Labor Force Participation and General Equilibrium Efficiency in Search and Matching Models

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Abstract

We provide a characterization of general-equilibrium efficiency in the standard labor search and matching framework. The efficiency condition we develop builds on the well-known Hosios condition for labor-market efficiency, which is derived in partial-equilibrium models of the labor market. What makes our analysis general equilibrium is that we consider a labor force participation decision, a margin absent in many models of the labor market. The efficiency condition we develop has a simple interpretation in terms of marginal rates of substitution and marginal rates of transformation; it also provides a criterion by which general equilibrium search models can measure the attainment of efficiency, as well as provides a new basis for empirical tests of labor-market efficiency.

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1 Introduction

In this note, we build on the partial equilibrium efficiency theorem of Hosios (1990) to provide a simple characterization of general equilibrium efficiency in labor search environments. Our general equilibrium efficiency condition is based on a labor force participation margin, which is absent in most specifications of search frameworks. The efficiency condition we develop thus links outcomes in labor markets to outcomes in goods markets. Considering activity across markets is what makes our efficiency condition general equilibrium in nature. In contrast, the well-understood Hosios (1990) condition is a partial equilibrium efficiency result in the sense that it takes as given, as does the basic search framework, outcomes in all markets other than labor markets. Our result can be used to gain insight into the results emerging in other general-equilibrium search frameworks, as a tool for measuring the attainment of efficiency in such models, as well as a guide for developing the proper set of policy instruments to offset inefficiencies in search environments. It potentially also provides a new basis for empirical tests of labor-market efficiency.

Our efficiency result is most pertinent for the recent vintages of macroeconomic models based on the search and matching frameworks of Pissarides (1985) and Mortensen and Pissarides (1994). General equilibrium search models have been applied to a wide variety of macroeconomic questions of late, even though much of the development of search and matching models has occurred in partial equilibrium. In this growing body of general-equilibrium applications of labor market theory, many studies have begun to ask normative questions regarding macroeconomic policy. Efficiency concerns lie at the heart of any normative study of optimal policy. As such, it is important to understand as fully as possible the basic efficiency properties of search models, both in partial equilibrium and in general equilibrium.

In neoclassical models, the labor supply margin is often the critical one for the consideration of efficiency and hence policy questions. For example, frictions such as proportional labor taxation, some types of monetary frictions, and monopoly power in goods markets manifest themselves as distortions in the labor supply margin of neoclassical models. These distortions are often very simple to see in a model's equilibrium conditions, showing up as — to use a term made popular by Chari, Kehoe, and McGrattan (2007) and Shimer (2008) — "wedges" between marginal rates of substitution and marginal rates of transformation. Labor force participation is a type of labor supply decision. By considering participation in a search and matching framework, we are able to interpret *search* efficiency in terms of marginal rates of substitution being equated to *search-based* marginal rates of transformation. As such, our result is easily understood through the lens of standard economic theory.

The rest of the note is organized as follows. Section 2 sketches the basic search and matching environment, which follows the textbook presentation in Pissarides (2000). In Section 3, we build on the Hosios (1990) result to describe general equilibrium efficiency. Section 4 provides discussion and frames our work against the broader literatures studying efficiency in partial-equilibrium search models and studying optimal policy in dynamic general equilibrium models. Section 5 concludes.

2 The Environment

We begin by sketching the basic search framework, which is by now well-known to many economists. We follow the textbook treatment in Pissarides (2000) and cast our analysis in continuous time and in a stationary environment, as well as adopt nearly identical notation.

2.1 Firms

Because labor markets are not neoclassical, a firm must decide whether or not to spend time and resources looking for a potential worker. The value to a firm of a job filled by a worker is

$$r\mathbf{J} = p - w - \lambda\mathbf{J}, \tag{1}$$

and the value to a firm of a position that has been posted but goes unfilled is given by

$$r\mathbf{V} = -\gamma + q(\theta)\left(\mathbf{J} - \mathbf{V}\right). \tag{2}$$

The notation is as follows: r is the discount rate, capturing the fact that there is a time delay between posting a vacancy and potentially finding a worker; λ is the constant probability that an existing match breaks apart; γ is the fixed per-unit cost of opening a vacancy regardless of whether or not it ultimately gets filled; p is the output produced in any filled job; w is the flow wage paid by the employer to the worker in any filled job; and $q(\theta)$ is the probability that an open vacancy becomes matched with a searching individual. Assuming a standard constant-returns-to-scale matching technology, this probability depends on only the relative number of searchers on the two sides of the labor market, denoted by the endogenous quantity $\theta \equiv v/u$, where v is the aggregate number of vacancies that have been opened in the economy and u is the aggregate number of individuals looking for a match. Because it depends on aggregate conditions, the matching probability $q(\theta)$ is taken parametrically by each firm.

Because there is no impediment to firms posting as many or as few vacancies as is optimal, the value of an unfilled vacancy is driven to zero, $r\mathbf{V} = 0$. This result is often described as arising due to free entry into the matching process on the part of firms.

2.2 Individuals

On the other side of the labor market, the standard assumption in search models is that individuals are either employed, or unemployed and searching for a match. The value to an individual who is employed is

$$r\mathbf{W} = w + \lambda\left(\mathbf{W} - \mathbf{U}\right), \tag{3}$$

and the value to an individual of searching for employment but not finding or consummating a match is

$$r\mathbf{U} = z + \theta q(\theta)\left(\mathbf{W} - \mathbf{U}\right), \tag{4}$$

with z the flow benefit that an individual receives while searching. The matching probability $\theta q(\theta)$ is, given constant-returns aggregate matching, the matching rate perceived by and taken parametrically by an individual.

2.3 Wage Setting with Fixed Participation

Wage payments are determined by Nash bargaining. Let η denote the exogenous bargaining power of workers and $1-\eta$ the bargaining power of firms. Choice of the wage payment w maximizes the generalized Nash product $\left(\mathbf{W} - \mathbf{U}\right)^{\eta}\left(\mathbf{J} - \mathbf{V}\right)^{1-\eta}$. The solution is characterized by the familiar Nash sharing rule, $(1-\eta)\left(\mathbf{W} - \mathbf{U}\right) = \eta\mathbf{J}$. Inserting the asset values above, the wage payment expressed in closed form is

$$w = \eta p + (1-\eta)z + \eta\gamma\theta. \tag{5}$$

The first two terms on the right hand side of (5) are a convex combination of the social payoff to a match (the output p) and the outside benefit that an unemployed individual receives (z), with weights given by the Nash bargaining shares.

The term $\gamma\theta$ is commonly interpreted as either the continuation value of the match or the value of saved hiring costs due to the existence of an additional matched worker, and $\eta\gamma\theta$ is thus often interpreted as the capitalized value built into the negotiated wage, shared according the worker's bargaining power. These interpretations are informative within the context of just the labor market. Below, we offer a new interpretation that has a general equilibrium foundation.

2.4 Labor Force Participation

The standard search model features exogenous labor supply — i.e., a fixed size of the labor force (see, for instance, Pissarides (2000), p. 17). If individuals, and not just firms, can optimally participate or not in the matching process — i.e., if labor force participation is fully endogenous, just as is labor demand in the form of vacancy creation — another condition impinges on the value equation (4). Specifically, there is a *labor force participation condition*,

$$r\mathbf{U} = mrs + z, \tag{6}$$

that must hold in addition to (4). In (6), mrs is a benefit that the individual receives from activities alternative to the formal labor market. Without loss of generality and because it will make clear our subsequent efficiency analysis, we have also included z as a benefit received from alternative activities, but this is not necessary for the arguments that follow.

The labor force participation condition — participation condition, for short — states that at the margin, the value of searching for a job equals the value of dropping out of the labor force altogether. For now, we leave unspecified from where mrs arises; for analysis of the labor market, it is simply taken as given.

We have written (6) as an equality, which implies optimization has occurred along the margin between labor-market search and outside-the-labor-force activities. If decisions along this margin were not optimal, then (6) would be a strict inequality, with the direction of the inequality governed by whether an individual preferred to move into or out of the labor force — that is, if participation were too low or too high. The rest of our analysis focuses on the case of (6) holding with equality, as would occur in any general equilibrium analysis. Finally, note that the assumption of optimal participation on the individuals' side of the labor market does not change conditions (1), (2), and (3).

3 General Equilibrium Efficiency

So far, our descriptions of both the fixed-participation and endogenous-participation environments are virtually the same as in Pissarides (2000, Chapter 1 and Chapter 7). We now build on Pissarides (2000) and Hosios (1990) to develop a general equilibrium notion of efficiency in a labor-search environment; the analyses in Pissarides (2000) and Hosios (1990), by contrast, are partial equilibrium on the labor market. In what follows, we first derive an explicit wage equation for the environment with endogenous participation, we then draw on the partial equilibrium efficiency results of Hosios (1990) to show how the wage outcome in the environment with endogenous participation must relate to the wage outcome in the environment with fixed participation, and we then present our main result, an efficiency characterization linking outcomes in labor markets to outcomes in alternative markets.

3.1 Wage Setting with Optimal Participation

The Nash wage outcome is still described by the sharing rule $(1-\eta)(\mathbf{W}-\mathbf{U}) = \eta\mathbf{J}$. Using the definition of $\mathbf{W}$ in (3) and the participation condition (6), we have

$$\mathbf{W} - \mathbf{U} = \frac{w}{r+\lambda} - \frac{mrs + z}{r+\lambda}, \tag{7}$$

which has the straightforward interpretation that the gain to an individual of moving from search unemployment to employment is the present value of his flow of wage payments net of the present value of the outside option, which is composed of a stream of $mrs + z$.

Inserting the above and (1) into the Nash sharing rule, the wage payment expressed in closed form is

$$w = \eta p + (1-\eta)z + (1-\eta)mrs. \tag{8}$$

Comparing the wage rule if participation is fixed (5) to the wage rule if participation is endogenous (8) allows us to draw our central result. We turn to this next.

3.2 General Equilibrium Efficiency

Our general equilibrium efficiency characterization centers on the wage rules (5) and (8) for the case of fixed participation and endogenous participation. We repeat them here for convenience; respectively,

$$w = \eta p + (1-\eta)z + \eta\gamma\theta \tag{9}$$

and

$$w = \eta p + (1-\eta)z + (1-\eta)mrs. \tag{10}$$

Hosios (1990) showed that in the standard search and matching model, which features fixed participation, the wage rule (9) decentralizes the efficient labor-market allocation if and only if the Nash weight η exactly equals the technological elasticity of the number of aggregate matches with respect to the number of individuals searching for jobs.[1] This parameter setting is now commonly referred to as the Hosios condition. In the rest of our analysis, we assume the Hosios condition holds. We now build on the Hosios condition to describe the condition under which the environment with endogenous participation achieves efficiency.

The wage rule (10) decentralizes the efficient labor-market allocation if and only if

$$\eta\gamma\theta = (1-\eta)mrs. \tag{11}$$

If this condition holds, then the wage rules (5) and (8) obviously coincide, and all of the arguments of Hosios (1990) hold. Condition (11) is thus, along with the Hosios parameterization, a necessary and sufficient condition for efficiency if participation is endogenous. It links returns inside the labor market to returns outside the labor market. Expressing the necessary and sufficient condition (11) in a slightly different way, for the environment with endogenous participation to achieve efficiency, it must be that

$$mrs = \frac{\eta}{1-\eta}\gamma\theta. \tag{12}$$

Viewed as either expression (11) or expression (12), this efficiency condition is our central result.

[1] This result assumes constant-returns-to-scale in matching, an assumption widely-used in labor-market theory both because of its empirical plausibility — see Petrongolo and Pissarides (2001) — and the theoretical underpinnings it provides for the Hosios (1990) theorem.

We interpret (12) (or, obviously, (11)) as a general equilibrium description of efficiency for a labor search model. It is general equilibrium in nature because it is based on the participation condition (6), which links the labor market activities of individuals to the payoffs available in other markets. More precisely, for the wage rule (10), which is based on endogenous participation, to achieve the same allocations as the wage rule (9), which is based on fixed participation, condition (12) is necessary and sufficient.

4 Discussion

The efficiency condition (12) is a new aspect of efficiency in search frameworks. We have suggestively used the notation mrs to denote the payoff outside the labor market. The marginal rate of substitution between consumption and leisure (where "leisure" is defined as activities that take place outside the formal labor market) is the natural outside option that arises in many macroeconomic models featuring explicit labor market activity and goods market activity *along with* optimization of behavior across markets (recall that we are considering (6) only holding with equality). This is why we refer to (12) as a general equilibrium characterization of search efficiency — it can only be entertained when optimization along the goods/labor margin has occurred. However, the margin could be that between labor-market activity and other activities, as well — for example, the margin between formal labor market activity and home production. The idea captured by (12) is quite general.

Applying the general principle that efficient outcomes entail marginal rates of substitution being equated to marginal rates of transformation, the right hand side of (12) can be interpreted as the marginal rate of transformation (MRT) between goods and labor. This MRT is a novel one in that it captures the search process as well as any production — streams of output p — that results from successful matching. Because in search models the matching process is almost always viewed as a *technology* of the economy, it is natural to want a broad notion of MRT that takes into account this technology; the right hand side of (12) captures exactly this. Another way to see this, based on only primitives, is to construct the marginal rate of technical substitution (MRTS) implied by a Cobb-Douglas matching function. Letting the matching technology be $m(u, v) = u^{\eta}v^{1-\eta}$ and $\theta \equiv v/u$, we have $\frac{m_u}{m_v} = \frac{\eta}{1-\eta}\gamma\theta$.

As noted above, the term $\eta\gamma\theta$ in (9) is often interpreted as the capitalized value of the continuation value of a match or the capitalized value of the saved hiring costs due to the existence of an additional matched worker. In the efficient general equilibrium outcome, we see from (11) that $\eta\gamma\theta$ also can be understood as the share of an individual's outside (-the-labor-force) payoff, mrs, that is built into the wage. Referring to (10), this makes good economic sense: with endogenous participation, the bargained wage is a convex combination of the output of a match, p, and the individual's payoff outside the labor market, $z + mrs$. Thus, in *general* equilibrium, $\eta\gamma\theta$ also takes on this third interpretation.

We think the general equilibrium efficiency condition (12) should be helpful in understanding results arising in dynamic general equilibrium models that feature search and matching frictions. In particular, the models used to study optimal macroeconomic policy in search frameworks must have some notion of (a possibly distorted version of) condition (12) in the background. As an example of a distortion in this condition, suppose that the payoff outside the labor market were $\frac{mrs}{1-\tau^n}$, where τ^n is a proportional tax on labor income. Replacing mrs with $\frac{mrs}{1-\tau^n}$ in (6), it is straightforward to see that the analog of (12) is

$$mrs = (1-\tau^n)\frac{\eta}{1-\eta}\gamma\theta, \tag{13}$$

which highlights the *wedge* between mrs and our search-based notion of MRT. In a given model, this wedge need not literally be induced by fiscal policy, but rather might arise from any number of frictions. At the very least, checking whether this condition hold in a model's equilibrium should shed some light on whether "good policy" is in place in general-equilibrium search models. Faia (2008), Thomas (2007), and Arseneau and Chugh (2008) are a few recent papers studying optimal monetary policy in labor-search environments; Domeij (2005) and Arseneau and Chugh (2009) are a couple of recent papers studying optimal fiscal policy in labor-search environments.

A couple of issues regarding our analysis suggest themselves. First, we assumed the presence of the payoff $z > 0$ in both (4) and (6). If one prefers, we could have instead set simply mrs as the outside option in the participation condition (6). All that matters, though, is mrs net of z, so we can then simply define $\widehat{mrs} \equiv mrs - z$. In the subsequent analysis — in particular, in the wage condition (8) — it would be $\widehat{mrs}$ that is relevant. That is, the wage condition would instead be $w = \eta p + (1-\eta)z + (1-\eta)\widehat{mrs}$, and our efficiency analysis in (11) and (12) would instead involve $\widehat{mrs}$. Because it is natural to assume that $mrs > z$ — that is, activities outside the labor market in which one can engage have a strictly higher payoff if they can be performed without the burden of also simultaneously searching for employment — our inclusion of z in the participation condition (6) is thus without loss of generality.

Another issue is the robustness of our efficiency condition to other wage-determination mechanisms besides Nash bargaining. One popular alternative to Nash bargaining is competitive search equilibrium, as developed by Moen (1997). In competitive search equilibrium, wages are posted by firms ex-ante, and individuals who are searching optimally direct their search based on these known wages.[2] If successful matching occurs (directed search does not get around the basic matching frictions), then the wage paid is the one that was posted; there are no ex-post negotiations.

[2]This is one common description of competitive search equilibrium; there are alternative, equivalent, descriptions. See Rogerson, Shimer, and Wright (2005, p. 972-973) for more discussion.

It is well understood (see, for example, the survey in Rogerson, Shimer, and Wright (2005)) that competitive search equilibrium and Nash bargaining under the Hosios condition deliver the same (efficient) labor-market outcomes. Hence, our analysis would go through as described if wages were determined by competitive search rather than by Nash bargaining. One may also wonder whether and to what extent our efficiency condition is robust to other matching specifications besides Cobb-Douglas. The efficiency properties of search models that feature something other than Cobb-Douglas matching and Nash bargaining (or competitive search) are even less well developed, so it is open question to what extent our general-equilibrium analysis would carry over.

5 Conclusion

Our contribution here was to provide a characterization of general-equilibrium efficiency in standard labor-search frameworks. Our characterization builds on the Hosios (1990) partial-equilibrium condition for search efficiency. What makes our condition general equilibrium in nature is that it considers endogenous labor force participation, hence linking activity *across* labor and other markets. The latter are absent in the baseline search and matching framework. A natural interpretation of these "other markets" is goods markets, and, indeed, in the recent vintage of DSGE labor search models, goods markets are the key "other markets."

Our characterization of efficiency has a simple interpretation in terms of standard economic theory: the requirement that efficiency entails some appropriate notion of a marginal rate of substitution being equated to an appropriately-defined marginal rate of transformation. This interpretation also naturally leads to a criterion by which to measure the attainment of efficiency in theoretical models studying optimal policy in search frameworks: simply measure the deviation of marginal rate of substitution from marginal rate of transformation. Finally, it has not escaped our notice that this criterion may also provide a novel basis for empirical tests of "wedges" in labor markets; it may offer a complementary view of the labor-wedge empirical findings of Chari, Kehoe, and McGrattan (2007), Shimer (2008), and Ohanion, Raffo, and Rogerson (2008).

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