Exogenous process**

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon^z_{t+1}$$

  - **TFP follows AR(1) (satisfies Markov property), with persistence ρ_z**
  - **Average productivity zbar; white noise process $\varepsilon \sim N(0, \sigma^2_z)$**
  - **Specification in logs implies fluctuations are in deviations of TFP from the average zbar**

PUTTING THE MODEL TOGETHER: EQUILIBRIUM

INDIVIDUALS' DECISIONS ARE OPTIMAL

- **Consumer decisions:**
  - **Taking as given the real wage and the rental price of capital, choices of consumption, investment, and labor solve utility maximization**
- **Firm decisions:**
  - **Taking as given the real wage and the rental price of capital, choices of labor and capital solve profit maximization**

ALL MARKETS CLEAR

- **Prices in goods, labor, and asset/savings markets adjust**
  - **Price of consumption normalized to one in every period**
  - **Prices (and thus decisions) depend on how TFP (and any other exogenous processes) evolves over time**

THE MODEL DETERMINES:

Allocations: **consumption, labor, savings/investment**
Prices: **real wage, rental rate of capital**

THE EQUATIONS AND VARIABLES

- **Equilibrium Conditions**

$$-\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = w_t$$
$$u_c(c_t,n_t) = \beta E_t\left\{u_c(c_{t+1},n_{t+1})(1+r_{t+1}-\delta)\right\}$$
$$c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t,n_t)$$
$$w_t = z_t f_n(k_t,n_t), \quad r_t = z_t f_k(k_t,n_t)$$
$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon^z_{t+1}$$

- **Exogenous variables (the inputs to the model):** $\{z_t\}_{t=0}^{\infty}$
- **Endogenous variables (the outputs of the model):** $\{c_t, n_t, k_{t+1}, w_t, r_t\}_{t=0}^{\infty}$
  - **Easy to express wage and rental rate as functions of *z*, *k*, and *n***

THE EQUATIONS AND VARIABLES

- **Equilibrium Conditions**

$$-\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = z_t f_n(k_t,n_t)$$ **Consumption-Labor Efficiency Condition**

$$u_c(c_t,n_t) = \beta E_t\left\{u_c(c_{t+1},n_{t+1})(1+z_{t+1} f_k(k_{t+1},n_{t+1})-\delta)\right\}$$ **Consumption-Investment Efficiency Condition**

$$c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t,n_t)$$ **Resource Constraint**

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon^z_{t+1}$$ **Law of motion for TFP**

- **Exogenous variables (the inputs to the model):** $\{z_t\}_{t=0}^{\infty}$
- **Endogenous variables (the outputs of the model):** $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$

HOW TO USE THE MODEL?

- **Need to specify/solve for FUNCTIONS (aka "decision rules") that describe how:**
  - **Consumers make choices based on prices and policies**
  - **Firms make choices based on prices and policies**
  - **Prices depend on state variables (capital, TFP, and all other exogenous variables)**
- **Except for very special cases, must turn to quantitative (i.e., numerical) methods**
  - **Because of the difference (differential) equation in the model:**

The Euler equation

APPROXIMATIONS

- **Looking for an equilibrium in which endogenous variables are time-invariant functions of the state of the model** $\mathbf{S}_t \equiv [k_t; z_t]$
  - **State describes the dynamic position of the model**
  - **So looking for** $c(\mathrm{S_t}), n(\mathrm{S_t}), k(\mathrm{S_t})$
- **Cannot solve difference equations analytically in general**
  - **These solutions are unknowable in general**
  - **Hence need to approximate – so look for**

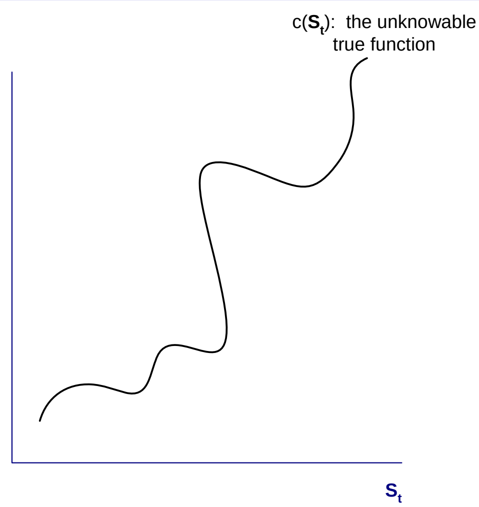
$$c^{approx}(\mathrm{S_t}), n^{approx}(\mathrm{S_t}), k^{approx}(\mathrm{S_t})$$

which are hopefully near the (unknowable…) truth…

TYPES OF APPROXIMATIONS

- **Global:** approximated functions are close to true functions "everywhere" (over a very broad range of states)
  - Hard to implement for medium- and large-scale models given current hardware capacity
  - Several popular methods
    - Chebyshev polynomials
    - Finite-element methods
    - Value function iteration
- **Local:** approximated functions are close to true functions only in a relatively small range of the state space
  - Much easier to implement
  - Based on Taylor approximations
    - Linear (first-order)
    - Quadratic (second-order)
    - Etc.

APPROXIMATIONS



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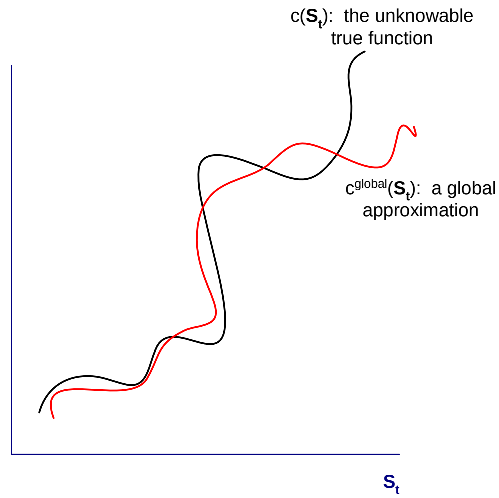
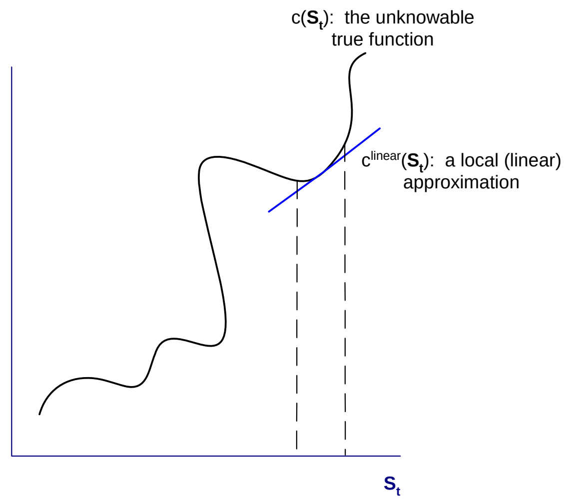


Quantitative Basics

APPROXIMATIONS



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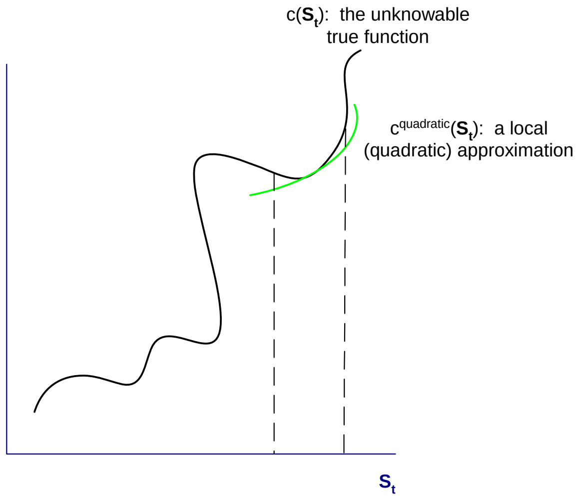

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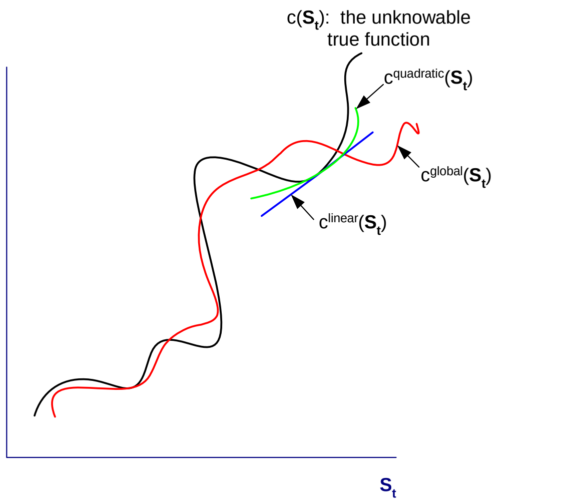

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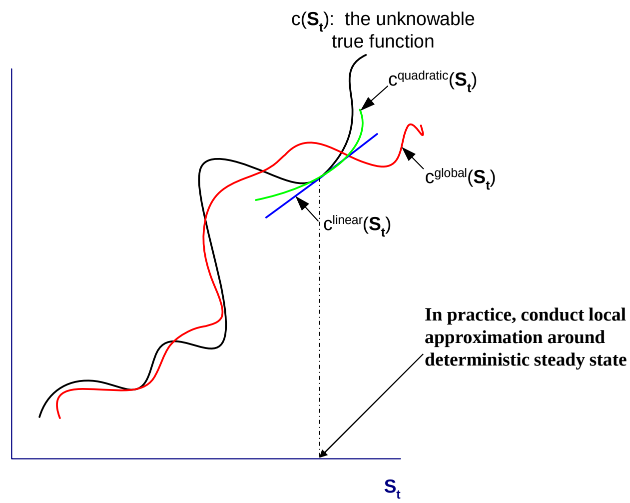

STEADY STATE

- **Shut down all shocks and set exogenous variables at their means**
- **Let model economy run for many (infinite) periods**
  - **Time eventually "doesn't matter" any more**
  - **Drop all time indices**

$$-\frac{u_n(c,n)}{u_c(c,n)} = \bar{z}f_n(k,n)$$

$$u_c(c,n) = \beta u_c(c,n)(1+\bar{z}f_k(k,n)-\delta)$$

$$c+\delta k = \bar{z}f(k,n)$$

- **(*c*, *n*, *k*) is a triple of scalars that are the steady state (aka long run) outcomes of the model economy**

- **Given functional forms and parameter values, solve for (*c*, *n*, *k*)**
  - **Conduct local approximation around this point**