Optimal Capital Taxation in an Economy with Capital Allocation Frictions*

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Abstract

We study optimal capital-income taxation in an economy in which search frictions in physical capital markets give rise to flows of economic profit. These profit flows are necessary compensation for sunk search costs of entry into the capital market. Viewed in this way, profits are quasi-rents. At any point in time, however, profit flows from existing matches can also be viewed as pure rents. Whether a Ramsey government considers profit flows as pure rents or as quasi-rents is crucial for whether and to what extent capital-income taxation should be used to tax profits. We prove that if the government treats profits as quasi-rents, the canonical long-run zero-capital-tax prescription arises. If profits are instead treated as pure rents, the long-run optimal capital-income tax is non-zero, with a calibrated version of this economy featuring a capital-income tax rate of over 30 percent. The sharply contrasting results are not due to any lack of commitment. Rather, because profit flows are explicitly linked to free-entry conditions, a Ramsey government has an economic basis for adopting either the pure-rent view or the quasi-rent view. In the long run, however, the quasi-rent equilibrium is welfare-superior.

*The views expressed here are solely those of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

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1 Introduction

We study optimal capital-income taxation in an environment in which search frictions in physical capital markets give rise to economic profits. In contrast to many macroeconomic models that feature positive economic profits, profits in our environment are the returns to capital demanders (firms) for sunk search costs of trying to locate the appropriate capital for their investment projects. Profit flows thus do not arise for reasons completely exogenous to the economy, as occurs in many popular macroeconomic models. In such models, profits represent pure rents. In contrast, profits in our environment are quasi-rents; i.e., rents that compensate for the ex-ante sunk costs necessary to overcome allocation frictions. We show that if the Ramsey government views all of these profits as quasi-rents, a zero capital-income tax is optimal in the long run. On the other hand, if the Ramsey government views profits from existing matches as pure rents, a positive capital-income tax is optimal in the long run. In a calibrated version of the model, the optimal capital-income tax rate under this latter view of profits is around 30 percent. Our analysis thus nests in one framework two main competing views of capital-income taxation: the celebrated result of Chamley (1986) and Judd (1985) of zero-capital-taxation on the one hand, and the notion that capital income taxes can and should be used to tax economic profits on the other.

It is widely-understood in tax theory that positive taxes on capital income can serve as an indirect means of taxing dividend income from economic profits. Indeed, the quite high capital tax rates observed in, say, the United States, are often interpreted to be a form of profit and dividend taxation. For example, Atkeson, Chari, and Kehoe (1999) begin their early review of the theory of capital-income taxation by stating as much. Underlying this view of capital-income taxation is the assumption that complete (100 percent) confiscation of dividend income is prohibited, which is the most empirically-relevant case for policy. Given less than complete confiscation of dividends, profit flows represent a potentially inelastic source of revenue for a government that needs to finance expenditures through distortionary taxation. Jones, Manuelli, and Rossi (1997) and Correia (1996) were the first to show in a dynamic macroeconomic context that positive capital income taxation can optimally serve as a means for taxing part of such rents. A pure-rent view of profits also describes the widely-used Dixit-Stiglitz framework of monopolistic competition, and Guo and Lansing (1999) have shown that positive capital-income taxation optimally taxes profit flows in such an environment, as well.

One common feature of all of these papers, however, is that the profits that are indirectly taxed exist because they are pure rents, imposed exogenously on the structure of the model economy. A natural question, then, is whether or not the resulting optimal policy prescription is robust to a modeling framework in which rents arise endogenously as a result of the optimizing behavior of private agents who face ex-ante entry costs.

We examine this question using the search-based general-equilibrium model of Kurmann and Petrosky-Nadeau (2007) in which the exchange of physical capital from households to firms requires both time and resources in order to find a trading partner. The model naturally implies that capital-firm relationships are long-lived and that at any given point in time, a fraction of existing capital remains unmatched. Both of these implications are readily observable in the data and cannot be captured by the neoclassical benchmark where, by definition, capital markets are frictionless.

The costly matching process for physical capital at the core of our model gives rise endogenously to match-specific rents. Both firms and households must make ex-ante entry decisions prior to production; as a result, profit flows carry both a quasi- as well as a pure-rent interpretation. Flow profits are quasi-rents in the sense that they are necessary for entry, however once entry is established the rents are essentially pure. This dual nature of profits turns out to be crucial for whether and to what extent capital-income taxation should be used to tax profits.

Under the pure-rent view, we prove analytically that the optimal capital-income-tax is positive. As the government commits to the optimal policy rule, it considers existing matches as a stock whose present value can be manipulated by tax policy. All the government is concerned about are subsequent flows in and out of the capital market. To the extent that these flows are small relative to the stock of existing matches, the welfare costs of distorting the intertemporal margin are outweighed by the gains resulting from improvements in labor market efficiency. A calibrated version of our economy show that the optimal capital income tax, at over 30 percent, is quantitatively quite large. The view that the capital tax should be used to indirectly tax rents is reminiscent of the intuition present in Jones, Manuelli, and Rossi (1997), Correia (1996), and Guo and Lansing (1999).

In contrast, under the quasi-rent view, we prove analytically that the canonical zero optimal capital-income tax result obtains in our environment despite the endogenous nature of profit flows. Even though the Ramsey government faces an existing stock of capital matches when it commits to the optimal policy rule, it understands that all of these matches eventually need to be replaced by new ones. In the long-run, all capital matches are subject to distortions that arise on the intertemporal margin from taxing quasi-rents indirectly thought the price of capital. The government therefore restricts itself from manipulating the present value of flow profits from any capital match.

One can interpret the results of our model as nesting — in a unified framework — both a Chamley-Judd type of intuition for zero-capital income taxation as well as the intuition present in models where the capital income tax is used to indirectly tax rents. This nesting allows us to make a welfare comparison between the two views: numerical results show that a quasi-rent view of profits is to be preferred. In this view, steady state lifetime welfare of the representative consumer is about three percent higher, in consumption terms, than under the optimal policy in which profits are viewed as pure rents.

It is worth explicitly noting that the Ramsey government fully commits to its policy rule in both the pure-rent and the quasi-rent equilibrium. The differences between the two optimal-policy prescriptions are thus not driven by discretionary expropriation of profits. Instead, how the government treats free-entry conditions admits two different equilibrium concepts. Armenter (2008) also makes this point in a model with a fixed, untaxed factor of production. He shows that if the government treats the present value of the rents arising from this factor as predetermined, the canonical Chamley-Judd result arises. If instead, the government does not impose this restriction, the optimal capital-income tax is generally non-zero. In Armenter's (2008) framework, this restriction is hard to rationalize because rents are imposed exogenously on the model. Our analysis with explicit entry costs and quasi-rents, by contrast, provides a clear economic interpretation for the two cases.

A broad lesson that emerges from our results is that it is absolutely crucial to understand the nature of profits. If profits, or a fraction thereof are pure rents (e.g., from a natural monopoly) taxing capital income may be optimal. If, however, profits are quasi-rents — the case that arises in virtually any environment with sunk entry costs — then taxing capital income is suboptimal in the long-run

The remainder of the paper is organized as follows. The next section describes the model. Section 3 discusses the efficiency properties of the model, and Section 4 lays out the Ramsey problem. Section 5 presents our main results, derived analytically, and a brief discussion linking our results to existing literature. Section 6 provides a quantitative assessment of the optimal capital tax in a calibrated version of our economy. Section 7 concludes.

2 The Economy

The economy is populated by a continuum of identical, infinitely-lived households, a continuum of identical firms, and a government. There is no uncertainty. Each household is comprised of a continuum of individuals who discount the future at rate β and have preferences $u(c_t, n_t)$ over consumption c_t and leisure $1 - n_t$. The household makes all intertemporal decisions for its members by allocating after-tax labor income and after-tax capital income to consumption, new investment in physical capital and a full set of state-contingent government bonds. Firms, in turn, operate a constant-returns-to-scale (CRTS) technology $f(k_t, n_t)$ to produce a homogenous good with physical capital k_t and labor n_t.

Both the goods and the labor market are frictionless, as in the neoclassical one-sector growth model. The market for physical capital, by contrast, is subject to allocation frictions. Following Kurmann and Petrosky-Nadeau (2008, KP henceforth), we posit that the exchange of physical capital from households to firms requires time and resources in order to find a trading partner. Once matched, it is thus costly for both a household and a firm to abandon an ongoing capital-market relationship. This match specificity gives rise to economic rents that households and firms split. These shared rents distinguish our model's capital allocation friction from other types of reduced-form capital-market common in macroeconomic models, such as time-to-build lags or investment adjustment costs.

Capital allocation and production occurs in two phases. In the first phase, firms (i.e., capital demanders) open new *projects* at flow cost γ per project in an effort to attract available capital. Households (i.e. capital suppliers) make available *liquid capital* in an effort to place with a firm that has an open project some of its resources that will not be spent in the current period. Let V denote the total number of new projects posted by all firms in the current period, and L the total number of liquid units of capital. Assuming that capital suppliers do not direct their capital towards any particular (group of) firms, let $\theta = \frac{V}{L}$ denote capital market tightness, and total additions to the capital stock are governed by a matching process $m(V, L) \leq \min(V, L)$. Each firm and household considers the aggregates V and L, and thus also θ, as exogenous when making its decisions. Following most of the random matching literature, we assume that $m(V, L)$ is CRTS and strictly concave in its arguments. Accordingly, each project matches with a unit of liquid capital with probability $p(\theta) = \frac{m(V,L)}{V}$, and each unit of liquid capital matches with a project with probability $q(\theta) = \frac{m(V,L)}{L}$: CRTS implies $p(\theta)\theta = q(\theta)$. Moreover, it will be useful to define the elasticity of the aggregate number of matches with respect to liquid capital as $\frac{\partial m(V,L)}{\partial L}\frac{L}{m(V,L)} = -\frac{p'(\theta)\theta}{p(\theta)} = \epsilon(\theta)$ and the elasticity of the aggregate number of matches with respect to projects as $\frac{\partial m(V,L)}{\partial V}\frac{V}{m(V,L)} = \frac{q'(\theta)\theta}{q(\theta)} = 1 - \epsilon(\theta)$.

In the second phase, projects with matched capital become productive and, together with the existing capital stock, yield output. After production has taken place, some exogenous fraction s of the capital stock separates from the firm. The separated capital is returned to the supplier net of depreciation; that is, households recover $(1 - \delta)sk_t$ of their matched capital at the end of each period for consumption or reconversion into new liquid capital. Non-separated capital net of depreciation $(1-\delta)(1-s)k$ remains matched with the firm for production in the next period. Finally, unmatched capital units remain idle (without any return) in the hands of the capital suppliers and are available for consumption, investment in government bonds, or redeployment as liquid capital in the following period. Given these assumptions, from the perspective of the representative firm that takes matching probabilities as given, the evolution of its productive capital stock is described by

$$k_{t+1} = (1 - \delta)(1 - s)k_t + p(\theta_t)v_t, \tag{1}$$

where v_t denotes the firm's project postings. From the perspective of the representative household, the same evolution of matched capital takes is perceived as

$$k_{t+1} = (1-\delta)(1-s)k_t + q(\theta_t)l_t, \tag{2}$$

where l_t denotes the number of liquid capital units available supplied to the matching market. Finally, from the perspective of the household, the evolution of idle capital is

$$u_{t+1} = [1-q(\theta_t)]l_t, \tag{3}$$

where u_{t+1} denotes the number of previously-unmatched capital units with which the household enters period $t+1$. The temporal distinction between k_{t+1} and u_{t+1} on the one hand, and k_t, v_t, l_t, and θ_t on the other, emphasizes the two phases of capital allocation and production: investment in new projects, the provision of liquid capital, and matching between new projects and liquid capital must occur *before* production occurs.

2.1 Households

The representative household enters period zero with a pre-existing stock of matched capital k_0 and unmatched capital u_0. Taking as given the stream of real wages $\{w_t\}_{t=0}^{\infty}$ and rental rates for physical capital $\{r_t\}_{t=0}^{\infty}$, the household maximizes discounted lifetime utility by choosing state-contingent processes for consumption, labor, liquid capital provision, productive capital provision, and state-contingent government bonds $\{c_t, n_t, l_t, k_{t+1}, b_t\}_{t=0}^{\infty}$. The constraints on household maximization are the sequence of perceived laws of motion of the household's capital stock (2) and the sequence of budget constraints that state that, in any period, outlays for consumption, liquid capital, and government bonds cannot exceed revenues from after-tax labor income, after-tax returns from investment in capital, government bonds, and dividends from firms.[1]

Formally, the household's problem is

$$\max_{\{c_t,n_t,l_t,k_{t+1},b_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{4}$$

subject to the infinite sequence of flow budget constraints

$$\begin{aligned} c_t + l_t + b_t &= (1-\tau_t^n)w_t n_t + (1-\tau_t^k)r_t k_t + \tau_t^k \delta k_t \\ &+(1-\delta)s k_t + (1-q(\theta_{t-1}))l_{t-1} + R_{t-1}b_{t-1} + d_t \end{aligned} \tag{5}$$

and the sequence of capital accumulation constraints (2). Denote by $V(k_0, u_0)$ the maximized value of this problem. In the budget constraint (5), w_t denotes the pre-tax real wage rate, r_t denotes the

[1] We assume implicitly that households send out each capital supplier with one unit of liquid capital; each of the family members takes as given the actions of every other family member. Thus, we abstract from any coalition-building in the setting of rental rates with capital demanders. This "small-capital-supplier" assumption means that the rental rate is viewed as exogenous at the stage of household optimization.

pre-tax real rental rate of capital, d_t denotes dividends received in lump-sum manner from firms, and R_{t-1} denotes the state-contingent gross return on one-period real government securities held between $t-1$ and t. The labor income tax rate is τ_t^n, and the capital-income tax rate is τ_t^k. Note that, as is standard in models of capital taxation, we allow for a capital depreciation allowance, captured by the term $\tau_t^k \delta k_t$. We rule out taxation of dividend receipts because that would akin to a lump-sum tax.

Using the sequence of Lagrange multipliers $\{\lambda_t\}_{t=0}^{\infty}$ and $\left\{\mu_t^h\right\}_{t=0}^{\infty}$ for the sequence of budget constraints and capital accumulation constraints, respectively, the first-order conditions with respect to c_t, n_t, b_t, k_{t+1}, and l_t are

$$u_{ct} - \lambda_t = 0, \tag{6}$$

$$u_{nt} + \lambda_t(1-\tau_t^n)w_t = 0, \tag{7}$$

$$-\lambda_t + \beta\lambda_{t+1}R_t = 0, \tag{8}$$

$$-\mu_t^h + \beta\left[\lambda_{t+1}\left((1-\tau_{t+1}^k)r_{t+1} + \tau_{t+1}^k\delta + (1-\delta)s\right) + (1-\delta)(1-s)\mu_{t+1}^h\right] = 0, \tag{9}$$

and

$$-\lambda_t + \mu_t^h q(\theta_t) + \beta\left[\lambda_{t+1}(1-q(\theta_t))\right] = 0. \tag{10}$$

Conditions (6) through (8) are standard. Condition (9) states that the marginal value of a matched unit of capital equals its return net of taxes next period plus its contination value. Condition (10), finally, is a no-arbitrage condition, which states that the marginal value of consumption is equal to the expected discounted marginal value of investing in liquid capital.

Combining (9) and (10) to eliminate μ_t^h and using (6), we obtain

$$\frac{u_{ct} - (1-q(\theta_t))\beta u_{ct+1}}{q(\theta_t)} = \beta\left\{\begin{array}{c} u_{ct+1}\left((1-\tau_{t+1}^k)r_{t+1} + \tau_{t+1}^k\delta + (1-\delta)s\right) \\ +\frac{(1-\delta)(1-s)\left[u_{ct+1}-(1-q(\theta_{t+1}))\beta u_{ct+2}\right]}{q(\theta_{t+1})} \end{array}\right\}. \tag{11}$$

We call this equation the capital-supply schedule because it describes the intertemporal link between the household's opportunity cost of saving and the return from setting aside a unit of resources as capital. In fact, for $s=1$ (all capital is free to be reallocated every period) and $q(.)=1\ \forall t$ (every unit of capital offered for sale will be put to use with probability one), we recover the familiar capital-supply schedule of the one-sector growth model $u_{ct} = \beta E_t\left[u_{ct+1}\left(1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right)\right]$.

The first-order condition with respect to state-contingent bond holdings gives rise to, when coupled with (6), the standard bond-Euler equation

$$u_{c_t} = \beta u_{ct+1}R_t. \tag{12}$$

Finally, because both goods markets and labor markets are frictionless, (6) and (7) imply a standard consumption-leisure optimality condition,

$$-\frac{u_{nt}}{u_{ct}} = (1-\tau_t^n)w_t. \tag{13}$$

2.2 Firms

The representative firm begins period zero with matched capital k_0. Taking both the sequence of real wages $\{w_t\}_{t=0}^{\infty}$ and the sequence of rental rates $\{r_t\}_{t=0}^{\infty}$ as given, the firm choses a state-contingent sequence of labor, project postings, and future capital stocks $\{n_t, v_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize the discounted present value of its flow profits, taking into account that its perceived law of motion for its capital stock, (1).[2] At the end of each period, firms rebate that period's flow profits to shareholders. Period-t flow profits are given by

$$d_t = f(k_t, n_t) - w_t n_t - r_t k_t - \gamma v_t. \tag{14}$$

Because households own a perfectly diversified portfolio of claims to firms' shares, the relevant discount factor for the firm is $\beta^t u_{ct}/u_{c0} = \prod_{i=0}^{t-1} R_i^{-1}$, by (12). Formally, the firm's problem is thus

$$\max_{\{n_t, v_t, k_{t+1}\}} \sum_{t=0}^{\infty} \prod_{i=0}^{t-1} R_i^{-1} \left[f(k_t, n_t) - w_t n_t - r_t k_t - \gamma v_t\right] \tag{15}$$

subject to the sequence of constraints (1). Denote by $J(k_0)$ the maximized value of this problem. Attaching the sequence of multipliers $\left\{\mu_t^f\right\}_{t=0}^{\infty}$ to the sequence of constraints, the resulting first-order conditions are

$$w_t = f_n(k_t, n_t), \tag{16}$$

$$\prod_{i=0}^{t-1} R_i^{-1} \gamma = \mu_t^f p(\theta_t), \tag{17}$$

and

$$\mu_t^f = \prod_{i=0}^{t} R_i^{-1} \left[f_k(k_{t+1}, n_{t+1}) - r_{t+1}\right] + \mu_{t+1}^f (1-\delta)(1-s). \tag{18}$$

Eliminating μ_t^f between (17) and (18) gives

$$\frac{\gamma}{p(\theta_t)} = R_t^{-1} \left[f_k(k_{t+1}, n_{t+1}) - r_{t+1} + \frac{(1-\delta)(1-s)\gamma}{p(\theta_{t+1})}\right], \tag{19}$$

which is the model's capital demand schedule. This condition states that the expected posting cost of successfully attracting a matched unit of capital equals the discounted marginal product of matched capital net of rental costs plus the expected continuation value in case the match survives exogenous separation.

As with the capital-supply schedule (11), we recover the simple static capital-demand equation $f_k(k_t, n_t) = r_t$ from (19) by setting $s = 1$ (all capital is free to be reallocated every period) and

[2] The assumption that firms take the rental rate as given is acceptable here because of constant-returns in the production technology coupled with competitive labor markets; this makes holdup problems irrelevant despite the capital-allocation friction. See Kurmann and Petrosky-Nadeau (2008) for further details.

$\gamma = 0$ (capital does not have to be "attracted" to a particular location), which in turn yields $p(.) = 1 \;\forall t$ (every unit of capital demanded will be obtained with probability one because firms post an infinity of project). If this were the case, Euler's theorem would imply zero flow profits because $f(k, n)$ is CRTS.

Our model with capital allocation frictions thus distinguishes itself from the neoclassical one-sector growth model in two crucial aspects: first, the inherent dynamic nature of the firm's optimal capital decisions; and second, the existence of flow profits despite a production technology that is CRTS in capital and labor. These flow profits are essential for the operation of the economy in that they compensate the firm for the fixed up-front project posting cost. This can be nicely illustrated by combining the discounted sum of the firm's flow profits from $t = 1$ onward with the law of motion of the capital stock and the first-order conditions (16)-(18):[3]

$$\begin{aligned}
\sum_{t=1}^{\infty}\prod_{i=0}^{t-1} R_i^{-1} d_t &= \sum_{t=1}^{\infty}\prod_{i=0}^{t-1} R_i^{-1}\left[f(k_t, n_t) - w_t n_t - r_t k_t - \gamma v_t\right] \\
&= \sum_{t=0}^{\infty} \frac{\mu_{t+1}^f p(\theta_{t+1})}{\gamma}[f_k(k_{t+1}, n_{t+1}) - r_{t+1}]k_{t+1} - \sum_{t=0}^{\infty}\mu_{t+1}^f p(\theta_{t+1}) v_{t+1} \\
&= \sum_{t=0}^{\infty}\left[\mu_t^f - \mu_{t+1}^f(1-\delta)(1-s)\right]k_{t+1} - \sum_{t=0}^{\infty}\mu_{t+1}^f p(\theta_{t+1})v_{t+1} \\
&= \sum_{t=0}^{\infty}\mu_t^f\left[(1-\delta)(1-s)k_t + p(\theta_t)v_t\right] - \sum_{t=0}^{\infty}\mu_{t+1}^f[(1-\delta)(1-s)k_{t+1} + p(\theta_{t+1})v_{t+1}] \\
&= \mu_0^f\left[(1-\delta)(1-s)k_0 + p(\theta_0)v_0\right] = \frac{\gamma}{p(\theta_0)}k_1;
\end{aligned}$$

or, equivalently,

$$\frac{\gamma}{p(\theta_0)} = \frac{\sum_{t=1}^{\infty}\prod_{i=0}^{t-1} R_i^{-1} d_t}{k_1}. \tag{20}$$

This condition states that the period-0 expected cost of obtaining a matched unit of capital must be exactly equal to the discounted present value of flow profits per unit of capital from period 1 onwards. Hence, profits are essential for the economy because they induce firms to enter the market for capital by opening projects; it is only after the capital allocation process can production occur.

2.3 Rental Rate Determination

It remains to specify how the rental payment r_t is determined. Because it is familiar in the Pissarides class of models, we start with a scheme that has a Nash bargaining foundation, even though we recognize that bargaining may not be the most palatable assumption for price determination in asset markets. As Appendix E shows, however, the main results of the paper remain unchanged with respect to alternative bargaining arrangements and competitive pricing schemes.

[3] This derivation is analogous to Domeji's (2005) derivation in a model with labor matching frictions.

Denoting by $\phi \in (0,1)$ the share of a capital-match surplus accruing to capital suppliers (households) and by $1-\phi$ the share accruing to capital demanders (firms), the Nash bargained rental rate solves the following problem

$$\max_{r_t} \left(\frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t} \right)^{\phi} J_k(k_t)^{1-\phi}. \tag{21}$$

The term $(V_k(k_t, u_t) - V_u(k_t, u_t))/\lambda_t$ is the household's marginal value of a matched unit of capital, measured in units of numeraire (the consumption good), net of the marginal value in case the unit remains unmatched. The term $J_k(k_t)$ is the firm's marginal value of a matched unit of capital. As was defined above, the functions $V(k_t, u_t)$ and $J(k_t)$ represent, respectively, the value of the optimal plan for households and firms at the start of period t. We show in Appendix A that the resulting rental rate is

$$r_t = \phi f_k(k_t, n_t) + (1-\phi)\delta + \phi \left[\frac{(\tau^k_{t+1} - \tau^k_t) + q(\theta_t)(1-\tau^k_{t+1})}{(1-\tau^k_t)} (1-\delta)(1-s) \frac{\gamma}{p(\theta_t)} \right]. \tag{22}$$

Two distinct forces are at work on the bargained rental-rate: one that reflects standard static components of a Nash-bargaining outcome, and one that is a consequence of dynamic policy-setting. The first part of (22) shows that a firm's gain from a successful capital placement, $f_k(k_t, n_t)$, and the household's opportunity cost of not placing a unit of capital in the match, underpins the rental rate. These components are from standard Nash bargaining theory. A continuation value component of the bargained price is also standard in Nash bargaining, but, as (22) shows, *changes in* the capital-income tax rate affect this component of the time-t rental rate in our model.[4] This type of dynamic effect of taxation on Nash-bargained prices was first pointed out by Arseneau and Chugh (2006).

2.4 Government

The government faces an exogenous, stochastic stream of government spending $\{g_t\}_{t=0}^{\infty}$ that must be financed out of the sequence of flow budget constraints

$$\tau^n_t w_t n_t + \tau^k_t (r_t - \delta) k_t + b_t = g_t + R_{t-1} b_{t-1}. \tag{23}$$

The government's sources of financing are proportional taxation of labor income, proportional taxation of (productive) capital income, and state-contingent debt issuance.

2.5 Aggregation

Because the production technology is CRTS, the capital demand equation (19) and the rental rate (22) depend on k_t/n_t and not on k_t and n_t independently. Firm size is therefore irrelevant and

[4] Note that if $\tau^k_{t+1} = \tau^k_t$, the term in brackets in (22) would collapse to $\phi q(\theta_t)(1-\delta)(1-s)\gamma\theta_t$, which is the standard (i.e., no-taxation) Nash-bargaining outcome.

aggregate variables are identical to firm-level variables; i.e. $L_t = l_t$ and $V_t = v_t$. The law of motion for the aggregate capital stock is therefore

$$k_{t+1} = (1-\delta)(1-s)k_t + m(l_t, v_t), \tag{24}$$

where $m(l_t, v_t)$ now describes aggregate net investment. Furthermore, the economy-wide resource constraint can be derived by combining the household's budget constraint (5) with the definition of flow profits (14) and the government's budget constraint (23), which gives

$$f(k_t, n_t) = c_t + g_t + [l_t + \gamma v_t] - [(1-\delta)sk_t + (l_{t-1} - m(l_{t-1}, v_{t-1}))]. \tag{25}$$

2.6 Private-Sector Equilibrium

A private-sector symmetric equilibrium is, for a given government policy $\{b_t, g_t, \tau_t^n, \tau_t^k\}_{t=0}^{\infty}$, a state-contingent sequence of feasible allocations $\{c_t, n_t, k_{t+1}, l_t, v_t, \theta_t\}_{t=0}^{\infty}$ and a price system $\{R_t, r_t, w_t\}_{t=0}^{\infty}$, that satisfies the household's budget constraint (5), the capital-supply condition (11), the labor-supply condition (13), the labor-demand condition (16), the capital-demand condition (19), the Nash bargained rental rate (22), the evolution of the aggregate capital stock (24), the resource constraint (25), and the definition of capital market tightness $\theta_t = v_t/l_t$.

The ensuing optimal taxation analysis is concerned with steady state allocations where $x_t = x$, $\forall t$ for any variable of the model. For this case, the following proposition holds.

Proposition 1. *There exists a unique steady-state equilibrium characterized by the following capital market tightness:*

$$\frac{1}{\theta^E} = \frac{\gamma}{1-\beta}\frac{\phi}{1-\phi}(1-\tau^k). \tag{26}$$

This value of θ^E, in turn, pins down a unique capital-labor ratio $(k/n)^E$.

PROOF: See Appendix B.

3 Intertemporal Efficiency

To understand the optimal tax results that emerge from the Ramsey problem, it is useful to first discuss the conditions that characterize the constrained efficient, or first-best steady state allocation $\{c, n, k, l, v, \theta\}$ a social planner would choose in the absence of distortionary taxation. Our notion of constrained efficiency is one that takes the matching process for physical capital as a technology of the economy. To facilitate the analysis of efficiency, we develop a general notion of the intertemporal marginal rate of transformation for our search-theoretic model of the capital market.

The social planner's problem is described by

$$\max_{\{c_t,n_t,v_t,l_t,k_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{27}$$

subject to the sequence of capital accumulation constraints (24) and resource constraints (25). Rather than deriving the full solution of this problem, we focus on intertemporal efficiency for capital accumulation, as summarized by the following proposition.

Proposition 2. *There exists a unique constrained-efficient (first-best) steady state equilibrium characterized by the following equation for capital market tightness*

$$\frac{1}{\theta^{1st}} = \frac{\gamma}{1-\beta} \frac{\epsilon(\theta^{1st})}{1-\epsilon(\theta^{1st})}. \tag{28}$$

The value of θ^{1st}*, in turn, pins down a unique capital-labor ratio* $(k/n)^{1st}$*.*

The formal proof of this proposition is detailed in Kurmann (2008). Here, we simply provide intuition by reconstructing condition (28) from the basic tenet that under intertemporal efficiency, the intertemporal marginal rate of substitution (IMRS) must equal the intertemporal marginal rate of transformation (IMRT). The IMRS in our model is straightforward, and standard, to define. It measures how many consumption goods in t a household is willing to give up in order to obtain one more unit of consumption in $t+1$ while remaining on the same intertemporal indifference curve. Given the subjective discount factor β, we have that this IMRS is measured, as in any standard intertemporal model, by $u_{ct}/\beta u_{ct+1}$.[5] In steady state, $u_{ct+1} = u_{ct} = u_c$, and thus

$$IMRS = \frac{1}{\beta}. \tag{29}$$

Constructing the IMRT is more involved due to the capital allocation friction. Consider the capital accumulation constraint (24) and the economy's resource constraint (25). There are two ways the economy can transform a unit of consumption in period t into a unit of consumption in period $t+1$. We thus trace the IMRT in these two possible ways, and then connect them.

One way for the economy to achieve transformation of c_t into c_{t+1} is to first trade off one unit of c_t for one unit of liquid capital l_t. The extra unit of l_t increases the number of aggregate capital matches by $\partial m(l_t, v_t)/\partial l_t \equiv m_{lt}$, which in turn increases output (and hence increases consumption) in future periods by $[(f_{kt+1} + (1-\delta)s) + (1-\delta)(1-s)(f_{kt+2} + (1-\delta)s) + ...]m_{lt}$. In steady state, this increase in production is equal to $[f_k + (1-\delta)s]/[1-(1-\delta)(1-s)]$. At the same time, a one-unit increase in l_t affects the amount of *un*matched capital carried forward into period $t+1$ by $(1-m_{lt})$, which, all else equal, would *decrease* c_{t+1}. Transforming one unit of c_t into one unit of l_t thus yields

$$IMRT = m_l \left(\frac{f_k + (1-\delta)s}{1-(1-\delta)(1-s)} \right) + (1-m_l) \tag{30}$$

[5]More formally, we have along an indifference curve that $-\frac{dc_{t+1}}{dc_t} = \frac{MU_t(c_t)}{MU_t(c_{t+1})} = \frac{u_{ct}}{u_{ct+1}}$.

extra units of c_{t+1} in steady state.

Alternatively, the economy can achieve transformation of c_t into c_{t+1} by first trading off one unit of c_t for project postings v_t; doing so yields $1/\gamma$ additional project postings, as (25) shows. As a consequence, the aggregate number of capital matches increases by $(\partial m(l_t, v_t)/\partial v_t)(1/\gamma) \equiv m_{vt}/\gamma$, which in turn increases production in future periods by $[(f_{kt+1} + (1-\delta)s) + (1-\delta)(1-s)(f_{kt+2} + (1-\delta)s) + ...] * m_{vt}/\gamma$. In steady state, this increase in production is equal to $[f_k + (1-\delta)s]/[1-(1-\delta)(1-s)] * m_v/\gamma$. Attendant with the rise in matched capital is a decrease in unmatched capital by $-m_{vt}/\gamma$ units. In steady state, the total effect of transforming one unit of c_t into $1/\gamma$ units of v_t is that

$$IMRT = \frac{m_v\left(\frac{f_k+(1-\delta)s}{1-(1-\delta)(1-s)}\right) - m_v}{\gamma} \tag{31}$$

extra units of c_{t+1} can be consumed.

For the economy to be on its production possibilities frontier, the two ways of transforming consumption intertemporally must be equivalent. This is satisfied if $\left(\frac{f_k+(1-\delta)s}{1-(1-\delta)(1-s)}\right) = \frac{m_v+(1-m_l)\gamma}{m_v-\gamma m_l}$ and thus we can collapse the two alternative representations of $IMRT$ into just one,

$$\begin{aligned} IMRT &= m_l\left(\frac{m_v+(1-m_l)\gamma}{m_v-\gamma m_l}\right) + (1-m_l)1 = \frac{m_v}{m_v-\gamma m_l} \\ &= \frac{1-\epsilon(\theta)}{1-\epsilon(\theta)[1+\gamma\theta]}. \end{aligned}$$

The second line uses the fact that $m_v/m_l = (1-\epsilon(\theta))/(\theta\epsilon(\theta))$ by definition of the matching function.

This conceptualization of the IMRT is novel. It compactly describes the *two* technologies — the matching technology $m(.)$ and the production technology $f(.)$ — that must operate for consumption to be transformed across time. As we discuss below, this IMRT is crucial for understanding our results. More broadly, as search-theoretic frameworks become increasingly popular in dynamic general equilibrium models, it is useful to be able to describe transformation frontiers and the MRTs implied by them in general ways.[6] Notice that our notion of the IMRT encompasses that in a standard RBC model. As we noted above, one can recover a simple neoclassical capital market by assuming project costs are zero ($\gamma = 0$) (in which case, because they are costless, the number of vacancies posted is infinite), and all capital is "returned to households" at the end of every period ($s = 1$). With an infinity of project postings, $m_l = 1$ and $m_v = 0$. Imposing these assumptions and results in the preceding logic, we obtain $IMRT = f_k + 1 - \delta$, obviously identical to an RBC model.

Regardless of whether or not we are considering a neoclassical environment, intertemporal efficiency naturally requires $IMRS = IMRT$. In our environment, we thus have

$$\frac{1}{\beta} = \frac{1-\epsilon(\theta)}{1-\epsilon(\theta)[1+\gamma\theta]}, \tag{32}$$

[6] In a general-equilibrium *labor*-search framework, Arseneau and Chugh (2008b) develop an analogous search-based notion of the MRT between consumption and leisure.

which, after some rearrangement, is equivalent to (28). Comparing (28) with the private-sector equilibrium solution in (26), we see that the (steady-state of the) decentralized economy achieves intertemporal efficiency only in some special cases that are summarized by the following proposition.

Proposition 3. *The private-sector equilibrium achieves intertemporal efficiency under the following conditions:*

1. $\phi = \epsilon(\theta)$ *and* $\tau^k = 0$*;*

2. $\phi \geq \epsilon(\theta)$ *and* $\tau^k = 1 - \frac{1-\phi}{1-\epsilon(\theta^{1st})}\frac{\epsilon(\theta^{1st})}{\phi}$.

The intuition for this proposition is straightforward. Consider condition (i). When $\phi = \epsilon(\theta^{1st})$, the capital supplier's share of the match surplus ϕ fully internalizes its relative contribution to the market for physical capital allocation $\epsilon(\theta^{1st}) \equiv m_l$. This is equivalent to the well-known Hosios (1990) efficiency condition for random matching models of the labor market.[7] In this case, any tax on capital will drive a wedge between the IMRS and the IMRT, thus leading to dynamic distortions in capital accumulation.

Alternatively, consider condition (ii) and more particularly a situation where $\phi > \epsilon(\theta^{1st})$. In this case, the capital supplier's share of the match surplus exceeds its relative contribution, which would, in the absence of capital taxes, lead to an excessive supply of liquid capital; i.e. $\theta^E < \theta^{SP}$. Capital taxes can therefore be used to correct for this externality. Specifically, for $\phi > \epsilon(\theta^{1st})$, we have $\tau^k = 1 - \frac{1-\phi}{1-\epsilon(\theta^{1st})}\frac{\epsilon(\theta^{1st})}{\phi} > 0$ such as to discourage the supply of liquid capital, thus restoring intertemporal efficiency.[8]

In what follows, we focus exclusively on equilibria that satisfy the usual Hosios parameter restriction $\phi = \epsilon(\theta^{1st})$. This case is interesting for two reasons. First, absent any proportional taxation whatsoever, this case is equivalent to an alternative competitive search environment (as in, say, Moen (1997), for the labor market) in which firms post rental rates in advance and where capital suppliers can direct their capital towards a particular firm.[9] Second, and more importantly for our purposes, this case describes an economy where, in the absence of distortionary taxation, positive flow profits can co-exist with a dynamically efficient capital stock. As the following analysis shows, the existence of such flow profits provides the Ramsey planner with a powerful motive to tax capital.

[7]Kurmann (2008) provides the formal analysis translating the Hosios (1990) efficiency condition to a search-theoretic view of capital markets.

[8]Similar results on optimal taxation in the presence of externalities have been derived, for example, by Domeji (2005) in the context of a model with labor search; or Bilbiie, Ghironi and Melitz (2007) in the context of a model with monopolistic competition and endogenous entry.

[9]See Kurmann (2008) for a formal derivation. Appendix E sketches the competitive search environment and provides robustness checks.

4 Ramsey Problem

Given initial bond holdings b_0, an initial matched capital stock k_0, and an inital set of unmatched capital units $(1-q_{-1})l_{-1}$, the government's problem is to choose tax rates $\{\tau_t^n, \tau_t^k\}_{t=0}^{\infty}$ so as to maximize the economy's welfare subject to the following constraints: (i) the resulting allocation is supportable as a private-sector equilibrium as defined above; and (ii) the government's budget constraint is satisfied for an exogenous stream of expenditures $\{g_t\}_{t=0}^{\infty}$. Following Lucas and Stokey (1983) and Chari and Kehoe (1999), we implement the Ramsey problem with the primal approach, in which we cast the Ramsey government's problem as one of choosing only allocations. As in standard Ramsey models, we use the present-value household budget constraint with prices and taxes substituted out using equilibrium conditions. Unlike in standard Ramsey models, however, this present-value budget constraint does *not* capture (apart from the resource constraint) all of the equilibrium conditions of the decentralized economy. In particular, we must separately impose firms' project-posting conditions, cast purely in terms of allocations, as a constraint on the Ramsey problem. With the Ramsey allocation in hand, we then compute the optimal tax rates using the private economy's equilibrium conditions.

Proposition 4. *Given initial states b_0, k_0, $(1-q_{-1})l_{-1}$ and initial rental and capital tax rates r_0 and τ_0^k, the private-sector equilibrium can be characterized by the evolution of the aggregate capital stock (24), the resource constraint (25), the implementability constraint (IC)*

$$\sum_{t=0}^{\infty} \beta^t \left[u_{ct}c_t + u_{nt}n_t - u_{ct}d_t\right] \geq A_0 \tag{33}$$

with

$$A_0 \equiv u_{c0}\left[R_0 b_{-1} + (1-q(\theta_{-1}))l_{-1}\right] + V_{k0}k_0,$$

and reformulated expressions for dividends and capital demand, respectively,

$$d_t = (1-\phi)\left[f_k(k_t,n_t)-\delta\right]k_t - \phi\left[1-\left[1-q(\theta_t)\right]\left(\frac{1-\beta\frac{u_{ct+1}}{u_{ct}}}{1-\beta\frac{u_{ct}}{u_{ct-1}}}\right)\frac{\theta_{t-1}}{\theta_t}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}k_t - \gamma\theta_t l_t, \tag{34}$$

$$\frac{\gamma}{p(\theta_t)} = \beta\frac{u_{ct+1}}{u_{ct}}\left[\frac{d_{t+1}+\gamma\theta_{t+1}l_{t+1}}{k_{t+1}} + (1-\delta)(1-s)\frac{\gamma}{p(\theta_{t+1})}\right]. \tag{35}$$

PROOF: See appendix.

As discussed in Section 2, our model differs from the neoclassical one-sector growth model in two crucial aspects, and both of these differences show up in the reformulated private-sector equilibrium. First, initial asset wealth in the IC in (33) is augmented by the presence of dividends

$\sum_{t=0}^{\infty} \beta^t u_{ct} d_t$. These dividends, defined in (34), are rents that compensate firms for their ex-ante project posting costs and are transferred lump-sum each period to the households who are the ultimate shareholders. Second, the capital demand in (35) describes optimal project entry and imposes an additional constraint: the expected posting costs per match equals the marginal discounted returns from productive capital.

We formulate the Ramsey problem for two polar cases, depending on whether the government considers rents from existing capital matches in the initial period as a taxable object or not. In the first case, the government takes existing capital matches as given. It thus considers the flow profits from these matches as pure rents whose present value is influenced by taxes. Accordingly, we define this case as the *pure-rent view.*

Definition 1. *The Ramsey problem under the pure-rent view is*

$$\max_{\{c_t, n_t, k_{t+1}, \theta_t, l_t, d_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

subject to the constraints of the private sector equilibrium (24)-(25) and (33)-(35), for a given exogenous expenditure stream $\{g_t\}_{t=0}^{\infty}$ *and initial conditions* b_0, k_0, $(1-q_{-1})l_{-1}$, r_0, *and* τ_0^k.

As this formulation of the Ramsey problem makes clear, the government in terms of capital accumulation is only concerned with entry of new projects; i.e. the discounted *marginal* value of new matches from $t = 0$ onward. Whether the discounted *average* return from total capital matches covers expected entry costs does not figure as a constraint because, as described above, the government considers the flow profits from existing matches as pure rents that can be influenced by tax policy.

The second, alternative case we consider is where the government does not take as given existing capital matches in the initial period. Instead, the government realizes that all flow profits that the firm receives are ultimately quasi rents that compensate for ex-ante project costs – as embodied in condition (20). According to this long-term or *quasi-rent view,* the government takes (20) as an explicit constraint in its optimization problem, which, allows us to rewrite the initial asset wealth from dividends in the IC as

$$\begin{aligned}
\sum_{t=0}^{\infty} \beta^t u_{ct} d_t &= u_{c0} d_0 + \sum_{t=1}^{\infty} \beta^t u_{ct} d_t = u_{c0} d_0 + u_{c0} \sum_{t=1}^{\infty} \prod_{i=0}^{t-1} R_i^{-1} d_t \\
&= u_{c0}[(f_{k0} - r_0) k_0 - \gamma v_0] + u_{c0} \frac{\gamma k_1}{p(\theta_0)} \\
&= u_{c0} \left[f_{k0} - r_0 + (1-\delta)(1-s) \frac{\gamma}{p(\theta_0)} \right] k_0 \\
&= u_{c0} \frac{\gamma}{p(\theta_{-1})} R_{-1}.
\end{aligned}$$

The inital asset wealth from dividends, or equivalently the discounted sum of total quasi-rents from capital matches, is thus predetermined in the sense that the government restricts itself from manipulating this wealth when setting tax policy. Integrating this result into the IC, we obtain the following definition of the Ramsey problem under the quasi-rent view.[10]

Definition 2. *The Ramsey problem under the quasi-rent view is*

$$\max_{\{c_t,n_t,k_{t+1},\theta_t,l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

subject to the constraints

$$\sum_{t=0}^{\infty} \beta^t \left[u_{ct}c_t + u_{nt}n_t\right] \geq u_{c0}\left[R_0 b_{-1} + (1 - q(\theta_{-1}))l_{-1} + u_{c0}\frac{\gamma}{p(\theta_{-1})}R_{-1}\right] + V_{k0}k_0 = A_0^{'}$$

,

$$f(k_t, n_t) + (1 - \delta)sk_t + (1 - q(\theta_{t-1}))l_{t-1} \geq c_t + g_t + (1 + \gamma\theta_t)l_t$$

$$(1 - \delta)(1 - s)k_t + q(\theta_t)l_t \geq k_{t+1}$$

,for a given exogenous expenditure stream $\{g_t\}_{t=0}^{\infty}$*, and initial conditions* b_0*,* k_0*,* $(1 - q_{-1})l_{-1}$*,* r_0*, and* τ_0^k*.*

In short, the pure-rent and the quasi-rent view of the Ramsey problem are based on exactly the same sequences of constraints imposed by the private-sector equilibrium. The only difference is how the government considers flow profits from existing capital matches. As the optimal tax policy results in the next section show, this difference has important implications.

As is standard in Ramsey taxation problems, we assume full commitment. Thus, we emphasize that none of our results is driven by the use of a discretionary policy. Finally, throughout our analysis, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior.

5 Optimal Tax Policy

We characterize the optimal tax policy for the two views presented in the previous section. We only consider the limiting case for which the economy converges to a steady state. Numerical results for a calibrated version of the pure-rent case are presented in Section 6.

[10]Since the same expression for $\sum_{t=0}^{\infty} \beta^t u_{ct} d_t$ is obtained with (34)-(35), the two constraints become redundant and can be dropped from the private-sector equilibrium.

5.1 Optimal tax policy under the pure-rent view

We first consider the Ramsey problem under the pure-rent view in Definition 1. We obtain the following result.

Proposition 5. *Under the pure-rent view in Definition 1, the Ramsey-optimal steady-state capital-market tightness is*

$$\frac{1}{\theta^{2nd}} = \frac{\gamma}{1-\beta}\frac{\epsilon(\theta^{2nd})}{1-\epsilon(\theta^{2nd})} + wedge, \tag{36}$$

with

$$wedge \equiv \frac{\gamma}{1-\beta}\frac{1}{[1-\epsilon(\theta^{2nd})]}\left(-\epsilon(\theta^{2nd})\frac{\mu^{rd}}{\mu^{rb}} - 2\epsilon(\theta^{2nd})\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k} + \frac{\mu^{rd}}{\mu^{rb}}\frac{\partial d}{\partial\theta^{2nd}}\right).$$

The variables μ^{rb}, μ^{rd}, and μ^{rp} denote the steady state values of the Lagrangian multipliers for the resource constraint (25), the definition of dividends (34), and capital demand (35), respectively. Because the Lagrangian multipliers μ^{rb}, μ^{rd}, μ^{rp} are in general non-zero and the partial derivative of flow profits with respect to capital market tightness $\partial d/\partial\theta < 0$, the optimal capital income tax τ^k that implements this allocation is in general non-zero.

PROOF: See Appendix D.

To provide some intuition about this result, note that the government faces the following trade-off. On the one hand, it wants to set liquid capital l_t and project postings v_t such as not the distort the intertemporal margin; i.e., $IMRS = IMRT$. This is reflected in the fact that the first term on the right-hand-side of (36) is identical to the first-best solution in (28) that implies a zero capital income tax.

On the other hand, the government considers the dividends from existing matches as pure rents that it would like to confiscate in order to reduce tax distortions; i.e. the lower initial asset wealth $\sum_{t=0}^{\infty}\beta^t u_{ct} d_t$ in the IC, the smaller the utility cost from distortionary taxes. Absent dividend taxation, which is akin to lump-sum taxes that are by definition ruled out in Ramsey problems, the government influences rents in (34) through l_t and v_t. This is reflected in the term *wedge* in (36). Specifically, as long as physical capital and thus l_t and v_t are complements for rents, this wedge is negative and thus, the optimal capital tax implied by comparison of (26) with (36) is positive.

The result stands in marked contrast to the canonical zero capital tax result in Chamley (1986) and Judd (1985). The sole reason for this difference is the existence of rents in our model, which occur as a natural consequence of ex-ante project costs that lead to frictions in the allocation of physical capital. As these project costs go to zero (i.e. $\gamma \to 0$), rents from capital matches disappear and the optimal capital income tax returns to Chamley's and Judd's limiting case.

5.2 Optimal tax policy under quasi-rent view

The optimal tax policy under the alternative quasi-rent view is quite different, as summarized in the following proposition.

Proposition 6. *Under the quasi-rent view in Definition 2, the Ramsey-optimal steady-state capital-market tightness is characterized by*

$$\frac{1}{\theta^{2nd'}} = \frac{\gamma}{1-\beta}\frac{\epsilon(\theta^{2nd'})}{1-\epsilon(\theta^{2nd'})}. \tag{37}$$

The steady-state capital income tax that implements this allocation is $\tau^k = 0$.

PROOF: See Appendix D.

Intuitively, the government under the quasi-rent view restricts itself from manipulating initial asset wealth from dividends, $\sum_{t=0}^{\infty} \beta^t u_{ct} d_t$, because it realizes that they all represent quasi-rents that compensate for previously incurred project costs. Doing so would distort match formation in the capital market in the long run. As a result, the trade-off described above falls away. All the government cares about is $IMRS = IMRT$, which is reflected by the fact that (36) is equivalent to (28), and thus $\tau^k = 0$. Similar to Chamley's and Judd's canonical case, the Ramsey problem under the quasi-rent view thus attains the first-best solution along the intertemporal margin, and all the distortions are concentrated on the intratemporal consumption-labor margin.

We turn now to a more in-depth discussion of the contrasting results under the quasi-rent case and the pure-rent case.

5.3 Discussion

Rents always exist in our model. What drives the starkly different uses of capital-income taxation as a way of taxing these rents is the view the Ramsey government adopts of their nature. In this section, we interpret the different views and draw comparisons to the existing literature.

Consider first the pure-rent view, captured in Definition 1. As the government commits to the optimal policy rule, it considers existing capital matches from successful search activity in the past as a *stock* whose present value can be manipulated by tax policy. Here, the government is concerned only about the subsequent *flows* in and out of the capital market — that is, about the formation of new productive capital relationships that replace depreciation and exogenous break-ups of existing matches that occur at rate s. Because the stock of existing capital matches is large compared to the flow of new matches (i.e., investment), setting a positive capital-income tax only slightly distorts the intertemporal margin, at the gain of being able to relax the distortion in the static consumption-leisure margin.

Consider now the quasi-rent view, captured in Definition 2. Even though the Ramsey government faces an existing stock of capital matches at time zero when it commits to the optimal policy rule, it understands that eventually all of these matches need to be replaced by new ones. In the long-run, all capital matches are eventually subject to distortions that arise on the intertemporal margin from taxing quasi-rents indirectly through the price of capital. The government therefore restricts itself from manipulating the present value of flow profits from any capital match — a policy that in the long-run is clearly superior to the optimal policy under the pure-rent view.

On a conceptual level, the positive capital tax result under the pure-rent view can be interpreted through the lens of the uniform commodity taxation theorem well-known in the (static) theory on optimal taxation. As described in, say, Stiglitz and Dasgupta (1971) or the monograph treatment in Atkinson and Stiglitz (1980), if certain restrictions on preferences are satisfied and all sources of factor income, including profit income, can be taxed, it is optimal to tax all consumption goods at the same rate.[11] In an intertemporal setting, taxing consumption goods across time uniformly requires setting a zero tax rate on capital income. Otherwise, the real interest rate, which links consumption prices at different dates, is distorted. In this sense, Chamley's (1986) and Judd's (1985) zero-capital-tax result is an intertemporal application of the uniform commodity tax theorem in an economy without profits. If, by contrast, the economy generates profits that *cannot* be taxed at a 100-percent rate, then the optimality of uniform commodity taxation breaks down. In this case, the government can use capital income taxes, through their distortion of the real interest rate, to tax consumption in different time periods at a non-uniform rate.

This analogy with the uniform commodity taxation theorem has been pointed out in earlier work by Jones, Manuelli, and Rossi (1997) and underlies much of the literature on Ramsey models with incomplete factor taxation. For example, in both Correia (1996) and Jones, Manuelli, and Rossi (1997), the government is restricted from taxing every factor of production at the rate of choice. This generates rents for households that, if left untaxed, affect their behavior only through an income effect.[12] As long as capital is complementary to the untaxed factor(s) and thus to the rents firms transfer to households, the government finds it optimal to depress these rents by taxing capital-income so as to reduce the accumulation of capital. Armenter (2008) makes this point very nicely in a recent paper, and the same mechanism is also at the heart of our positive capital taxation result under the pure-rent view.

[11] The most germane of these preference restrictions is homotheticity in all consumption goods, an assumption present in any macro model employing balanced-growth preferences.

[12] As Correia (1996) states, one can think of a model with incomplete factor taxation alternatively as a setting where the production function exhibits decreasing returns to scale and where the thus generated profits are not taxed on the household side. As mentioned above, a 100-percent tax on profits transferred to households is akin to a lump-sum tax, which we generally rule out in the Ramsey approach.

In contrast to the existing literature, however, our results do not stem from either arbitrary restrictions about the available menu of tax instruments or an exogenous assumption of profit. Rather, use of the capital tax achieves taxation of the profits that arise *endogenously* as the reward to some type of activity to gain access to a market.[13] Familiar examples are product marketing or research and development activities, in which profits (or at least a limited time span in which to earn them) are a *necessary* compensation for upfront sunk costs. Our model is of course simpler than these real-life situations but we think our formalization of entry as a purposeful *search* decision captures the same idea. Most importantly, rents in our model cannot be taxed away (other than lump-sum) because this would rule out entry and thus production. The only way rents and thus positive capital taxes disappear in our model is for the limiting case when project posting (i.e. search) costs go to zero.

The explicit formalization of entry and the endogenous nature of rents also distinguishes our analysis from studies on optimal taxation in the presence of monopolistic competition (e.g. Dixit-Stiglitz goods differentiation). For example, Judd (1997, 2002) argues that monopoly frictions in and of themselves call for *negative* capital taxes because the resulting markup in goods markets leads to inefficiently low factor demand, including the demand for capital. Guo and Lansing (1999) extend Judd's analysis in a fully articulated macro public finance framework and show that the existence of rents arising from monopoly power can reverse Judd's negative capital tax result.[14] The mechanism behind their result is exactly the same as in our model under the pure-rent view.

In sharp contrast to our analysis, however, there is no explicit entry decision in Guo and Lansing's (1999) economy, and profits are exogenously imposed by the Dixit-Stiglitz structure. Hence, the concept of quasi-rents is absent. As suggested by Armenter's (2008) general discussion of incomplete factor taxation models, this absence of quasi-rents in and of itself does not prevent the government from considering the present-value of flow profits as a predetermined variable that cannot be manipulated by policy. However, in a setting without explicit entry and quasi-rents, there is simply no meaningful way to understand why the government should tie its hands in that way. Our analysis with explicit entry, by contrast, provides a clear economic rationale why the pure-rent view is suboptimal and why, within the confines of our model, it is still a bad idea to tax capital income.

More generally, our analysis affords two conclusions that, we believe, are new for the Ramsey

[13] Of course, we could add other distortionary taxes or subsidies to the capital market; for example a tax on entry (i.e. project postings), a tax on corporate profits or an allowance for capital expenditures (e.g. Abel, 2007). While this would affect some results about specific taxes, it would not change allocations for it is the *effective* tax rate on the intertemporal margin that matters for private-sector decisions.

[14] In the context of monetary models featuring Dixit-Stiglitz monopoly power, Schmitt-Grohe and Uribe (2005) and Chugh (2007) also can be interpreted as analyzing the Judd's argument in a macro public-finance framework.

taxation literature. First, even within the class of proportional taxation under full commitment, a range of optimal tax policy concepts may apply. Second, for optimal tax policy, it is absolutely crucial to understand the nature of profits. If profits or a fraction thereof are pure rents (e.g. from a natural monopoly) taxing capital income may be optimal. If, however, profits are quasi-rents – the case that arises in virtually any environment with sunk entry costs – then taxing capital income is suboptimal.

6 Quantitative Results

In this section, we computationally solve for the steady-state Ramsey allocations in a calibrated version of our economy in order to assess the quantitative significance of the optimal capital tax under the pure-rent view. In computing the optimal policy under the pure-rent view, we also assume the time-zero allocation is the same as the asymptotic steady-state Ramsey allocation, thus endogenizing the initial condition of the economy.[15] This assumption in and of itself does not affect whether or not a zero capital-income tax is optimal in the steady-state.

6.1 Calibration

We assume the instantaneous utility function is

$$u(c_t, n_t) = \ln c_t + \frac{\zeta}{1-\nu}(1-n_t)^{1-\nu}. \tag{38}$$

The unit of time in the model is meant to be a quarter, so we set the subjective discount factor to $\beta = 0.99$, yielding an annual real interest rate of about four percent. For the leisure subutility function, we fix the parameter ζ such that the average fraction of hours worked equals $n = 0.2$ in the socially efficient allocation. Together with $\nu = 4$, this results in a Frisch elasticity of labor supply of 1. When solving for the Ramsey steady states under both the pure and quasi-rent view of flow profits, we hold these values of ζ and ν, along with all other model parameters, constant.

For production, we assume a Cobb-Douglas function with constant returns to scale of the form

$$f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}. \tag{39}$$

We set the share of capital in the production function to $\alpha = 0.3$, and the rate of depreciation of capital to $\delta = 0.025$.

For the parameterization of the capital market, we follow Kurmann and Petrosky-Nadeau (2008) who fit a very similar model to Compustat data. Based on their calculations, we set the quarterly separation rate to $s = 0.01$. Together with $\delta = 0.025$, this implies a ratio of total investment to

[15] Thus, we analyze policy not only under full commitment, but also under the "timeless" perspective.

capital of 18 percent per year, and a ratio of used investment to total investment of 28 percent. Both of these ratios are roughly in line with Compustat statistics reported by Ramey and Shapiro (1998) and Eisfeldt and Rampini (2007), respectively.[16] Furthermore, we fix $\epsilon = \phi = 0.5$, which achieves the Hosios (1990) parameterization for search efficiency in the absence of taxation, and set γ such that $q(\theta) = 0.25$. In a business cycle context with productivity and separation shocks, these parameter values generate the procyclical and volatile nature of used capital reallocation that observed in the Compustat data. Finally, we choose steady-state government purchases $\bar{g}$ so that they constitute ten percent of total output, and the steady state value of government debt is assumed to be zero, so that $b = 0$.

6.2 Quantitative Assessment of the Optimal Capital Tax

Results are presented in Table 1. The first row shows the constrained-efficient (first-best) allocations. Examining first the key macroeconomic ratios, the consumption to output ratio ($c/y = 0.70$) is in line with data, but the ratio of the steady-state capital stock capital to output, k/y, is somewhat lower. We note, however, that capital in our model is not consistent with the way the capital stock is calculated in the NIPA accounts, thus one might expect at least some difference. Equilibrium profits play a large role in our model, thus it is comforting to see that the ratio of dividends to output, d/y, is quite low but comparable with that found in existing literature; see, for example, Basu and Fernald (1997) or Rotemberg and Woodford (1995). Overall, our calibrated model seems to replicate the key aggregate macroeconomic ratios observed in the data reasonably well.

As stated in Proposition 2, the unique value of market tightness, $\theta = 3$ given our calibration, pins down the socially-efficient capital-labor ratio, $k/n = 18.759$. These efficient values for capital-market tightness and the capital-labor ratio serve as the benchmark against which to compare the two alternative Ramsey equilibria.

The next two rows of Table 1 present allocations for the two alternative formulations of the Ramsey problem presented in Section 4. If flow profits are treated as quasi-rents, we know from Proposition 6 that the optimal capital tax is zero. The first column of the second row of Table 1 confirms this numerically. The Ramsey planner finances all government expenditure through the use of the labor-income tax, setting τ^n to about 14 percent. Although the labor tax pushes labor input below its socially-efficient level, efficiency in the capital market is achieved because $\theta = 3$ and

[16] An annual investment-capital ratio of 17 percent may appear high compared to the empirical counterparts obtained from the NIPA tables, which average around 10 percent annually. However, NIPA tables only measure investment flows of new capital goods and then infer aggregate capital stocks as the sum of current and past investment flows less depreciation. NIPA investment-capital ratios thus underestimate the true investment-capital ratio for two reasons: first, because they do not take into account investment in used capital; and second, because the inferred capital stocks do not take into account loss due to reallocation.

	τ^k	τ^n	r	θ	k/n	c	n	y	k/y	d/y	Welfare
Social efficiency	–	–	0.037	0.333	18.759	0.343	0.200	0.486	7.785	0.003	–
Ramsey (quasi-rents)	0	0.136	0.037	0.333	18.759	0.317	0.188	0.453	7.785	0.003	-1.42
Ramsey (pure rents)	0.354	0.084	0.042	0.516	15.138	0.311	0.191	0.431	6.670	0.004	-4.76

Table 1: Steady state results. Welfare reported in the last column is the percentage additional consumption the representative household requires in the given allocation to be just as well off as in the socially-efficient steady-state allocation.

$k/n = 18.759$.

However, if flow profits are viewed as pure rents, Proposition 5 establishes that optimal capital tax is non-zero. The third row of Table 1 shows that the optimal capital-income tax rate is about 35 percent. The second column of the table shows that the additional revenue raised from the capital tax is used to finance a reduction in the proportional labor tax, hence labor input under the pure rent view is a bit closer to its socially efficient level relative to the quasi-rent case. However, $\tau^k > 0$ introduces an inefficiency into the capital market. We have that both capital-market tightness, $\theta = 1.938$, and the capital-to-labor ratio, $k/n = 15.138$, are lower than in the socially-efficient allocation. Thus, in comparing the Ramsey equilibria in the pure- versus quasi-rent views, the positive capital tax delivers efficiency gains in the labor market at the expense of distorting capital accumulation.

The final column of the table shows the welfare loss of the Ramsey allocations relative to the socially-efficient outcome. We measure welfare loss as the percentage additional consumption the representative household requires in the Ramsey steady state to be just as well off as in the socially-efficient allocation.[17] The table shows that if flow profits are viewed as quasi-rents, the welfare cost of the Ramsey allocations relative to the socially efficient allocations is 1.4 percent of steady state consumption. The welfare cost associated with the Ramsey outcome in which flow profits are viewed as pure rents, at near 5 percent, is much larger. The difference, of over 3 percent, thus represents the steady-state welfare gain of switching from a Ramsey government that views profits as pure rents to a Ramsey government that views profits as quasi-rents.

[17] We do not take into account transition costs, which could attenuate these comparisons. We leave this for future research.

7 Conclusion

We have investigated the extent to which entry decisions in capital markets affect long-run capital-income tax prescriptions when the capital tax has the potential to tax profit flows. We found that if the Ramsey government considers these profit flows as pure rents, it is optimal to set a positive capital-income tax. In contrast, if the Ramsey government considers these profit flows as quasi-rents — compensation for the upfront sunk costs of entry — a zero capital-income tax is optimal. Our model thus nests in one environment two competing views of optimal capital taxation: the Chamley-Judd zero-capital-tax view on the one hand, and the idea that capital taxes can and should be used to tax profit flows on the others. The critical economic phenomenon linking these two views is free-entry.

What distinguishes our model and results from these two branches of the existing literature is that there is an economically meaningful way for a government to entertain the two possible views. Our analysis shows that one treatment of profits — the quasi-rent view — is clearly welfare-superior to the other. However, if a government did want to entertain a pure-rent view of profits, our model articulates features of the economy that rationalize doing so. Though welfare-inferior, the pure-rent view is not an ad-hoc view, but rather an alternative way of viewing the private-sector equilibrium. More broadly, our results suggest that it is important to understand the nature of profits as well as explicit entry considerations when designing capital-tax policy.

A Nash Bargaining

The Nash-bargained rental rate solves the following problem

$$\max_{r_t} \left(\frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t} \right)^{\phi} J_k(k_t)^{1-\phi},$$

where, by the envelope condition, the household's marginal value of matched capital equals[18]

$$V_k(k_t, u_t) = \lambda_t \left[(1 - \tau_t^k) r_t + \tau_t^k \delta + \varphi(1 - \delta)s \right] + (1 - \delta)(1 - s)\beta V_k(k_{t+1}, u_{t+1}),$$

the household's marginal value of unmatched capital equals

$$V_u(k_t, u_t) = \lambda_t,$$

and the firm's marginal value of matched capital equals

$$J_k(k_t) = f_k(k_t, n_t) - r_t + (1 - \delta)(1 - s) R_t^{-1} J_k(k_{t+1}).$$

The first-order conditions of the Nash bargaining problem are

$$\begin{aligned} \phi \left(\frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t} \right)^{\phi-1} J_k(k_t)^{1-\phi} \frac{1}{\lambda_t} \left(\frac{\partial V_k(k_t, u_t)}{\partial r_t} - \frac{\partial V_u(k_t, u_t)}{\partial r_t} \right) & \\ + (1 - \phi) \left(\frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t} \right)^{\phi} J_k(k_t)^{-\phi} \frac{\partial J_k(k_t)}{\partial r_t} &= 0. \end{aligned}$$

Since $\frac{\partial V_k(k_t,u_t)}{\partial r_t} = \lambda_t(1 - \tau_t^k)$, $\frac{\partial V_u(k_t,u_t)}{\partial r_t} = 0$ and $\frac{\partial J_k(k_t)}{\partial r_t} = -1$, this condition reduces to

$$\phi J_k(k_t) = (1 - \phi) \left(\frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t (1 - \tau_t^k)} \right). \tag{40}$$

Analogous to the analysis in a labor search environment by Arseneau and Chugh (2008a), taxes drive a wedge into the standard Nash bargaining solution.

To derive an explicit solution for the rental rate, we start by defining $S_t = J_k(k_t) + \frac{V_k(k_t,u_t) - V_u(k_t,u_t)}{\lambda_t}$, which we interpret as the total surplus from the marginal capital match. Combining this definition with the generalized bargaining formula in (40), we obtain

$$\begin{aligned} S_t &= \frac{(1 - \phi)}{\phi} \left(\frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t (1 - \tau_t^k)} \right) + \frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t (1 - \tau_t^k)} \\ \frac{\phi(1 - \tau_t^k)}{(1 - \phi) + \phi(1 - \tau_t^k)} S_t &= \frac{V_k(k_t, u_t) - V_u(k_t, u_t)}{\lambda_t} \end{aligned} \tag{41}$$

[18] Note that $\beta V_k(k_t, u_t)$ is, by definition equal to the multiplier μ_{t-1}^h; i.e. the value of a marginal unit of matched capital in t from the view $t - 1$. Hence, this equation is equivalent to the household first-order condition with respect to k_{t+1}.

Likewise,

$$
\begin{aligned}
S_t &= J_k(k_t) + \frac{\phi(1-\tau_t^k)}{(1-\phi)} J_k(k_t) \\
\frac{(1-\phi)}{(1-\phi)+\phi(1-\tau_t^k)} S_t &= J_k(k_t) \qquad (42)
\end{aligned}
$$

And using the definitions of μ_t^h and μ_t^f above, we express the surplus as

$$
\begin{aligned}
S_t &= f_k(k_t, n_t) - r_t + (1-\delta)(1-s)\beta R_t^{-1} J_k(k_{t+1}) \\
&\quad + \left[(1-\tau_t^k) r_t + \tau_t^k \delta + \varphi(1-\delta)s\right] + (1-\delta)(1-s)\beta \frac{V_k(k_{t+1}, u_{t+1})}{\lambda_t} - 1 \\
&= f_k(k_t, n_t) - \tau_t^k (r_t - \delta) + \varphi(1-\delta)s - 1 \\
&\quad + (1-\delta)(1-s) R_t^{-1} \left(J_k(k_{t+1}) + \frac{V_k(k_{t+1}, u_{t+1}) - \lambda_{t+1}}{\lambda_{t+1}} \right) \\
&\quad + (1-\delta)(1-s) R_t^{-1}.
\end{aligned}
$$

where we used the household's Euler equation with respect to bond holdings to express $R_t^{-1} = \beta\lambda_{t+1}/\lambda_t$. Now, we use (10) and rewrite it as[19]

$$
R_t^{-1} = 1 - q(\theta_t) R_t^{-1} \left(\frac{V_k(k_{t+1}, u_{t+1}) - \lambda_{t+1}}{\lambda_{t+1}} \right),
$$

and from (41), we know that

$$
\frac{V_k(k_{t+1}, u_{t+1}) - \lambda_{t+1}}{\lambda_{t+1}} = \frac{\phi(1-\tau_{t+1}^k)}{(1-\phi)+\phi(1-\tau_{t+1}^k)} S_{t+1}.
$$

Hence, the surplus becomes

$$
\begin{aligned}
S_t &= f_k(k_t, n_t) - \tau_t^k (r_t - \delta) + \varphi(1-\delta)s - 1 \\
&\quad + (1-\delta)(1-s) R_t^{-1} S_{t+1} \\
&\quad + (1-\delta)(1-s) \left[1 - q(\theta_t) R_t^{-1} \left(\frac{\phi(1-\tau_{t+1}^k)}{(1-\phi)+\phi(1-\tau_{t+1}^k)} S_{t+1} \right) \right] \\
&= f_k(k_t, n_t) - \tau_t^k (r_t - \delta) + \varphi(1-\delta)s - 1 + (1-\delta)(1-s) \\
&\quad (1-\delta)(1-s) R_t^{-1} \left[1 - q(\theta_t) \left(\frac{\phi(1-\tau_{t+1}^k)}{(1-\phi)+\phi(1-\tau_{t+1}^k)} \right) \right] S_{t+1} \\
&= f_k(k_t, n_t) - \tau_t^k (r_t - \delta) + \varphi(1-\delta)s - 1 + (1-\delta)(1-s) \\
&\quad + (1-\delta)(1-s) R_t^{-1} \left[\frac{(1-\phi) + (1-q(\theta_t))\phi(1-\tau_{t+1}^k)}{(1-\phi)+\phi(1-\tau_{t+1}^k)} S_{t+1} \right] \qquad (43)
\end{aligned}
$$

Next, consider the firm's capital demand equation (19), which, by definition of $J_k(k_t)$ above can be rewritten as

$$
\frac{\gamma}{p(\theta_t)} = R_t^{-1} J_k(k_{t+1}).
$$

[19] Again using fact that $\beta V_k(k_{t+1}, u_{t+1}) = \mu_t^h$.

Using this condition together with (42), the surplus in (43) can be expressed as

$$\begin{aligned} S_t &= f_k(k_t,n_t) - \tau_t^k(r_t-\delta) + \varphi(1-\delta)s - 1 + (1-\delta)(1-s) \\ &\quad +(1-\delta)(1-s)\frac{(1-\phi)+(1-q(\theta_t))\phi(1-\tau_{t+1}^k)}{(1-\phi)}\frac{\gamma}{p(\theta_t)} \end{aligned}$$

Likewise, we can combine the definition of $J_k(k_t)$ above with (19) and (42) to obtain

$$\frac{(1-\phi)}{(1-\phi)+\phi(1-\tau_t^k)}S_t = f_k(k_t,n_t) - r_t + (1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}.$$

Combining the two equations to substitute out for the surplus, we end up with

$$\begin{aligned} & (1-\phi)\left[f_k(k_t,n_t) - \tau_t^k(r_t-\delta) + \varphi(1-\delta)s - 1 + (1-\delta)(1-s)\right] \\ & +(1-\delta)(1-s)\left[(1-\phi)+(1-q(\theta_t))\phi(1-\tau_{t+1}^k)\right]\frac{\gamma}{p(\theta_t)} \\ = & \left[(1-\phi)+\phi(1-\tau_t^k)\right]\left[f_k(k_t,n_t) - r_t + (1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}\right] \end{aligned}$$

which, after some rearrangement, yields

$$\begin{aligned} \left[(1-\phi)+\phi(1-\tau_t^k)-(1-\phi)\tau_t^k\right]r_t &= \phi(1-\tau_t^k)f_k(k_t,n_t) \\ &\quad +\phi\left[(1-\tau_t^k)-(1-q(\theta_t))(1-\tau_{t+1}^k)\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)} \\ &\quad -(1-\phi)\left[\tau_t^k\delta + \varphi(1-\delta)s - 1 + (1-\delta)(1-s)\right] \end{aligned}$$

or equivalently

$$\begin{aligned} r_t &= \phi f_k(k_t,n_t) \\ &\quad +\phi\left[\frac{(\tau_{t+1}^k-\tau_t^k)+q(\theta_t)(1-\tau_{t+1}^k)}{(1-\tau_t^k)}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)} \\ &\quad +(1-\phi)\left[\delta + \frac{(1-\varphi)(1-\delta)s}{(1-\tau_t^k)}\right] \end{aligned}$$

Note that for $\tau_t^k = \tau_{t+1}^k = \tau^k$, this solution reduces to

$$\begin{aligned} r_t &= \phi f_k(k_t,n_t) \\ &\quad +\phi(1-\delta)(1-s)\gamma\theta_t \\ &\quad +(1-\phi)\left[\delta + \frac{(1-\varphi)(1-\delta)s}{(1-\tau^k)}\right], \end{aligned}$$

and for $\tau^k = 0$, we recover exactly the rental rate equation in Kurmann and Petrosky-Nadeau (2008)

$$\begin{aligned} r_t &= \phi\left[f_k(k_t,n_t) + (1-\delta)(1-s)\gamma\theta_t\right] \\ &\quad +(1-\phi)\left[\delta + (1-\varphi)(1-\delta)s\right]. \end{aligned}$$

B Steady-State Private-Sector Equilibrium

Consider equation (10) in steady state, rewritten as

$$\lambda = \beta \left[(1 - q(\theta)\lambda + q(\theta)V_k \right],$$

where we used again the fact that $\mu^h = \beta V_k$. We re-express this condition as

$$1 - \beta = q(\theta)\beta \left(\frac{V_k - \lambda}{\lambda} \right).$$

But, from the Nash bargaining solution, we know that

$$\phi J_k = (1 - \phi) \left(\frac{V_k - \lambda}{\lambda(1 - \tau^k)} \right)$$

and thus, the above condition becomes

$$1 - \beta = \frac{\phi(1 - \tau^k)}{1 - \phi} q(\theta)\beta J_k.$$

Now, consider the firm's capital demand equation (19), which, by definition of J_k above and the fact that in steady state $\beta = R^{-1}$, can be rewritten as

$$\frac{\gamma}{p(\theta)} = \beta J_k.$$

Combining this express with the above equation to eliminate βJ_k yields

$$1 - \beta = \frac{\phi(1 - \tau^k)}{1 - \phi} \gamma \theta,$$

or equivalently

$$\theta = \frac{1 - \beta}{\gamma} \frac{1 - \phi}{\phi} \frac{1}{1 - \tau^k}.$$

Hence, there exists a unique steady state value of capital market tightness, which completes the first part of the proof of Proposition 1.

Once θ computed, we can derive the steady state equilibrium value of k/n by combining the steady state expression for (19)

$$f_k(k, n) - r = \frac{\gamma}{p(\theta)} \left[R - (1 - \delta)(1 - s) \right]$$

with the steady state expression of the rental rate equation (22)

$$\begin{aligned} r &= \phi \left[f_k(k, n) + (1 - \delta)(1 - s)\gamma\theta \right] \\ &\quad + (1 - \phi) \left[\delta + \frac{(1 - \varphi)(1 - \delta)s}{(1 - \tau^k)} \right], \end{aligned}$$

to eliminate r. We obtain

$$\begin{aligned}\frac{\gamma}{p(\theta)}\left[R-(1-\delta)(1-s)\right] &= (1-\phi)\left[f_k(k,n)-\delta-\frac{(1-\varphi)(1-\delta)s}{(1-\tau^k)}\right] \\ &\quad -\phi(1-\delta)(1-s)\gamma\theta,\end{aligned}$$

which implicitly defines $f_k(k,n)$ and thus k/n as a function of θ. From there, it is straightforward to use the remaining equations of the competitive equilibrium to find the steady state values of the other variables.

In Figure 1, we plot key long-run endogenous variables as we exogenously vary the long-run capital income tax rate, holding $\tau^n = 0$ and assuming a lump-sum tax in the background. Thus, the results in Figure 1 do not display Ramsey equilibria, but illustrate how variations in the capital-income tax rate affect the private sector equilibrium in the long-run.

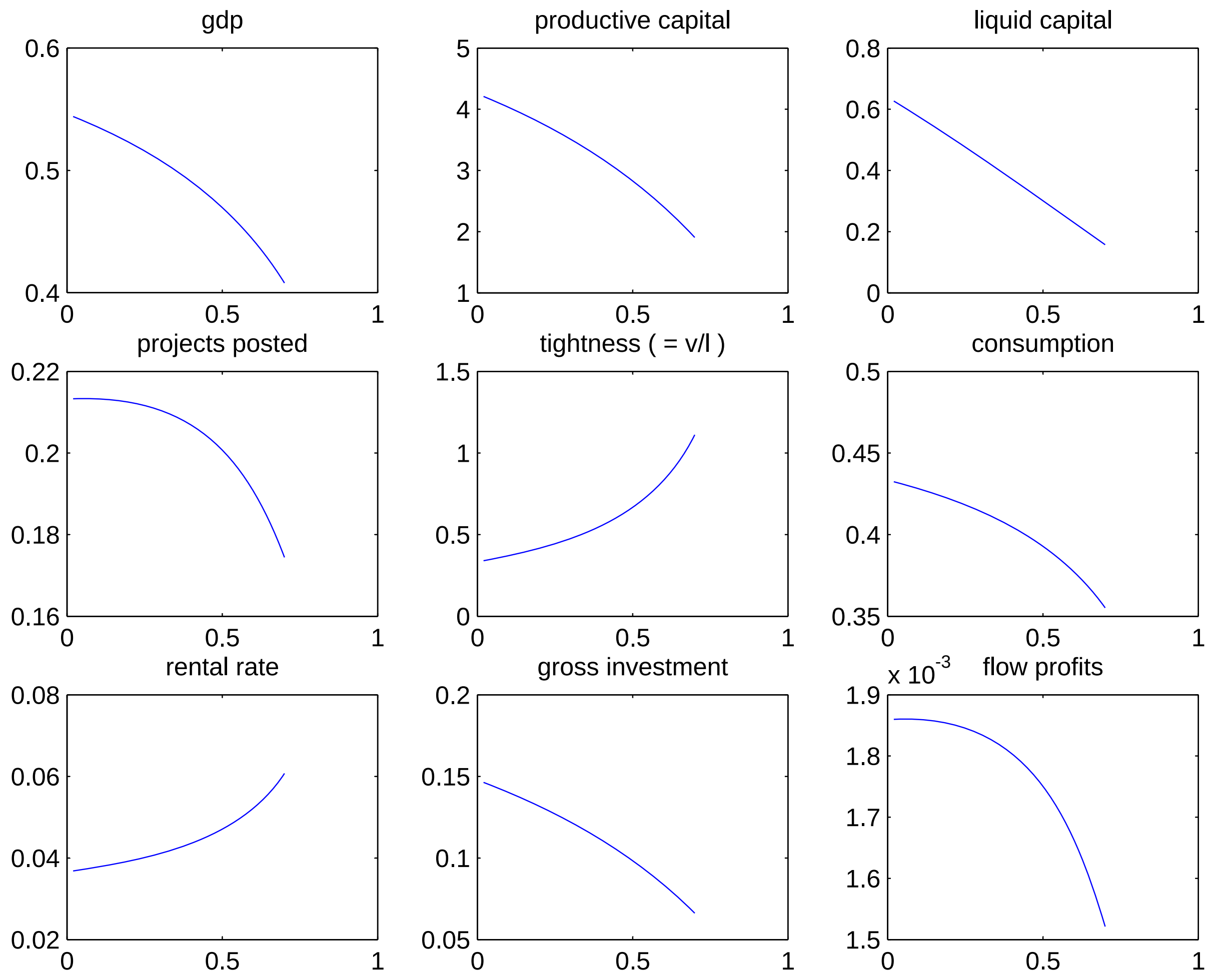

Figure 1: Long-run effects of exogenous capital-income taxation when rental rate is determined by Nash bargaining (τ^k plotted on horizontal axis).

C Derivation of Implementability Constraint

Start as usual with the time-t flow budget constraint of the household. Multiply it by $\beta^t u_{ct}$, which represents the Ramsey planner's perceived value of delivering a marginal unit of goods to the representative household (technically, we are multiplying by $\beta^t \lambda_t$, where λ_t is the household's multiplier on its budget constraint; in equilibrium, of course, $\lambda_t = u_{ct}$). Summing the resulting flow budget constraint from $t = 0$ to infinity, we have

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u_{ct} c_t + \sum_{t=0}^{\infty} \beta^t u_{ct} l_t + \sum_{t=0}^{\infty} \beta^t u_{ct} b_t = \sum_{t=0}^{\infty} \beta^t u_{ct}(1-\tau_t^n) w_t n_t + \sum_{t=0}^{\infty} \beta^t u_{ct} d_t \\
&+ \sum_{t=0}^{\infty} \beta^t u_{ct} \left[(1-\tau_t^k) r_t + \delta\tau_t^k + \varphi(1-\delta)s\right] k_t + \sum_{t=0}^{\infty} \beta^t u_{ct}(1-q(\theta_{t-1})) l_{t-1} + \sum_{t=0}^{\infty} \beta^t u_{ct} R_t b_{t-1}.
\end{aligned}$$

Note the absence of any expectations operators because in the end we focus only on deterministic steady states. As usual, first substitute for the u_{ct} term in the third summation on the left-hand-side using the bond Euler equation, $u_{ct} = \beta R_{t+1} u_{ct+1}$. After this substitution, this summation cancels with the analogous summation on the right-hand-side (the last term on the right-hand-side), leaving only the marginal value of the time-zero bond position:

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u_{ct} c_t + \sum_{t=0}^{\infty} \beta^t u_{ct} l_t = \sum_{t=0}^{\infty} \beta^t u_{ct}(1-\tau_t^n) w_t n_t + \sum_{t=0}^{\infty} \beta^t u_{ct} d_t \\
&+ \sum_{t=0}^{\infty} \beta^t u_{ct} \left[(1-\tau_t^k) r_t + \delta\tau_t^k + \varphi(1-\delta)s\right] k_t + \sum_{t=0}^{\infty} \beta^t u_{ct}(1-q(\theta_{t-1})) l_{t-1} + u_{c0} R_0 b_{-1}.
\end{aligned}$$

Next, we use the optimal capital-supply condition to substitute for the u_{ct} term in the second summation on the left-hand-side. Specifically, we use a compact representation of the capital-supply condition, $u_{ct} = q(\theta_t)\beta V_{kt+1} + (1-q(\theta_t))\beta u_{ct+1}$, in which $V_{kt} = u_{ct}\left[(1-\tau_t^k) r_t + \delta\tau_t^k + \varphi(1-\delta)s\right] + \beta(1-\delta)(1-s)V_{kt+1}$ is the marginal value to the household, in utility terms, of a pre-existing unit of physical (installed) capital at the beginning of period t (hence V_{kt+1} represents this value at the start of period $t+1$):

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u_{ct} c_t + \sum_{t=0}^{\infty} \beta^{t+1} q(\theta_t) l_t V_{kt+1} + \sum_{t=0}^{\infty} \beta^{t+1} u_{ct+1}(1-q(\theta_t)) l_t = \sum_{t=0}^{\infty} \beta^t u_{ct}(1-\tau_t^n) w_t n_t + \sum_{t=0}^{\infty} \beta^t u_{ct} d_t \\
&+ \sum_{t=0}^{\infty} \beta^t u_{ct} \left[(1-\tau_t^k) r_t + \delta\tau_t^k + \varphi(1-\delta)s\right] k_t + \sum_{t=0}^{\infty} \beta^t u_{ct}(1-q(\theta_{t-1})) l_{t-1} + u_{c0} R_0 b_{-1}.
\end{aligned}$$

The third summation on the left-hand-side cancels with the analogous summation on the right-hand-side (the last summation on the right-hand-side), yielding

$$\begin{aligned}
&\sum_{t=0}^{\infty} \beta^t u_{ct} c_t + \sum_{t=0}^{\infty} \beta^{t+1} q(\theta_t) l_t V_{kt+1} = \sum_{t=0}^{\infty} \beta^t u_{ct}(1-\tau_t^n) w_t n_t + \sum_{t=0}^{\infty} \beta^t u_{ct} d_t \\
&+ \sum_{t=0}^{\infty} \beta^t u_{ct} \left[(1-\tau_t^k) r_t + \delta\tau_t^k + \varphi(1-\delta)s\right] k_t + u_{c0}\left[R_0 b_{-1} + (1-q(\theta_{-1})) l_{-1}\right].
\end{aligned}$$

Next, use the law of motion for the productive capital stock, $q(\theta_t)l_t = k_{t+1} - (1-\delta)(1-s)k_t$ to replace in the second summation on the left-hand-side:

$$\begin{aligned} &\sum_{t=0}^{\infty}\beta^t u_{ct}c_t + \sum_{t=0}^{\infty}\beta^{t+1}V_{kt+1}k_{t+1} - \sum_{t=0}^{\infty}\beta^{t+1}(1-\delta)(1-s)V_{kt+1}k_t = \sum_{t=0}^{\infty}\beta^t u_{ct}(1-\tau_t^n)w_t n_t + \sum_{t=0}^{\infty}\beta^t u_{ct}d_t \\ &+ \quad \sum_{t=0}^{\infty}\beta^t u_{ct}\left[(1-\tau_t^k)r_t + \delta\tau_t^k + \varphi(1-\delta)s\right]k_t + u_{c0}\left[R_0 b_{-1} + (1-q(\theta_{-1}))l_{-1}\right]. \end{aligned}$$

Then, substitute for the V_{kt+1} term in the second summation on the left-hand-side using the envelope condition $V_{kt+1} = u_{ct+1}\left[(1-\tau_{t+1}^k)r_{t+1} + \delta\tau_{t+1}^k + \varphi(1-\delta)s\right] + \beta(1-\delta)(1-s)V_{kt+2}$,

$$\begin{aligned} &\sum_{t=0}^{\infty}\beta^t u_{ct}c_t + \sum_{t=0}^{\infty}\beta^{t+1}u_{ct+1}\left[(1-\tau_{t+1}^k)r_{t+1} + \delta\tau_{t+1}^k + \varphi(1-\delta)s\right]k_{t+1} + \sum_{t=0}^{\infty}\beta^{t+2}(1-\delta)(1-s)V_{kt+2}k_{t+1} \\ &- \quad \sum_{t=0}^{\infty}\beta^{t+1}(1-\delta)(1-s)V_{kt+1}k_t = \sum_{t=0}^{\infty}\beta^t u_{ct}(1-\tau_t^n)w_t n_t + \sum_{t=0}^{\infty}\beta^t u_{ct}d_t \\ &+ \quad \sum_{t=0}^{\infty}\beta^t u_{ct}\left[(1-\tau_t^k)r_t + \delta\tau_t^k + \varphi(1-\delta)s\right]k_t + u_{c0}\left[R_0 b_{-1} + (1-q(\theta_{-1}))l_{-1}\right]. \end{aligned}$$

Several terms now cancel: the second summation on the left-hand-side cancels with the third summation on the right-hand-side, and the third summation on the left-hand-side cancels with the fourth summation on the left-hand-side, which leaves

$$\begin{aligned} &\sum_{t=0}^{\infty}\beta^t u_{ct}c_t = \sum_{t=0}^{\infty}\beta^t u_{ct}(1-\tau_t^n)w_t n_t + \sum_{t=0}^{\infty}\beta^t u_{ct}d_t \\ &\quad + \quad u_{c0}\left[R_0 b_{-1} + \left((1-\tau_0^k)r_0 + \delta\tau^k + \varphi(1-\delta)s\right)k_0 + (1-q(\theta_{-1}))l_{-1}\right] + \beta(1-\delta)(1-s)V_{k1}k_0. \end{aligned}$$

Finally, using the household's consumption-leisure optimality condition, $-u_{nt} = u_{ct}(1-\tau_t^n)w_t$, we have the present-value implementability constraint

$$\begin{aligned} &\sum_{t=0}^{\infty}\beta^t\left[u_{ct}c_t + u_{nt}n_t - u_{ct}d_t\right] \\ &\quad = \quad u_{c0}\left[R_0 b_{-1} + \left((1-\tau_0^k)r_0 + \delta\tau^k + \varphi(1-\delta)s\right)k_0 + (1-q(\theta_{-1}))l_{-1}\right] + \beta(1-\delta)(1-s)V_{k1}k_0 \\ &\quad = \quad u_{c0}\left[R_0 b_{-1} + \left((1-\tau_0^k)r_0 + \delta\tau^k + \varphi(1-\delta)s\right)k_0 + (1-q(\theta_{-1}))l_{-1}\right] \\ &\qquad\quad +V_{k0}k_0 - u_{c0}\left[(1-\tau_0^k)r_0 + \delta\tau_0^k + (1-\delta)s\right]k_0 \\ &\quad = \quad u_{c0}\left[R_0 b_{-1} + (1-q(\theta_{-1}))l_{-1}\right] + V_{k0}k_0 \end{aligned}$$

which is equation (33) in the main text.

D Ramsey Planner Solution and Optimal Capital Taxation

Consider the reformulated Ramsey problem

$$\max_{\{c_t,n_t,k_{t+1},\theta_t,l_t,d_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t,n_t)$$

subject to

$$\sum_{t=0}^{\infty} \beta^t \left[u_{ct}c_t + u_{nt}n_t - u_{ct}d_t\right] \geq A_0$$

$$f(k_t,n_t) + (1-\delta)sk_t + (1-q(\theta_{t-1}))l_{t-1} \geq c_t + g_t + (1+\gamma\theta_t)l_t$$

$$(1-\delta)(1-s)k_t + q(\theta_t)l_t \geq k_{t+1}$$

$$d_t \geq (1-\phi)\left[f_k(k_t,n_t)-\delta\right]k_t - \phi\left[1-\left[1-q(\theta_t)\right]\left(\frac{1-\beta\frac{u_{ct+1}}{u_{ct}}}{1-\beta\frac{u_{ct}}{u_{ct-1}}}\right)\frac{\theta_{t-1}}{\theta_t}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}k_t - \gamma\theta_t l_t$$

$$\beta\frac{u_{ct+1}}{u_{ct}}\left[\frac{d_{t+1}+\gamma\theta_{t+1}l_{t+1}}{k_{t+1}} + (1-\delta)(1-s)\frac{\gamma}{p(\theta_{t+1})}\right] \geq \frac{\gamma}{p(\theta_t)}$$

Defining $V(c_t,n_t,d_t,\Phi) = u(c_t,n_t) + \Phi\left[u_{ct}c_t + u_{nt}n_t - u_{ct}d_t\right]$, the Lagrangian of this problem can be formulated as

$$\begin{aligned}
\pounds \;=\; & \sum_{t=0}^{\infty}\beta^t V(c_t,n_t,d_t,\Phi) \\
& +\beta^t\mu_t^{rb}\left[\begin{array}{c} f(k_t,n_t)+(1-\delta)sk_t+(1-q(\theta_{t-1}))l_{t-1} \\ -c_t-g_t-(1+\gamma\theta_t)l_t \end{array}\right] \\
& +\beta^t\mu_t^{rk}\left[(1-\delta)(1-s)k_t+q(\theta_t)l_t-k_{t+1}\right] \\
& +\beta^t\mu_t^{rd}\left[\begin{array}{c} d_t-(1-\phi)\left[f_k(k_t,n_t)-\delta\right]k_t \\ +\phi\left[1-\left[1-q(\theta_t)\right]\left(\frac{1-\beta\frac{u_{ct+1}}{u_{ct}}}{1-\beta\frac{u_{ct}}{u_{ct-1}}}\right)\frac{\theta_{t-1}}{\theta_t}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}k_t+\gamma\theta_t l_t \end{array}\right] \\
& +\beta^t\mu_t^{rp}\left[\beta\frac{u_{ct+1}}{u_{ct}}\left[\frac{d_{t+1}+\gamma\theta_{t+1}l_{t+1}}{k_{t+1}}+(1-\delta)(1-s)\frac{\gamma}{p(\theta_{t+1})}\right]-\frac{\gamma}{p(\theta_t)}\right]-\Phi A_0.
\end{aligned}$$

The idea of the proof is to use the first-order conditons for θ_t and l_t and then to combine them to obtain an expression for θ in steady state that can be compared to the steady state value of θ for the private sector equilibrium. Consider the first-order conditons for θ_t and l_t:

$$\begin{aligned}
\theta_t \;:\; & -\beta\mu_{t+1}^{rb}q'(\theta_t)l_t-\mu_t^{rb}\gamma l_t+\mu_t^{rk}q'(\theta_t)l_t \\
& +\mu_t^{rd}\phi\left[q'(\theta_t)\left(\frac{1-\beta\frac{u_{ct+1}}{u_{ct}}}{1-\beta\frac{u_{ct}}{u_{ct-1}}}\right)\frac{\theta_{t-1}}{\theta_t}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}k_t \\
& -\beta\mu_{t+1}^{rd}\phi\left[\left[1-q(\theta_{t+1})\right]\left(\frac{1-\beta\frac{u_{ct+2}}{u_{ct+1}}}{1-\beta\frac{u_{ct+1}}{u_{ct}}}\right)\frac{1}{\theta_{t+1}}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_{t+1})}k_{t+1} \\
& +\mu_t^{rd}\phi\left[\left[1-q(\theta_t)\right]\left(\frac{1-\beta\frac{u_{ct+1}}{u_{ct}}}{1-\beta\frac{u_{ct}}{u_{ct-1}}}\right)\frac{\theta_{t-1}}{\theta_t^2}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta_t)}k_t
\end{aligned}$$

$$-\mu_t^{rd}\phi\left[1-[1-q(\theta_t)]\left(\frac{1-\beta\frac{u_{ct+1}}{u_{ct}}}{1-\beta\frac{u_{ct}}{u_{ct-1}}}\right)\frac{\theta_{t-1}}{\theta_t}\right](1-\delta)(1-s)\frac{\gamma p'(\theta_t)}{p(\theta_t)^2}k_t+\mu_t^{rd}\gamma l_t$$

$$+\frac{\mu_{t-1}^{rp}}{\beta}\beta\frac{u_{ct}}{u_{ct-1}}\left[\frac{\gamma l_t}{k_t}-(1-\delta)(1-s)\frac{\gamma p'(\theta_t)}{p(\theta_t)^2}\right]+\mu_t^{rp}\frac{\gamma p'(\theta_t)}{p(\theta_t)^2}=0$$

and

$$l_t:\beta\mu_{t+1}^{rb}(1-q(\theta_t))-\mu_t^{rb}(1+\gamma\theta_t)+\mu_t^{rk}q(\theta_t)+\mu_t^{rd}\gamma\theta_t+\frac{\mu_{t-1}^{rp}}{\beta}\beta\frac{u_{ct}}{u_{ct-1}}\frac{\gamma\theta_t}{k_t}=0$$

In steady state, these two conditions simplify considerably. Redefining $\mu^{rk}\equiv\beta R_k$ (the discounted marginal value of an additional unit of matched capital next period), the first FOC can be rewritten as

$$\begin{aligned}\theta_t\quad:\quad & q'(\theta)\beta\left[R_k-\mu^{rb}\right]l=\left[\mu^{rb}-\mu^{rd}\right]\gamma l\\ & -\mu^{rd}\phi\left[q'(\theta)-\frac{\beta(1-q(\theta))}{\theta}+\frac{(1-q(\theta))}{\theta}-[1-(1-q(\theta))]\frac{p'(\theta)}{p(\theta)}\right](1-\delta)(1-s)\frac{\gamma}{p(\theta)}k\\ & -\mu^{rp}\frac{\gamma l}{k}+\mu^{rp}[(1-\delta)(1-s)-1]\frac{\gamma p'(\theta)}{p(\theta)^2}\end{aligned}$$

Likewise, the second FOC can be rewritten as

$$l_t:\beta\left[q(\theta)R_k+(1-q(\theta))\mu^{rb}\right]=\mu^{rb}(1+\gamma\theta)+\mu^{rd}\gamma\theta-\mu^{rp}\frac{\gamma\theta}{k}\tag{44}$$

It will turn out to be useful to rewrite the first FOC as

$$\theta_t:q'(\theta)\beta\left[R_k-\mu^{rb}\right]=\left[\mu^{rb}-\mu^{rd}\right]\gamma+\mu^{rd}\gamma D_\theta+\mu^{rp}\gamma P_\theta\tag{45}$$

with

$$D_\theta=-\phi\left[q'(\theta)-\frac{\beta(1-q(\theta))}{\theta}+\frac{(1-q(\theta))}{\theta}-[1-(1-q(\theta))]\frac{p'(\theta)}{p(\theta)}\right](1-\delta)(1-s)\frac{1}{p(\theta)}\frac{k}{l}<0$$

and

$$\begin{aligned}P_\theta &= -\frac{1}{k}+\frac{[1-(1-\delta)(1-s)]}{l}\frac{p'(\theta)}{p(\theta)^2}\\ &= -\frac{[1-(1-\delta)(1-s)]}{q(\theta)l}+\frac{[1-(1-\delta)(1-s)]}{l}\frac{p'(\theta)}{p(\theta)^2}\\ &= \frac{[1-(1-\delta)(1-s)]}{l}\left[\frac{p'(\theta)\theta}{p(\theta)}\frac{1}{q(\theta)}-\frac{1}{q(\theta)}\right]\\ &= \frac{[1-(1-\delta)(1-s)]}{l}\left[-\epsilon(\theta)\frac{1}{q(\theta)}-\frac{1}{q(\theta)}\right]=\frac{1}{k}\left[-\epsilon(\theta)-1\right]<0\end{aligned}$$

It seems quite intuitive that these two terms are negative: an increase in $\theta=v/l$ decreases marginal profits ($D_\theta<0$) and decreases the marginal match value for a firm ($P_\theta<0$). Now, divide both sides of (45) by $p(\theta)=q(\theta)/\theta$ and rewrite the condition as

$$\begin{aligned}\frac{\theta}{q(\theta)}q'(\theta)\beta\left[R_k-\mu^{rb}\right] &= \frac{\mu^{rb}\gamma}{p(\theta)}\left[1+\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\\ [1-\epsilon(\theta)]\beta\left[R_k-\mu^{rb}\right] &= \frac{\mu^{rb}\gamma}{p(\theta)}\left[1+\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\end{aligned}\tag{46}$$

Likewise, rewrite (44) as

$$\begin{aligned}\beta q(\theta)[R_k-\mu^{rb}] &= \mu^{rb}(1-\beta)+\mu^{rb}\gamma\theta\left[1+\frac{\mu^{rd}}{\mu^{rb}}-\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}\right]\\ \beta[R_k-\mu^{rb}] &= \frac{\mu^{rb}}{q(\theta)}(1-\beta)+\frac{\mu^{rb}\gamma}{p(\theta)}\left[1+\frac{\mu^{rd}}{\mu^{rb}}-\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}\right]\end{aligned}$$

Next, we use the first condition to substitute out for the term $\frac{\mu^{rb}\gamma}{p(\theta)}$ in the second condition. We obtain

$$\beta[R_k-\mu^{rb}]=\frac{\mu^{rb}}{q(\theta)}(1-\beta)+[1-\epsilon(\theta)]\beta\left[R_k-\mu^{rb}\right]\frac{\left[1+\frac{\mu^{rd}}{\mu^{rb}}-\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}\right]}{\left[1+\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]}$$

and after rearranging

$$\begin{aligned}\left[1+\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\beta[R_k-\mu^{rb}] &= \frac{\mu^{rb}}{q(\theta)}(1-\beta)\left[1+\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\\ &\quad+[1-\epsilon(\theta)]\beta\left[R_k-\mu^{rb}\right]\left[1+\frac{\mu^{rd}}{\mu^{rb}}-\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}\right]\end{aligned}$$

$$\begin{aligned}&\epsilon(\theta)\beta[R_k-\mu^{rb}]+\beta\left[R_k-\mu^{rb}\right]\left\{[1-\epsilon(\theta)]\left[\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}-\frac{\mu^{rd}}{\mu^{rb}}\right]+\left[\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\right\}\\ =\;&\frac{\mu^{rb}}{q(\theta)}(1-\beta)\left[1+\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\end{aligned}$$

Finally, dividing this equation by (46) yields

$$\frac{\epsilon(\theta)}{1-\epsilon(\theta)}+\left\{\left[\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}-\frac{\mu^{rd}}{\mu^{rb}}\right]+\frac{1}{[1-\epsilon(\theta)]}\left[\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\right\}=\frac{p(\theta)}{\gamma q(\theta)}(1-\beta)$$

or as a solution of θ

$$\frac{1}{\theta}=\frac{\gamma}{1-\beta}\frac{\epsilon(\theta)}{1-\epsilon(\theta)}+\frac{\gamma}{1-\beta}\left\{\left[\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}-\frac{\mu^{rd}}{\mu^{rb}}\right]+\frac{1}{[1-\epsilon(\theta)]}\left[\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\right\}.$$

The first part $\frac{1}{\theta}=\frac{\gamma}{1-\beta}\frac{\epsilon(\theta)}{1-\epsilon(\theta)}$ is just the inverse of the IMRS=IMRT condition that would obtain in the absence of distortionary taxation. Under the Hosios condition, this would imply a zero optimal capital-income tax. The second part in curly brackets is a wedge due to distortionary taxation that implies a non-zero optimal capital tax. Specifically, comparing this solution with the private-sector equilibrium, rewritten as

$$\frac{1}{\theta}=\frac{\gamma}{1-\beta}\frac{\phi}{1-\phi}(1-\tau^k)$$

it is clear that for $t^k>0$, it needs to be the case that

$$\left\{\left[\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}-\frac{\mu^{rd}}{\mu^{rb}}\right]+\frac{1}{[1-\epsilon(\theta)]}\left[\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\right\}<0$$

To investigate whether this is true, rewrite this term as

$$
\begin{aligned}
& \frac{1}{[1-\epsilon(\theta)]}\left\{[1-\epsilon(\theta)]\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}-[1-\epsilon(\theta)]\frac{\mu^{rd}}{\mu^{rb}}+\left[\frac{\mu^{rd}}{\mu^{rb}}(D_\theta-1)+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\right\} \\
= \quad & \frac{1}{[1-\epsilon(\theta)]}\left\{-\epsilon(\theta)\frac{\mu^{rd}}{\mu^{rb}}+[1-\epsilon(\theta)]\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}+\left[\frac{\mu^{rd}}{\mu^{rb}}D_\theta+\frac{\mu^{rp}}{\mu^{rb}}P_\theta\right]\right\}
\end{aligned}
$$

Inserting the above definition of P_θ, we obtain

$$
\frac{1}{[1-\epsilon(\theta)]}\left\{-\epsilon(\theta)\frac{\mu^{rd}}{\mu^{rb}}-2\epsilon(\theta)\frac{\mu^{rp}}{\mu^{rb}}\frac{1}{k}+\frac{\mu^{rd}}{\mu^{rb}}D_\theta\right\}<0
$$

as long as the three lagrangian multipliers are positive (which, intuitively, seems to make sense...but we should also be able to show this analytically).

E Alternative Rental Rate Determination Mechanisms

E.1 Proportional Bargaining

As we show in Appendix A, the Nash-bargained rental rate in (22) satisfies

$$\frac{\mathbf{V}_{kt} - \mathbf{V}_{ut}}{1 - \tau_t^k} = \frac{\phi}{1 - \phi} \mathbf{J}_{kt}, \tag{47}$$

where $\mathbf{V}_{kt} - \mathbf{V}_{ut}$ is the surplus accruing to the household from moving the marginal unit of capital from an unmatched state to a matched state, and $\mathbf{J}_{kt}$ is the surplus accruing to the firm of installing a unit of capital. As shown in Arseneau and Chugh (2006, 2008a), proportional taxation in equilibrium changes parties' bargaining powers. In our model, households' bargaining power is effectively reduced by $1 - \tau_t^k$ because they are the ones that are formally obliged to pay the capital income tax. Because equilibrium splits of match surpluses take into account future equilibrium splits of match surpluses under Nash bargaining, time-variation in tax rates leads to what Arseneau and Chugh (2008a) identify as a *dynamic bargaining power effect*; this is captured by the presence of both τ_t^k and τ_{t+1}^k in the determination of r_t in the Nash condition (22).

An alternative axiomatic bargaining solution, proportional bargaining, does away with this dynamic bargaining power effect and instead makes taxes have only a static effect on the equilibrium.[20] The proportional bargaining solution satisfies

$$\mathbf{V}_{kt} - \mathbf{V}_{ut} = \frac{\phi}{1 - \phi} \mathbf{J}_{kt}; \tag{48}$$

when we use the value equations defined in Appendix A, we get

$$r_t \left[1 - \tau_t^k (1 - \phi)\right] = \phi \left[f_k(k_t, n_t) + (1 - \delta)(1 - s)\gamma\theta_t\right] + (1 - \phi) \left[(1 - s)\delta + s\left[1 - \varphi(1 - \delta)\right] - \tau_t^k \delta\right], \tag{49}$$

which replaces (22). All other equilibrium conditions are identical to that in the baseline model. The question of which bargaining environment is a "better" empirical description is outside the scope of our analysis. We only mean to show that how capital taxes affect and are affected by capital allocations frictions depends on the precise manner by which asset-market prices are determined; this issue obviously cannot be asked in a simple Ramsey environment that assumes fundamentally neoclassical capital markets.

For this bargaining mechanism, Figure 2 plots key long-run endogenous variables as we exogenously vary the long-run capital income tax rate, holding $\tau^n = 0$ and assuming a lump-sum tax in the background. Just as in Figure 1, we are thus not plotting Ramsey equilibria here. The major difference compared to Figure 1 is that under proportional bargaining, the capital tax does not at all affect capital market tightness, hence does not affect matching probabilities.

[20] See Kalai (1977) as the original reference on proportional bargaining, and Aruoba, Rocheteau, and Waller (2007) for a recent application to monetary economics.

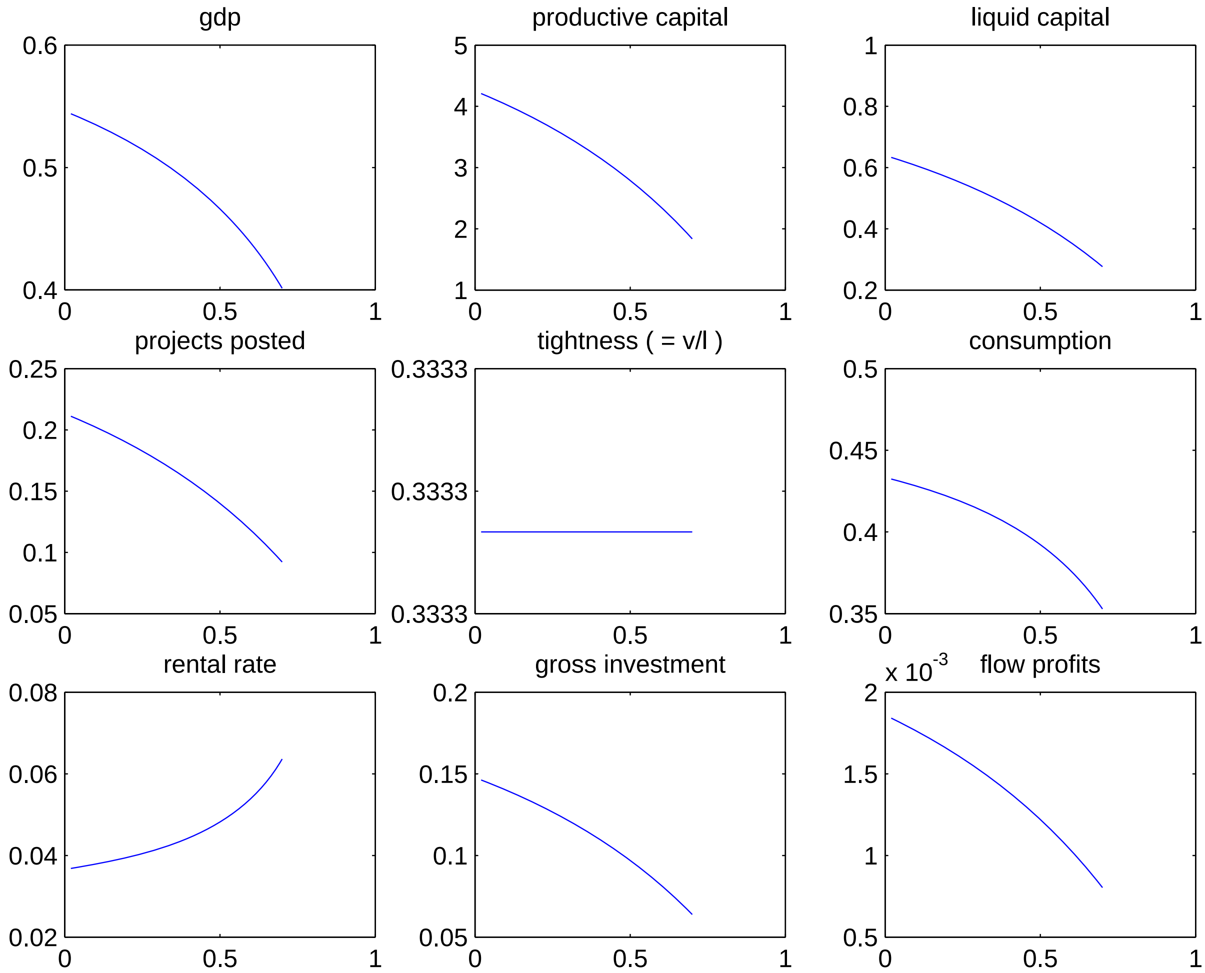

Figure 2: Long-run effects of exogenous capital-income taxation when rental rate is determined by proportional bargaining (τ^k plotted on horizontal axis).

E.2 Competitive Search

An alternative to bilateral bargaining altogether is a decentralized price-setting mechanism, competitive search equilibrium, in which all capital suppliers and capital demanders take as given the rental rate when making their search decisions. We follow Moen (1997) in how we implement a competitive search equilibrium (CSE). More details also appear in Kurmann (2008).

In capital submarket j, the capital demand condition is

$$\gamma = p(\theta_{ijt})\Xi_{t+1|t}\left[f_k(k_{it+1}, n_{it+1}) - r_{ijt+1} + \frac{(1-\delta)(1-s)\gamma}{p(\theta_{jt+1})}\right], \tag{50}$$

and the capital supply condition is

$$\frac{u_{ct} - (1-q(\theta_{ijt}))\beta u_{ct+1}}{q(\theta_{ijt})} = \beta\left\{u_{ct+1}\left((1-\tau^k_{t+1})r_{ijt+1} + \tau^k_{t+1}\delta + (1-\delta)s\right) + \frac{(1-\delta)(1-s)\left[u_{ct+1} - (1-q(\theta_{jt+1}))\beta u_{ct+2}\right]}{q(\theta_{jt+1})}\right\}. \tag{51}$$

The CSE rental rate and market tightness (p_{ijt}, θ_{ijt}) maximizes the value of a filled investment project, taking as constraint the capital supply (directed search) condition. The period $t+1$ rental rate is determined in period t. Formally, r_{ijt+1} is the result of the following optimization problem:

$$\max p(\theta_{ijt})\left[f(k_{jt+1}, n_{jt+1}) - r_{ijt+1} + \frac{(1-\delta)(1-s)\gamma}{p_(\theta_{jt+1})}\right] \tag{52}$$

subject to

$$u_{ct} - (1-q(\theta_{ijt}))\beta u_{ct+1} = q(\theta_{ijt})\beta\left\{u_{ct+1}\left[(1-\tau^k_{t+1})r_{ijt+1} + \tau^k_{t+1}\delta + (1-\delta)s\right] + \frac{(1-\delta)(1-s)\left[u_{ct+1} - (1-q(\theta_{jt+1}))\beta u_{ct+2}\right]}{q(\theta_{jt+1})}\right\}. \tag{53}$$

Denoting by $\mu_t \equiv \frac{p(\theta_{ijt})}{q(\theta_{ijt})}\frac{\Xi_{t+1|t}}{\beta\left[u_{ct+1}(1-\tau^k_{t+1})\right]}$, the CSE rental rate r_{ijt} is characterized by

$$p'(\theta_{ijt})\left[f(k_{jt+1}, n_{jt+1}) - r_{ijt+1} + \frac{(1-\delta)(1-s)\kappa}{p(\theta_{jt+1})}\right] = \tag{54}$$

$$\mu_t q'(\theta_{ijt})\beta u_{ct+1} - \beta u_{ct+1}\left[(1-\tau^k_{t+1})r_{ijt+1} + \tau^k_{t+1}\delta + (1-\delta)s\right] \tag{55}$$

$$- \quad \frac{\beta(1-\delta)(1-s)\left[u_{ct+1} - (1-q(\theta_{jt+1}))\beta u_{ct+2}\right]}{q(\theta_{jt+1})}.$$

A few details of the formal maximization problem are worth pointing out. First, the discount factor $\Xi_{t+1|t}$ does not appear in the objective (52). Second, we attach the multiplier $\beta\mu_t/u_{ct+1}$ to the constraint (53): the subjective discount factor β appears as part of the multiplier because determination of the rental rate occurs one period in advance, while the period-$t+1$ marginal utility of wealth appears because $t+1$ is the period of valuation.[21] With these details in mind,

[21] Technically, it is of course the multiplier on the household budget constraint that is the appropriate here; however, in equilibrium, all of the details of which the Ramsey planner internalizes, the multiplier on the household budget constraint equals the marginal utility of consumption.

optimization is with respect to the r_{ijt+1} and θ_{ijt}. Following optimization, we impose symmetry across firms and submarkets, and the rental rate is characterized by

$$\begin{aligned} & \frac{p\prime(\theta_t)\kappa}{p(\theta_t)} \\ + \quad & \mu_t q'(\theta_t)\beta\left\{(1-\tau^k_{t+1})r_{t+1}+\tau^k_{t+1}\delta+(1-\delta)s+\frac{(1-\delta)(1-s)\left[u_{ct+1}-(1-q(\theta_{t+1}))\beta u_{ct+2}\right]}{u_{ct+1}q(\theta_{t+1})}\right\} \\ - \quad & \beta\mu_t q'(\theta_t)=0, \end{aligned} \tag{56}$$

with the multiplier satisfying $\mu_t = \frac{p(\theta_t)}{q(\theta_t)\beta(1-\tau^k_{t+1})}$. In the first term in (56), we used the project-posting condition to simplify the first-order condition with respect to θ_{ijt+1}. Furthermore, in the second term, we can use the capital supply condition (53) to rewrite as

$$\frac{p'(\theta_t)\kappa}{p(\theta_t)}+\frac{\mu_t q'(\theta_t)\beta}{u_{ct+1}}\frac{u_{ct}-(1-q(\theta_t))\beta u_{ct+1}}{q(\theta_t)}-\beta\mu_t q'(\theta_t)=0. \tag{57}$$

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