Ramsey Meets Hosios:
The Optimal Capital Tax and Labor Market Efficiency*

David M. Arseneau †
Federal Reserve Board

Sanjay K. Chugh‡
University of Maryland

First Draft: February 2006
This Draft: June 26, 2009

Abstract

This paper studies optimal capital income taxation in an economy where labor markets are subject to search and matching frictions. We show analytically that, provided the government is constrained to capital and labor income taxation, inefficient labor force participation gives rise to a non-zero optimal capital tax in the long run. In a calibrated version of our economy, we demonstrate quantitatively that the optimal capital income tax is extremely sensitive to how far above or below the participation rate is from its efficient level. Thus, even seeming small inefficiencies in participation may call for large capital income taxes (or subsidies).

JEL Classification: E24, E62, H21, J64

Keywords: Optimal fiscal policy, labor search, Hosios condition

*The views expressed here are solely those of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.

†email address: david.m.arseneau@frb.gov.

‡email address: chughs@econ.umd.edu.

1 Introduction

An enduring question in the theory of fiscal policy is that of the optimal long-run capital income tax rate. In this paper, we study optimal capital income taxation when labor markets are subject to search and matching frictions. Our main result is that the standard Chamely/Judd optimal fiscal policy prescription of a zero capital tax rate in the long run does not, in general, carry over to our environment with frictional labor markets. This result arises for two distinct reasons. First, echoing a result previously reported by Domeij (2005), we show that a non-zero optimal capital tax obtains if a well-known condition for efficient search and matching, the Hosios (1990) condition, is not satisfied. In addition, we show that regardless of whether or not the Hosios condition is in place, a non-zero capital tax obtains if labor force participation—that is, entry by individuals into labor market activity—is inefficient. This later result is the primary contribution of our paper. Intuitively, in either case the capital tax is used to correct the underlying labor market distortion by indirectly influencing search activity of private agents in the labor market.

Most models examining policy issues in a general equilibrium setting with search and matching frictions in the labor market assume a fixed participation rate. One justification for this assumption is that fluctuations along the participation margin observed in the data are quite small. Furthermore, and perhaps more substantially, assuming a fixed participation margin helps the model deliver a Beveridge curve—the empirically-observed negative cyclical relationship between job vacancies and unemployment. In contrast, models that feature endogenously determined participation rates, such as Shi and Wen (1999) and Domeij (2005), do not display a Beveridge Curve. They instead counterfactually predict a positive cyclical relationship between job vacancies and unemployment. It appears, then, that there may be good empirical reasons for assuming fixed participation.

The focus of our study here is not on short-run issues, but rather on the long-run capital income tax, so while the Beveridge Curve does not play a direct role in our analysis, we nevertheless think it useful to study long-run capital taxation in an environment that also has attractive short-run properties. The Beveridge Curve is a short-run outcome with which virtually all labor-market models aim to be consistent, and a fixed participation rate delivers this outcome. A given (fixed) participation rate, however, is not likely to be the efficient participation rate. In light of this, one of the key contributions of this paper is to show that there are first order policy implications that can arise from imposing the fixed participation assumption.

We demonstrate our point by formulating a dynamic general equilibrium (DGE) labor-search model in such a way that it nests both the fixed- as well as the efficient-participation framework in a single model. We nest the two frameworks by defining a shadow value of adjustment along the participation margin which, in turn, appears in several key equilibrium conditions of the model. If participation is efficient, this shadow value is by construction zero; however, if participation is inefficient, as generically is the case for a fixed participation rate, we show that this shadow value drives a wedge into the equilibrium of the labor market. We show analytically that the presence of this wedge generates a non-zero optimal capital tax.

Intuitively, the capital tax is used by the Ramsey planner to alleviate the distortion generated by inefficient participation by indirectly influencing the job creation decision of the firm. We present quantitative evidence to demonstrate that when participation is inefficiently low, a capital tax can be used to increase the firm's incentive to create jobs. As firms post more vacancies it becomes easier for searching individuals to find employment and the unemployment rate falls. With more workers in the economy engaged in productive jobs, the total distortion to output resulting from the inefficiently low participation rate is mitigated. Conversely, a capital subsidy can be used to restrain job creation when labor force participation is inefficiently high. Ultimately, our results stem from the fact that the Ramsey planner is assumed to have access to an incomplete set of tax instruments and thus must the capital tax to indirectly influence outcomes in the labor market.[1] Arseneau and Chugh (2008) show that the inflation tax can be used in a similar manor to influence labor market outcomes in a DSGE labor search model with nominal wage rigidities.

A more closely related paper is Domeij (2005), which uses a labor search model with households that optimize over search activity to demonstrate that, provided the Hosios (1990) efficiency condition is satisfied, the optimal capital income tax is zero in the long run. With the standard assumptions in the literature of Cobb-Douglas matching and Nash bargaining as the wage-setting protocol, Hosios (1990) showed that a simple parameter restriction in the decentralized economy eliminates the market-tightness externality inherent in search environments, thus rendering the labor market efficient.[2] Because the Hosios condition is obtained in a model of only labor markets and takes all other activity - in particular, participation decisions - as given, it can be interpreted as a partial-equilibrium efficiency condition. Domeij's (2005) result can thus be understood as the partial equilibrium Hosios condition carrying over to a general equilibrium setting provided participation is efficient. With the Hosios condition delivering labor-market efficiency for a given participation rate and with participation, in turn, determined efficiently through optimal search

[1] In our baseline analysis, the missing instrument is one that directly encourages or discourages participation. See Ljungqvist and Sargent (2004, p. 478) for discussion about using the capital tax as part of an incomplete tax system, or also Correia (1996), Jones, Manuelli, and Rossi (1997), and Armenter (2008).

[2] The specific parameter restriction is that that the bargaining power of workers equals the elasticity of workers' input into the aggregate matching process. In search environments, a market-tightness externality arises from the fact that one additional job-seeker in the market increases the probability that a firm will match with a worker but decreases the probability that job-seekers already in the market will match with a firm. Equivalently, the externality can be thought of as arising from the fact that one additional firm with a vacancy increases the probability that a job-seeker will match with a firm but decreases the probability that firms already in the market will match with a job-seeker.

behavior, there is no role for capital income taxation. Because Domeij (2005) is nested as a special case of our modeling framework, we are able to replicate his finding analytically. We then extend his analysis a more general setting with fixed participation and demonstrate analytically that non-zero capital taxation is an even more general result. In particular, we show a non-zero capital tax is optimal when participation is fixed, regardless of whether or not the Hosios condition is in place.

We use a quantitative version of our model to first numerically illustrate the zero capital tax result when the household chooses labor force participation optimally and the Hosios condition holds. We then use the model to show that, with participation determined optimally, deviations from the Hosios condition result in a capital income tax rate that is small, ranging from -11 percent to 11 percent. This numerical result mirrors Domeij (2005). In contrast, if participation is assumed to be exogenous, as is the case in much of the DSGE labor search literature, the magnitude of the capital tax (in an absolute sense) is much larger depending on how far above or below the participation rate is from its efficient level. Furthermore, this is true regardless of whether or not the Hosios condition is in place. Thus, even seemingly “small” inefficiencies in participation may call for large capital-income taxes (or subsidies).

The rest of the paper is organized as follows. Section 2 lays out the model, in which we are able to capture in nested form both an environment that does feature and an environment that does not feature a labor force participation margin. In Section 3, we describe the social planning solution in this environment and present a key efficiency result. Section 4 presents the Ramsey problem. Section 5 proves the conditions under which a zero long-run capital tax is optimal. Section 6 presents a parameterized version of the model to quantify the optimal capital tax when participation is either inefficiently high or inefficiently low. Section 7 concludes.

2 Model

As many other recent studies have done, we embed the baseline Pissarides (2000) textbook search model with capital into a deterministic dynamic general equilibrium (DGE) framework. The baseline Pissarides (2000) framework features a fixed size of the labor force. The large majority of the recent vintage of DGE models adopting a search-theoretic foundation for labor markets has inherited this structure. As mentioned in the introduction, this structure preserves a Beveridge Curve when studying short-run issues. A few DGE studies, on the other hand, have endogenized the participation decision — for example, Shi and Wen (1999) and Domeij (2005). However, endogenous participation, while perhaps attractive for some issues, is inconsistent with the existence of a cyclical Beveridge Curve.[3]

[3]As noted in the introduction, a model displays a short-run Beveridge Curve if vacancies and unemployment move in opposite directions over the business cycle. If participation is endogenous in a model, then the Beveridge relation

To encompass both types of models found in the literature, we allow for both the possibility that the number of individuals in the labor force (defined as individuals either working or actively seeking employment) is fixed and one in which it is endogenous. Regardless of whether participation is endogenous or exogenous, however, transitions into employment are always subject to search and matching frictions in our model. We present these two versions of our search model in a unified, nested way. Specifically, rather than construct essentially two different environments, we construct one very general framework that seems like it "always" allows for an endogenous participation decision but that, by imposing one particular constraint, can also allow for exogenous participation. As described below, nesting the two environments in this way allows us to clearly isolate the incentives relevant for optimal policy. These incentives turn out to be captured by the shadow value of the constraint that allows us to move between the exogenous- and the endogenous-participation versions of our model.

We present in turn the choice problems of the representative firm, the representative household, the determination of wages, the actions of the government, and the definition of private-sector equilibrium.

2.1 Production

The production side of the economy features a representative firm that must open vacancies, which entail costs, in order to hire workers and produce. The representative firm is "large" in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs.

The firm requires labor and capital to produce its output. The firm must engage in costly search for a worker to fill each of its job openings. In each job j that will produce output, the worker and firm bargain over the pre-tax real wage w_{jt} paid in that position. Output of any job j is given by $y_{jt} = f(k_{jt})$, where $f(.)$ exhibits diminishing marginal product. The capital k_{jt} used in job j is rented by the firm in a spot capital market at rental rate r_t after it is known that an employee will operate job j — that is, capital is rented only after it is certain there is an employee to operate it, hence there are no issues about idle capital in our model.

Any two jobs j_a and j_b at the firm are identical, so from here on we suppress the second subscript and denote by w_t the real wage in any job, and so on. Total output of the firm thus depends on

does not arise. To understand why, consider an exogenous improvement in productivity. Following such a shift, firms would try to hire more workers, which requires them to post additional vacancies. At the same time, improved labor-market prospects, in the sense that the probability of finding a job has increased, causes movement of some individuals from outside to inside the labor force, which raises measured unemployment. Hence, because the returns to search have increased for both sides of the market, vacancies and unemployment move in the *same* direction. This does not happen if the labor force is assumed fixed.

the technology realization and the measure of matches n_t^f that produce,

$$y_t = n_t^f f(k_t). \tag{1}$$

The total real wage bill of the firm is the sum of wages paid at all of its positions, $n_t^f w_t$, and the total capital rental bill is $n_t^f r_t k_t$.

The firm begins period t with employment stock n_{t-1}^f. Its period-t productive employment stock, n_t^f, depends on its period-t vacancy-posting choices as well as the random matching process described below.[4] With probability $q(\theta)$, taken as given by the firm, a vacancy will be filled by a worker. Labor-market tightness is $\theta \equiv v/u$, where u denotes the number of individuals searching for jobs. Matching probabilities for both firms and households depend only on aggregate market tightness given the Cobb-Douglas matching function assumed below.

Our baseline wage-determination mechanism is Nash bargaining, as described below. In the firm's profit maximization problem, the wage-setting protocol is taken as given. We employ Nash bargaining because it is familiar from baseline search models and has become common in DGE search models, thus enhancing comparability with existing studies.

The firm thus chooses vacancies to post v_t and a current employment stock n_t^f to maximize discounted profits starting at date t,

$$\sum_{t=0}^{\infty} \beta^t \left\{ \Xi_{t|0} \left[n_t^f f(k_t) - w_t n_t^f - r_t k_t n_t^f - \gamma v_t \right] \right\}, \tag{2}$$

where $\Xi_{t|0}$ is the period-0 value to the representative household of period-t goods, which we assume the firm uses to discount profit flows because households are the ultimate owners of firms.[5] In period t, the firm's problem is thus to choose v_t and n_t^f to maximize (2) subject to a sequence of perceived laws of motion for its employment level,

$$n_t^f = (1 - \rho^x) n_{t-1}^f + v_t q(\theta_t). \tag{3}$$

Firms incur the real cost γ for each vacancy created, and job separation occurs with exogenous fixed probability ρ^x. Note the timing of events embodied in the law of motion (3). Period t begins with employment stock n_{t-1}^f, some of whom separate from the firm before period-t production

[4] Labor-market matching thus occurs within a period, which, given the quarterly calibration we will pursue, is empirically descriptive of U.S. labor-market flows — see, for example, the evidence of Davis, Faberman, and Haltiwanger (2006). This so-called "instantaneous-hiring" view of labor-market flows has recently become widely used in this class of models, employed by, among others, Blanchard and Gali (2007, 2008) and Krause, Lopez-Salido, and Lubik (2007).

[5] Technically, of course, it is the real interest rate with which firms discount profits, and in equilibrium the real interest rate between time zero and time t is measured by $\Xi_{t|0}$. Because there will be no confusion using this equilibrium result "too early," we skip this intermediate level of notation and structure.

occurs; the firm posts vacancies and hires a flow of new employees $q(\theta_t)v_t$, which depends on both the firm's decisions and market conditions; the employment stock n_t^f then engages in production.

The firm's first-order conditions with respect to v_t and n_t^f yield the job-creation condition

$$\frac{\gamma}{q(\theta_t)} = f(k_t) - w_t - r_t k_t + (1-\rho^x)\left[\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right], \tag{4}$$

where $\Xi_{t+1|t} \equiv \Xi_{t+1|0}/\Xi_{t|0}$ is the household discount factor (again, technically, the real interest rate) between period t and $t+1$. The job-creation condition states that at the optimal choice, the vacancy-creation cost incurred by the firm is equated to the discounted value of profits from a match. Profits from a match take into account match output, the wage cost of the match, the capital rental cost of the match, and the asset value of having a pre-existing relationship with an employee in period $t+1$. This condition is a free-entry condition in the creation of vacancies and is a standard equilibrium condition in a labor search and matching model.

Finally, because capital rental occurs in a spot market after the employment stock has been resolved, optimal capital rental implies

$$r_t = f'(k_t). \tag{5}$$

2.2 Households

There is a representative household in the economy. Each household consists of a continuum of measure one of family members, and each individual family member is classified as either inside the labor force or outside the labor force. An individual family member that is outside the labor force enjoys leisure. An individual family member that is part of the labor force is engaged in one of two activities: working, or not working but actively searching for a job. The convenience of an "infinitely-large" household is that we can naturally suppose that each individual family member experiences the same level of consumption regardless of his personal labor-market status. This tractable way of modeling perfect consumption insurance in general-equilibrium search-theoretic models of labor markets has been common since Andolfatto (1996) and Merz (1995). We use the terms "individual" and "family member" interchangeably from here on. Given the basics of the environment, we also use the terms "leisure" and "outside the labor force" interchangeably from here on.

As noted above, we construct our model in a flexible way to allow for two cases regarding labor-force participation. In the first case, which corresponds to the benchmark Pissarides (2000) model, the size of the labor force is exogenously fixed at the measure $n_t^h + u_t^h = \bar{l} < 1$, with $\bar{l}$ a parameter of the decentralized economy. With the labor-force participation rate fixed at $\bar{l}$, the fixed measure $1-\bar{l} = 1 - u_t^h - n_t^h$ of family members thus enjoy leisure in every period. For reasons that will become clear immediately below, we label this version of our model the *pseudo-labor-force-participation model*, or pseudo-LFP model for short. In the second case, households each period optimally choose the size of $n_t^h + u_t^h$; the labor-force participation rate is thus endogenous in every period t. We label this version of the model the *labor-force-participation model* (LFP model). As the descriptions below show, with the exception of one constraint, the analysis of the household problem is identical in both the pseudo-LFP model and the LFP model.

2.2.1 Pseudo-LFP Model

Although the labor-force participation rate is fixed in the baseline environment, we formulate the household problem *as if there were free choice over the participation rate*, but impose a constraint that ensures it is always $\bar{l}$. This formulation motivates our label "pseudo" LFP model. We emphasize that while this formulation allows *analysis* of the household problem in a way that is easily comparable with the full LFP model described below, in no sense do we mean that households actually "make a participation decision" in the pseudo-LFP model. In the pseudo-LFP, it is always the case that the size of the labor force is $n_t^h + u_t^h = \bar{l}$. The advantage this formulation of the problem delivers is that the shadow value of this size restriction directly enters the problem; this shadow value sheds light on the mechanism driving our optimal-policy results.

Thus, we analyze the household as if it maximized its expected lifetime discounted utility

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1 - u_t^h - n_t^h) \right] \tag{6}$$

subject to a sequence of flow budget constraints

$$c_t + K_{t+1} + b_t = n_t^h (1 - \tau_t^n) w_t + \left[1 + (1 - \tau_t^k)(r_t - \delta) \right] K_t + R_t b_{t-1} + (1 - \tau^d) d_t, \tag{7}$$

a sequence of perceived laws of motion for the measure of family members that are employed,

$$n_t^h = (1 - \rho^x) n_{t-1}^h + u_t^h p(\theta_t), \tag{8}$$

and the exogenous restriction on the size of the labor force

$$n_t^h + u_t^h = \bar{l}. \tag{9}$$

The functions $u(.)$ and $g(.)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and leisure, respectively.

The rest of the notation is as follows. The household's holdings of government bonds at the end of period t is b_t, each unit of which pays a gross real return R_{t+1} at the beginning of $t+1$, K_t denotes the household's capital holdings at the start of period t, and δ is the depreciation rate of capital. As already defined above, each employed individual's wage is w_t, and the capital rental rate is r_t. Household labor income is taxed at the rate τ_t^n, and capital income net of depreciation is taxed at the rate τ_t^k. The household takes as given the probability $p(\theta)$ that one of its unemployed and searching individuals will find employment. As with matching probabilities for firms, p depends only on aggregate labor-market tightness given the assumption of a Cobb-Douglas matching technology. Of course, because this is a Ramsey-taxation model, there are no lump-sum taxes or transfers between the government and the private sector.

Due to firms' sunk resource and time costs of finding employees, firms earn positive flows of economic profits. These profits are transferred to households at the end of each period in lump-sum fashion: d_t is the household's receipts of firms' flow profits. We permit government taxation of households' receipts of dividends at the fixed tax rate τ^d. As is well-understood in the Ramsey literature, flows of untaxed dividends received by households in and of themselves affect capital-taxation prescriptions.[6] To abstract from profit-taxation issues, our main analysis is conducted assuming $\tau^d = 1$. The consequence of this assumption is that any predictions made by our model regarding optimal capital-income taxation cannot be due to incentives to tax profits.

The formal analysis of this problem is presented in Appendix A; here we simply describe intuitively the optimality conditions that emerge. Two conditions are standard and thus deserve no further discussion: a bond demand condition,

$$u'(c_t) = \beta u'(c_{t+1}) R_{t+1}, \tag{10}$$

and a capital supply condition,

$$u'(c_t) = \beta u'(c_{t+1}) \left[1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta)\right]. \tag{11}$$

As usual, the bond demand condition defines the stochastic discount factor, $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which firms, in equilibrium, discount profit flows.

More central to the analysis of the pseudo-LFP model is the household's *pseudo-labor-force participation condition*

$$\frac{g'(1-u^h_t-n^h_t)}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)} = u'(c_t)(1-\tau^n_t)w_t - g'(1-u^h_t-n^h_t) - \phi_t + \beta(1-\rho^x)\left[\frac{g'(1-u^h_{t+1}-n^h_{t+1})}{p(\theta_{t+1})} + \frac{\phi_{t+1}}{p(\theta_{t+1})}\right], \tag{12}$$

which is derived in detail in Appendix A. In (12), ϕ_t denotes the Lagrange multiplier associated with the constraint (9). Suppose for a moment that $\phi_t = 0$, $\forall t$, which means there is no restriction on the participation rate. With $\phi_t = 0$, condition (12) has a straightforward interpretation: at the optimum, the household each period sends a measure u^h_t of family members to search for jobs until the expected cost of search — the left-hand-side of (12) — is equated to the expected benefit of search — the right-hand-side of (12). The expected cost of search is measured by the marginal utility of leisure (each unit of search involves forgoing one unit of leisure). The expected benefit of

[6] See, for example, Stiglitz and Dasgupta (1971), Jones, Manuelli, and Rossi (1997), Schmitt-Grohe and Uribe (2004), Siu (2004), Albanesi and Armenter (2007), Armenter (2008), and Arseneau, Chugh, and Kurmann (2008) for examples in various contexts of this type of taxation incentive.

search is the marginal utility value of after-tax wage income net of the marginal disutility of work (the first two terms on the right-hand-side of (12)), along with the asset value to the household of having an additional family member engaged in an ongoing employment relationship (the last term on the right hand side of (12)). This asset value reflects the value to the household of sending one fewer family member out to look for a job in the future.

With $\phi_t = 0$, condition (12) naturally has the interpretation of a free-entry condition into the labor force. However, because in the pseudo-LFP model the size of the labor force is fixed at $\bar{l} < 1$, the shadow value ϕ_t is non-zero. The shadow value ϕ_t measures the value to the household of being able to freely adjust its participation rate; as such, it can be interpreted as the price of participation. In equilibrium, this shadow price adjusts so that condition (12) results in *no* net entry into or exit from the labor force. Hence the terminology *pseudo* labor-force participation condition to refer to (12). The shadow price ϕ_t is crucial in driving our optimal policy results below.

We again emphasize that in this version of the model, the participation rate $n_t^h + u_t^h$ is indeed fixed at $\bar{l}$ every period. Hence, our pseudo-LFP formulation delivers the same household decisions as if we had taken the more standard approach of specifying the household problem as one of choosing only c_t subject to the budget constraint (7), completely dropping constraints (8) and (9), setting $g(.)$ to a constant, and setting, as is common in DGE search models, $n_t^h + u_t^h = \bar{l} = 1\ \forall t$. It is not the underlying optimal choices of the household that we change by specifying the model this way, it is only the way we analyze the household problem that is different. The important gain our formulation brings to the analysis is being able to measure the shadow value ϕ_t.

2.2.2 LFP Model

If households were instead *actually* free to choose the labor-force participation rate, the household's decision problem would be exactly as just described, except of course constraint (9) would not bind household behavior. Household optimal choices would thus be characterized by the bond demand condition (10), the capital supply condition (11), and the "true" labor-force participation condition,

$$\frac{g'(1-u_t^h-n_t^h)}{p(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) + \beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}^h-n_{t+1}^h)}{p(\theta_{t+1})}\right\}, \quad (13)$$

which of course is simply condition (12) with $\phi_t = 0\ \forall t$. Except for differences in timing, which of course do not matter in the long run, this participation condition appears in Domeij (2005, p. 627), as well.

Despite the search frictions and long-lived nature of employment relationships, the labor-force participation condition (13) has the same interpretation as the labor-supply function in a simple neoclassical labor market, as exists in standard Ramsey models. Indeed, we can recover a neoclassical labor market by setting $\rho^x = 1$ (all employment "relationships" are one-period, spot, transactions) and fixing the probability of "finding a job" to $p(\theta) = 1$ (because in a neoclassical market there of course is no friction in "finding a job"). Imposing these assumptions and logic on (13), we obtain $\frac{g'(.)}{u'(.)} = (1 - \tau_t^n)w_t$, which defines the labor-supply function in a neoclassical market.

With matching frictions that create a meaningful separation of the labor force into those individuals that are employed and those individuals that are unemployed, condition (13) defines transitions of individuals from outside the labor force (leisure, in our model) into the pool of searching unemployed, from where the aggregate matching process will pull some individuals into employment. In the pseudo-LFP model, condition (12) plays the same conceptual role, but, because the participation rate is exogenous, what it pins down is the shadow price ϕ_t rather than the participation rate. We thus speak of (13) and (12) as labor-force participation functions, rather than labor-supply functions.

2.3 Wage Bargaining

The wage-determination mechanism is Nash bargaining. Specifically, we assume that wages of all workers, whether newly-hired or not, are set in period-by-period Nash negotiations. This assumption is common in DGE search models. A detailed derivation of the wage-bargaining problem is presented in Appendix B. We point out that the Nash wage function is the same for both the pseudo-LFP model and the LFP model, reflecting the fact that the shadow value ϕ_t is not a price that either firms or households in the economy observe, nor is it a payoff any party can ever receive; it thus plays no role in the bargaining process. Thus, ϕ_t is truly a *shadow* price in our analysis.

In what follows, we simply present the bargaining outcome. Assuming that $\eta \in (0, 1)$ is a worker's Nash bargaining power and $1 - \eta$ a firm's Nash bargaining power, the Nash wage outcome is given by

$$w_t = \eta \left[f(k_t) - r_t k_t\right] + (1 - \eta)\frac{g'(1 - u_t^h - n_t^h)}{u'(c_t)(1 - \tau_t^n)} + \eta(1 - \rho^x)\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\left[\frac{\tau_{t+1}^n - \tau_t^n}{1 - \tau_t^n}\right]. \tag{14}$$

The first two terms in (14) show that part of the period-t wage payment is a convex combination of the contemporaneous values to the firm and the household, given, respectively, by the net revenue $(f(k_t) - r_t k_t)$ generated for the firm by a new employment match and the after-tax MRS between consumption and leisure. The last term in (14) captures the forward-looking, relationship, aspect of employment, whose value is also capitalized in the period-t wage payment.

In a dynamic analysis, the forward-looking change in tax rates, $\tau_{t+1}^n - \tau_t^n$, affects the period-t bargained wage. If labor tax rates were instead constant at $\tau_t^n = \bar{\tau}^n \ \forall t$, the wage outcome would be truly static: $w_t = \eta \left[f(k_t) - r_t k_t\right] + (1 - \eta)\frac{g'(1-u_t-n_t)}{u'(c_t)(1-\bar{\tau}^n)} \ \forall t$. In this case, the presence of the labor tax only changes firms' effective bargaining power in a static manner: $\bar{\tau}^n > 0$ causes

$(1-\eta)/(1-\bar{\tau}^n) > 1-\eta$. However, steady state cannot be imposed before construction of the Ramsey problem, so the fully-dynamic wage rule (14) is part of the definition of a private-sector equilibrium presented below.

2.4 Government

The government finances an exogenous stream of spending $\{g_t\}$ by collecting labor income taxes, capital income taxes, dividend income taxes, and real debt. The period-t government budget constraint is

$$\tau_t^n w_t n_t + \tau_t^k (r_t - \delta) K_t + \tau^d d_t + b_t = g_t + R_t b_{t-1}. \tag{15}$$

2.5 Matching Technology

In equilibrium, $n_t = n_t^f = n_t^h$, so we now refer to employment simply as n_t. Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a constant-returns matching technology, $m(u_t, v_t)$, where u_t is the number of searching individuals and v_t is the number of posted vacancies. A fraction ρ^x of matches that produced in period $t-1$ are exogenously destroyed before period t. All newly-formed matches produce at least once before possibly dissolving. The evolution of aggregate employment is thus given by

$$n_t = (1-\rho^x) n_{t-1} + m(u_t, v_t). \tag{16}$$

2.6 Private-Sector Equilibrium

Whether considering the pseudo-LFP model or the LFP model, a symmetric private-sector equilibrium is made up of endogenous processes $\{c_t, K_{t+1}, w_t, k_t, n_t, v_t, u_t, R_t, b_t\}_{t=0}^{\infty}$ that satisfy the vacancy-posting condition (4), the capital-demand condition (5), the bond-demand condition (10), the capital-supply condition (11), the Nash wage outcome (14), the government budget constraint (15), the law of motion for the aggregate stock of employment (16), the aggregate resource constraint of the economy

$$c_t + g_t + K_{t+1} - (1-\delta) K_t + \gamma v_t = n_t f(k_t), \tag{17}$$

and the relationship between the quantity of aggregate capital and the quantity of capital used in any given match

$$K_t = n_t k_t. \tag{18}$$

Note that in (17), total costs of posting vacancies γv_t are a resource cost for the economy. Also note the nature of capital-market clearing. The aggregate capital stock K_t, which in equilibrium is accumulated by the representative household, is pre-determined in period t. However, because employment n_t is not pre-determined (recall, with our empirically-supported timing assumption, that matching and the onset of production occurs within the same period), the allocation of aggregate capital across the n_t productive employment relationships is also not pre-determined.

To complete the set of conditions characterizing equilibrium, in the LFP model, we need the labor-force participation condition (13), while in the pseudo-LFP model, we need the pseudo-LFP condition (12) as well as the exogenous restriction on the total size of the labor force

$$n_t + u_t = \bar{l}. \tag{19}$$

In the pseudo-LFP model, the endogenous process of shadow prices $\{\phi_t\}_{t=0}^{\infty}$ is also added to the list of endogenous equilibrium processes. Finally, in either the pseudo-LFP model or the LFP model, the private sector takes as given the sequence $\{g_t, \tau_t^n, \tau_t^k\}_{t=0}^{\infty}$.

3 Efficient Allocations

To understand the optimal taxation results that emerge from the Ramsey problem, it is useful to first present the conditions that characterize the constrained-efficient allocation that would be chosen by a social planner that is restricted by the matching technology and that is able to efficiently allocate individuals across participation and non-participation. The social planner chooses decision rules for the allocation $\{c_t, K_{t+1}, k_t, u_t, n_t, v_t\}$ to maximize the lifetime utility (6) of the representative household subject to the law of motion of the employment stock (16), the goods-resource constraint (17), and the relation between match-level capital and aggregate capital (18).

After imposing deterministic steady state on the first-order-conditions of this problem, two simple efficiency conditions describe the constrained-efficient outcome. We summarize these conditions in the following proposition:

Proposition 1. ***(Efficient Allocations)*** *There exists a constrained-efficient allocation in which steady-state efficiency along the consumption-leisure margin is characterized by*

$$\frac{g'(1-n-u)}{u'(c)} = \gamma\theta\frac{\xi_u}{1-\xi_u}, \tag{20}$$

and steady-state efficiency along the capital accumulation margin is characterized by

$$\frac{1}{\beta} - 1 + \delta = f'(k). \tag{21}$$

Proof. See Appendix C.

Proposition 1 will help us understand the conditions under which the Ramsey government implements a non-zero capital tax in the long run. We now turn to the description of the Ramsey problem.

4 Ramsey Problem

The Ramsey government raises revenue through labor, capital, and dividend income taxes in such a way that maximizes the welfare of the representative household, subject to implementation as a decentralized search equilibrium. As we mentioned in the description of the household problem, the Ramsey government trivially would like to tax dividend income as heavily as possible because it represents a lump-sum source of income for households. Thus, we simply fix $\tau^d = 1$, the largest possible (sensible) rate; this setting has the virtue of removing any motive to use the capital income tax to indirectly tax profits.

In principle, the Ramsey problem is to maximize (6) subject to all of the equilibrium conditions listed in Section 2.6. However, we can condense many of the equilibrium conditions into more compact summary descriptions. In standard Ramsey fashion, we condense the equilibrium conditions (5) (capital demand by firms), (10) (bond demand by households), (11) (capital supply by households), and the government budget constraint (15) in a single present-value implementability constraint (PVIC)

$$\sum_{t=0}^{\infty} \beta^t \left\{ u'(c_t)c_t - \left[g'(1-u_t-n_t)+\phi_t\right](n_t+u_t) - u'(c_t)(1-\tau^d)d_t \right\} = A_0, \tag{22}$$

where A_0 represents the time-zero liabilities of the government. The PVIC is expressed in terms of only allocations and has the interpretation that the initial liabilities of the government must be covered by the present value of all fiscal surpluses starting from date zero. A detailed derivation is presented in Appendix D.[7]

We can also condense the three conditions that characterize equilibrium in the labor market in period-t, the vacancy-posting condition (4), the pseudo-LFP condition (12), and the Nash wage outcome (14), into a single constraint.[8]

$$\Psi(c_t, c_{t+1}, u_t, u_{t+1}, n_t, n_{t+1}, v_t, v_{t+1}, k_t, \tau_t^n, \tau_{t+1}^n) = 0, \tag{23}$$

which is fully described and derived in Appendix E. Note that this summary description of the labor-market equilibrium depends on both period-t and period-$t+1$ outcomes. The intuition of the proofs we present below goes through this $\Psi(\cdot)$ function.

The Ramsey problem is to choose state-contingent functions for the process $\{c_t, K_{t+1}, k_t, u_t, n_t, v_t, \tau_t^n\}$, taking as given an exogenous stream of government purchases $\{g_t\}$ and initial values n_{-1}, k_{-1}, K_0, such that they maximize the discounted lifetime utility of the representative household, (6), subject to the PVIC (22), the sequence of labor-market equilibria (23), and the sequence of resource

[7] Also note that we substitute into the PVIC the equilibrium expression for flow dividends, $d_t = z_t n_t - w_t n_t - \gamma v_t$; however, this will not matter for the Ramsey equilibrium because of our assumption $\tau^d = 1$.

[8] We construct the $\Psi(.)$ function using the pseudo-LFP condition (12) because by simply setting $\phi_t = 0$ it also nests the LFP condition (13).

constraints (16), (17) and (18). If participation is assumed fixed, the Ramsey problem must also take as a constraint the sequence of size restrictions (19).

In solving the Ramsey problem computationally, we adopt a "timeless" perspective by assuming that the time-zero Ramsey allocation is the same as the asymptotic steady state Ramsey allocation, thus endogenizing the initial condition of the economy. Finally, throughout our analysis, we assume that the first-order conditions of the Ramsey problem are necessary and sufficient and that all allocations are interior.

5 Optimal Capital Taxation: Analytical Results

We first characterize analytically the conditions under which a zero long-run capital income tax is optimal. We begin with the case in which labor force participation is chosen optimally by the household, after which we turn to the case in which participation is fixed.

5.1 Efficient Labor Force Participation

Suppose that the household is free to choose its labor force participation, $n+u$, optimally, in which case we are in the LFP environment. The following proposition presents sufficient conditions for the optimality of a zero long-run capital tax.

Proposition 2. ***(Efficient LFP and Zero Capital Taxation)*** *If labor force participation is determined optimally in the decentralized economy (which implies $\phi = 0$) and if the Hosios condition ($\eta = \xi_u$) holds, the Ramsey steady state features $\tau^k = 0$.*

Proof. See Appendix E.

The logic of the proof is as follows. When $\eta = \xi_u$, the resulting efficient split of the surplus allows the constraint imposed by the equilibrium in the labor market to simplify considerably to the following expression

$$\Psi(\cdot) = \frac{g'(1 - n_t - u_t)}{u'(c_t)} - (1 - \tau_t^n)\gamma\theta_t \frac{\xi_u}{1 - \xi_u} = 0. \tag{24}$$

In this case, $\Psi(\cdot)$ is virtually identical, up to the presence of the labor income tax, to the efficiency condition given by equation (20). Furthermore, it is independent of the sequence of capital stocks (either match-level or aggregate capital stocks). From a technical perspective, this implies that labor market outcomes in no way constrain the choice of the optimal capital tax; hence, the Ramsey planner is free to set the capital tax in such a way as to minimize the distortion to the capital accumulation margin. The result is a zero optimal capital tax for exactly the same reason as in the original Chamely (1986) analysis.

In the more general case in which $\eta \neq \xi_u$, the labor market constraint $\Psi(\cdot)$ does not simplify, rather it remains a complicated nonlinear function of the capital stock. The implication is that the Ramsey planner faces a tradeoff. There is, on the one hand, an incentive to use the capital tax to help minimize the labor market distortion resulting from the inefficient division of the surplus. Doing so, however, comes at the expense of disrupting the capital accumulation margin. We confirm the previous finding of Domeij (2005) that this tradeoff resolves in a non-zero capital tax.

The core intuition is identical to that described in Domeij (2005). When $\eta < \xi_u$, the share of the surplus that accrues to the firm is too high relative to what is privately efficient. Households will respond by searching too little and firms will search too much, resulting in a θ that is inefficiently high. The Ramsey planner can raise the capital tax, which, by lowering the capital stock, indirectly decreases the marginal product of labor making job creation less profitable for firms. Decreased vacancy posting then pushes θ closer to its (second-best) efficient level. The opposite intuition holds when $\eta > \xi_u$. In section 6, we use a calibrated version of our economy to quantitatively demonstrate the relationship between bargaining power and the optimal capital tax.[9]

5.2 Fixed Labor Force Participation

A common assumption in much of the literature studying labor search frictions in a general equilibrium setting is that the participation rate is fixed, $n + u = \bar{l}$. The following proposition summarizes the conditions for the optimality of a zero long-run capital income tax under this assumption.

Proposition 3. ***(Inefficient LFP and Non-Zero Capital Taxation)*** *If labor force participation is fixed at $n + u = \bar{l}$ (which implies $\phi \neq 0$), then the Ramsey steady state features $\tau^k = 0$ if at least one of the following conditions holds:*

1. *The labor force participation rate is exogenously parameterized, so that $g'(\bar{l}) = -\phi$; or*
2. *The Hosios condition holds ($\eta = \xi_u$) and all matches are destroyed at the end of every period, so that $\rho^x = 1$.*

Proof. See Appendix E.

[9] While Domeij (2005) is able to analytically sign the optimal capital tax as a function of bargaining power, we are not and thus must rely on quantitative evidence. The difference is due to alternative assumptions regarding wage determination across the two papers. In this paper, the wage is determined based on Nash bargaining over a surplus that is explicitly defined by value functions determined via envelope conditions. Continuation values play a prominent role in the resulting Nash wage expression; it is these continuation values that prohibit a clean analytic solution to the optimal capital tax for all but the special cases outlined in propositions 2 and 3. In contrast, Domeij (2005) uses a simple split of the contemporaneous surplus, which effectively ignores continuation values. (A proof that Domeij's wage satisfies the zero capital tax in our model setup is available from the authors upon request.)

The logic of the proof is similar to that of proposition 2. With the exception of the two special cases outlined in the proposition, the equilibrium in the labor market imposes a binding constraint for the choice of the optimal capital income tax; hence, τ^k is, in general, nonzero when $\phi \neq 0$. This is true regardless of whether or not the Hosios parameterization is assumed. The intuition is similar to that behind proposition 2. When labor force participation is inefficiently high, the Ramsey planner can implement a negative capital tax, thereby increasing the profitability of hiring. Higher vacancy posting pushes θ back closer to its (second-best) efficient level. The opposite intuition holds when labor force participation is inefficiently low. We explore the quantitative relationship between ϕ and the optimal capital tax in greater detail in section 6.

We turn now to the two unique exceptions, outlined in proposition 3, to our general characterization of the optimality of the nonzero capital income tax. The first case is the one in which labor force participation is parameterized such that the marginal utility of leisure is exactly offset by the shadow cost to the household of not optimizing along the labor force participation margin, so that $g'(1-n-u) = -\phi$. The intuition behind why optimal capital tax is zero in this special case is that the assumption $g'(1-n-u) = -\phi$ implies that τ^n is a lump sum tax instrument. Thus, regardless of whether of not the Hosios condition holds, the Ramsey planner will always find it optimal to finance all government expenditures with the non-distortionary labor tax. Appendix E offers a proof that the labor tax is a lump sum instrument in this special case. The second case is the one in which $\rho^x = 1$. When matches are destroyed at the end of every period, they have no continuation value and the capital tax cannot be used to influence match formation. For the most part, we view both of these parameterizations as largely uninteresting, knife-edge cases.

5.3 Discussion

A brief summary of our analytical results is as follows. Proposition 2 simply restates the previous finding of Domeij (2005) that the Hosios condition delivers a zero optimal capital tax provided labor force participation is efficient. The primary contribution of this paper, summarized in proposition 3, is to show that this result does not generalize to a modeling framework in which labor force participation is assumed exogenous and, therefore, is not necessarily efficient.

Given that much of the existing literature studying optimal policy in a labor search and matching environment assumes fixed participation, we think our more general result is an important one to stress to the broader literature. Optimal policy prescriptions in the search and matching modeling framework are highly sensitive to assumptions regarding adjustment along the labor force participation margin. In this paper, we show that that it matters from one of the most basic principles of optimal fiscal policy: the Chamely/Judd prescription of a zero optimal capital tax.

Finally, it is useful to tie our results in more closely with recent developments in the literature on optimal capital taxation. In particular, our proofs of propositions 2 and 3 fit squarely into the general sufficient conditions provided by Albanesi and Armenter (2008) in their synthesis of various capital-taxation results. One simple way to understand their result is that if the non-technological constraints faced by the Ramsey government do *not* depend on capital stocks, then a zero capital tax is optimal in the long run. Both of our propositions fit squarely into their general result.

6 Optimal Capital Taxation: Quantitative Results

In this section, we use an calibrated version of our model to examine the quantitative magnitude of the optimal capital tax result found in the previous section. The following subsection describes the parameterization of the model and the results are presented in section 6.2

6.1 Parameterization

For utility, standard functional forms are used,

$$u(c_t) = \frac{1}{1-\sigma} c_t^{1-\sigma} \tag{25}$$

and

$$g(x_t) = \frac{\kappa}{1-\iota} x_t^{1-\iota}. \tag{26}$$

The time unit is meant to be a quarter, so we set the subjective discount factor to $\beta = 0.99$, yielding an annual real interest rate of about four percent. In the baseline analysis we set the curvature parameter with respect to consumption to $\sigma = 1$, but also consider the sensitivity of our optimal taxation results to higher levels of risk aversion.

For the subutility function over leisure, we set $\iota = 1$ in the baseline analysis, which corresponds to a unit elasticity of labor-force participation. This value is a common compromise in DGE models because it lies between macro evidence that suggests a very high elasticity of labor supply and micro evidence that suggests a very low elasticity of labor supply. We emphasize, however, that ι measures the elasticity of labor-force participation in our environment, not the elasticity of labor supply per se, due to the search frictions. We nonetheless set $\iota = 1$ to achieve some comparability with standard DGE models that study optimal policy, but also consider the sensitivity of our results to more elastic participation.

To emphasize the consequences of inefficient labor force participation for the capital income tax, we start with a parametrization in which the long-run (deterministic) Ramsey steady state is *identical* in *both* the LFP model and the pseudo-LFP model. By then perturbing parameters around this knife-edge case, we will be able to isolate the role of inefficient participation. To inform these experiments, note that the average labor-force participation rate in the U.S. during the period

1988-2007 was 66 percent, so we set $\bar{l} = 0.66$ as the measure of individuals in the representative household that are in the labor force on average in the LFP model.[10] The parameter κ is then chosen so that $n+u = \bar{l} = 0.66$ in the Ramsey steady-state of the LFP model. The resulting value is $\kappa = 0.41$. Because this setting for κ makes $\bar{l} = l^* = 0.66$ endogenously optimal in the LFP model, it has the consequence that in the *pseudo*-LFP model, in which we exogenously impose $n+u = \bar{l} = l^*$, the equilibrium value of the multiplier ϕ is exactly zero. It is thus this precise setting for κ that, in concert with the rest of the calibration, makes the Ramsey steady-states identical in the two cases.

In the pseudo-LFP model, however, the optimality of $\bar{l} = l^* = 0.66$ is a knife-edge case. Our main quantitative experiment is thus to vary $\bar{l}$ both above and below l^* to illustrate how the Ramsey optimal capital tax responds to the inefficiency in the participation margin. For our quantitative exercises we consider participation rates that are 5 percent above ($\bar{l} > l^*$) and 5 percent below ($\bar{l} > l^*$) the optimal level.

The production function displays diminishing returns in capital,

$$f(k_t) = k_t^{\alpha}. \tag{27}$$

and we set capital's share to $\alpha = 0.3$. The quarterly depreciation rate of capital is $\delta = 0.02$.

The matching function is Cobb-Douglas, $m(u, v) = \psi u_t^{\xi_u} v_t^{1-\xi_u}$, with $\xi_u = 0.4$, in line with the evidence in Blanchard and Diamond (1989), and ψ chosen so that the quarterly job-finding rate of a searching individual is 90 percent. The resulting value is $\psi = 0.9$. Given the Cobb-Douglas matching specification, this also directly fixes the matching rate for an open vacancy at the same value, $q(\theta) = 0.90$.

For our baseline results, we fix the Nash bargaining weight at $\eta = 0.40$ so that it satisfies the usual Hosios (1990) condition $\eta = \xi_u$ for partial-equilibrium search efficiency. We use this as our baseline setting because our main focus is on the consequences of inefficient participation *even if* the Hosios parametrization is in place. To show how our results reconcile with Domeij (2005), however, we also conduct some robustness exercises with respect to this parameter.

6.2 Quantitative Results

The upper panel of table 1 presents key steady-state allocations and policy variables under the Ramsey plan for three alternative parameterizations of the model, each of which correspond to a different level of labor force participation. The middle row presents allocations and policy variables from our baseline calibration in which participation is fixed at its efficient level, $\bar{l} = l^* = 0.66$. The

[10] We focus on just this latter time period because by the late 1980's the trend rise in women's labor force participation had largely played out, so that this latest period perhaps represents a true steady state participation rate.

	τ^k	τ^n	ϕ	u	n	w	c	K	y	c/y	K/y	$\gamma v/y$	g/y
							Ramsey allocations						
$\bar{l} < l^*$	0.271	0.297	0.253	0.028	0.582	1.322	0.843	13.150	1.484	0.569	8.864	0.007	0.202
$\bar{l} = l^*$	0	0.300	0	0.066	0.594	1.614	0.888	15.852	1.591	0.558	9.966	0.004	0.189
$\bar{l} > l^*$	−0.072	0.319	−0.304	0.168	0.542	1.762	0.789	14.946	1.466	0.538	10.197	0.002	0.205
							Socially-efficient allocations						
	—	—	0	0.073	0.655	1.614	1.011	17.494	1.755	0.576	9.966	0.004	0.171

Table 1: Steady-state Ramsey and socially-efficient allocations.

top row presents the same information with an exogenously imposed labor force participation rate that is *below* the efficient level ($\bar{l} = 0.61 < l^*$) so that leisure is inefficiently high, and the bottom row imposes a fixed labor force participation rate that is *above* the efficient level ($\bar{l} = 0.71 > l^*$) so that leisure is inefficiently low. For each case, we assume the Hosios parameterization, $\eta = \xi_u$, holds. Finally, the lower panel of the table presents the allocations in the socially efficient equilibrium, as outlined in section 3.

Comparing the middle row of the top panel to the socially efficient allocations, the Ramsey allocations differ from the socially efficient allocations because the Ramsey planner is forced to finance government spending via labor income taxation. Relative to the socially efficient outcome, the distortion generated by the labor income tax causes the household to substitute out of both work and search activity and into leisure, with u and n falling in equal proportion. Consumption, the aggregate capital stock, and output are all, of course, lower as a result. Note, though, that in this case the K/y ratio, the Nash wage, and labor-market tightness (not shown) are identical in both the Ramsey and socially-efficient allocations. Finally, the fact that the capital income tax is zero in the Ramsey equilibrium follows directly from proposition 1.

In order to shed light on how deviations in labor force participation from the second best Ramsey equilibrium, l^*, influence the baseline results, compare the middle row of the top panel ($\bar{l} = l^*$) to the top row ($\bar{l} < l^*$). When participation is constrained to be inefficiently low, the household over-consumes leisure implying that the shadow value of adjustment along the participation margin is non-zero, $\phi = 0.253$. In absence of a more direct fiscal instrument to affect participation, the Ramsey planner can use the capital tax to indirectly ease the resulting labor market distortion. The basic intuition builds on the fact that the household is exogenously forced to consume more leisure than it would otherwise want to in absence of the fixed participation constraint. With too few workers in the eocnomy, output is also pushed below its constrained efficient level. The Ramsey planner can help mitigate this distortion by increasing the capital tax. The higher capital tax increases the hiring incentive of firms by lowering the Nash wage through a reduction of the capital stock (we have that $\frac{\partial w}{\partial k} = -\eta f''(k)k > 0$ in the steady state version of equation 14). Increased vacancy posting on the part of firms helps to lower the unemployment rate $(u/(u+n))$, thereby putting more searching individuals into productive jobs which mitigates the total distortion to output. A similar, but opposite, intuition applies when labor force participation is inefficiently high, as shown in the bottom row.

Table 2 shows how the optimal tax rates in the Ramsey equilibrium vary with bargaining power across the three different parameterizations for labor force participation. The middle column replicates the optimal tax rates presented in table 1 at the Hosios condition, $\eta = \xi_u$, for each of the three parameterizations of labor force participation. We then vary bargaining power in the neighborhood of the Hosios condition to examine the sensitivity of the optimal tax results.

	$\xi_u = 0.4$				
	$\eta = 0.2$	$\eta = 0.3$	$\eta = 0.4$	$\eta = 0.5$	$\eta = 0.6$
	Inefficiently Low LFP, ($\bar{\mathbf{l}} < \mathbf{l}^*$)				
τ^k	50.8	38.6	27.1	15.7	3.8
τ^n	28.5	28.9	29.7	30.7	32.0
	Efficient LFP, ($\bar{\mathbf{l}} = \mathbf{l}^*$)				
τ^k	10.7	5.1	0	-5.3	-11.6
τ^n	28.8	29.3	30.0	30.9	32.2
	Inefficiently High LFP, ($\bar{\mathbf{l}} > \mathbf{l}^*$)				
τ^k	-2.9	-4.8	-7.2	-10.4	-14.7
τ^n	30.6	31.1	31.9	32.8	34.1

Table 2: Optimal tax rates

With efficient labor force participation ($\bar{l} = l^*$) the middle panel of the table tells a similar story to the results presented in Domeij (2005): the optimal capital tax remains fairly low, varying between -11 and 11 percent as η varies between 0.2 and 0.6. Looking at the upper and lower panels of the middle column, even with efficient bargaining, the table shows that efficiency along the labor force participation margin has a significant quantitative effect on the optimal capital tax. Furthermore, this impact grows even larger when the participation distortion interacts with the bargaining distortion (moving either to the left or the right of the middle column). Thus, a key thing to take from this table is that efficiency along the labor force participation margin is an important determinate of the optimal capital tax; quantitatively, it is at least as important as bargaining efficiency.

Finally, table 3 shows how sensitive our results are to alternative assumptions regarding preferences. The top panel considers higher risk aversion, with $\sigma = 3$ as opposed to $\sigma = 1$ in the baseline and the bottom panel considers a higher elasticity of labor force participation, with $\iota = 2.5$ as opposed to $\iota = 1$ in the baseline.

				$\xi_u = 0.4$		
		$\eta = 0.2$	$\eta = 0.3$	$\eta = 0.4$	$\eta = 0.5$	$\eta = 0.6$
			Risk aversion, $\sigma = 3$			
Inefficiently Low LFP, $(\bar{l} < l^*)$	τ^k	20.1	11.5	4.5	−2.9	−8.9
	τ^n	24.9	26.3	27.5	28.6	29.7
Efficient LFP, $(\bar{l} = l^*)$	τ^k	10.7	5.1	0	−5.3	−11.6
	τ^n	26.4	27.2	28.0	28.9	30.0
Inefficiently High LFP, $(\bar{l} > l^*)$	τ^k	−2.9	−4.9	−7.4	−10.6	−15.1
	τ^n	28.7	29.1	29.5	30.1	30.9
			Labor force participation elasticity, $\iota = 2.5$			
Inefficiently Low LFP, $(\bar{l} < l^*)$	τ^k	53.6	46.0	36.8	26.4	14.3
	τ^n	63.6	58.2	54.8	52.9	52.3
Efficient LFP, $(\bar{l} = l^*)$	τ^k	10.7	5.1	0	−5.3	−11.6
	τ^n	45.7	45.5	46.0	46.8	48.3
Inefficiently High LFP, $(\bar{l} > l^*)$	τ^k	−2.8	−4.6	−6.8	−9.7	−13.8
	τ^n	47.4	48.2	49.2	50.5	52.4

Table 3: Sensitivity of optimal tax rates to preference parameters

Provided the model is efficient along the labor force participation margin, the optimal capital tax is essentially insensitive to these two preference parameters. This echoes the sensitivity results presented in Domeij (2005). In contrast, the optimal capital tax appears quite sensitive to preference assumptions in the presence of an inefficiency in the participation margin. Regardless of whether participation is too high or too low, both the Ramsey optimal capital and labor income tax rates vary considerably with η for different values of either risk aversion or the elasticity of labor force participation.

7 Conclusion

We study optimal capital income taxation in a DGE model with search and matching frictions in the labor market. Our main result is to show analytically that inefficient labor force participation gives rise to a non-zero capital income taxation in the long run. If participation is inefficiently low, a capital tax can be implemented to stifle vacancy postings through a reduction in the profitability of job creation. Doing so pushes labor market tightness closer to its constrained efficient level. The opposite intuition holds when participation is inefficiently high. In a calibrated version of our economy, we show that the quantitative magnitude of the optimal capital income tax is quite sensitive to the size of the inefficiency in the participation margin. By highlighting the importance of this margin for the most basic optimal fiscal policy prescription, our work raises an important cautionary note for typical formulations of DSGE labor search models, especially as they are increasingly applied to study policy questions.

A Deriving the Pseudo-Labor-Force Participation Condition

In the pseudo-LFP model, we view the household as facing the problem of choosing state-contingent processes for $\{c_t\}$, $\{K_{t+1}\}$, $\{b_t\}$, $\{u_t^h\}$, and $\{n_t^h\}$ to maximize

$$\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1 - u_t^h - n_t^h) \right] \tag{28}$$

subject to sequences of flow budget constraints

$$c_t + K_{t+1} + b_t = (1 - \tau_t^n) w_t n_t^h + \left[1 + (1 - \tau_t^k)(r_t - \delta) \right] K_t + R_t b_{t-1} + (1 - \tau^d) d_t, \tag{29}$$

perceived laws of motion for the employment stock

$$n_t^h = (1 - \rho^x) n_{t-1}^h + u_t^h p(\theta_t), \tag{30}$$

and the restriction that the size of the labor force is fixed

$$n_t^h + u_t^h = \bar{l}. \tag{31}$$

As discussed in Section 2, this formulation is an analytical tool. The households in this environment do not actually make any decisions regarding the stocks of u^h and n^h; rather, by casting the model in this way, we are able to track the shadow price on the restriction $n^h + u^h = \bar{l}$ that *would* make it optimal for the household to always choose, if it could do so, the sum $n^h + u^h$.

Denote by $\{\lambda_t\}$, $\{\mu_t^h\}$, and $\{\phi_t\}$ the sequences of Lagrange multipliers on the sequences of these constraints, respectively. The first-order conditions with respect to c_t, K_{t+1}, b_t, u_t^h and n_t^h are, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{32}$$

$$-\lambda_t + \beta R_t \lambda_{t+1} = 0, \tag{33}$$

$$-\lambda_t + \beta \lambda_{t+1} \left[1 + (1 - \tau_{t+1}^k)(r_{t+1} - \delta) \right] = 0, \tag{34}$$

$$-g'(1 - u_t^h - n_t^h) + \mu_t^h p(\theta_t) - \phi_t = 0, \tag{35}$$

and

$$-\mu_t^h + \lambda_t (1 - \tau_t^n) w_t - g'(1 - u_t^h - n_t^h) - \phi_t + \beta (1 - \rho^x) \mu_{t+1}^h = 0. \tag{36}$$

Conditions (32) and (33) clearly yield a standard bond-demand condition, which is expression (10) in the main text. Conditions (32) and (34) yield a standard capital-supply condition, which is expression (11) in the main text.

To obtain the pseudo-labor-force-participation (LFP) condition, start with (35) and (36). Solving (35) for μ_t^h,

$$\mu_t^h = \frac{g'(1 - u_t^h - n_t^h)}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)}, \tag{37}$$

in which we have used the result $\lambda_t = u'(c_t)$, which follows from (32). Using this expression and its period $t+1$ analog in (36), we have

$$\frac{g'(1-u_t^h-n_t^h)}{p(\theta_t)}+\frac{\phi_t}{p(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h)-\phi_t+\beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}^h-n_{t+1}^h)}{p(\theta_{t+1})}+\frac{\phi_{t+1}}{p(\theta_{t+1})}\right\}, \tag{38}$$

which is the pseudo-LFP condition presented in condition (12) in the main text.

B Nash Bargaining

The Nash bargaining problem and hence the wage rule does not depend on whether or not labor force participation is fixed or endogenous. The bargaining-relevant value equations are defined using the household-level envelope conditions. A household's state variable is its beginning-of-period-t employment stock, n^h_{t-1}. Regardless of whether or not participation is fixed or endogenous, the household perceives that its employment evolves according to

$$n^h_t = (1-\rho^x)n^h_{t-1} + p(\theta_t)u^h_t. \tag{39}$$

Define $\mathbf{V}(n^h_{t-1})$ as the value function associated with the optimal plan that solves the household problem. The envelope condition is thus $\mathbf{V}'(n^h_{t-1}) = (1-\rho^x)\mu^h_t$, where μ^h_t is the value to the household of having one more family member employed in period t. In turn, this value is given by

$$\mu^h_t = \lambda_t(1-\tau^n_t)w_t - g'(1-u^h_t-n^h_t) + \beta(1-\rho^x)\mu^h_{t+1} \tag{40}$$

because an additional employed member brings to the household an after-tax wage (measured in utility — λ_t is the marginal utility of wealth), incurs disutility for the household by decreasing its leisure, and has a continuation value from the perspective of the household. Using this value μ^h_t, we can express the envelope condition as

$$\frac{\mathbf{V}'(n^h_{t-1})}{1-\rho^x} = \lambda_t(1-\tau^n_t)w_t - g'(1-u^h_t-n^h_t) + \beta(1-\rho^x)\left(\frac{\mathbf{V}'(n^h_t)}{1-\rho^x}\right), \tag{41}$$

in which we have normalized by $1-\rho^x$ due to the timing of events.

To express things in units of goods, define $\mathbf{W_t}$ as

$$\begin{aligned}\mathbf{W_t} \equiv \frac{\mathbf{V}'(n^h_{t-1})}{\lambda_t(1-\rho^x)} &= -\frac{g'(1-u^h_t-n^h_t)}{\lambda_t} + (1-\tau^n_t)w_t + (1-\rho^x)\left(\frac{\beta\mathbf{V}'(n^h_{t-1})}{\lambda_t(1-\rho^x)}\right)\\ &= -\frac{g'(1-u^h_t-n^h_t)}{\lambda_t} + (1-\tau^n_t)w_t + (1-\rho^x)\left(\Xi_{t+1|t}\mathbf{W_{t+1}}\right).\end{aligned} \tag{42}$$

The second line makes use of the definition of the one-period-ahead stochastic discount factor, $\Xi_{t+1|t} \equiv \beta\lambda_{t+1}/\lambda_t$.

In the pseudo-LFP model, $n^h_{t-1} + u^h_{t-1} = \bar{l}$, which means that, given we have used n^h_{t-1} as the state variable of the household problem, we do not need to also specify u^h_{t-1} as a state. Hence, the value to the household of an unemployed family member is $\mathbf{U_t} \equiv \left(\partial\mathbf{V_t}/\partial u^h_{t-1}\right)/\lambda_t = 0$.

On the firm side, the value of having an additional employee is

$$\mathbf{J_t} = f(k_t) - w_t - r_t k_t + (1-\rho^x)\left(\Xi_{t+1|t}\mathbf{J_{t+1}}\right); \tag{43}$$

for use below, note that $\mathbf{J_t} = \frac{\gamma}{q(\theta_t)}$.

In generalized Nash bargaining, the parties choose w_t in every period t to maximize

$$(\mathbf{W_t} - \mathbf{U_t})^{\eta}\, \mathbf{J_t}^{1-\eta}. \tag{44}$$

The solution to this problem gives the time-t generalized Nash sharing rule, $\frac{\mathbf{W_t}}{1-\tau_t^n} = \frac{\eta}{1-\eta}\mathbf{J_t}$.

Now proceed to derive an explicit expression for w_t. Inserting the definition of $\mathbf{W_t}$ into the Nash sharing rule,

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}\left(\Xi_{t+1|t}\mathbf{W_{t+1}}\right) = \frac{\eta}{1-\eta}\mathbf{J_t}, \tag{45}$$

and then using the time-$t+1$ Nash sharing rule,

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}\left(\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\mathbf{J_{t+1}}\right) = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{46}$$

Make the substitution $\mathbf{J_t} = \frac{\gamma}{q(\theta_t)}$, and similarly for $\mathbf{J_{t+1}}$, which yields

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}\left(\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\frac{\gamma}{q(\theta_{t+1})}\right) = \frac{\eta}{1-\eta}\frac{\gamma}{q(\theta_t)}. \tag{47}$$

Next, use the job-creation condition $\frac{\gamma}{q(\theta_t)} = f(k_t) - w_t - r_t k_t + (1-\rho^x)\left(\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right)$ to substitute on the right-hand-side, which gives

$$-\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + w_t + \frac{1-\rho^x}{1-\tau_t^n}\left(\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\eta}{1-\eta}\frac{\gamma}{q(\theta_{t+1})}\right) = \frac{\eta}{1-\eta}\left[f(k_t) - w_t - r_t k_t + (1-\rho^x)\left(\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right)\right]. \tag{48}$$

Grouping terms involving w_t,

$$\begin{aligned} w_t\left[1+\frac{\eta}{1-\eta}\right] &= \frac{\eta}{1-\eta}(f(k_t) - r_t k_t) + \frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} \\ &- \frac{\eta}{1-\eta}\frac{1-\rho^x}{1-\tau_t^n}\left(\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{q(\theta_{t+1})}\right) + \frac{\eta}{1-\eta}(1-\rho^x)\left(\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right). \end{aligned} \tag{49}$$

Multiplying by $1-\eta$ gives

$$\begin{aligned} w_t &= \eta(f(k_t) - r_t k_t) + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} \\ &- \eta\frac{1-\rho^x}{1-\tau_t^n}\left(\Xi_{t+1|t}(1-\tau_{t+1}^n)\frac{\gamma}{q(\theta_{t+1})}\right) + \eta(1-\rho^x)\left(\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right). \end{aligned} \tag{50}$$

Finally, because this is a deterministic environment, combine the terms on the last line to arrive at

$$w_t = \eta(f(k_t) - r_t k_t) + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + \eta(1-\rho^x)\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\left[\frac{\tau_{t+1}^n - \tau_t^n}{1-\tau_t^n}\right], \tag{51}$$

which is expression (14) in the main text, given that $\lambda_t = u'(c_t)$. Note that if $\tau_{t+1}^n = \tau_t^n = \tau^n$, the last two terms cancel with each other and the wage collapses to a simple static split,

$$w_t = \eta(f(k_t) - r_t k_t) + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau^n)}. \tag{52}$$

C Social Planner Problem

We prove the efficiency condition presented in Proposition 1. The social planner's problem is

$$\max_{c_t,u_t,n_t,v_t,k_t,K_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1 - n_t - u_t)\right] \tag{53}$$

subject to the sequences of resource constraints,

$$c_t + g_t + K_{t+1} - (1-\delta)K_t + \gamma v_t = n_t f(k_t), \tag{54}$$

law of motion for the employment stock,

$$n_t = (1-\rho^x)n_{t-1} + m(u_t, v_t), \tag{55}$$

and relationship between match-level capital and aggregate capital,

$$K_t = n_t k_t. \tag{56}$$

Let λ_t^1, λ_t^2, and λ_t^3 be the sequences of Lagrange multipliers on these constraints, respectively. The first order conditions with respect to c_t, u_t, n_t, v_t, k_t, and K_{t+1}, respectively, are

$$u'(c_t) - \lambda_t^1 = 0, \tag{57}$$

$$-g'(1-n_t-u_t) + \lambda_t^2 m_u(u_t, v_t) = 0, \tag{58}$$

$$-g'(1-n_t-u_t) + \lambda_t^1 f(k_t) - \lambda_t^2 + \beta(1-\rho_x)\lambda_{t+1}^2 + \lambda_t^3 k_t = 0, \tag{59}$$

$$-\gamma\lambda_t^1 + \lambda_t^2 m_v(u_t, v_t) = 0, \tag{60}$$

$$n_t \lambda_t^1 f'(k_t) + n_t \lambda_t^3 = 0, \tag{61}$$

and

$$-\lambda_t^1 + \beta(1-\delta)\lambda_{t+1}^1 - \beta\lambda_{t+1}^3 = 0. \tag{62}$$

Because we are concerned only with the long-run Ramsey equilibrium, impose deterministic steady state on conditions (57) through (62). Taking the ratio of equations (58) and (60) and then substituting equation (57) along with the fact that, due to the Cobb-Douglas matching technology, $\frac{m_u(u_t,v_t)}{m_v(u_t,v_t)} = \theta_t \frac{\xi_u}{1-\xi_u}$, into the resulting expression, we obtain the following expression for steady state efficiency in the search labor market

$$\frac{g'(1-n-u)}{u'(c)} = \gamma\theta\frac{\xi_u}{1-\xi_u} \tag{63}$$

which is equation (20) in the text.

Next, solving equation (61) for λ_t^3, substituting the resulting expression into equation (62) and imposing steady state yields the following expression for efficiency along the capital accumulation margin

$$\frac{1}{\beta} - 1 + \delta = f'(k), \tag{64}$$

which is equation (21) in the text.

D Derivation of Implementability Constraint

The derivation of the implementability constraint follows that laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). The derivation presented here nests that for the both the pseudo-LFP model and the LFP model — obtaining the implementability constraint for the latter from the former requires setting the shadow value on the exogenous labor-force size restriction to zero, as has been discussed in the text.

From the pseudo-labor-force participation model, we will be able to use the household's (pseudo) perceived law of motion for employment,

$$n_t = (1-\rho^x)n_{t-1} + p(\theta_t)u_t^h, \tag{65}$$

the size restriction

$$n_t + u_t^h = \bar{l}, \tag{66}$$

and the pseudo-labor-force-participation condition

$$\frac{g'(1-u_t^h-n_t^h)}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) - \phi_t + \beta(1-\rho^x)E_t\left\{\frac{g'(1-u_{t+1}^h-n_{t+1}^h)}{p(\theta_{t+1})} + \frac{\phi_{t+1}}{p(\theta_{t+1})}\right\}. \tag{67}$$

Start with the household flow budget constraint in equilibrium

$$c_t + K_{t+1} + b_t = (1-\tau_t^n)w_t n_t + \left[1+(1-\tau_t^k)(r_t-\delta)\right]K_t + R_t b_{t-1} + (1-\tau^d)d_t. \tag{68}$$

Multiply by $\beta^t u'(c_t)$ and sum over dates and states starting from $t=0$,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} + \sum_{t=0}^{\infty}\beta^t u'(c_t)b_t = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t n_t$$
$$+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]K_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)R_t b_{t-1} + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t.$$

Use the household's bond-demand condition, $u'(c_t) = \beta u'(c_{t+1})R_{t+1}$, to substitute for $u'(c_t)$ in the term on the left-hand-side involving b_t,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} + \sum_{t=0}^{\infty}\beta^{t+1} u'(c_{t+1})R_{t+1}b_t = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t n_t$$
$$+\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]K_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)R_t b_{t-1} + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t.$$

Canceling terms in the third summation on the left hand side with the third summation on the right hand side leaves only the time-zero bond position,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)K_{t+1} = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau_t^n)w_t n_t$$
$$+\quad\sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1+(1-\tau_t^k)(r_t-\delta)\right]K_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1}.$$

Next, use the capital supply condition $u'(c_t) = \beta u'(c_{t+1})\left[1 + (1-\tau^k_{t+1})(r_{t+1}-\delta)\right]$, to substitute for $u'(c_t)$ in the second summation on the left hand side,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t + \sum_{t=0}^{\infty}\beta^{t+1}u'(c_{t+1})\left[1 + (1-\tau^k_{t+1})(r_{t+1}-\delta)\right]K_{t+1} = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^n_t)w_t n_t$$
$$+ \quad \sum_{t=0}^{\infty}\beta^t u'(c_t)\left[1 + (1-\tau^k_t)(r_t-\delta)\right]K_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1}.$$

Canceling terms in the second summation on the left hand side with the second summation on the right hand side leaves only the time-zero capital position,

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^n_t)w_t n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)\left\{R_0 b_{-1} + \left[1 + (1-\tau^k_0)(r_0-\delta)\right]K_0\right\}. \tag{69}$$

Next, use (67) to substitute for the sequence of terms $u'(c_t)(1-\tau^n_t)w_t$ in the first summation on the right hand side, which gives

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t\left[\frac{g'_t}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)}\right]n_t + \sum_{t=0}^{\infty}\beta^t\left[g'_t + \phi_t\right]n_t$$
$$- \quad (1-\rho^x)\sum_{t=0}^{\infty}\beta^{t+1}\left[\frac{g'_{t+1}}{p(\theta_{t+1})} + \frac{\phi_{t+1}}{p(\theta_{t+1})}\right]n_t + \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t$$
$$+ \quad u'(c_0)\left\{R_0 b_{-1} + \left[1 + (1-\tau^k_0)(r_0-\delta)\right]K_0\right\}.$$

To further conserve on notation, we now use g'_t to stand for $g'(1-u^h_t-n^h_t)$.

Next, use $n_t = (1-\rho^x)n_{t-1} + p(\theta_t)u^h_t$ to substitute for the sequence of n_t terms that appear in the first summation on the right hand side, which gives

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = (1-\rho^x)\sum_{t=0}^{\infty}\beta^t\left[\frac{g'_t}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)}\right]n_{t-1}$$
$$+ \quad \sum_{t=0}^{\infty}\beta^t\left[\frac{g'_t}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)}\right]p(\theta_t)u^h_t + \sum_{t=0}^{\infty}\beta^t\left[g'_t + \phi_t\right]n_t - (1-\rho^x)\sum_{t=0}^{\infty}\beta^{t+1}\left[\frac{g'_{t+1}}{p(\theta_{t+1})} + \frac{\phi_{t+1}}{p(\theta_{t+1})}\right]n_t$$
$$+ \quad \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)\left\{R_0 b_{-1} + \left[1 + (1-\tau^k_0)(r_0-\delta)\right]K_0\right\}.$$

The first summation on the right hand side cancels with the fourth summation on the right hand side, leaving only the time-zero term:

$$\sum_{t=0}^{\infty}\beta^t u'(c_t)c_t = \sum_{t=0}^{\infty}\beta^t\left[\frac{g'_t}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)}\right]p(\theta_t)u^h_t + \sum_{t=0}^{\infty}\beta^t\left[g'_t + \phi_t\right]n_t$$
$$+ \quad \sum_{t=0}^{\infty}\beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)\left\{R_0 b_{-1} + \left[1 + (1-\tau^k_0)(r_0-\delta)\right]K_0\right\} + (1-\rho^x)\left[\frac{g'_0}{p(\theta_0)} + \frac{\phi_0}{p(\theta_0)}\right]n_{-1}.$$

Expanding and rearranging the first summation on the right-hand-side,

$$\sum_{t=0}^{\infty} \beta^t u'(c_t)c_t = \sum_{t=0}^{\infty} \beta^t \left[g'_t + \phi_t\right] u_t^h + \sum_{t=0}^{\infty} \beta^t \left[g'_t + \phi_t\right] n_t$$
$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)\left\{R_0 b_{-1} + \left[1 + (1-\tau_0^k)(r_0 - \delta)\right] K_0\right\} + (1-\rho^x)\left[\frac{g'_0}{p(\theta_0)} + \frac{\phi_0}{p(\theta_0)}\right] n_{-1}.$$

Finally, regrouping some terms, we have the present-value implementability constraint

$$\sum_{t=0}^{\infty} \beta^t \left\{u'(c_t)c_t - \left[g'(1 - u_t^h - n_t) + \phi_t\right](n_t + u_t^h) - u'(c_t)(1-\tau^d)d_t\right\} = A_0, \tag{70}$$

where

$$A_0 \equiv u'(c_0)\left\{R_0 b_{-1} + \left[1 + (1-\tau_0^k)(r_0 - \delta)\right] K_0\right\} + (1-\rho^x)\left[\frac{g'_0}{p(\theta_0)} + \frac{\phi_0}{p(\theta_0)}\right] n_{-1}. \tag{71}$$

E Proof of Propositions 2 and 3

We prove Propositions 2 and 3 in several steps. First, we describe the $\Psi(\cdot)$ function (equation (23), which is key for the results. We then construct the Ramsey first-order conditions and consider the deterministic steady state. Using these first-order conditions, we show that a zero capital income tax is optimal if the derivative of $\Psi(\cdot)$ with respect to the capital stock is zero (this was the idea of the discussion in the text following Proposition 3). Once this is established, the last steps are to show that the derivative of $\Psi(\cdot)$ with respect to the capital stock is indeed zero for each of the cases described in Propositions 2 and 3.

E.1 The $\Psi(\cdot)$ function

Three conditions describe thE period-t equilibrium of the labor market: the vacancy-creation condition

$$\frac{\gamma}{q(\theta_t)} = f(k_t) - w_t - r_t k_t + (1-\rho^x)\left[\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right], \tag{72}$$

the Nash wage equation

$$w_t = \eta(f(k_t) - r_t k_t) + (1-\eta)\frac{g'(1-u_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} + \eta(1-\rho^x)\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\left[\frac{\tau_{t+1}^n-\tau_t^n}{1-\tau_t^n}\right], \tag{73}$$

and the pseudo-LFP condition

$$\frac{g'(1-u_t^h-n_t^h)}{p(\theta_t)} + \frac{\phi_t}{p(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - g'(1-u_t^h-n_t^h) - \phi_t + \beta(1-\rho^x)\left[\frac{g'(1-u_{t+1}^h-n_{t+1}^h)}{p(\theta_{t+1})} + \frac{\phi_{t+1}}{p(\theta_{t+1})}\right]. \tag{74}$$

We summarize the labor-market equilibrium by the function $\Psi(\cdot)$. To construct $\Psi(\cdot)$, divide equation (72) by equation (74), observe that $\theta = p(\theta)/q(\theta)$ because of constant-returns matching, and define the following two terms:

$$\Delta_t^1 \equiv (1-\eta)\left(f(k_t) - f'(k_t)k_t - \frac{g'(1-u_t-n_t)+\phi_t}{u'(c_t)(1-\tau_t^n)}\right) + (1-\rho^x)\Xi_{t+1|t}\left[\eta\frac{\tau_t^n-\tau_{t+1}^n}{1-\tau_t^n} + \frac{\gamma}{q(\theta_{t+1})}\right] \tag{75}$$

$$\Delta_t^2 \equiv \eta\left(f(k_t) - f'(k_t)k_t - \frac{g'(1-u_t-n_t)+\phi_t}{u'(c_t)(1-\tau_t^n)}\right) \tag{76}$$

$$+ \quad (1-\rho^x)\Xi_{t+1|t}\left[\eta\frac{\tau_t^n-\tau_{t+1}^n}{1-\tau_t^n} + \frac{g'(1-u_{t+1}-n_{t+1})+\phi_{t+1}}{u'(c_{t+1})}\right]. \tag{77}$$

Also, impose the market clearing conditions $r_t = f'(k_t)$ and $K_t = n_t k_t$ to substitute out r_t and K_t, and recognize that the equilibrium value of the discount factor is $\Xi_{t+1|t} = \frac{\beta u'(c_{t+1})}{u'(c_t)}$. We then define $\Psi(c_t, c_{t+1}, u_t, u_{t+1}, n_t, n_{t+1}, v_t, v_{t+1}, k_t, \tau_t^n, \tau_{t+1}^n)$ as

$$\Psi(\cdot) = \left(\frac{g'(1-u_t-n_t)+\phi_t}{u'(c_t)}\right)\frac{\Delta_t^1}{\Delta_t^2} - (1-\tau_t^n)\gamma\theta_t. \tag{78}$$

This function captures (72), (73), and (74) because we obtained it by first dividing (72) by (74), and decomposing (73) into the Δ_t^1 and Δ_t^2 functions, each of which is a component of $\Psi(\cdot)$.

As noted in the discussion following Proposition 2, if households optimally choose their participation rate (which implies $\phi_t = 0 \ \forall t$) and the Hosios parametrization $\eta = \xi_u$ holds, $\Psi(\cdot)$ takes a much simpler form

$$\Psi(\cdot) = \frac{g'(1-n_t-u_t)}{u'(c_t)} - (1-\tau_t^n)\gamma\theta_t\frac{\xi_u}{1-\xi_u}, \tag{79}$$

which is independent of the capital stock. We prove this below.

E.2 Ramsey First-Order Conditions

The Ramsey government's problem is

$$\max_{c_t,u_t,n_t,v_t,K_{t+1},k_t,\tau_t^n} \sum_{t=0}^{\infty} \beta^t \left[u(c_t) + g(1-n_t-u_t)\right] \tag{80}$$

subject to the PVIC

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{u'(c_t)c_t - \left[g'(1-u_t-n_t)+\phi_t\right](n_t+u_t) - u'(c_t)(1-\tau^d)d_t\right\} = A_0, \tag{81}$$

and the sequences of constraints

$$c_t + g_t + K_{t+1} - (1-\delta)K_t + \gamma v_t = n_t f(k_t), \tag{82}$$

$$n_t = (1-\rho^x)n_{t-1} + m(u_t, v_t), \tag{83}$$

$$\Psi(c_t, c_{t+1}, u_t, u_{t+1}, n_t, n_{t+1}, v_t, v_{t+1}, k_t, \tau_t^n, \tau_{t+1}^n) = 0, \tag{84}$$

$$K_t = n_t k_t, \tag{85}$$

and, if participation in the decentralized economy is fixed at $\bar{l}$,

$$\bar{l} = n_t + u_t. \tag{86}$$

Let λ^1 be the Lagrange multiplier on the PVIC, and λ_t^2, λ_t^3, λ_t^4, λ_t^5, and λ_t^6 be the multipliers on the other constraints, respectively. In the following, recall that we are considering fixed $\tau^d = 1$.

The first-order-conditions with respect to, c_t, u_t, n_t, v_t, K_{t+1}, k_t, and τ_t^n are, respectively:

$$u'(c_t) - \lambda^1\left((u''(c_t)c_t + u'(c_t)) + u''(c_t)(1-\tau^d)d_t\right) - \lambda_t^2 - \frac{1}{\beta}\lambda_{t-1}^4\frac{\partial\Psi_{t-1}}{\partial c_t} - \lambda_t^4\frac{\partial\Psi_t}{\partial c_t} = 0 \tag{87}$$

$$-g_t' - \lambda^1\left(g_t'' - (g_t' + \phi_t)\right) + \lambda_t^3 m_u(u_t, v_t) - \frac{1}{\beta}\lambda_{t-1}^4\frac{\partial\Psi_{t-1}}{\partial u_t} - \lambda_t^4\frac{\partial\Psi_t}{\partial u_t} + \lambda_t^6 = 0 \tag{88}$$

$$\begin{aligned} & -g_t' - \lambda^1\left(g_t'' - (g_t' + \phi_t)\right) + \lambda_t^2 f(k_t) \\ -\lambda_t^3 + \beta(1-\rho^x)\lambda_{t+1}^3 - \frac{1}{\beta}\lambda_{t-1}^4\frac{\partial\Psi_{t-1}}{\partial n_t} - \lambda_t^4\frac{\partial\Psi_t}{\partial n_t} + \lambda_t^5 k_t + \lambda_t^6 \quad &= \quad 0 \end{aligned} \tag{89}$$

$$-\gamma\lambda_t^2 + \lambda_t^3 m_v(u_t, v_t) - \frac{1}{\beta}\lambda_{t-1}^4 \frac{\partial \Psi_{t-1}}{\partial v_t} - \lambda_t^4 \frac{\partial \Psi_t}{\partial v_t} = 0 \tag{90}$$

$$-\lambda_t^2 + \beta(1-\delta)\lambda_{t+1}^2 - \beta\lambda_{t+1}^5 = 0 \tag{91}$$

$$-n_t f'(k_t)\lambda_t^2 - \lambda_t^4 \frac{\partial \Psi_t}{\partial k_t} + n_t \lambda_t^5 = 0 \tag{92}$$

$$-\frac{1}{\beta}\lambda_{t-1}^4 \frac{\partial \Psi_{t-1}}{\partial \tau_t^n} - \lambda_t^4 \frac{\partial \Psi_t}{\partial \tau_t^n} = 0. \tag{93}$$

The partial derivatives $\frac{\partial \Psi_{t-1}}{\partial c_t}$, $\frac{\partial \Psi_t}{\partial c_t}$, $\frac{\partial \Psi_{t-1}}{\partial u_t}$, $\frac{\partial \Psi_t}{\partial u_t}$, $\frac{\partial \Psi_{t-1}}{\partial n_t}$, $\frac{\partial \Psi_t}{\partial n_t}$, $\frac{\partial \Psi_{t-1}}{\partial v_t}$, $\frac{\partial \Psi_t}{\partial v_t}$, $\frac{\partial \Psi_{t-1}}{\partial \tau_t^n}$, $\frac{\partial \Psi_t}{\partial \tau_t^n}$, and $\frac{\partial \Psi_t}{\partial k_t}$ are all defined in the final section of this appendix.

E.3 $\frac{\partial \Psi(\cdot)}{\partial k_t} = 0$ and Zero Capital Tax

We now establish that $\frac{\partial \Psi(\cdot)}{\partial k_t} = 0$ is a sufficient condition for the optimality of a zero long-run capital tax. The interpretation of the condition $\frac{\partial \Psi(\cdot)}{\partial k_t} = 0$ is that the labor-market equilibrium is independent of the capital stock.

Using the steady state version of (92) to solve for λ^5 and inserting the resulting expression into (91) yields

$$\lambda^2 \left(\frac{1}{\beta} - 1 + \delta + f'(k) \right) - \frac{1}{n}\lambda^4 \frac{\partial \Psi}{\partial k} = 0. \tag{94}$$

From the household's steady-state Euler equation for capital, $1 = \beta \left[1 + (1-\tau^k)(r-\delta) \right]$, we can replace the term in parentheses in the previous expression to get

$$\lambda^2 \left(\tau^k (\delta - f'(k)) \right) - \frac{1}{n}\lambda^4 \frac{\partial \Psi}{\partial k} = 0. \tag{95}$$

This last expression can be solved for the optimal capital tax,

$$\tau^k = \frac{1}{n(\delta - f'(k))} \frac{\lambda^4}{\lambda^2} \frac{\partial \Psi}{\partial k}. \tag{96}$$

Note that $\lambda^2 \neq 0$ because the resource constraint always binds given standard Inada conditions on the consumption subutility function. Furthermore, the only circumstance under which $\lambda^4 = 0$ is if the labor market equilibrium achieves the constrained-efficient outcome — that is, if condition (20) holds. It will be clear below that this is not the case in the Ramsey equilibrium because $\tau^n > 0$. Thus, equation (96) tells us that in the Ramsey equilibrium, the only circumstance under which $\tau^k = 0$ will be if $\frac{\partial \Psi}{\partial k} = 0$. This concludes the first part of the proof.

E.4 Conditions Under Which $\frac{\partial \Psi(\cdot)}{\partial k} = 0$

We turn now to the particular conditions, outlined in propositions 2 and 3, to show that for each of the cases described there, $\frac{\partial \Psi}{\partial k} = 0$, hence $\tau^k = 0$ in the Ramsey steady state.

E.4.1 Proposition 2

In order to conclude our proof of proposition 2, begin by explicitly writing out the Δ_t^1 and Δ_t^2 terms in the definition of $\Psi(\cdot)$:

$$\Psi(\cdot) = \frac{g_t' + \phi_t}{u'(c_t)} \frac{f(k_t) - w_t - f'(k_t)k_t + (1-\rho^x)\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}}{(1-\tau_t^n)w_t + \frac{g_t'+\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{p(\theta_{t+1})}\frac{g_{t+1}'+\phi_{t+1}}{u'(c_{t+1})}} - \gamma\theta_t. \tag{97}$$

The strategy is to first guess that, as long as the Hosios condition holds and labor force participation is efficient in the decentralized economy, $\Psi(\cdot)$ reduces to:

$$\Psi(\cdot) = \frac{g'(1-u_t-n_t)}{u'(c_t)} - (1-\tau_t^n)\frac{\xi_u}{1-\xi_u}\gamma\theta_t. \tag{98}$$

We now proceed to verify that this guess is indeed correct under the conditions outlined in proposition 2.

In order for equation (97) to coincide with equation (98), it must be that the second fraction to the right of the equals sign satisfies the following condition

$$\frac{f(k_t) - w_t - f'(k_t)k_t + (1-\rho^x)\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}}{(1-\tau_t^n)w_t + \frac{g_t'+\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{p(\theta_{t+1})}\frac{g_{t+1}'+\phi_{t+1}}{u'(c_{t+1})}} = \frac{1}{1-\tau^n}\frac{1-\xi_u}{\xi_u} \tag{99}$$

Solving this expression for w_t yields

$$w_t = \xi_u\left(f(k_t) - f'(k_t)k_t + (1-\rho^x)\Xi_{t+1|t}\frac{\gamma}{q(\theta_{t+1})}\right) \tag{100}$$

$$+(1-\xi_u)\frac{1}{1-\tau_t^n}\left(-\frac{g_t'+\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{p(\theta_{t+1})}\frac{g_{t+1}'+\phi_t}{u'(c_t)}\right) \tag{101}$$

Note that the first term in parentheses is $\mathbf{J}_t + w_t$ while the second term in parentheses is $\mathbf{W}_t - \mathbf{U}_t - (1-\tau_t^n)w_t - \frac{\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{q(\theta_{t+1})}\frac{\phi_{t+1}}{u'(c_{t+1})}$. Inserting these into the immediately previous equation above and rearranging yields

$$w_t = \xi_u\mathbf{J}_t + \xi_u w_t - (1-\xi_u)\frac{1}{1-\tau_t^n}(\mathbf{W}_t - \mathbf{U}_t) + (1-\xi_u)w_t \tag{102}$$

$$-\frac{\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{q(\theta_{t+1})}\frac{\phi_{t+1}}{u'(c_{t+1})}. \tag{103}$$

Eliminating the w_t term reduces this to

$$0 = \xi_u\mathbf{J}_t - (1-\xi_u)\frac{1}{1-\tau_t^n}(\mathbf{W}_t - \mathbf{U}_t) - \frac{\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{q(\theta_{t+1})}\frac{\phi_{t+1}}{u'(c_{t+1})} \tag{104}$$

$$0 = \frac{1}{1-\tau_t^n}(\mathbf{W}_t - \mathbf{U}_t)\left(\xi_u\frac{1-\eta}{\eta} - (1-\xi_u)\right) - \frac{\phi_t}{u'(c_t)} + (1-\rho^x)\Xi_{t+1|t}\frac{1}{q(\theta_{t+1})}\frac{\phi_{t+1}}{u'(c_{t+1})}, \tag{105}$$

where in the second step we have used the Nash sharing rule, $\eta \mathbf{J}_t = \frac{1}{1-\tau_t^n}(1-\eta)(\mathbf{W}_t - \mathbf{U}_t)$. Clearly, this equation is satisfied as long as the Hosios condition holds ($\eta = \xi_u$) and labor force participation is set optimally (which implied $\phi = 0$). In this case, the we have verified the original conjecture that $\Psi(\cdot)$ collapses to expression (98).

It follows that equation (98) is not a function of the capital stock, so that $\frac{\partial \Psi_t}{\partial k_t} = 0$. This, along with equation (96) in the previous section, tells us that $\tau^k = 0$ in the Ramsey equilibrium when ($\eta = \xi_u$) and $\phi = 0$. This completes the proof of proposition 2.

E.4.2 Proposition 3 (Case One)

Now consider the cases presented in Proposition 3. For the special case in which $g'(\bar{l}) = -\phi$, the labor market constraint reduces to

$$\Psi(\cdot) = -(1 - \tau_t^n)\gamma\theta_t \tag{106}$$

It is clear that $\frac{\partial \Psi_t}{\partial k_t} = 0$ in the special case in which the subutility function over leisure is exogenously parameterized in such a way that $g'(\bar{l}) = -\phi$. Furthermore, imposing steady state on the Ramsey first order condition on τ^n in this special case yields

$$-\lambda^4 \gamma\theta = 0. \tag{107}$$

It follows that $\lambda^4 = 0$. In other words, the labor market does not impose any constraint on the Ramsey planner when setting optimal policy.

This proves special case one in proposition 3 and provides analytical support to our contention that the labor tax acts as a lump sum instrument in this special case.

E.4.3 Proposition 3 (Case Two)

For the special case in which $\rho^x = 1$, we have that $\Delta_t^1 = \Delta_t^2$, so that the labor market constraint reduces to

$$\Psi(\cdot) = \left(\frac{g'(1 - u_t - n_t) + \phi_t}{u'(c_t)}\right)\frac{1-\eta}{\eta} - (1 - \tau_t^n)\gamma\theta_t \tag{108}$$

In the special case in which matches dissolve after every period, so that $\rho^x = 1$, we have that $\frac{\partial \Psi_t}{\partial k_t} = 0$. This proves special case two in proposition 3.

E.5 Partial Derivatives

In this subsection, we define the partial derivatives $\frac{\partial \Psi_{t-1}}{\partial c_t}$, $\frac{\partial \Psi_t}{\partial c_t}$, $\frac{\partial \Psi_{t-1}}{\partial u_t}$, $\frac{\partial \Psi_t}{\partial u_t}$, $\frac{\partial \Psi_{t-1}}{\partial n_t}$, $\frac{\partial \Psi_t}{\partial n_t}$, $\frac{\partial \Psi_{t-1}}{\partial v_t}$, $\frac{\partial \Psi_t}{\partial v_t}$, $\frac{\partial \Psi_{t-1}}{\partial \tau_t^n}$, $\frac{\partial \Psi_t}{\partial \tau_t^n}$, and $\frac{\partial \Psi_t}{\partial k_t}$ in the Ramsey first order conditions.

$$\frac{\partial \Psi_{t-1}}{\partial c_t} = \frac{g'_{t-1} + \phi_{t-1}}{u'(c_{t-1})} \left(\frac{\frac{\partial \Psi_{1,t-1}}{\partial c_t}}{\Delta^2_{t-1}} + \frac{\Delta^1_{t-1}}{\Delta^2_{t-1}} \frac{\frac{\partial \Delta^2_{t-1}}{\partial c_t}}{\Delta^2_{t-1}} \right) \tag{109}$$

where:

$$\frac{\partial \Delta^1_{t-1}}{\partial c_t} = (1-\rho^x)\beta \frac{u''(c_t)}{u'(c_{t-1})} \left(\eta \frac{\tau_{t-1} - \tau_t}{1 - \tau_{t-1}} + \gamma p \left(\frac{v_t}{u_t} \right) \right) \tag{110}$$

$$\frac{\partial \Delta^2_{t-1}}{\partial c_t} = (1-\rho^x)\beta \left(\frac{u''(c_t)}{u'(c_{t-1})} \left(\eta \frac{\tau_{t-1} - \tau_t}{1 - \tau_{t-1}} + \frac{g'_t + \phi_t}{u'(c_t)} \right) + \frac{u'(c_t)}{u'(c_{t-1})} \left(\frac{g'_t + \phi_t}{u'(c_t)} \frac{u''(c_t)}{u'(c_t)} \right) \right) \tag{111}$$

$$\frac{\partial \Psi_t}{\partial c_t} = \frac{g'_t + \phi_t}{u'(c_t)} \frac{u''(c_t)}{u'(c_t)} \frac{\Delta^1_t}{\Delta^2_t} + \frac{g'_t + \phi_t}{u'(c_t)} \left(\frac{\frac{\partial \Delta^1_t}{\partial c_t}}{\Delta^2_t} + \frac{\Delta^1_t}{\Delta^2_t} \frac{\frac{\partial \Delta^2_t}{\partial c_t}}{\Delta^2_t} \right) \tag{112}$$

where

$$\frac{\partial \Delta^1_t}{\partial c_t} = (1-\eta) \frac{g'_t + \phi_t}{u'(c_t)(1-\tau^n_t)} \frac{u''(c_t)}{u'(c_t)} - (1-\rho^x)\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{u''(c_t)}{u'(c_t)} \left(\eta \frac{\tau^n_t - \tau^n_{t+1}}{1 - \tau^n_{t-1}} + \gamma p \left(\frac{v_{t+1}}{u_{t+1}} \right) \right) \tag{113}$$

$$\frac{\partial \Delta^2_t}{\partial c_t} = \eta \frac{g'_t + \phi_t}{u'(c_t)(1-\tau^n_t)} \frac{u''(c_t)}{u'(c_t)} - (1-\rho^x)\beta \frac{u'(c_{t+1})}{u'(c_t)} \frac{u''(c_t)}{u'(c_t)} \left(\eta \frac{\tau^n_t - \tau^n_{t+1}}{1 - \tau^n_{t-1}} + \frac{g'_{t+1} + \phi_{t+1}}{u'(c_{t+1})} \right) \tag{114}$$

$$\frac{\partial \Psi_{t-1}}{\partial n_t} = \frac{g'_{t-1} + \phi_{t-1}}{u'(c_{t-1})} \left(\frac{\Delta^1_{t-1}}{\Delta^2_{t-1}} \frac{(1-\rho^x)\beta \frac{g''_t}{u'(c_t)} \frac{u'(c_t)}{u'(c_{t-1})}}{\Delta^2_{t-1}} \right) \tag{115}$$

$$\frac{\partial \Psi_t}{\partial n_t} = \frac{g''_t + \phi_t}{u'(c_t)} \frac{\Delta^1_t}{\Delta^2_t} + \frac{g'_t + \phi_t}{u'(c_t)} \left(\frac{(1-\eta)\frac{g''_t}{u'(c_t)(1-\tau^n_t)}}{\Delta^2_t} + \frac{\Delta^1_t}{\Delta^2_t} \frac{\eta \frac{g''_t}{u'(c_t)(1-\tau^n_t)}}{\Delta^2_t} \right) \tag{116}$$

$$\frac{\partial \Psi_{t-1}}{\partial u_t} = \frac{g'_{t-1} + \phi_{t-1}}{u'(c_{t-1})} \left(\frac{-(1-\rho^x)\beta \frac{u'(c_t)}{u'(c_{t-1})} \gamma p' \left(\frac{v_t}{u_t} \right) \frac{v_t}{u_t^2}}{\Delta^2_{t-1}} + \frac{\Delta^1_{t-1}}{\Delta^2_{t-1}} \frac{(1-\rho^x)\beta \frac{g''_t}{u'(c_t)} \frac{u'(c_t)}{u'(c_{t-1})}}{\Delta^2_{t-1}} \right) \tag{117}$$

$$\frac{\partial \Psi_t}{\partial u_t} = \frac{g''_t + \phi_t}{u'(c_t)} \frac{\Delta^1_t}{\Delta^2_t} + \frac{g'_t + \phi_t}{u'(c_t)} \left(\frac{(1-\eta)\frac{g''_t}{u'(c_t)(1-\tau^n_t)}}{\Delta^2_t} + \frac{\Delta^1_t}{\Delta^2_t} \frac{\eta \frac{g''_t}{u'(c_t)(1-\tau^n_t)}}{\Delta^2_t} \right) + (1-\tau^n_t)\gamma \frac{v_t}{u_t} \frac{1}{u_t} \tag{118}$$

$$\frac{\partial \Psi_{t-1}}{\partial v_t} = \frac{g'_{t-1} + \phi_{t-1}}{u'(c_{t-1})} \left(\frac{(1-\rho^x)\beta \frac{u'(c_t)}{u'(c_{t-1})} \gamma p' \left(\frac{v_t}{u_t} \right) \frac{1}{u_t}}{\Delta^2_{t-1}} \right) \tag{119}$$

$$\frac{\partial \Psi_t}{\partial v_t} = \frac{g'_t + \phi_t}{u'(c_t)} \frac{\Delta^1_t}{\Delta^2_t} - (1-\tau^n_t)\gamma \frac{1}{u_t} \tag{120}$$

$$\frac{\partial \Psi_t}{\partial k_t} = \frac{g'_t + \phi_t}{u'(c_t)} \frac{\Delta^1_t}{\Delta^2_t} - \frac{g'_t + \phi_t}{u'(c_t)} \left(\frac{(1-\eta) f_{kk}(k_t,)}{\Psi_{2,t}} + \frac{\Psi_{1,t}}{\Psi_{2,t}} \frac{\eta f_{kk}(k_t,)}{\Delta^2_t} \right) \tag{121}$$

$$\frac{\partial \Psi_{t-1}}{\partial \tau^n_t} = -\frac{g'_{t-1} + \phi_{t-1}}{u'(c_{t-1})} \left(\frac{(1-\rho^x)\beta \frac{u'(c_t)}{u'(c_{t-1})} \eta \frac{1}{1-\tau^n_t}}{\Delta^2_{t-1}} + \frac{\Delta^1_{t-1}}{\Delta^2_{t-1}} \frac{(1-\rho^x)\beta \frac{u'(c_t)}{u'(c_{t-1})} \eta \frac{1}{1-\tau^n_t}}{\Delta^2_{t-1}} \right) \tag{122}$$

$$\begin{aligned} \frac{\partial \Psi_t}{\partial \tau^n_t} \quad = \quad & \frac{g'_t + \phi_t}{u'(c_t)} \frac{\Psi_{1,t}}{\Psi_{2,t}} + \frac{g'_t + \phi_t}{u'(c_t)} \left(\frac{\eta \frac{g'_t + \phi_t}{u'(c_t)(1-\tau^n_t)^2} + (1-\rho^x)\beta \frac{u'(c_t)}{u'(c_{t-1})} \eta \frac{1-\tau^n_{t+1}}{(1-\tau^n_t)^2}}{\Delta^2_t} \right) \\ & + \frac{g'_t + \phi_t}{u'(c_t)} \left(\frac{\Delta^1_t}{\Delta^2_t} \frac{\eta \frac{g'_t + \phi_t}{u'(c_t)(1-\tau^n_t)^2} + (1-\rho^x)\beta \frac{u'(c_t)}{u'(c_{t-1})} \eta \frac{1-\tau^n_{t+1}}{(1-\tau^n_t)^2}}{\Delta^2_t} \right) + \gamma \frac{v_t}{u_t} \end{aligned} \tag{123}$$

References

Albanesi, Stefania and Roc Armenter. 2007. "Intertemporal Distortions in the Second Best." NBER Working Paper 13629.

Andolfatto, David. 1996. "Business Cycles and Labor-Market Search." *American Economic Review,* Vol. 86, pp. 112-132.

Armenter, Roc. 2008. "A Note on Incomplete Factor Taxation." *Journal of Public Economics.* Forthcoming.

Arseneau, David M., and Sanjay K. Chugh. 2008. "Optimal Fiscal and Monetary Policy with Costly Wage Bargaining." *Journal of Monetary Economics* , Vol. 55 , pp. 1401-1414.

Arseneau, David M., Sanjay K. Chugh, and Andre Kurmann. 2008. "Optimal Capital Taxation in an Economy with Capital Allocation Frictions." University of Maryland.

Blanchard, Olivier and Peter Diamond. 1989. "The Beveridge Curve." *Brookings Papers on Economic Activity,* Vol. 1, pp. 1-76.

Blanchard, Olivier and Jordi Gali. 2007. "Real Wage Rigidities and the New Keynesian Model." *Journal of Money, Credit, and Banking,* Vol. 39, pp. 35-65.

Blanchard, Olivier and Jordi Gali. 2008. "A New Keynesian Model with Unemployment." MIT.

Chamley, Christophe. 1986. "The Welfare Cost of Capital Income Taxation in a Growing Economy." *Econometrica,* Vol. 54, pp. 607-622.

Chari V. V., and Patrick J. Kehoe. 1999. "Optimal Fiscal and Monetary Policy. In *Handbook of Macroeconomics,* edited by John B. Taylor and Michael Woodford, Vol. 1C. Elsevier.

Correia, Isabel. 1996. "Should Capital Income be Taxed in the Steady State?." *Journal of Public Economics,* Vol. 60, pp. 147-151.

Davis, Steven J, R. Jason Faberman, and John Haltiwanger. 2006. "The Flow Approach to Labor Markets: New Data Sources and Micro-Macro Links." *Journal of Economic Perspectives,* Vol. 20, pp. 3-26.

Domeij, David. 2005. "Optimal Capital Taxation and Labor Market Search." *Review of Economic Dynamics,* Vol. 8, pp. 623-650.

Gertler, Mark and Antonella Trigari. 2009. "Unemployment Fluctuations with Staggered Nash Bargaining." *Journal of Political Economy,* Vol. 117(1), pp. 38-86.

Hagedorn, Marcus and Iourii Manovskii . 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited ." *American Economic Review* , Vol. 98(4), pp. 1692-1706.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics,* Vol. 16, pp. 309-327.

Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies,* Vol. 57, pp. 279-298.

Jones, Larry E., Rodolfo E. Manuelli, and Peter E. Rossi. 1997. "On the Optimal Taxation of Capital Income." *Journal of Economic Theory,* Vol. 73, pp. 93-117.

Judd, Kenneth L. 2002. "Capital Income Taxation with Imperfect Competition." *American Economic Review,* Vol. 92, pp. 417-421.

Krause, Michael U., David Lopez-Salido, and Thomas A. Lubik. 2008. "Do Search Frictions Matter for Inflation Dynamics." *European Economic Review,* Vol. 52(8), pp. 1464-1479.

Krause, Michael U. and Thomas A. Lubik. 2007. "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions." *Journal of Monetary Economics,* Vol. 54, pp. 706-727.

Ljungqvist, Lars and Thomas J. Sargent. 2004. *Recursive Macroeconomic Theory.* MIT Press.

Lucas, Robert E. Jr. and Nancy L. Stokey. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics,* Vol. 12, pp. 55-93.

Merz, Monika. 1995. "Search in the Labor Market and the Real Business Cycle." *Journal of Monetary Economics,* Vol. 36, pp. 269-300.

Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory.* MIT Press.

Rogerson, Richard. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics,* Vol. 21, pp. 3-16.

Rogerson, Richard, Robert Shimer, and Randall Wright. 2005. "Search-Theoretic Models of the Labor Market: A Survey." *Journal of Economic Literature,* Vol. 43, pp. 959-988.

Schmitt-Grohe, Stephanie and Martin Uribe. 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." NBER Macroeconomics Annual 2005.

Shi, Shouyong and Quan Wen. 1999. "Labor Market Search and the Dynamic Effects of Taxes and Subsidies." *Journal of Monetary Economics,* Vol. 43, pp. 457-495.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review,* Vol. 95, pp. 25-49.

Trigari, Antonella. 2009. "Equilibrium Unemployment, Job Flows and Inflation Dynamics." *Journal of Money, Credit, and Banking,* Vol. 41(1), pp. 1-33.

Walsh, Carl E. 2005. "Labor Market Search, Sticky Prices, and Interest Rate Policies." *Review of Economic Dynamics,* Vol. 8, pp. 829-849.