CALIBRATING THE RBC MODEL

SEPTEMBER 15, 2011

STEADY STATE

- **Deterministic steady state the natural point of approximation**
- **Shut down all shocks and set exogenous variables at their means**
- **The idea: let economy run for many (infinite) periods**
  - **Time eventually "doesn't matter" any more**
  - **Drop all time indices**

$$-\frac{u_n(\bar{c},\bar{n})}{u_c(\bar{c},\bar{n})} = \bar{z}F_n(\bar{k},\bar{n})$$

$$u_c(\bar{c},\bar{n}) = \beta u_c(\bar{c},\bar{n})\left[\bar{z}F_k(\bar{k},\bar{n}) + 1 - \delta\right]$$

$$\bar{c} + \delta\bar{k} = \bar{z}F(\bar{k},\bar{n})$$

$$\ln\bar{z} = (1-\rho_z)\ln\bar{z} + \rho_z\ln\bar{z} \Rightarrow \bar{z} = \bar{z} \quad \text{(a parameter of the model)}$$

- **Given functional forms and parameter values, solve for** $(\bar{c},\bar{n},\bar{k})$
  - **The steady state of the model**
  - **Taylor expansion around this point**

CALIBRATION – PHILOSOPHY

- An economic model is a measuring device
- If model makes "believable" predictions along some important dimensions (i.e., "matches some key data")…
- …then maybe its predictions are "believable" along the novel dimensions of the model
- Getting some "partial derivatives" of the model in known directions correct…
- …may build credibility that its "partial derivatives" in novel directions are at least not grossly incorrect

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- Make model match some data of interest – often long-run (i.e., time-averaged data) growth facts
  - Preferably well-accepted "stylized facts"
  - Solow growth model in the background
  - Natural candidate: Kaldor growth facts
- Calibration vs. Estimation

CALIBRATION OF BASELINE RBC MODEL

- Must take a stand on three (related) points
  - Which data do we want model to match? (even constructing data is challenging…)
  - Functional forms (utility, production)
  - Parameter values
- Choose functional forms consistent with "Kaldor-plus facts"
  - (K1) Capital income share and labor income share of GDP are stationary
  - (K2) All real quantity variables grow at same rate in the long run
  - (K3) Real interest rate is stationary
  - (K4) Hours per worker are stationary
  - (K5) (K2) requires trend productivity to be labor-augmenting (Phelps 1966)

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- Often start with RBC model that abstracts from long-run growth
- But "true" calibration begins with model featuring ***only*** long-run growth
  - Puts restrictions on instantaneous utility and production forms
  - Use (K1)-(K5) to obtain these restrictions
- Richer models: more calibration targets and/or treating data differently
  - Monopoly markups (e.g., Dixit-Stiglitz and sticky price models)
  - Probability of finding a job (e.g., labor search models)
  - Durable consumption vs. non-durable consumption

RBC MODEL WITH GROWTH

- **Absent shocks, TFP grows at deterministic rate γ**
- **Planner problem/perfect competition**

$$\max E_0 \sum_{t=0}^{\infty} b^t u(C_t, n_t)$$ **subject to**

Trend productivity is labor-augmenting (Harrod-neutral) (Makes use of fact (K5))

$$C_t + K_{t+1} - (1-\delta)K_t = z_t F(K_t, n_t X_t)$$ **Flow resource constraint**

$$X_t = \gamma X_{t-1}, \qquad \gamma \geq 1$$ **Evolution of deterministic component of productivity**

Red indicates variables or parameters that will be modified when detrending the model

given stochastic process for evolution of z_t and (K_0, z_0, X_0)

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given stochastic process for evolution of z_t and (K_0, z_0, X_0)

- **Suppose z_t = 1 always, so only deterministic growth**
- **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_{t+1}) governed by**

(1) $$-\frac{u_n(C_t, n_t)}{u_c(C_t, n_t)} = X_t F_2(K_t, n_t X_t)$$ **Labor supply function (aka consumption-labor optimality)**

(2) $$\frac{u_C(C_t, n_t)}{b u_C(C_{t+1}, n_{t+1})} = F_1(K_{t+1}, n_{t+1} X_{t+1}) + 1 - \delta$$ **Capital supply function (aka consumption-savings optimality)**

(3) $$C_t + K_{t+1} - (1-\delta)K_t = F(K_t, n_t X_t)$$

(4) $$X_t = \gamma X_{t-1}$$ **Normalize X_0 = 1**

RESTRICTIONS ON FUNCTIONAL FORMS

- **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_t) governed by**

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- **(K1) Capital income share and labor income share of GDP are stationary**
  And viewing economic profits as zero

$$\Rightarrow F(K,nX) = K^{\alpha}(nX)^{1-\alpha} \qquad (\alpha \approx 0.4)$$

Calibration of RBC Model with Growth

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(2) $$\frac{u_C(C_t,n_t)}{bu_C(C_{t+1},n_{t+1})} = \alpha\left(\frac{K_{t+1}/X_{t+1}}{n_{t+1}}\right)^{\alpha-1} + 1 - \delta$$

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- (K2) All real quantity variables grow at same rate in the long run

$$\Rightarrow\frac{Y_{t+1}}{Y_t}=\frac{C_{t+1}}{C_t}=\frac{K_{t+1}}{K_t}=\frac{X_{t+1}}{X_t}=\gamma,\quad\forall t$$

$$\Rightarrow y_t\equiv\frac{Y_t}{X_t}=\bar{y},\quad k_t\equiv\frac{K_t}{X_t}=\bar{k},\quad c_t\equiv\frac{C_t}{X_t}=\bar{c},\quad\forall t$$

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- And scale (3) by X_t to make stationary

Note long run growth rate affects capital accumulation even in stationary representation!

$$-\frac{u_n(C_t,n_t)}{u_c(C_t,n_t)}=(1-\alpha)X_t\left(\frac{k_t}{n_t}\right)^{\alpha}$$

$$c_t+\gamma k_{t+1}-(1-\delta)k_t=k_t^{\alpha}n_t^{1-\alpha}$$

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- **(K4) Hours per worker are stationary**

$$\Rightarrow n_t=\bar{n}$$

along deterministic path.
BUT $\bar{n}$ is endogenous…

RESTRICTIONS ON FUNCTIONAL FORMS

- **Deterministic dynamics of C_t governed by**

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- **Implied already (RHS of (2)) is**
  - **(K3) Real interest rate is stationary**
- **Final step – functional form for utility?**
- **Observations**
  - **Optimal choice of labor ($\bar{n}$) must be independent of X_t (from (1))**
    - **Requires offsetting income and substitution effects of wages (long-run productivity) on labor supply**
  - **IMRS can only depend on C_{t+1}/C_t (from (2)), which in turn = γ**

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- **Two requirements together imply**

$$u(C_t,n_t)=\begin{cases}\dfrac{[C_t v(n_t)]^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma>0,\ \sigma\neq 1\\ \ln C_t+v(n_t) & \text{if } \sigma=1\end{cases}$$

King, Plosser, Rebelo (1988 *JME*)

STEADY STATE

- **Steady state $(\bar{c},\bar{n},\bar{k})$ solves (1), (2), (3)**
- **A dynamic phenomenon!**
  - **Not static!**
  - **Economy is moving exactly along its long-run (i.e., deterministic) growth path**
  - **Balanced growth path**
- **Scale of absolute quantity outcomes within model is meaningless**
  - **What does, e.g., $\bar{c}=1.56$ mean?**

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- **Scale of absolute quantity outcomes within model is meaningless**
  - **What does, e.g., $\bar{c} = 1.56$ mean?**
- **Relative quantity outcomes are interpretable**
  - **Provide calibration targets**
  - **e.g., $\bar{c}/\bar{y} = 0.70, \quad \bar{k}/\bar{y} = 2.5$ (if annual measurement)**
- **Time use and intertemporal price outcomes within model are interpretable**
  - **Provide calibration targets**
  - **$\bar{n}$ is fraction of time spent in paid market work**
    - **Empirical: $n \approx 0.30$**
  - **Return on capital $\alpha \left(\frac{\bar{k}}{\bar{n}} \right)^{\alpha-1} + 1 - \delta$**
- **Use ss calibration targets to set parameter values, given functional forms**

RBC Model Without Growth

- **Often start instead with**

$$u(c_t, n_t) = \begin{cases} \dfrac{[c_t v(n_t)]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0,\ \sigma \neq 1 \\ \ln c_t + v(n_t) & \text{if } \sigma = 1 \end{cases} \quad \text{and} \quad \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

$c_t = C_t / X_t$

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- **For this transformed model to deliver same steady state (relative quantities, time use, and r), require**
  - **Resource constraint** $c_t + \gamma k_{t+1} - (1-\delta)k_t = z_t k_t^{\alpha} n_t^{1-\alpha}$
  - **Subjective discount factor** $\beta \equiv b\gamma^{1-\sigma}$ **(King and Rebelo, p. 945)**
- **Typical assumption γ = 1 omits growth altogether**
- **What if trend growth rate fluctuates, γ_t?**
  - **Typical representation cannot accommodate trend shocks because γ = 1**
  - **Trend shocks fairly common in small-open-economy models**
  - **Affects discount factor and capital accumulation equation**

BASELINE RBC MODEL

- **Assuming γ = 1…**
- **…complete calibration?**
- **Data: long-run labor income share of GDP ≈ 0.60**
  - **Cobb-Douglas $F(.)$ implies**

$$\frac{wn}{F(k,n)}=\frac{(1-\alpha)k^{\alpha}n^{1-\alpha}}{k^{\alpha}n^{1-\alpha}}=1-\alpha \quad \Rightarrow \quad \alpha=0.40$$

- **Data: long-run ratio of (annual) gross investment to capital stock ≈ 0.07**

$$\frac{k-(1-\delta)k}{k}=\frac{\delta k}{k}=0.07 \quad \Rightarrow \quad \delta=0.07 \ \text{(annual) or } 0.018 \ \text{(quarterly)}$$

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- **Data:** long-run ratio of (annual) output to capital stock ≈ 0.4
  - Steady-state Euler equation

$$1 = \beta\left[\frac{\alpha k^{\alpha}n^{1-\alpha}}{k} + 1 - \delta\right] = \beta\left[\frac{\alpha F(k,n)}{k} + 1 - \delta\right] \qquad \Rightarrow \qquad \beta = 0.95 \text{ (annual) or } 0.99 \text{ (quarterly)}$$

OR

- **Data:** avg. net real return on capital ≈ 5% per year (e.g, return on S&P500)
  - Steady-state Euler equation

$$f_k(k,n) = \frac{1}{\beta} - 1 + \delta \qquad \Rightarrow \qquad \beta = 0.96 \text{ (annual) or } 0.99 \text{ (quarterly)}$$

BASELINE RBC MODEL

- Utility parameters

$$u(c_t,n_t) = \begin{cases} \dfrac{[c_t v(n_t)]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0,\ \sigma \neq 1 \\ \ln c_t + v(n_t) & \text{if } \sigma = 1 \end{cases}$$

- **Data:** IES is around unity(?) or lower
  - Implies σ > 1
  - (Recall: IES = 1/σ for time-separable CRRA utility)
  - σ = 1 a conventional value

- Labor subutility
  - Common form

$$v(n) = -\frac{\psi}{1+1/\eta} n^{1+1/\eta}$$

  - η measures Frisch elasticity of labor supply (use C-L optimality condition)
  - Calibrate Ψ to hit $\bar{n} \approx 0.3$
  - Empirical evidence on Frisch elasticity?

BASELINE RBC MODEL

- **Labor supply elasticity "controversial"**

- **Micro evidence: very low – η (substantially) smaller than one**
- **Macro evidence: very high – η (substantially) larger than one**

- **"Tension" between macro and micro evidence not useful way to frame the "controversy"**

- **Micro studies pick up intensive margin of labor supply**
- **Macro studies pick up (mostly) extensive margin of labor supply**
  - **And other frictions in allocation of workers to jobs…**

- **Chetty (2010): uses both macro and micro evidence to put bounds on labor supply elasticity**

- **Common in DSGE models: $\eta > 1$**

BASELINE RBC MODEL

- **Exogenous process for TFP (deviations from long-run trend productivity)**

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}$$

$$\varepsilon_{t+1} \sim \text{iid } N(0, \sigma_z^2)$$

- **Normalize $\bar{z} = 1$**
  - **Only governs absolute scale of model, which is arbitrary**
  - **What does, e.g., $\bar{c} = 1.56$ mean?**

- **Construct time-series for z_t using**
  - **Data on labor, (detrended) capital, and (detrended) output**

- **AR(1) estimation**
  - **Quarterly frequency**

$$\Rightarrow \rho_z = 0.95 \quad \text{and} \quad \sigma_z = 0.006$$

USING THE RBC (OR ANY DSGE) MODEL

1. **Dream up/construct/write fully-articulated model**
   - ❒ **Ideally to answer questions motivated by data and with hypotheses**
2. **Choose parameter values**
   - ❒ **Perhaps extremely rigorously, if goal is to match certain empirical facts very precisely**
   - ❒ **Perhaps adopting generally-accepted values, if goal is to illustrate some insight**
3. **Solve for deterministic steady state (balanced growth path)**
4. **Solve for dynamic decision rules (e.g., linear approximation, second-order approximation, global approximation)**
5. **Conduct informative battery of experiments (impulse responses, simulations, etc.) to try to falsify hypotheses**
6. **Tabulate results, write a (good!) paper, get it published**