

SIMPLE DSGE MODELS OF "MONEY"

PART I

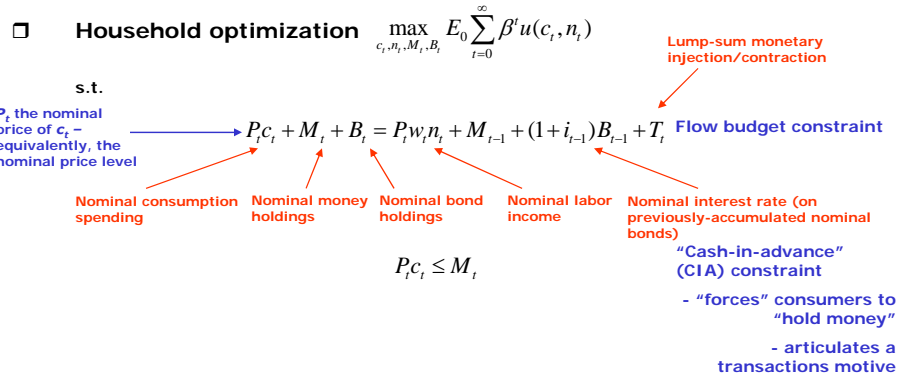
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Introduction

BASIC ISSUES

- ☐ Money/monetary policy issues an enduring fascination in macroeconomics
- ☐ How can/should central bank "control" the economy? Should it/can it at all?
- ☐ Roles of "money"
 - ☐ Medium of exchange (transactions role) ← Highlighted in CIA, MIU, and money-search approaches
 - ☐ Unit of account (numeraire role) ← Highlighted in New Keynesian approach
 - ☐ Store of value (asset role)
- ☐ How to "model money" in DSGE environment?
 - ☐ Which role to model?
 - ☐ Which role is tractable to model?
 - ☐ Which role is most relevant for conduct of monetary policy?

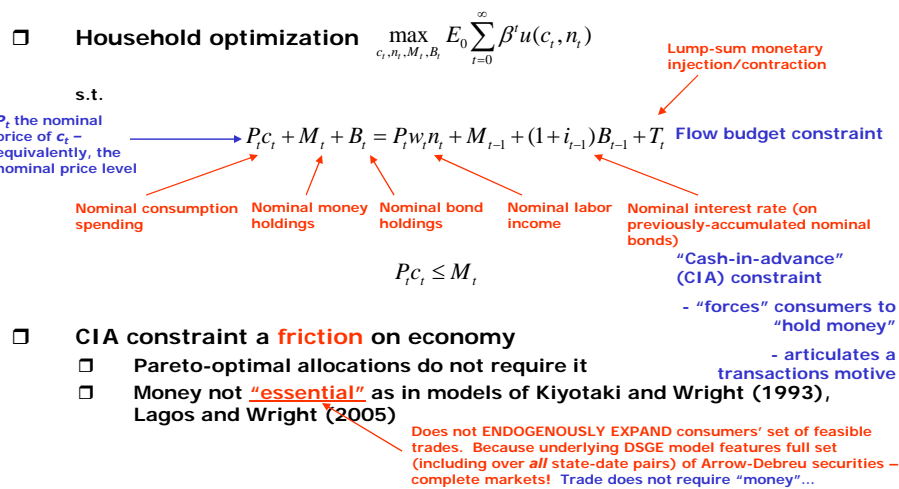
HOUSEHOLDS



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SIMPLE POLICY ANALYSIS

- ❑ Removing monetary friction...
 - ❑ ...requires an allocation that features a **zero multiplier on CIA constraint...**
 - ❑ ...implies **zero nominal interest rate**
- ❑ **Friedman Rule**
 - ❑ Benchmark result in monetary theory
 - ❑ Completely relaxing “monetary friction” requires eliminating any (opportunity) cost of holding money

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 - ❑ **Friedman Rule**
 - ❑ Benchmark result in monetary theory
 - ❑ Completely relaxing “monetary friction” requires eliminating any (opportunity) cost of holding money
 - ❑ **Other Interpretations**
 - ❑ Eliminate the wedge between alternative nominal assets: $i = 0$ makes money and nominal bonds equivalent assets (in terms of their cost and benefit properties)
 - ❑ Eliminate the wedge in the consumption-leisure optimality condition
 - ❑ **Are monetary frictions empirically important?...and thus, is the Friedman Rule of practical use for advising monetary policy?**
- Really the same thing...

SIMPLE POLICY ANALYSIS

□ Household optimality conditions

hh multiplier on CIA constraint hh multiplier on budget constraint

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

No-arbitrage between money and nominal bonds

(Assumption: i_t in the information set of time t)

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$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

Consumption-leisure optimality condition

- relative price depends on w_t AND i_t

Efficiency requires
C-L optimality
depends on real
wage....

...but not on
monetary
aspects of
economy (non-
technology)

Friedman Rule achieves
Pareto efficiency along
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Note disutility of labor
appears in intertemporal
MRS...

If monetary friction were
"shut down," would have
 u_c here "as usual."

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t \left[1 + \frac{i_t}{1+i_t} \right]^{-1}$$

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Either through Friedman Rule or through
"cashless" New Keynesian environment
(later...)

Friedman Rule achieves
Pareto efficiency along
this margin

$$\frac{u_n(c_t, n_t)}{w_t} = (1+i_t) \beta E_t \left[\frac{u_n(c_{t+1}, n_{t+1})}{w_{t+1}} \cdot \frac{P_t}{P_{t+1}} \right]$$

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation!)

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SIMPLE POLICY ANALYSIS

Household optimality conditions (continued)

$$\phi_t = \lambda_t \left[\frac{i_t}{1+i_t} \right]$$

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(aka bond Euler equation)
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$$c_t = \frac{M_t}{P_t}$$

Binding CIA constraint

Obvious if $i_t > 0$ (why hold excess money?)

Also assume it even in states where $i_t = 0$:
pins down a monetary equilibrium level of M_t ,
hence is an equilibrium selection device

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SIMPLE POLICY ANALYSIS

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Rest of the environment

w_t = marginal product of labor (linear production + competitive factor market)

Govt budget: $T_t = M_t - M_{t-1} = (1+\mu_t)M_{t-1}$ Resource constraint: $c_t = z_t n_t$

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SIMPLE POLICY ANALYSIS

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Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

Combine t and $t-1$ (binding)
CIA constraints

$$\frac{c_t}{c_{t-1}} = \frac{1+\mu_t}{1+\pi_t}$$

Equilibrium link between money growth and inflation

Articulates a quantity-theoretic channel

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Examine steady-state equilibrium

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SIMPLE POLICY ANALYSIS

Household optimality conditions in deterministic steady state

$$\phi = \lambda \left[\frac{i}{1+i} \right]$$

No-arbitrage between money and nominal bonds

$$-\frac{u_n(c, n)}{u_c(c, n)} = w \left[1 + \frac{i}{1+i} \right]^{-1}$$

Consumption-leisure optimality condition

Friedman Rule: $i = 0 \rightarrow n = \beta - 1$ $1 + \pi = \beta(1+i)$

BUT ONLY IN STEADY STATE!

NOT (necessarily) dynamically....

Consumption-savings optimality condition
(aka bond Euler equation)
(aka Fisher equation)

$$1 = \frac{1+\mu}{1+\pi}$$

Equilibrium link between money growth and inflation

...and optimal policy calls for $\mu = \beta - 1$
(i.e., SHRINK nominal money supply!)

Articulates a quantity-theoretic channel

Rest of the environment

- ☐ w = marginal product of labor (linear production + competitive factor market)
- ☐ Govt budget: $T/P = (1+\mu)(M/P)(1/(1+\pi))$ Resource constraint: $C = n$

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OTHER ANALYSIS

Other aspects of equilibrium

Imply $\phi < 0$,
i.e., money
NOT valued
for exchange

- ☐ $\mu < \beta - 1$ (in steady-state!) inconsistent with monetary equilibrium
- ☐ Dynamic analog: $i_t < 0$ inconsistent with monetary equilibrium
 - ☐ Zero-lower-bound constraint

Model's "policy rate" typically identified with a (short-run Euler equation) market interest rate

- ☐ Whether CIA models, MIU models, New Keynesian models, money search models
- ☐ Model mechanism: change in policy rate (potentially) affects intertemporal incentives (i.e., the real interest rate)
- ☐ A valid empirical identification? Term-structure issues? Other issues? See Canzoneri, Cumby, and Diba (2007 JME)...

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OTHER VARIANTS OF CIA

- ❑ Cash good/credit good model
 - ❑ Lucas and Stokey (1983)
 - ❑ Foundation for Ramsey models of optimal fiscal and monetary policy – see Chari and Kehoe (1999 *Macro Handbook* chapter)
 - ❑ Subset of goods (c_1) require “cash in advance”
 - ❑ Subset of goods (c_2) do not require cash in advance

$$\frac{u_{c_1}}{u_{c_2}} = 1 + i \quad \text{MRS}_{\text{cash/credit}} = \text{gross nominal interest rate}$$

Monetary policy creates a STATIC wedge!....

- ❑ Investment in CIA constraint
 - ❑ Stockman (1981): long-run inflation lowers long-run capital stock

$$c_t + k_{t+1} - (1 - \delta)k_t \leq \frac{M_t}{P_t}$$

- ❑ Basic Idea: Positive nominal interest rate taxes whatever is in the CIA constraint

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ALTERNATIVE MONETARY MODELS

- ❑ Alternatives to CIA
 - ❑ Money in the utility function (MIU) models

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t, \frac{M_t}{P_t} \right)$$

Feenstra (1986 *JME*) shows conditions under which CIA, MIU, shopping-time are equivalent
 - ❑ Shopping-time & transactions costs models
 - ❑ Nominal money holdings reduce “cost” of acquiring goods
 - ❑ Go “cashless”
 - ❑ New Keynesian models don’t model “money demand” at all (or, at best, as an appendage separate from the “main” equilibrium)
 - ❑ Go for deep micro-foundations
 - ❑ Kiyotaki and Wright (1989, 1993)
 - ❑ Lagos and Wright (2005), Aruoba, Waller and Wright (2006)

Can think of as
“Friedman Rule
running in the
background”

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