

A SIMPLE DSGE MODEL OF "WORK"

SEPTEMBER 20, 2011

Introduction

TIME USE?

- ❑ "Labor" and "leisure" the only time uses in standard macro models
- ❑ Aguiar and Hurst's (2007 *QJE*) time-use [classifications](#)
- ❑ Are other uses of time important for macro issues?
- ❑ Diamond's (1981) monograph *On Time*
- ❑ Rogerson (1988 *JME*) and Hansen (1985 *JME*) issue:
 - ❑ Can "indivisibility" in labor (i.e., binary individual labor/leisure outcome) be tractably modeled?
 - ❑ **ANSWER: YES** – through "randomization" over WHO actually works
 - ❑ **Aggregation result:** representative-agent preferences quasi-linear in labor

LABOR AGGREGATION – MAIN RESULTS

- Total equilibrium labor

$$n = eh$$

Number (measure) of individuals working
 ↓
 $n = eh$
 ↑ ↓
 Total hours Hours per worker

- Normalize $h = 1$ for employed individuals (so $n = e$)

- Suppose non-convexity

- Measure e of individuals work $h = 1$ hours
- Measure $1-e$ of individuals work $h = 0$ hours
- An INDIVIDUAL'S decision problem is not convex
 - Must choose $\{0, 1\}$ (work/don't work)

- Economy populated by individuals each with $u(c) + \begin{cases} v(1-n=0) & \text{if work} \\ v(1-n=1) & \text{if don't work} \end{cases}$

equivalent (in terms of aggregate outcomes) to a representative-agent economy with quasi-linear preferences

$$u(c) - \lambda n$$

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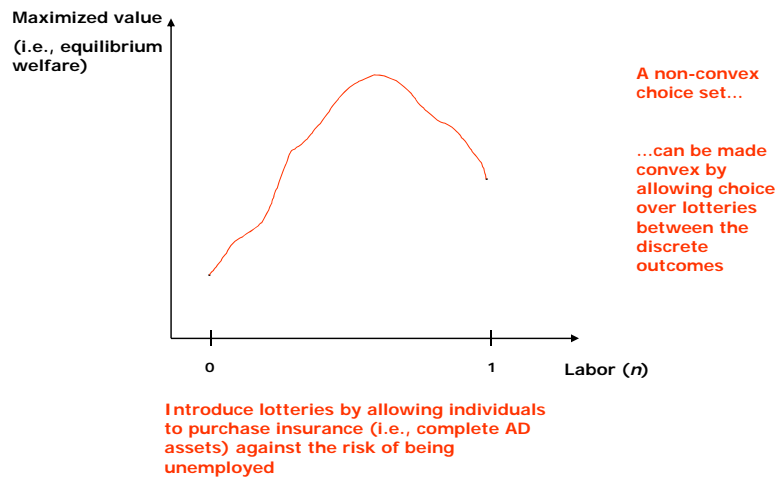
LABOR AGGREGATION – MAIN RESULTS

- **Theoretical attraction:** makes "low substitution" (of labor across time periods) economy at the micro level a "high substitution" (of labor across time periods) economy at the macro level
 - Which helps make aggregate hours more volatile over time in a DSGE model – can illustrate with two-period model
- Interpretation(s)
 - **Underlying market structure:** individuals choose "lotteries" over possibility of being employed, rather than whether or not to work
 - **Insurance:** individuals can (and do) purchase (actuarially fair) full insurance against employment risk
 - **"Risk-neutrality":** representative consumer "doesn't care" how many hours he works in a given period – because of full risk sharing!
- Gain in DSGE model performance?
- Intuitive plausibility?
- Empirical relevance?

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THE SIMPLE MICROECONOMICS



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THE SIMPLE MICROECONOMICS

- Consider simple **static** problem

$$\max_{c_1, c_2, y} e[u(c_1) + v(1-n=0)] + (1-e)[u(c_2) + v(1-n=1)] \quad \text{Expected utility}$$

subject to

$$c_1 + py = w$$

State 1: work (probability e)

$$c_2 + py = y$$

State 2: don't work (probability $1-e$)

$$n = e$$

Subject to exogenous randomness

Utility in principle depends on n , so would like to optimize on n ...

...but n is trivially exogenous at the optimal choice!

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Probability e of working

Equivalently, measure e of individuals work

State 1: work (probability e)

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State 2: don't work (probability $1-e$)

- **Insurance**

- Quantity y purchased by consumer
- (Competitive) price p
- Only pays off (y units) in the event the consumer doesn't work

- FOCs yield $u'(c_1) = \lambda_1$ and $u'(c_2) = \lambda_2$
along with $-\lambda_1 e p - \lambda_2 (1-e)p + \lambda_2 (1-e) = 0$

- **Conjecture** $\lambda = \lambda_1 = \lambda_2 \rightarrow c = c_1 = c_2$

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THE SIMPLE MICROECONOMICS

- Perfectly competitive representative insurance firm

$$\max_y py - (1-e)y$$

$$\rightarrow p = (1-e)$$

Actuarially fair: competitive price of one unit of insurance that pays off in the event "don't work" = probability of event

THE SIMPLE MICROECONOMICS

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- **Theorem:** Risk-averse consumer + actuarially fair insurance contract \rightarrow consumer will choose to **fully insure** against loss ($y = w$ in this case) (See, e.g., Varian (1992) text)

\rightarrow Consumer's TOTAL INCOME/WEALTH (including any insurance payoff) **not** a function of his employment status

$$\therefore \lambda = \lambda_1 = \lambda_2 \rightarrow c = c_1 = c_2 \quad \text{Verifies conjecture}$$

A consequence of complete markets:
consumption equated across states
(intratemporal consumption smoothing)

THE SIMPLE MICROECONOMICS

- Perfect competition in output production $\rightarrow w = f'(E)$
- Labor-market clearing $E = \int_0^1 h(i) di$
 - Recall either $h = 0$ or $h = 1$
- Equilibrium: $E = e$ and resource constraint $c = f(e)$

Aggregate labor
hired by firm

Use equilibrium results to construct equivalent
alternative problem that yields same aggregates

$$\begin{aligned} \max_{c, n} & e[u(c) + v(0)] + (1-e)[u(c) + v(1)] \\ \text{s.t.} & ec + (1-e)c + epy + (1-e)py = ew + (1-e)y \end{aligned}$$

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Aggregate labor
hired by firm

Use equilibrium results to construct equivalent
alternative problem that yields same aggregates
In particular, use $n = e$

$$\begin{aligned} \max_{c, n} & n[u(c) + v(0)] + (1-n)[u(c) + v(1)] \\ \text{s.t.} & nc + (1-n)c + npy + (1-n)py = nw + (1-n)y \end{aligned}$$

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$$\begin{aligned} & \max_{c,n} n[u(c) + v(0)] + (1-n)[u(c) + v(1)] \\ & \text{s.t. } nc + (1-n)c + npy + (1-n)py = nw + (1-n)y \\ & y = w \text{ (fully insure)} \quad \downarrow \quad p = 1 - e (= 1 - n) \text{ (competitive insurance price)} \\ & \max_{c,n} u(c) + nv(0) + (1-n)v(1) \\ & \text{s.t. } c = wn \end{aligned}$$

THE SIMPLE MICROECONOMICS

$$\begin{aligned} & \max_{c,n} u(c) + [v(0) - v(1)]n + v(1) \\ & \text{s.t. } c = wn \\ & \quad \downarrow \quad \begin{array}{l} \text{Drop } v(1) \text{ because constant} \\ \text{Define } A = v(1) - v(0) \text{ (simply a constant!)} \end{array} \\ & \max_{c,n} u(c) - An \\ & \text{s.t. } c = wn \end{aligned}$$

THE SIMPLE MICROECONOMICS

$$\max_{c,n} u(c) + [v(0) - v(1)]n + v(1)$$

$$\text{s.t. } c = wn$$

↓ Drop $v(1)$ because constant
Define $A = v(1) - v(0)$ (simply a constant!)

$$\max_{c,n} u(c) - An$$

$$\text{s.t. } c = wn$$

Quasi-linear utility:
"risk-neutral" in labor
because of full insurance!

□ Equilibrium: $w = f'(E)$, $E = n$, and $c = f(n)$

□ Rogerson Result

□ Aggregates (c, n) in this economy identical to those from the indivisible labor economy with lotteries/full insurance

□ An application of perfect risk sharing / representative consumer results

□ If embedded in dynamic model (Hansen 1985), individuals **do not care** (i.e., are risk neutral with respect to) whether they work more in the present or the future

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BUSINESS CYCLE IMPLICATIONS

- Embed quasi-linear preferences into standard RBC model
- Can approximate and simulate using "usual" methods
 - Hansen uses LQ (linear-quadratic) approximation

□ Hansen results

Table 1
Standard deviations in percent (a) and correlations with output (b) for U.S. and artificial economies.

| Series | Quarterly U.S. time series ^a (55,3-84,1) | | Economy with divisible labor ^a | | Economy with indivisible labor ^a | |
|---------------|--|------|--|-------------|--|-------------|
| | (a) | (b) | (a) | (b) | (a) | (b) |
| Output | 1.76 | 1.00 | 1.35 (0.16) | 1.00 (0.00) | 1.76 (0.21) | 1.00 (0.00) |
| Consumption | 1.29 | 0.85 | 0.42 (0.06) | 0.89 (0.03) | 0.51 (0.08) | 0.87 (0.04) |
| Investment | 8.60 | 0.92 | 4.24 (0.51) | 0.99 (0.00) | 5.71 (0.70) | 0.99 (0.00) |
| Capital stock | 0.63 | 0.04 | 0.36 (0.07) | 0.06 (0.07) | 0.47 (0.10) | 0.05 (0.07) |
| Hours | 1.66 | 0.76 | 0.70 (0.08) | 0.98 (0.01) | 1.35 (0.16) | 0.98 (0.01) |
| Productivity | 1.18 | 0.42 | 0.68 (0.08) | 0.98 (0.01) | 0.50 (0.07) | 0.87 (0.03) |

The main successes claimed: in particular, RATIO of S.D. much higher than basic RBC model; but, at 2.7, TOO high!

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THE VERDICT?

"This description of the employment allocation mechanism strains credibility and is at odds with the micro evidence on individual employment histories."

Browning, Hansen, and Heckman (1999 *Macro Handbook* p. 602)

Lotteries model predicts an individual's employment status is *iid* over time.

Micro evidence shows it is highly persistent over time.

"Rogerson's aggregation result is every bit as important as the one giving rise to the aggregate production function."

Prescott (2004 *Nobel Lecture* p. 385)