Monopolistic Competition in a DSGE Model: Part I

September 29, 2011

Empirical and Theoretical Considerations

- Evidence supports existence of markups in goods markets (i.e., $p > mc$)
  - Basu and Fernald (1997 *JPE*) often-cited source
- Evidence also supports positive (but small?...) pure economic profits
- Are firms always price-takers?
  - If not, must endow them with market power
- If increasing returns in production exist, a model without market power does not admit an equilibrium with increasing returns
- Introduce imperfect competition
  - Typically monopolistic competition…
  - …a building block of modern sticky-price models

WORKHORSE MODEL

- Dixit-Stiglitz (1977 ***AER***) model
  - Most common specification of imperfect competition in macro models
  - (Near-) universal building block of modern sticky price models
  - Basic idea: imperfectly-substitutable goods combined yield an aggregate good

$$c_t = \left[\sum_{i=1}^{N_t} c_{it}^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Discrete number of differentiated goods

$$c_t = \left[\int_0^{N_t} c_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Continuum of differentiated goods

ε the constant elasticity of substitution between any pair of differentiated goods

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  - Important properties of aggregator
    - Symmetric in all arguments ← Drives efficiency/optimal policy results (later…)
    - Strictly increasing in all arguments
    - Strictly concave in all arguments
    - Homogenous of degree one

TWO EQUIVALENT IMPLEMENTATIONS

- **A consumption aggregator** $c_t = \left[\int_0^1 c_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$
  - "First-stage" problem: **Consumer chooses c_t…** ← A standard utility-maximization problem
  - "Second-stage" problem: **…then chooses each of the $c_t(i)$** ← A cost-minimization problem
  - **Each differentiated good i produced by a unique producer**
    - **KEY: takes as given the demand function it faces**

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- **A production aggregator** $y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$ **DS MODEL II**
  - **Final-goods producer chooses $y_t(i)$…**
  - **…to sell a composite final good y_t to consumers**
  - **Each differentiated good i produced by a unique intermediate-goods producer**
    - **KEY: takes as given the demand function it faces**

Market Organization

- **Differentiated producer *i* production technology** $y_{it} = \overbrace{z_t f(k_{it}, n_{it})}^{\text{Usual CRS}} - \Phi$
  - "Net-of-fixed-factor production technology" exhibits IRS (i.e., marginal cost < average cost)
  - Φ: Some fixed production factor – primarily useful for calibrating profit share
  - **See Rotemberg and Woodford (*Frontiers* chapter) for details on "materials cost" foundations**

Dixit-Stiglitz Model

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- **Differentiated producer *i* hires inputs on perfectly-competitive markets…**
- **…and sells its output on its own *monopolistically-competitive* market**
  - **Sells "directly" to consumers…** DS MODEL I
  - **…or to final-goods firms** DS MODEL II
- **Common assumption: Φ = 0 (→ mc = ac assuming CRS)**

FINAL-GOODS FIRMS

- **DS MODEL II**
  - **(Representative) final goods producer**

$$\max_{\{y_{it}\}_{i=0}^{1}} y_t - \int_0^1 p_{it} y_{it}\, di$$

NOTE: final output serving as numeraire

Substitute in CES final-goods aggregator

$$\max_{\{y_{it}\}_{i=0}^{1}} \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{it} y_{it}\, di$$

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- **Takes as given all p_{it}**

- **Profit-maximization leads to demand functions for each underlying differentiated good *i***

$$y_{it} = p_{it}^{-\varepsilon} \cdot y_t$$

TAKEN AS GIVEN BY DIFFERENTIATED FIRM *i*

Each differentiated firm *i* chooses its p_i to maximize profit

Relative price of firm *i*'s output

Aggregate output a ***shifter*** of firm *i*'s demand function

DIFFERENTIATED-GOODS FIRMS

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  - **Differentiated goods producer *i***

$$\max_{p_{it}} p_{it}y_{it} - w_t n_{it} - r_t k_{it}$$

Substitute in demand function

$$\max_{p_{it}} p_{it}p_{it}^{-\varepsilon}y_t - w_t n_{it} - r_t k_{it}$$

- A "two-stage" optimization problem
  - Stage 1: Choose optimal p_i
  - (Intermediate "stage"): "choose" to produce the y_i corresponding to the optimal choice of p_i
  - Stage 2: Choose factor inputs to produce y_i at minimum cost

i.e., total production y_i is simply "read off the demand curve"

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GIVEN 1) CRS $f(k, n)$ and 2) $\Phi = 0$

→ mc = ac = CONSTANT (with respect to quantity)

STAGE-1 PROBLEM

$$\max_{p_{it}} p_{it}p_{it}^{-\varepsilon}y_t - mc_t y_{it}$$

Substitute in demand function

$$\max_{p_{it}} p_{it}p_{it}^{-\varepsilon}y_t - mc_t p_{it}^{-\varepsilon}y_t$$

DIFFERENTIATED-GOODS FIRMS

- **DS MODEL I or II**
  - **Differentiated goods producer *i* optimal choice of p_i**

$$p_{it} = \underbrace{\frac{\varepsilon}{\varepsilon - 1}}\cdot mc_t$$

Gross product-market markup

Linked *only* to degree of substitutability

RBC model: ε = infinity (perf. comp.)

Monopoly model requires ε > 1 and ε < infinity

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- **Stage 2: cost-minimization**
  - Given **optimal (p_i, y_i)**

$$\max_{k_{it}, n_{it}} p_{it} z_t f(k_{it}, n_{it}) - w_t n_{it} - r_t k_{it}$$

↓ substitute $p_{it} = \left[z_t f(k_{it}, n_{it})\right]^{-1/\varepsilon} y_t^{1/\varepsilon}$ from dmd. fct.

$$\max_{k_{it}, n_{it}} \left[z_t f(k_{it}, n_{it})\right]^{1-1/\varepsilon} y_t^{1/\varepsilon} - w_t n_{it} - r_t k_{it}$$

NOTE: cost-minimization equivalent to profit-maximization *GIVEN* (p_i, y_i) -- i.e., DUAL PROBLEM

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  - Factor demands (k_i, n_i) solve

$$\frac{\varepsilon - 1}{\varepsilon} p_{it} z_t f_k(k_{it}, n_{it}) = r_t \qquad \frac{\varepsilon - 1}{\varepsilon} p_{it} z_t f_n(k_{it}, n_{it}) = w_t$$

BUILDING THE EQUILIBRIUM

- DS MODEL I or II
  - Putting things together – impose symmetry across all *i*

$$\frac{\varepsilon - 1}{\varepsilon} p_t z_t f_k(k_t, n_t) = r_t \quad \& \quad \frac{\varepsilon - 1}{\varepsilon} p_t z_t f_n(k_t, n_t) = w_t \quad \& \quad p_t = \frac{\varepsilon}{\varepsilon - 1} \cdot mc_t$$

↓ implies

$$mc_t = \frac{w_t}{z_t f_n(k_t, n_t)} = \frac{r_t}{z_t f_k(k_t, n_t)}$$

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Symmetric equilibrium *relative price* of an intermediate good? Substitute demand functions into DS aggregator and compute…

$$p_t=1$$

With measure one of intermediate firms, can think of as a normalization…but what if measure [0, N_t] of firms?

Dixit-Stiglitz Model

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MONOPOLISTICALLY-COMPETITIVE EQUILIBRIUM

- **Equilibrium Conditions (symmetric across all differentiated goods)**
  - **Consumption-leisure optimality condition**
  - **Consumption-savings optimality condition**
  - **Aggregate resource constraint**

$$c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t)$$

  - **(Market clearing in labor, capital, and goods markets)**
  - $mc_t = \frac{\varepsilon - 1}{\varepsilon} \quad \forall t$ **(< 1 with ε > 1)**
  - **Factor prices a *markdown* of marginal products**

$$w_t = \frac{\varepsilon - 1}{\varepsilon} \cdot z_t f_n(k_t, n_t), \; k_t = \frac{\varepsilon - 1}{\varepsilon} \cdot z_t f_k(k_t, n_t)$$

THE LABOR WEDGE