

# Tax Smoothing in Frictional Labor Markets <sup>\*</sup>

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## Abstract

The optimality of tax smoothing is re-examined from the point of view of frictional labor markets. The main result is that, in a calibrated matching model that generates empirically-relevant labor-market fluctuations conditional on exogenous fiscal policy, the Ramsey-optimal policy calls for extreme labor-tax-rate volatility. Purposeful tax volatility induces dramatically smaller, but efficient, fluctuations of labor markets by keeping distortions constant over the business cycle. We relate the results to standard Ramsey theory by developing welfare-relevant concepts of efficiency and distortions that take into account primitive matching frictions and that can be applied to any general-equilibrium matching model. Although the basic Ramsey principles of “wedge-smoothing” and zero intertemporal distortions hold in a matching framework, whether or not they imply tax smoothing depends on whether or not wages are set efficiently.

**Keywords:** labor market frictions, optimal taxation, labor wedge, zero intertemporal distortions

**JEL Classification:** E24, E50, E62, E63

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# 1 Introduction

We have two main aims in this paper. The first is to re-examine a classic issue in the theory of fiscal policy — the optimality of labor-income tax smoothing — from the point of view of frictional labor markets. Since Barro’s (1979) partial-equilibrium intuition, Lucas and Stokey’s (1983) general-equilibrium analysis, and continuing through to today’s quantitative DSGE models used to study optimal fiscal policy, the prescription that governments ought to hold labor tax rates virtually constant in the face of aggregate shocks is well-known to macroeconomists.

We show that this benchmark optimal policy prescription does not carry over to a general-equilibrium matching model that, conditional on exogenous fiscal policy, is calibrated to generate empirically-observed labor market fluctuations. The optimal degree of tax-rate variability is orders of magnitude larger than the cornerstone tax-*smoothing* result of the Ramsey literature. Purposeful tax volatility induces efficient fluctuations in labor markets by keeping distortions constant over the business cycle. Thus, while the goal of optimal policy is to “smooth wedges” just as in standard Ramsey models, the very nature of “wedges” and how they map into taxes depends on the nature of wage determination in a model of search and matching frictions and, in particular, whether or not wages are set efficiently.

The second aim of our work is thus to develop a welfare-relevant notion of efficiency for general-equilibrium matching models. As part of the recent widespread application of DSGE models with matching frictions in labor markets, many studies have focused on the transmission channels of policy and the determination of optimal policy. Efficiency concerns lie at the heart of any model studying policy. In light of this, another central contribution of the paper is to develop a welfare-relevant concept of efficiency that not only clearly shows the conditions under which tax-smoothing is and is not optimal, but also is likely to be helpful in interpreting other results in the literature.

The starting point of the analysis is a general-equilibrium search and matching model that incorporates a labor force participation decision. Modeling participation in search frameworks has recently attracted attention because it may be an important margin of adjustment in labor markets.<sup>1</sup> The model is calibrated so that it generates labor-market fluctuations similar along many key dimensions to those observed in U.S. data. The baseline model’s fluctuations are conditional on exogenous productivity, government spending, and labor tax-rate processes, each of which is also calibrated to U.S. data. This baseline exogenous-policy model generates a Beveridge curve and matches well the cyclical volatilities of employment, unemployment, vacancies, and participation.

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<sup>1</sup>While not as large as cyclical employment fluctuations in the U.S., at one-third the volatility of employment, fluctuations in participation are not trivial. In a variety of applications, Veracierto (2008), den Haan and Kaltenbrunner (2009), Krusell, Mukoyama, Rogerson, and Sahin (2008), and Ebell (2010), among others, have introduced participation margins into matching models.

The parameters that are most important for the exogenous-policy model’s cyclical properties are the wage elasticity of participation, low bargaining power on the part of workers, and large unemployment transfers. The latter two features embody a calibration approach for matching models used by Hagedorn and Manovskii (2008) (henceforth, HM). The HM calibration effectively delivers a rigid (pre-tax) real wage under Nash bargaining which, as pointed out by Hall (2005) and Shimer (2005), is required for a matching model to generate labor-market fluctuations in line with those observed in U.S. data.

Keeping fixed the structural parameters from the exogenous-policy model, we then endogenize tax policy by solving the Ramsey problem. The Ramsey-optimal labor income tax rate is very volatile over the business cycle, orders of magnitude more volatile than benchmark tax-smoothing results in the Ramsey literature. The labor-market dynamics induced by optimal taxes are vastly different from their counterparts in the exogenous-tax model: the Beveridge curve disappears, and the volatilities of employment, unemployment, and vacancies are all much smaller. The *low* volatility of *quantities* induced by *large* volatility of *taxes* is efficient in a sense we make precise. The Ramsey government can thus be understood as using labor tax volatility to ensure efficient labor-market fluctuations. From a policy perspective, the results suggest that, when viewed through the lens of the calibrated model, actual U.S. tax policy has produced too smooth a labor tax rate, resulting in suboptimally-large labor market volatility. In contrast, the results show that the optimal policy calls for considerably more labor tax volatility so that the after-tax real wage can more efficiently cushion the labor market from underlying shocks.

The optimality of tax volatility can be understood through the lens of standard Ramsey theory. As applied to dynamic macroeconomic models, two basic principles of normative tax theory are that distortions affecting static margins should be kept constant across the business cycle and that intertemporal distortions should be set to zero.<sup>2</sup> We show that these two basic principles also characterize Ramsey-optimal policy in a general-equilibrium matching model. Moreover, achieving so-called “wedge-smoothing” is necessary and sufficient for the Ramsey government to induce efficient labor-market fluctuations. These basic Ramsey insights thus have nothing to do with whether or not markets feature inherent frictions that even a social planner cannot transcend, as matching frictions are typically viewed in the literature. The standard RBC notions of efficiency and wedges do not apply, however, so wedge-smoothing does not imply tax smoothing. What is necessary for application of basic Ramsey insights is welfare-relevant notions of efficiency and distortions.

Towards this end, we develop precise notions of static and intertemporal marginal rates of

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<sup>2</sup>The former insight traces back to Barro (1979) in a partial-equilibrium analysis, Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991) in general-equilibrium frameworks, and was recently renewed by Werning (2007) for models featuring some types of heterogeneity. The latter insight is the foundation of the well-known Chamley (1986) and Judd (1985) zero-capital-tax result, which was recently generalized by Albanesi and Armenter (2011).

transformation (MRT) that explicitly treat matching as a primitive. These concepts of MRTs allow us to characterize efficiency for general-equilibrium matching models using only the elementary principle that MRTs should be equated to their associated marginal rates of substitution (MRS) along both the static and intertemporal margins. Our notion of efficiency builds on the well-known Hosios (1990) condition for (partial-equilibrium) search efficiency and nests as a special case the standard RBC notions of the static MRT between leisure and output and the intertemporal MRT between consumption in different time periods.

This characterization of efficiency allows us to define a novel, explicitly search-based, *labor wedge* between MRSs and MRTs that has both static and intertemporal dimensions. This welfare-relevant notion of the labor wedge nests as a special case the RBC notion of the labor wedge emphasized by Chari, Kehoe, and McGrattan (2007) and Shimer (2009). As search-theoretic frameworks become increasingly common in general equilibrium analysis, it is useful to be able to describe transformation frontiers and the MRTs implied by them in general ways.

Applying this view of labor wedges to the fiscal policy questions of our model allows us to pinpoint the conditions under which tax smoothing is or is not optimal, even though labor markets are always frictional. Specifically, the two aspects of the HM-style calibration that generate the real wage rigidity required for the model to match the data — low worker bargaining power and high unemployment transfers, both of which influence wages through Nash bargaining — result in inefficiently-time-varying components of static and intertemporal wedges. The labor tax is then used to generate sufficient flexibility in the after-tax real wage so that wedges are stabilized and the labor market is insulated from the underlying shocks. The optimal degree of labor tax volatility that achieves this is large: the standard deviation is about one percent around a long-run tax rate of about 10 percent. For comparison, the Ramsey literature’s conventional tax-smoothing result entails optimal tax-rate volatility of 0.1 percent or less.<sup>3</sup> Thus, optimal labor tax rates are one or more orders of magnitude more volatile in our baseline matching framework than in frictionless models. Nonetheless, the welfare-relevant wedges we identify are stabilized to the same virtually complete degree as in simple Ramsey models.

The mapping from the dynamics of distortions to the dynamics of taxes depends on the nature of wage determination. This is the crucial difference between Ramsey analysis in frictionless models versus in search models. In the former, conditional on the tax system not being overdetermined, there is a unique decentralization of Ramsey allocations using market prices (wages) and government policy. In the latter, because prices themselves are indeterminate in equilibrium — the key insight of Hall (2005) and a point re-emphasized by Rogerson and Shimer (2011) — any inefficiency implied

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<sup>3</sup>For example, see Chari and Kehoe (1999) or Werning (2007). These baseline tax smoothing results are obtained in environments in which government debt repayments are fully state-contingent, which means that not only does the Ramsey government *want* to smooth taxes but is *able* to do so.

by pricing can in principle be completely undone by policy. In our calibrated model, optimal tax volatility ultimately stems from inefficiencies in the way the surplus is split in the decentralized economy. We derive this result in a model with matching frictions using Nash bargaining as the wage determination mechanism implemented with a particular rigid wage calibration (the HM calibration). Thus, in the calibrated model, optimal tax volatility is ultimately driven by an insufficiently flexible real wage. It is important to emphasize, however, that neither matching frictions nor Nash bargaining are necessary to generate this result. What is required is some friction that generates a surplus — matching frictions in our model — that is then split in some inefficient way — real wage rigidity in our model, regardless of whether that rigidity stems from Nash bargaining with a particular calibration or otherwise.

This paper contributes to the growing literature on optimal policy in search-and-matching models. Regarding fiscal policy, Domeij (2005) and Arseneau and Chugh (2006) study long-run optimal capital-income taxation, and their main result is that non-zero capital-income taxes can offset wage-setting distortions. Arseneau and Chugh (2008) study the dynamic properties of both optimal fiscal and monetary policy, but they focus on the effects of nominal wage rigidities; nominal wage rigidities is also the focus of Thomas (2008). Faia (2008) studies optimal monetary policy in a New Keynesian environment with matching frictions, as do Ravenna and Walsh (2011) and Blanchard and Gali (2010); the latter two derive linear-quadratic formulations standard in New Keynesian analysis that show why labor-market stabilization may explicitly be part of a central bank’s objectives. Ravenna and Walsh (2011) and Blanchard and Gali (2010) both find, as we do, that wage-setting frictions call for active policy intervention. In addition to our focus on the dynamics of fiscal policy, an important contribution of our paper is to make explicit the margins and wedges that are the basis for the policy insights that emerge from search-based models.

The rest of our work is organized as follows. Section 2 lays out the decentralized search and matching economy. Section 3 calibrates a non-Ramsey version of the model that matches well basic cyclical properties of labor markets. Using the calibrated model, Section 4 then studies the Ramsey problem and shows that optimal tax dynamics and their implied labor market dynamics are vastly different compared to the data and compared to the basic Ramsey literature. To parse the results, Section 5 develops our static and intertemporal notions of MRTs and efficiency, and Section 6 shows which features of the decentralized search and bargaining economy disrupt efficiency. Section 7 uses these concepts of efficiency and distortions to discuss several aspects of the model and results, including how tax smoothing could in principle be optimal despite the presence of matching frictions. Section 8 concludes.

## 2 Model

The model features matching frictions in the labor market that impede transitions of individuals from search unemployment to employment, as well as a labor-force participation decision on the part of households. The model thus depicts individuals in three labor-market states: employment, search unemployment, and outside the labor force (which we interchangeably refer to as “leisure”).

### 2.1 Labor Market Accounting

The model uses the “instantaneous hiring” view of transitions between search unemployment and employment, in which new hires begin working right away, rather than with a one-period delay. This timing is empirically descriptive of U.S. labor-market flows at a quarterly frequency (see, for example, the evidence in Davis, Faberman, and Haltiwanger (2006)), and has recently become fairly common in general-equilibrium matching models, used by, among others, Blanchard and Gali (2010) and Krause, Lopez-Salido, and Lubik (2007).

To introduce some basic notation of the model, suppose that  $n_{t-1}$  individuals worked in period  $t-1$ . At the beginning of any period  $t$ , a fraction  $\rho$  of employment relationships that were active in period  $t-1$  experiences separations. Some of these newly-separated individuals may immediately enter the period- $t$  job-search process, as may some individuals who were non-participants in the labor market in period  $t-1$ ; these two groups taken together constitute the measure  $s_t$  of individuals searching for jobs in period  $t$ . Of these  $s_t$  individuals,  $(1-p_t)s_t$  individuals turn out to be unsuccessful in their job searches, where  $p_t$  is the probability that a searching individual finds a job, which is a market-determined variable. The measure  $n_t = (1-\rho)n_{t-1} + s_t p_t$  of individuals are thus employed and produce in period  $t$ . Each of the  $(1-p_t)s_t$  individuals who does not find a job receives an unemployment transfer  $\chi$  from the government. With these definitions and timing events, the measured labor force in period  $t$  is  $lfp_t = n_t + (1-p_t)s_t$ . Figure 1 summarizes the timing of the model.

### 2.2 Households

There is a representative household in the economy. Each household consists of a continuum of measure one of family members, and each individual family member is classified as either inside the labor force or outside the labor force. With the notation introduced above,  $lfp_t$  individuals participate in the labor force in period  $t$ , and  $1-lfp_t$  are non-participants. An individual family member that is outside the labor force enjoys leisure. An individual family member that is part of the labor force is engaged in one of two activities: working, or not working but actively searching for a job. The convenience of an “infinitely-large” household is that we can naturally suppose

that each individual family member experiences the same level of consumption regardless of his personal labor-market status. This tractable way of modeling perfect consumption insurance in general-equilibrium search-theoretic models of labor markets has been common since Andolfatto (1996) and Merz (1995). We use the terms “individual” and “family member” interchangeably from here on.

For periods  $t = 0, 1, \dots$ , the representative household chooses state-contingent decision rules for consumption  $c_t$ , bond holdings  $b_t$ , search activity  $s_t$ , and the desired stock of employment  $n_t^h$  to maximize expected lifetime discounted utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - h((1 - p_t)s_t + n_t^h) \right] \quad (1)$$

subject to a sequence of flow budget constraints

$$c_t + b_t = n_t^h(1 - \tau_t^n)w_t + (1 - p_t)s_t\chi + R_t b_{t-1} + (1 - \tau^d)d_t \quad (2)$$

and a sequence of perceived laws of motion for the measure of family members that are employed,

$$n_t^h = (1 - \rho)n_{t-1}^h + s_t p_t. \quad (3)$$

The job-finding probability  $p_t$  is taken as given by the household. The function  $u(\cdot)$  is a standard strictly-increasing and strictly-concave subutility function over consumption, and the function  $h(\cdot)$  is strictly increasing and strictly convex in the size of the labor force.<sup>4</sup>

Due to firms’ sunk resource and time costs of finding employees, firms earn positive flows of economic profits. These profits are transferred to households at the end of each period in lump-sum fashion:  $d_t$  is the household’s receipts of firms’ flow profits, which are taxed at the fixed tax rate  $\tau^d$ . As is well-understood in the Ramsey literature, flows of untaxed dividends received by households in and of themselves affect optimal-policy prescriptions.<sup>5</sup> To make our results as comparable as possible to baseline models that prescribe labor-tax-rate smoothing, in which there are zero economic profits/dividends, our main analysis is conducted assuming  $\tau^d = 1$ . The consequence of this assumption is that any predictions made by our model regarding optimal labor-income taxation cannot be due to incentives to tax profits, either in the long-run or the short-run. Robustness tests presented in Appendix G consider the opposite extreme of  $\tau^d = 0$  and show that the main optimal-policy results are unaffected.

The rest of the notation and primitives of the household maximization problem are as follows. The pre-tax real wage each employed individual earns is  $w_t$ , and the after-tax real wage is  $(1 - \tau_t^n)w_t$ .

<sup>4</sup>Given the definitions presented above, sometimes we will write  $h(lfp_t)$ .

<sup>5</sup>See, for example, Stiglitz and Dasgupta (1971), Jones, Manuelli, and Rossi (1997), Schmitt-Grohe and Uribe (2004), and Siu (2004) for examples in various contexts of this type of taxation incentive.

As described below, the wage-determination mechanism is Nash bargaining; households take the wage-setting protocol as given. The household’s holdings of a state-contingent one-period real government bond at the end of period  $t - 1$  are  $b_{t-1}$ , each of which has gross state-contingent payoff  $R_t$  at the beginning of period  $t$ . Finally, because this is a Ramsey-taxation model, there are no lump-sum taxes or transfers between the government and the private sector.<sup>6</sup>

We frequently will make analogies with the RBC model. To this end, it is helpful to interpret the measure  $1 - lfp_t$  of individuals outside the labor force as enjoying leisure. Hence, we use the terms leisure and non-participation interchangeably. A utility value of leisure is sometimes what is meant when partial-equilibrium labor-search models refer casually to the “outside benefit of not working.” Our model formalizes this idea. Our model also has a second notion of the outside benefit, the unemployment benefit  $\chi$ , which is assumed to be time-invariant.

The derivation of the household’s optimality conditions is presented in Appendix A; here we simply intuitively describe the outcomes. One condition arising from household optimization is a standard consumption-savings condition,

$$u'(c_t) = E_t [\beta u'(c_{t+1})R_{t+1}]. \quad (4)$$

As usual, this condition defines the one-period-ahead stochastic discount factor,  $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$ , with which firms, in equilibrium, discount profit flows.

The other optimality condition is the household’s *labor-force participation (LFP) condition*,

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t \left[ (1 - \tau_t^n)w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \left( \frac{1 - p_{t+1}}{p_{t+1}} \right) \left( \frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right] + (1 - p_t)\chi. \quad (5)$$

The LFP condition has straightforward interpretation: at the optimum, the household sends a fraction of individuals to search for jobs such that the MRS between participation and consumption is equated to the expected payoff of searching. The payoff is either an unemployment benefit  $\chi$  in the event of unsuccessful search (which happens with probability  $1 - p_t$ ) or, if search is successful, an immediate after-tax wage plus an expected discounted continuation value. The LFP condition is thus simply a free-entry condition on the part of households into the labor force.

The LFP condition has a similar interpretation to the labor-supply function in a neoclassical labor market, which is the foundation for baseline Ramsey models used to study the optimality of tax smoothing. In our model, a neoclassical labor-supply function is recovered by setting  $\rho = 1$  (all employment relationships are one-period transactions), setting  $\chi = 0$  (because there is no notion of “unemployment” hence no notion of “unemployment benefits” in a neoclassical model), and fixing the probability of “finding a job” to  $p_t = 1 \forall t$  (because in a neoclassical market there is no friction

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<sup>6</sup>When we consider how the model economy behaves in response to exogenous fiscal policy in Section 3, we do temporarily allow for lump-sum taxation because there we are not studying government financing issues. For the Ramsey analysis in Section 4, lump-sum taxes are fixed to zero.

in “finding a job”). Imposing these restrictions on (5) gives  $\frac{h'(\cdot)}{u'(\cdot)} = (1 - \tau_t^n)w_t$ , which defines the labor-supply function in a neoclassical market.

With matching frictions that create a meaningful separation of the labor force into those individuals that are employed and those individuals that are unemployed, the LFP condition (5) defines transitions of individuals from outside the labor force (leisure) into the pool of searching unemployed, from where the aggregate matching process pulls some individuals into employment.

### 2.3 Firms

On the production side of the economy, there is a measure one of identical firms, so we can consider the representative firm. The representative firm is “large” in the sense that it operates many jobs and consequently has many individual workers attached to it through those jobs. The firm requires only labor to produce its output. The firm must engage in costly search for a worker to fill each of its job openings. In each job  $k$  that will produce output, the worker and firm bargain over the pre-tax real wage  $w_{kt}$  paid in that position. Output of any job  $k$  is given by  $y_{kt} = z_t$ , which is subject to a common technology realization  $z_t$ .

Any two jobs  $k_a$  and  $k_b$  at the firm are identical, so from here on we suppress the second subscript and denote by  $w_t$  the real wage in any job, and so on. Total output of the firm thus depends on the technology realization and the measure of workers  $n_t^f$  that produce,

$$y_t = z_t n_t^f. \quad (6)$$

The total real wage bill of the firm is the sum of wages paid to all of its employees,  $n_t^f w_t$ .

The firm begins period  $t$  with employment stock  $n_{t-1}^f$ . Its period- $t$  productive employment stock,  $n_t^f$ , depends on its period- $t$  vacancy postings as well as the random matching process, as depicted in Figure 1. With probability  $q_t$ , taken as given by the firm, a given vacancy is filled by a worker. As described below, matching probabilities for both firms and households ( $p$  and  $q$ , respectively) depend only on aggregate labor-market conditions given the Cobb-Douglas matching function assumed below, which is standard in this class of models.

For  $t = 0, 1, \dots$ , the representative firm chooses state-contingent decision rules for vacancies  $v_t$  and desired stocks of labor  $n_t^f$  to maximize discounted profits,

$$E_0 \sum_{t=0}^{\infty} \left\{ \Xi_{t|0} \left[ z_t n_t^f - w_t n_t^f - (1 - \tau_t^s) \gamma v_t \right] \right\}, \quad (7)$$

in which  $\gamma$  is the per-vacancy posting cost,  $\Xi_{t|0}$  is the period-0 value to the representative household of period- $t$  goods, which the firm uses to discount profit flows because households are the ultimate owners of firms. As with households, firms take the wage-setting protocol as given.

Vacancy costs are subsidized by the government at the proportional rate  $\tau_t^s$ . This subsidy is important for the Ramsey analysis because its inclusion, along with the labor income tax, ensures that the tax system is complete, in the sense that there is at least one independent tax instrument along each unique equilibrium margin of the model. This issue is discussed further in Section 7.

In period  $t$ , the firm's problem is thus to choose  $v_t$  and  $n_t^f$  to maximize (7) subject to a sequence of perceived laws of motion for its employment level,

$$n_t^f = (1 - \rho)n_{t-1}^f + v_t q_t. \quad (8)$$

The firm's first-order conditions with respect to  $v_t$  and  $n_t^f$  yield a standard job-creation condition

$$\frac{\gamma(1 - \tau_t^s)}{q_t} = z_t - w_t + (1 - \rho)E_t \left[ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right], \quad (9)$$

in which  $\Xi_{t+1|t} \equiv \Xi_{t+1|0}/\Xi_{t|0}$  is the household discount factor (again, technically, the real interest rate) between period  $t$  and  $t + 1$ . The job-creation condition states that at the optimal choice, the (after-tax) vacancy-creation cost incurred by the firm is equated to the discounted expected value of profits from a match. Total profits from a match take into account the contemporaneous marginal profit from the match and the asset value of having a pre-existing relationship with an employee in period  $t + 1$ . This condition is a standard free-entry condition in the creation of vacancies.

## 2.4 Wage Determination

The baseline wage-determination mechanism is Nash bargaining. Specifically, wages of all workers, whether newly-hired or not, are set in period-by-period Nash negotiations, a common assumption in search-based DSGE models.<sup>7</sup> The solution to the formal wage-bargaining problem is presented in Appendix B. In what follows, we simply present the bargaining outcomes.

Assuming that  $\eta \in (0, 1)$  is a worker's bargaining power and  $(1 - \eta)$  a firm's bargaining power, the wage outcome is given by

$$w_t = \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau_t^n} + \eta(1 - \rho)E_t \left\{ \Xi_{t+1|t} \left[ 1 - (1 - p_{t+1}) \frac{1 - \tau_{t+1}^n}{1 - \tau_t^n} \right] \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}. \quad (10)$$

The first two terms of (10) show that part of the period- $t$  wage payment is a convex combination of the contemporaneous values to the firm and the household, given by the marginal product of a new employee  $z_t$  and the tax-adjusted value of unemployment benefits, respectively. The last term on the right-hand side of (10) captures the forward-looking aspect of employment, whose value is also capitalized in the period- $t$  wage payment. Changes in taxes affect this forward-looking component.

<sup>7</sup>Although Pissarides (2009) recently highlights that it may be important to distinguish between the wages of new hires and wages of ongoing employees in search models, we use the conventional assumption that all workers receive the same wage.

The fact that both period- $t$  and expectations of period- $(t+1)$  tax rates affect the period- $t$  wage outcome will be important in understanding the optimal-policy results. If both labor taxes and vacancy subsidies were constant at  $\tau_t^n = \tau^n$  and  $\tau_t^s = \tau^s \forall t$ , the wage outcome would simplify to  $w_t = \eta \left[ z_t + (1 - \tau^s)(1 - \rho)E_t \left\{ \Xi_{t+1|t} \gamma \theta_{t+1} \right\} \right] + (1 - \eta) \frac{\chi}{u'(c_t)(1 - \bar{\tau}^n)}$ . In this case, taxes only changes parties' effective bargaining power in a static manner:  $\tau^n > 0$  and  $\tau^s > 0$  cause  $(1 - \eta)/(1 - \tau^n) > 1 - \eta$  and  $\eta(1 - \tau^s) > \eta$ . This kind of static bargaining power effect underpins the results in Arseneau and Chugh (2006). In the model here, a purely static effect of taxes would render policy unable to manage cyclical fluctuations. It is instead tax-rate variability that is important for the cyclical properties of optimal policy in the model, a channel dubbed the “dynamic bargaining power effect” in the monetary policy study of Arseneau and Chugh (2008).

## 2.5 Government

The government finances an exogenous stream of spending  $\{g_t\}$  by collecting labor income taxes, dividend income taxes, and issuing real state-contingent debt. As describes above, it also provides vacancy subsidies and unemployment benefits. The period- $t$  government budget constraint is

$$\tau_t^n w_t n_t + \tau^d d_t + b_t = g_t + R_t b_{t-1} + (1 - p_t) s_t \chi + \tau_t^s \gamma v_t. \quad (11)$$

As noted in the introduction, the fact that the government is able to issue fully state-contingent real debt means that none of the optimal policy results is driven by incompleteness of debt markets or ad-hoc limits on government assets.

Payment of unemployment benefits is included as a government activity for two reasons. First, it is empirically descriptive to view the government as providing such insurance. Second, including  $(1 - p_t) s_t \chi$  in the government budget constraint means that  $\chi$  does *not* appear in the economy-wide resource constraint (presented below). In DSGE labor-search models, it is common to include unemployment benefits in the household budget constraint but yet exclude them from the economy-wide resource constraint — see, for example, Krause and Lubik (2007) or Faia (2008). In such models, the government budget constraint is a residual object due to the presence of a lump-sum tax. In contrast, we rule out lump-sum taxes in order to conduct our Ramsey analysis and thus cannot treat the government's budget as residual. To make our model setup as close as possible to existing ones that study tax smoothing, we explicitly model payment of unemployment benefits as a transfer between the government and households.<sup>8</sup>

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<sup>8</sup>A Ramsey problem requires specifying both the resource constraint and either the government or household budget constraint as equilibrium objects, and this requires us to take a more precise stand on the source of unemployment benefits than usually taken in the literature.

## 2.6 Matching

In equilibrium,  $n_t = n_t^f = n_t^h$ , so we now refer to employment simply as  $n_t$ . Matches between unemployed individuals searching for jobs and firms searching to fill vacancies are formed according to a constant-returns matching technology,  $m(s_t, v_t)$ , and  $s_t$  and  $v_t$  are now considered to be economy-wide aggregates (due to the assumptions of unit measures of identical households and identical firms). Consistent with the timing depicted in Figure 1, aggregate employment evolves according to

$$n_t = (1 - \rho)n_{t-1} + m(s_t, v_t). \quad (12)$$

## 2.7 Private-Sector Equilibrium

A symmetric private-sector equilibrium is made up of endogenous processes  $\{c_t, w_t, n_t, v_t, s_t, R_t, b_t\}_{t=0}^{\infty}$  that satisfy the consumption-savings optimality condition (4), the labor-force participation condition (5), the vacancy-posting condition (9), the period-by-period Nash wage outcome (10), the government budget constraint (11), the law of motion for the aggregate stock of employment (12), and the aggregate resource constraint of the economy

$$c_t + g_t + \gamma v_t = z_t n_t. \quad (13)$$

In (13), total costs of posting vacancies  $\gamma v_t$  are a resource cost for the economy. As discussed above, unemployment benefits  $\chi$  do not absorb any part of market output. The private sector takes as given stochastic processes  $\{z_t, g_t, \tau_t^n, \tau_t^s\}_{t=0}^{\infty}$  and the fixed parameters  $(\tau^d, \chi)$ .

# 3 Exogenous Fiscal Policy

Before studying the model's implications for optimal tax policy, we study its cyclical properties under an exogenous fiscal policy. In this section, we set the vacancy subsidy to  $\tau_t^s = 0 \forall t$  because it is the implicit assumption in virtually all DSGE matching models. Given this, we calibrate the model so that it generates empirically-relevant business-cycle fluctuations, especially along important labor-market dimensions, when driven by empirically-relevant government spending and labor-income tax rate processes.

## 3.1 Data Targets

At a minimum, we intend for the exogenous-policy model to do a reasonable job in explaining the cyclical volatilities of vacancies, search activity, and labor-market tightness, which have received much empirical and theoretical attention since Shimer (2005). Because our model also includes a participation margin, we want it to capture reasonably well the cyclical volatility of participation

and the long-run rate of participation. Since Veracierto’s (2008) recent critique of three-state models of frictional labor markets, there has been growing interest in developing models that can successfully capture all of these and other dimensions of labor-market fluctuations simultaneously. Of independent interest from the optimal policy results presented in Section 4 is the result that our model, under exogenous fiscal policy, performs well along all of these dimensions simultaneously when appropriately calibrated.

Table 1 presents empirical facts regarding U.S. labor markets. The business-cycle statistics for GDP, search unemployment, vacancies, labor-market tightness, employment, and wages are taken from Gertler and Trigari (2009, Table 2). The cyclical correlation between search unemployment and vacancies is taken from Shimer (2005, Table 1) — the strong negative correlation is indicative of the Beveridge Curve. We take from Veracierto (2008) the long-run participation rate of 74 percent as well as the cyclical properties of participation.

We use the methodology of Jones (2002) to construct an empirical measure of the average U.S. labor income tax rate from 1947:Q1-2009:Q4.<sup>9</sup> The mean labor income tax rate over this period is about 20 percent. In terms of its cyclical properties, the first-order autocorrelation is 0.66, and the standard deviation of the cyclical component of the tax rate is 2.8 percent. The statistics allow us to compute the standard error of the shocks to the tax-rate process below.

### 3.2 Calibration

For utility, standard functional forms are used,

$$u(c_t) = \ln c_t \tag{14}$$

and

$$h(x_t) = \frac{\kappa}{1 + 1/\iota} x_t^{1+1/\iota}. \tag{15}$$

The parameter  $\iota$  is the elasticity of labor-force participation with respect to the real wage, which we set to  $\iota = 0.18$  in order to match the relative volatility of participation of 20 percent reported in Table 1. The scale parameter is set to  $\kappa = 7$  to deliver a steady-state participation rate of 67 percent.

Hagedorn and Manovskii (2008) — henceforth, HM — show that, given Nash bargaining, workers’ bargaining power and the unemployment replacement rate (the level of unemployment benefits relative to the after-tax wage rate) are important for dynamics in search models. We rely on an HM-style calibration to generate sufficiently rigid (pre-tax) real wages that produce empirically-reasonable fluctuations in the exogenous-policy model, which requires appropriately parameterizing

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<sup>9</sup>The source data are the NIPA accounts of the U.S. Bureau of Economic Analysis, and the methodology to construct the tax rate series is described in detail in Appendix B of Jones (2002).

$\eta$  and  $\chi$ . We set  $\eta = 0.05$ , the calibrated value of HM. We set  $\chi$  such that  $\frac{\chi}{(1-\tau^n)w} = 0.988$ , which is a bit higher than HM's calibration of a 95-percent replacement rate. This slightly higher setting allows our model, which of course differs in details from the HM model, to match the empirical volatilities of  $s$ ,  $v$ , and  $\theta$  given in Table 1. The calibrated value is  $\chi = 0.76$ .

The other parameter values are relatively standard in this class of models. The model is quarterly, so we set a subjective discount factor  $\beta = 0.99$ , which implies a steady-state real interest rate of about four percent. The quarterly job-separation rate is  $\rho = 0.10$ , a standard value in search and matching models and in line with the evidence in Davis, Faberman, and Haltiwanger (2006). The matching function is Cobb-Douglas,  $m(s, v) = \psi s^\xi v^{1-\xi}$ , with  $\xi = 0.4$ , in line with the evidence in Blanchard and Diamond (1989), and  $\psi$  chosen so that the quarterly job-filling rate of a vacancy is 90 percent, in line with Andolfatto (1996). The resulting value is  $\psi = 0.77$ . Given the Cobb-Douglas matching specification, this also directly fixes the matching rate for a searching individual to  $p = 0.61$ , in line with data and with calibrations such as Blanchard and Gali (2010). The fixed cost  $\gamma$  of opening a vacancy is set so that posting costs absorb 3 percent of total output in the steady state; the resulting value is  $\gamma = 0.27$ .

The three exogenous processes are productivity, government spending, and the labor tax rate, each of which follows an AR(1) process in logs:

$$\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z, \quad (16)$$

$$\ln g_t = (1 - \rho_g) \ln \bar{g} + \rho_g \ln g_{t-1} + \epsilon_t^g, \quad (17)$$

and

$$\ln \tau_t^n = (1 - \rho_{\tau^n}) \ln \bar{\tau}^n + \rho_{\tau^n} \ln \tau_{t-1}^n + \epsilon_t^{\tau^n}. \quad (18)$$

The innovations  $\epsilon_t^z$ ,  $\epsilon_t^g$ , and  $\epsilon_t^{\tau^n}$  are distributed  $N(0, \sigma_{\epsilon^z}^2)$ ,  $N(0, \sigma_{\epsilon^g}^2)$ , and  $N(0, \sigma_{\epsilon^{\tau^n}}^2)$  respectively, and are independent of each other. Matching the persistence and standard error for the empirical tax-rate series reported above requires setting  $\sigma_{\tau^n} = 0.02$ .

The steady-state level of government spending  $\bar{g}$  is calibrated so that it constitutes 17 percent of steady-state output; the resulting value is  $\bar{g} = 0.11$ . It is important to note that  $g$  is spending *not* including unemployment transfers. When including transfers, total government outlays are  $\bar{g} + \chi(1-p)s$  in the steady state; given our calibrated values, we have  $\frac{\bar{g} + \chi(1-p)s}{gdp} = 0.21$ . We choose parameters  $\rho_z = 0.95$ ,  $\rho_g = 0.97$ ,  $\sigma_{\epsilon^z} = 0.006$ , and  $\sigma_{\epsilon^g} = 0.027$ , consistent with the RBC literature and Chari and Kehoe (1999). Also regarding policy, we assume that the steady-state government debt-to-GDP ratio (at an annual frequency) is 0.4, in line with evidence for the U.S. economy and with the calibrations of Schmitt-Grohe and Uribe (2005), Chugh (2006), and Arseneau and Chugh (2008).

For the exogenous tax-rate series, we compute the percentage deviation of the tax rate from its quarterly HP-filter trend in each time period. We estimate an AR(1) in this detrended series because it corresponds to the model’s tax-rate process (18). The estimated parameters are  $\rho_{\tau^n} = 0.70$  and  $\sigma_{\tau^n} = 0.02$ . Finally, for the exogenous-policy experiments conducted here, the government is also assumed to have available a lump-sum tax/transfer vis-a-vis households, which allows us to ignore, for the exogenous-policy experiments, government financing issues.

The deterministic steady-state equilibrium is computed using a nonlinear numerical solution. To study dynamics, we compute a first-order approximation of the equilibrium conditions around the deterministic steady-state.<sup>10</sup> We use the first-order accurate decision rules to simulate time-paths of the equilibrium in the face of TFP, government spending, and labor tax realizations, the shocks to which we draw according to the parameters of the laws of motion described above. We conduct 1000 simulations, each 200 periods long. For each simulation, we then compute first and second moments and report the medians of these moments across the 1000 simulations.

### 3.3 Results

Table 3 presents simulation results for the calibrated exogenous-policy model. The top panel presents dynamics when all three exogenous processes are active, and the bottom panel presents results conditional only on shocks to TFP. Compared with the empirical evidence presented in Table 1, the model performs quite well. In particular, the volatilities of  $s$ ,  $v$ ,  $\theta$ , and  $lfp$  are all in line with the data.

On the other hand, the volatility of wages is quite low compared to the data. This result, which is especially pronounced conditional on shocks to only TFP, is a consequence of the HM-style calibration, which dampens the transmission of productivity fluctuations into wage fluctuations, thus making wages “rigid.” In turn, the rigidity of wages provides firms powerful incentives to alter vacancy-posting behavior in the face of fluctuations. This transmission mechanism has been well-understood in matching models since Shimer (2005), Hall (2005), and HM.<sup>11</sup>

Another noteworthy result is that the model features a cyclical Beveridge curve. Although the contemporaneous correlation between search unemployment and vacancies (-0.55 conditional on all shocks, and -0.74 conditional on only TFP shocks) is not as strong as in the data (an empirical

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<sup>10</sup>Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004).

<sup>11</sup>More precisely, Shimer (2005) suggested that the basic problem in a standard calibration of a search model was that wages absorbed too much of fluctuations in productivity, thus almost entirely choking off changes in the profit signals that govern firms’ vacancy posting decisions, the central margin of a search model. Hall (2005) demonstrated in a simple way that low volatility of wages indeed can dramatically increase fluctuations of vacancy postings and, in turn, other labor-market quantities.

correlation of -0.89, shown in Table 1), the fact that it emerges at all in our three-state model is an interesting result because three-state models have a difficult time generating a Beveridge curve. Three-state models instead typically display a positive correlation between search unemployment and vacancies because expansionary shocks, say a positive TFP shock, create incentives for *both* sides of the market to increase match-creation activities — individuals to increase their job-search activity, and firms to increase recruiting activity. An early study that made this point was Tripier (2003).

The emergence of a Beveridge curve in the model turns out to hinge most critically on the high replacement rate, and thus low workers' share of match surpluses, that is emblematic of an HM-style calibration. The following experiment verifies that  $\chi$  is the critical parameter for this result. Holding all other parameters constant, we recalibrate  $\chi$  so that unemployment benefits replace 95 percent of after-tax wages (the HM calibration, although, again, the precise value is model-specific), rather than our baseline calibration of 98.7 percent of after-tax wages. With this lower setting of  $\chi$ , the correlation between search unemployment and vacancies becomes zero, so that the Beveridge relation completely disappears.<sup>12</sup> For even smaller values of  $\chi$ , the correlation turns positive. Because both a 95-percent replacement rate and a 98-percent replacement rate are extremely large, these results demonstrate that the slope of the Beveridge curve is extremely sensitive to the replacement rate in this range of  $\chi$ .<sup>13</sup> Nonetheless, it is interesting to know that the HM-style calibration is also useful in generating a Beveridge curve in three-state models. Ebell (2010) has also recently independently discovered this result, as have denHaan and Kaltenbrunner (2009, Section 3.4), although the latter are focused on a different range of issues.

## 4 Optimal Fiscal Policy

With the baseline calibration established, we now discard the exogenous process (18) for the labor income tax rate and instead endogenize tax policy (both income taxes and vacancy subsidies).<sup>14</sup> While taxes are now optimally chosen by a Ramsey government, government purchases continue to follow the exogenous process (17).<sup>15</sup>

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<sup>12</sup>And, correspondingly, the labor-market variables of the model become less volatile.

<sup>13</sup>While the precise numerical results depend on all the details of the model and parameterization, the basic intuition seems to be that a high replacement rate provides a strong incentive for firms to post vacancies following expansionary shocks, which all else equal, lowers unemployment. *Simultaneously*, the *low* setting for worker bargaining power ( $\eta = 0.05$ ) means that individuals do *not* have much incentive to increase search activity following expansionary shocks because very little of the surplus of a job accrues to them. Hence the model's ability to predict a negative correlation between  $s$  and  $v$ .

<sup>14</sup>We also return to the case of zero lump-sum taxes, required for a Ramsey analysis.

<sup>15</sup>Thus, we follow the standard convention in Ramsey analysis that spending is exogenous but the revenue side of fiscal policy is determined optimally.

## 4.1 Ramsey Problem

A standard approach in Ramsey models based on neoclassical markets is to capture in a single, present-value implementability constraint (PVIC) all equilibrium conditions of the economy apart from the resource frontier. The PVIC is the key constraint in any Ramsey problem because it governs the welfare loss of using non-lump-sum taxes to finance government expenditures.<sup>16</sup> As is standard, we can construct a PVIC starting from the household flow budget constraint (2) and using the household optimality conditions (4) and (5). However, because of the forward-looking aspects of *firm* optimization, it cannot capture all of the model's equilibrium conditions.<sup>17</sup> As shown in Appendix E, the PVIC is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u'(c_t)c_t - h'(lfp_t)lfp_t - u'(c_t)(1 - \tau^d)d_t \right\} = A_0, \quad (19)$$

with the time-zero assets of the private-sector given by

$$A_0 \equiv u'(c_0)R_0b_{-1} + (1 - \rho) \left( \frac{1 - p_0}{p_0} \right) [h'(lfp_0) - u'(c_0)\chi] n_{-1}. \quad (20)$$

Several observations about the PVIC are in order. First, because employment is a state variable, the household's "ownership" of the initial stock of employment relationships,  $n_{-1}$ , is part of its time-zero assets, as shown in  $A_0$ . Second, if labor markets were neoclassical,  $lfp_t$  would be interpreted as simply *labor* because there would be no notion of search unemployment. Third, as mentioned above, a spot, neoclassical labor market can be interpreted as featuring  $\rho = 1$  because there is no long-lived aspect to labor-market transactions. Fourth, with a constant-returns production technology in a neoclassical environment,  $d_t = 0 \forall t$ . Imposing the last three of these conditions collapses the PVIC (19), as well as the initial assets  $A_0$ , to that in a standard Ramsey model based on neoclassical markets.<sup>18</sup>

However, unlike in a neoclassical model, the PVIC (19) does not capture all equilibrium conditions of the decentralized economy. In particular, Ramsey allocations must also respect the vacancy-posting condition (9), the Nash wage outcome (10), and the law of motion for the aggregate employment stock (12). None of these restrictions is encoded in the PVIC (19).

<sup>16</sup>See, for example, Ljungqvist and Sargent (2004, p. 494) for more discussion. The PVIC is the household (equivalently, government) budget constraint expressed in intertemporal form with all prices and policies substituted out using equilibrium conditions. In relatively simple models, the PVIC encodes all the equilibrium conditions that must be respected by Ramsey allocations in addition to feasibility. In complicated environments that deviate substantially from neoclassical markets, however, such as Schmitt-Grohe and Uribe (2005), Chugh (2006), and Arseneau and Chugh (2008), it is not always possible to construct such a single constraint.

<sup>17</sup>A very similar, in form, construction of the Ramsey problem arises in Chugh and Ghironi (2012), who study optimal fiscal policy in a model with endogenous product creation.

<sup>18</sup>In particular, we would have  $E_0 \sum_{t=0}^{\infty} \beta^t [u'(c_t)c_t - h'(n_t)n_t] = u'(c_0)R_0b_{-1}$ . This PVIC is identical to that in Chari and Kehoe (1999) for an environment without physical capital.

The Ramsey problem is thus to choose state-contingent processes  $\{c_t, n_t, s_t, \theta_t, w_t, \tau_t^n, \tau_t^s\}_{t=0}^{\infty}$  to maximize (1) subject to the PVIC (19), the vacancy-posting condition (9), the Nash wage outcome (10), the law of motion for the aggregate employment stock (12), and the aggregate resource constraint (13).<sup>19</sup> For computational convenience, the bargained real wage, the labor tax rate, and the vacancy subsidy rate are left as explicit Ramsey choice variables. We instead could have used the Nash wage equation to eliminate the real wage; in any case, however, we would still be left with the two policy instruments as explicit Ramsey choice variables.<sup>20</sup> Finally, as is standard in Ramsey taxation problems, we assume full commitment. Thus, we emphasize that none of the results would change with the use of a discretionary policy.

## 4.2 Computational Issues

The first-order conditions of the Ramsey problem are assumed to be necessary and sufficient, and all allocations are assumed to be interior. As in the exogenous-policy baseline, we use a nonlinear numerical solution algorithm to compute the deterministic Ramsey steady-state equilibrium. As is common in the Ramsey literature, when characterizing asymptotic policy dynamics (that is, the dynamics of the Ramsey equilibrium implied by the Ramsey  $t > 0$  first-order conditions), we also make the auxiliary assumption that the initial state of the economy is the asymptotic Ramsey steady state.

More precisely, to study dynamics, we compute a first-order approximation of the Ramsey first-order conditions for time  $t > 0$  around the deterministic steady-state of these conditions. We then use the first-order accurate decision rules to simulate the Ramsey equilibrium in the face of TFP and government spending realizations. The TFP and government spending realizations used to conduct the Ramsey simulations are the same as those in the exogenous-policy experiments in Section 3, which means that any differences between the Ramsey equilibrium and exogenous-policy equilibrium are attributable entirely to the dynamics of tax policy.

## 4.3 Results

The main result, presented in Table 4, is that the optimal labor income tax rate is very volatile. More precisely, the optimal labor tax rate is about four times more volatile, in a relative sense,

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<sup>19</sup>The dividend tax rate  $\tau^d$  is omitted from the list of Ramsey choice variables for convenience. As described above, it is trivial to show that  $\tau^d = 1$  is optimal in every period because taxing households' receipts of lump-sum dividend payments is non-distortionary. Furthermore,  $\chi$  is viewed as an institutional parameter and hence not chosen during the course of "normal" fiscal policy.

<sup>20</sup>This further illustrates that the Ramsey problem here cannot be cast in pure primal form; in particular, the appearance of expectations of future tax rates in equilibrium conditions is what prevents formulation in pure primal form, because there is no way to eliminate them from the problem.

than in the exogenous policy model calibrated to U.S. tax data. Moreover, a comparison of Table 4 (the optimal policy results) with Table 3 (the exogenous policy results) shows that the volatilities of search unemployment, vacancies, and, especially, labor-market tightness are all much lower in the Ramsey equilibrium.

These results suggest that, when viewed through the lens of our model, actual U.S. tax policy has not been optimal. Actual policy has produced far too smooth a labor tax rate, which in turn results in suboptimal volatility in labor market allocations under our rigid wage calibration. In contrast, our results show that the optimal policy calls for considerably more labor tax volatility so that the after-tax wage can more efficiently insulate the broader labor market from shocks.

Our result is all the more striking when we compare our optimal tax volatility results to benchmark Ramsey theory. In a benchmark Ramsey model, which is based on a frictionless labor market, a well-known result is that zero or near-zero volatility of the tax rate is optimal over the business cycle.<sup>21</sup> The optimal labor tax rate in a model of empirically-relevant matching frictions is thus orders of magnitude more volatile than in the benchmark model.

It is useful to highlight a few additional aspects of optimal policy. First, the Beveridge curve that was apparent in the exogenous-policy case, reflected in the negative correlation between search activity and vacancy postings, disappears in the Ramsey equilibrium. As was discussed in Section 3, a positive correlation is the typical result in three-state labor models, because expansionary shocks provide incentives to both sides of the labor market to increase match creation activities. In our calibrated model, however, a positive correlation only emerges in the Ramsey equilibrium.

Second, the vacancy subsidy rate fluctuates over the business cycle, and the long-run vacancy subsidy (shown in the lower panel) is positive. The latter result indicates that, absent the Ramsey government's intervention, vacancy postings are on average inefficiently low. The large fluctuations in subsidies indicate that the vacancy creation margin is important for understanding the Ramsey dynamics.

A precise explanation of the incentives that shape Ramsey outcomes, as well as how they are decentralized, requires introducing several new concepts, which is done in Sections 5 and 6. Section 7 then uses these concepts to explain the optimal policy results in ways that connect naturally to both the matching literature and the Ramsey literature.

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<sup>21</sup>Werning (2007) proves for the case of isoelastic utility over employment that perfect tax smoothing is optimal when labor markets are neoclassical, which adds analytical insight to the quantitative results known in the DSGE Ramsey literature since Chari, Christiano, and Kehoe (1991).

## 5 Search Efficiency in General Equilibrium

Ramsey allocations trade off efficiency against market decentralization. As Figure 2 illustrates, characterizing efficient allocations is thus a necessary first step for understanding the optimal policy results. As is standard in search-based models, efficient allocations are understood to be restricted by the matching technology. We characterize efficiency for a general constant-returns matching technology, not just for the Cobb-Douglas form used in the quantitative experiments.

Efficient allocations  $\{c_t, s_t, v_t, n_t\}_{t=0}^{\infty}$  are characterized by four (sequences of) conditions

$$\frac{h'(lfp_t)}{u'(c_t)} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}, \quad (21)$$

$$\frac{\gamma}{m_v(s_t, v_t)} - z_t = (1 - \rho) E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\gamma}{m_v(s_{t+1}, v_{t+1})} (1 - m_s(s_{t+1}, v_{t+1})) \right\}, \quad (22)$$

$$c_t + g_t + \gamma v_t = z_t n_t, \quad (23)$$

and

$$n_t = (1 - \rho)n_{t-1} + m(s_t, v_t), \quad (24)$$

in which  $m_v(\cdot)$  and  $m_s(\cdot)$  are the marginal products of the matching function. The efficiency conditions (21) and (22) are obtained by maximizing household welfare, given by (1), subject to the technological frontier defined by the sequence of goods resource constraints (23) and laws of motion for employment (24). The formal analysis of this problem appears in Appendix C.

Condition (21) is a static dimension of efficiency and is analogous to static consumption-leisure efficiency in the RBC model. Condition (22) is an intertemporal dimension of efficiency, and it corresponds to the RBC model's Euler equation for efficient capital accumulation. Even though the search model does not have “physical capital” in the strict RBC sense, the creation of an employment match is an investment activity that yields a long-lasting asset. Employment thus inherently has both static and intertemporal dimensions in a matching framework. Together, conditions (21) and (22) define the two “zero-wedge” benchmarks for Ramsey allocations, both of which are statements about labor markets.

To highlight this “zero-wedges” aspect, it is useful to restate efficiency in terms of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). For the intertemporal condition, this restatement is most straightforward for the non-stochastic case, which allows an informative disentangling of the preference and technology terms inside the  $E_t(\cdot)$  operator in (22).

**Proposition 1. *Efficient Allocations.*** *The MRS and MRT for the pairs  $(c_t, lfp_t)$  and  $(c_t, c_{t+1})$  are defined by*

$$MRS_{c_t, lfp_t} \equiv \frac{h'(lfp_t)}{u'(c_t)} \quad MRT_{c_t, lfp_t} \equiv \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$$

$$IMRS_{c_t, c_{t+1}} \equiv \frac{u'(c_t)}{\beta u'(c_{t+1})} \quad IMRT_{c_t, c_{t+1}} \equiv \frac{(1 - \rho) \left( \frac{\gamma}{m_v(s_{t+1}, v_{t+1})} \right) (1 - m_s(s_{t+1}, v_{t+1}))}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}.$$

Static efficiency (21) is characterized by  $MRS_{c_t, l_{fp_t}} = MRT_{c_t, l_{fp_t}}$ , and (for the non-stochastic case) intertemporal efficiency (22) is characterized by  $IMRS_{c_t, c_{t+1}} = IMRT_{c_t, c_{t+1}}$ .

*Proof.* See Appendix C. □

Each MRS in Proposition 1 has the standard interpretation as a ratio of relevant marginal utilities. By analogy, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.<sup>22</sup> Efficient allocations are then characterized by an  $MRS = MRT$  condition along each optimization margin, implying zero distortion on each margin. These efficiency conditions are the welfare-relevant ones for the Ramsey government. However, rather than taking the efficiency conditions as *prima facie* justification that the expressions in Proposition 1 *are* properly to be understood as MRTs, each can be described conceptually from first principles, independent of the characterization of efficiency. Formal details of the following mostly intuitive discussion appear in Appendix C.

## 5.1 Static MRT

To understand the static MRT in Proposition 1,  $MRT_{c_t, l_{fp_t}}$ , consider how the economy can transform a unit of non-participation (leisure) in period  $t$  into a unit of consumption in period  $t$ , holding output constant. A unit reduction in leisure allows a unit increase in  $s_t$ , which in turn leads to  $m_s(s_t, v_t)$  new employment matches in period  $t$ . Each of these new matches, in principle, produces  $z_t$  units of output, and hence consumption. The overall marginal transformation between leisure and consumption described thus far is  $z_t m_s(s_t, v_t)$ .

However, in order to hold output constant in this transformation, the number of vacancies must be lowered by  $m_v(s_t, v_t)$  units, so that employment remains unchanged. The resulting reduction in matches lowers output by  $\frac{z_t m_v(s_t, v_t)}{\gamma}$  units, which translates directly into lower consumption.<sup>23</sup>

Hence, the overall within-period MRT between leisure and consumption is  $\gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}$ , as shown in Proposition 1. If we restrict attention to the case of Cobb-Douglas matching ( $m(s_t, v_t) = s_t^\xi v_t^{1-\xi}$ ) used in the quantitative experiments, then the MRT takes the form  $\gamma \theta_t \frac{\xi}{1-\xi}$ , in which case the static

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<sup>22</sup>We have in mind a very general notion of transformation frontier as in Mas-Colell, Whinston, and Green (1995, p. 129), in which every object in the economy can be viewed as either an input to or an output of the technology to which it is associated. Appendix C provides formal details.

<sup>23</sup>When the economy is on its resource frontier, the output tradeoff between  $s$  and  $v$  must be scaled by  $\frac{1}{\gamma}$  (see expression (77) in Appendix C.4). Intuitively, a change in household search activity translates only indirectly to a change in output via the matching function. In contrast, a change in vacancies alters output both directly and indirectly, the former by economizing on posting costs and the latter through the matching function.

efficiency condition (21) is expressed as

$$\frac{h'(lfp_t)}{u'(c_t)} = \gamma \theta_t \frac{\xi}{1 - \xi}. \quad (25)$$

## 5.2 Intertemporal MRT

Now consider the intertemporal MRT (IMRT) in Proposition 1. The IMRT measures how many additional units of  $c_{t+1}$  the economy can achieve if one unit of  $c_t$  is foregone, holding constant output in period  $t$  and  $t + 1$ .

If  $c_t$  is reduced by one unit,  $1/\gamma$  additional units of vacancy postings are possible, as (23) shows. Because of the model's timing assumption of instantaneous production, this additional flow of vacancy postings increases the number of aggregate employment matches in period  $t$  by  $m_v(s_t, v_t)/\gamma$ , which in turn would increase  $c_t$  by  $z_t m_v(s_t, v_t)/\gamma$  units. This latter effect must be netted out so that the resulting increase in period- $t$  consumption is  $1 - z_t m_v(s_t, v_t)/\gamma = \frac{\gamma - z_t m_v(s_t, v_t)}{\gamma}$  ( $< 1$ ).

Thus, in net terms, reducing period- $t$  consumption by one unit allows an additional  $\frac{1}{\gamma - z_t m_v(s_t, v_t)}$  units of vacancies. These vacancies in turn yield  $\frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$  additional matches in period  $t$ , which subsequently results in  $(1 - \rho) \frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)}$  matches in period  $t + 1$ .

However, to hold output in period  $t + 1$  constant in this transformation, household search must be lowered by  $m_s(s_{t+1}, v_{t+1})$  so that period- $t + 1$  employment remains constant. The overall marginal transformation from period  $t$  consumption into units of the capital good (employment) yields  $(1 - \rho) \left( \frac{m_v(s_t, v_t)}{\gamma - z_t m_v(s_t, v_t)} \right) (1 - m_s(s_{t+1}, v_{t+1}))$  net vacancies in period  $t + 1$ .

Finally, transforming these vacancies into period- $t + 1$  consumption yields  $\frac{m_v(s_{t+1}, v_{t+1})}{\gamma}$  units. Putting together this logic leads to the IMRT shown in Proposition 1.<sup>24</sup> The fully stochastic intertemporal efficiency condition can thus be represented as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[ \frac{(1 - \rho) \gamma (1 - m_s(s_{t+1}, v_{t+1}))}{\frac{m_v(s_{t+1}, v_{t+1})}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}} \right] \right\} = E_t \left\{ \frac{IMRT_{c_t, c_{t+1}}}{IMRS_{c_t, c_{t+1}}} \right\}. \quad (26)$$

## 5.3 Nesting the RBC Model

These search-based static and intertemporal MRTs apply basic economic theory to a general equilibrium search and matching model. They compactly describe the *two* technologies — the matching technology  $m(s_t, v_t)$  and the production technology  $z_t n_t$  — that must operate for the within-period transformation of leisure into consumption and the transformation of consumption across time. Due to the participation decision and the investment nature of both vacancy postings and job search, employment inherently features both static and intertemporal dimensions.

<sup>24</sup>The numerator and denominator of the expression in square brackets that appear in the next equation are the numerator and denominator, respectively, of expression (83) in Appendix C.4.

DSGE models based on the matching framework have largely ignored the static dimension of employment, focusing on the intertemporal aspect. While this focus may be natural in a matching model, it is a sharp departure from a key margin in RBC-style models, in which the static consumption-leisure dimension is the driver of an array of positive and normative business-cycle results. The pair of MRTs we develop are general in that they encompasses the MRT between leisure and consumption in a standard RBC model.

To see how the efficiency concepts developed here nest the RBC notion of consumption-leisure efficiency, suppose first that  $\rho = 1$ , which makes employment a one-period, though not a frictionless, phenomenon. With one-period employment relationships, the static and intertemporal conditions (21) and (22) reduce to the single within-period condition,

$$\frac{h'(lfp_t)}{u'(c_t)} = z_t m_s(s_t, v_t). \quad (27)$$

Viewed as a primitive, the “frictions” captured by the matching function are formally part of the MRT of the economy, even though a neoclassical “labor wedge accounting” exercise as in Shimer (2009), Chari, Kehoe, and McGrattan (2007), or Ohanian, Raffo, and Rogerson (2008) would regard them as wedges between the MRS and the marginal product  $z_t$  of the production technology.

Moving all the way to the RBC model also requires discarding matching frictions. The RBC model can be trivially viewed as featuring  $m(s_t, v_t) = s_t$  (in addition to  $\rho = 1$ ). The previous expression then reduces to the familiar  $\frac{h'(lfp_t)}{u'(c_t)} = z_t$ , with “participation” now interchangeably interpretable as “employment” because there is no friction between the two.

Next, we use the model-appropriate definition of efficiency developed here to show how two key features of the decentralized bargaining economy disrupt efficiency

## 6 Search-Based Equilibrium Wedges

With the model-appropriate characterizations of static and intertemporal efficiency just developed, equilibrium wedges are defined as the deviations of MRS from MRT that arise in the decentralized economy. These wedges measure inefficiencies, and, because the inefficiencies all relate to the allocation of labor, it may be informative to think of them jointly as a “labor wedge.” Understanding the determinants and consequences of these inefficiencies provides the foundation for understanding optimal policy.

## 6.1 Static Distortion

In the decentralized economy with Nash bargaining and Cobb-Douglas matching, the within-period (static) equilibrium margin can be expressed as

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= \chi + (1 - \tau_t^n)(1 - \tau_t^s)\gamma\theta_t \frac{\eta}{1 - \eta} \\ &= \gamma\theta_t \frac{\xi}{1 - \xi} \left[ \frac{\chi(1 - \xi)}{\gamma\xi\theta_t} + (1 - \tau_t^n)(1 - \tau_t^s) \frac{\eta(1 - \xi)}{\xi(1 - \eta)} \right]. \end{aligned} \quad (28)$$

The term in square brackets measures the static distortion.

Comparing (28) with the static efficiency condition (25), it is clear that sufficient conditions for the decentralized economy with Nash bargaining and Cobb-Douglas matching to achieve efficiency are: the decentralized economy features  $\eta = \xi$ ; the unemployment transfer is  $\chi = 0$ ; proportional labor income taxation is  $\tau_t^n = 0$ ; and the proportional vacancy subsidy is  $\tau_t^s = 0$ . These conditions are not necessary, however, because for any arbitrary ( $\eta \neq \xi, \chi \neq 0$ ), an appropriate setting for policy  $(\tau_t^n, \tau_t^s)$  achieves efficiency.

Important to note is that, conditional on Nash-bargained wages, the two features that are most critical in enabling a matching model to match U.S. labor market volatility — a low value of  $\eta$  and a high value of  $\chi$  — are thus exactly the two features (apart from tax rates) that *create* inefficiencies. Also note that, if  $\eta$  were zero, neither the long-run value of nor short-run fluctuations in tax rates would have any effect on the wedge. The bargaining share  $\eta$  thus governs how easily taxes affect the equilibrium in both the long run and the short run.

## 6.2 Intertemporal Distortion

In the decentralized economy with Nash bargaining and Cobb-Douglas matching, the intertemporal equilibrium margin can be expressed as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[ \frac{(1 - \rho) \frac{\gamma(1 - \tau_{t+1}^s)}{m_v(s_{t+1}, v_{t+1})} \left[ 1 - \eta + \frac{\eta(1 - \tau_{t+1}^n)}{1 - \tau_t^n} \left( 1 - \frac{m_s(s_{t+1}, v_{t+1})}{\xi} \right) \right]}{\frac{\gamma(1 - \tau_t^s)}{m_v(s_t, v_t)} - \left( \frac{1 - \eta}{1 - \xi} \right) \left( z_t - \frac{\chi}{1 - \tau_t^n} \right)} \right] \right\}. \quad (29)$$

Comparing the term in square brackets with the term in square brackets in the intertemporal efficiency condition (26) implicitly defines the intertemporal distortion.

Intuitively, condition (29) is simply the equilibrium version of the vacancy-creation condition, which will be helpful in describing in Section 7 how optimal policy operates.<sup>25</sup> Comparing (29) with (26), it is clear that the sufficient conditions for intertemporal efficiency in the decentralized

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<sup>25</sup>Given optimal participation on the part of households, it can equivalently be thought of as the equilibrium version of the LFP condition, as Appendix D shows. Because the vacancy-creation condition is a feature of any matching model, while optimal participation may or may not be, it is useful to think of it intuitively as the former.

equilibrium are the same as those that achieve static efficiency:  $\eta = \xi$ ;  $\chi = 0$ ;  $\tau_t^n = 0 \forall t$ ; and  $\tau_t^s = 0 \forall t$ . However, these conditions are not necessary: along the dynamic stochastic equilibrium path, appropriate combinations of  $\tau_t^n$  and  $\tau_t^s$  and, loosely speaking, expectations of  $\tau_{t+1}^n$  and  $\tau_{t+1}^s$  can potentially neutralize the distortionary effects of a given pair  $(\eta \neq \xi, \chi \neq 0)$ .

## 7 Analysis and Discussion of Optimal Taxation

Based on the welfare-relevant concepts of efficiency and wedges developed in Sections 5 and 6, it is now straightforward to explain the optimal policy results through the lens of basic Ramsey theory. In doing so, we also quantify the role of the two key features of the decentralized bargaining economy that disrupt efficiency and thus call for volatile taxes, as well as briefly discuss a few other aspects of the model and results.

### 7.1 Wedge Smoothing...

A basic result in dynamic Ramsey analysis is that the least distortionary way for a government to collect a present value of revenue through proportional taxes is to maintain *low volatility of distortions* — “wedge smoothing” — across time periods. Keeping distortions constant (or nearly constant) over time is the basic insight behind Barro’s (1979) partial-equilibrium tax-smoothing result, which carries over to quantitative general equilibrium models, as first shown by Chari, Christiano, and Kehoe (1991) and recently by Werning (2007). This basic Ramsey insight also applies to our model, as optimal policy keeps the volatility of both static and intertemporal distortions low.

The first row of Table 5 shows that the static wedge is two orders of magnitude less volatile in the Ramsey equilibrium than in the benchmark exogenous-policy equilibrium. In terms of volatility relative to that of GDP, volatility of the static wedge is 11.4 in the exogenous-policy economy, compared to 0.05 in the Ramsey economy. These quantitative results make it quite clear that the basic Ramsey principle of smoothing static distortions carries over to a matching model

Table 5 also shows that optimal policy smooths intertemporal wedges over the business cycle. This result is even more stark than for static wedges: intertemporal distortions are exactly zero at all points along the business cycle. Albanesi and Armenter (2011) recently showed that for a wide class of optimal-policy models, achieving zero intertemporal distortions is the primary goal. Their results generalize the well-known zero-capital-taxation results of Chamley (1986) and Judd (1985). Existing zero intertemporal distortions results apply only to the steady state, however; the result here is that intertemporal efficiency is achieved not only in the long run, but also along the business cycle.<sup>26</sup>

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<sup>26</sup>This difference arises from the fact that newly-matched employees begin working and producing output imme-

Our model does not include physical capital in the strict sense, but intertemporal efficiency is nonetheless a primary concern of policy due to the asset nature of employment. For the overall economy, employment is a form of capital; as Proposition 1 implies, employment matches are in fact the means by which consumption is transformed across time and hence the means by which the economy “saves.” The intertemporal efficiency insight of Ramsey analysis is thus not limited to a narrow notion of “physical capital,” but instead applies to any accumulation decision.<sup>27</sup>

## 7.2 ...Supports Efficient Labor-Market Fluctuations...

If static wedges are constant over time and intertemporal wedges are always zero, the decentralized economy achieves efficient fluctuations. To see this, first recall the characterization of efficient allocations in Proposition 1. Considering the deterministic case for clarity, the period  $t$  and period  $t + 1$  static efficiency conditions can be written in intertemporal form as

$$\begin{aligned} \frac{u'(c_t)}{\beta u'(c_{t+1})} &= \frac{h'(lfp_t)}{\beta h'(lfp_{t+1})} \frac{m_v(s_t, v_t)}{m_v(s_{t+1}, v_{t+1})} \frac{m_s(s_{t+1}, v_{t+1})}{m_s(s_t, v_t)} \\ &= \frac{h'(lfp_t)}{\beta h'(lfp_{t+1})} \frac{\theta_{t+1}}{\theta_t}, \end{aligned} \quad (30)$$

in which the second line follows from the assumption of constant-returns matching. Together with the intertemporal efficiency condition (22), the goods resource constraint (23), and the law of motion (24), this expression describes the efficient fluctuation of the economy between periods  $t$  and  $t + 1$ .

Now, in the decentralized economy, suppose that the static wedge, even if not zero, is constant across periods  $t$  and  $t + 1$ . Condition (28) shows that static wedge smoothing implies

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{h'(lfp_t)}{\beta h'(lfp_{t+1})} \frac{\theta_{t+1}}{\theta_t}, \quad (31)$$

which is identical to the implication (30) of period-by-period static efficiency. Thus, *fluctuations in the decentralized economy are efficient if static wedges are constant over time and intertemporal wedges are always zero.*

Table 5 showed that optimal policy achieves complete stabilization of intertemporal wedges and (virtually) complete stabilization of static wedges. The implication of wedge smoothing, then, is that Ramsey equilibria display efficient fluctuations. Figure 2 illustrates this through impulse 

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diately in our model, which implies a static component in forward-looking match-creation activities; whereas the standard assumption in RBC models is that, due to time-to-build lags, newly-created physical capital does not yield any contemporaneous output.

<sup>27</sup>Another recent example in which intertemporal efficiency is a central goal of policy, despite the absence of “physical capital,” is the model of dynamic product creation and destruction in which Chugh and Ghironi (2012) study optimal policy.

responses to a TFP shock.<sup>28</sup> It is not surprising that the *goal* of optimal policy is to induce efficient fluctuations in the labor market because at the core of any Ramsey problem is a social planning problem; what may be surprising is that the Ramsey government *is not constrained* in doing so.<sup>29</sup>

Figure 2 also makes clear that the calibrated exogenous-policy model, whose large fluctuations match the data well, displays highly *inefficient* fluctuations. Even more precisely, then, the goal of optimal policy is not just to induce efficient fluctuations, but to *stabilize* the labor market over the business cycle.

### 7.3 ...Which Requires Tax Volatility

The final step, then, is to describe how efficient fluctuations are decentralized using tax policy. In baseline Ramsey models, the mapping from the Ramsey-optimal intertemporal and static wedges to the *set* of available taxes is straightforward, and the mappings along each margin are almost always independent of each other: the Ramsey-optimal intertemporal wedge pins down the capital tax independently of the static wedge, and the Ramsey-optimal static wedge pins down the labor (or consumption) tax independently of the intertemporal wedge.<sup>30</sup>

In our model, the mapping from a given period- $t$  allocation to the pair of taxes  $(\tau_t^n, \tau_t^s)$  is defined by the wedge conditions (28) and (29). These two conditions *jointly* determine the period- $t$  tax policy  $(\tau_t^n, \tau_t^s)$  that supports the period- $t$  Ramsey allocation, rather than each wedge condition pinning down a single tax instrument in isolation.<sup>31</sup> Except for the special case of  $(\eta = \xi, \chi = 0)$ , the mapping from allocations to taxes is a complicated endogenous object that can only be approximated quantitatively, and it is apparent that *wedge* smoothing does not immediately imply *tax* smoothing, as it typically does in Walrasian-based Ramsey analysis.

Intuitively, it is useful to think of the mapping from fluctuations in wedges to fluctuations in taxes in the following way. Intertemporal efficiency is the paramount concern, which can be thought of as requiring the efficient fluctuation in market tightness  $\theta_t/\theta_{t-1}$  in every period  $t$ .<sup>32</sup> Given the period- $t$  state of the economy and, loosely speaking, expectations of period- $t + 1$  allocations, an appropriate subsidy  $\tau_t^s$  induces firms to open a quantity of vacancies that, for a given level of search unemployment, induces the efficient  $\theta_t$  and thus a zero intertemporal wedge. This argument is based on the intertemporal wedge condition (29), which, recall from above, is usefully thought

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<sup>28</sup>We could as well look at an impulse to government purchase shocks, but studying responses to TFP shocks is a standard convention in the matching literature.

<sup>29</sup>The efficient and Ramsey impulse responses in all panels of Figure 2 are *identical*.

<sup>30</sup>A caveat to this simple decentralization is if an *incomplete* tax system is in place; this point is discussed below.

<sup>31</sup>Aruoba and Chugh (2010) present another environment in which frictions (affecting monetary exchange) imply joint mappings from wedges to taxes.

<sup>32</sup>With constant-returns matching,  $\theta_t$  is a summary statistic of period- $t$  labor-market outcomes. Along the intertemporal margin,  $\theta_t/\theta_{t-1}$  is thus a summary of fluctuations of labor-market outcomes.

of as the equilibrium version of the vacancy-creation condition. Depending on parameter values and the size of realized shocks, the vacancy subsidy  $\tau_t^s$  may differ substantially from  $\tau_{t-1}^s$ ; Table 4 shows that fluctuations in  $\tau^s$  are indeed large for the baseline parameters.

The effect just described takes as given the measure of searching unemployed individuals and, by implication, the labor-force participation rate. Now viewing vacancy postings instead as given, variation in  $\tau^s$  between period  $t - 1$  and  $t$  causes an inefficient fluctuation of search activity and, in turn, market tightness. Given the period- $t$  state of the economy, an appropriate labor tax rate  $\tau_t^n$  induces a rate of participation that, for a given quantity of vacancies, induces the efficient  $\theta_t$  and thus a static wedge unchanged from period  $t - 1$ . This argument is based on the static wedge condition (28), which is usefully thought of as the equilibrium version of the LFP condition. Again depending on parameter values and the size of realized shocks, the tax rate  $\tau_t^n$  may differ sharply from  $\tau_{t-1}^n$ .

An appropriate combination of time-varying labor taxes and vacancy subsidies thus jointly achieves zero intertemporal distortions and static wedge smoothing, which is tantamount to stabilizing the two-dimensional notion of the “labor wedge.” More generally, the mapping from wedge smoothing to the dynamics of taxes in the model depends on whether or not the non-tax components of the wedges fluctuate efficiently. If they do not, then tax variability offsets inefficient fluctuations in the wedge; if they do, then tax variability is unnecessary.<sup>33,34</sup>

## 7.4 Restoring the Optimality of Tax Smoothing

The non-tax components of the wedges that make tax variability optimal are inefficiently-low worker bargaining power and the existence of positive unemployment transfers, as the preceding analysis and discussions make clear. The second, third, and fourth rows of Table 5 document the volatility of Ramsey-optimal wedges and taxes when, respectively, the Hosios condition ( $\eta = \xi$ ) is restored, unemployment transfers are assumed to be zero ( $\chi = 0$ ), or both. Each of these experiments is conducted keeping all other parameters fixed at their baseline settings; the aim of these experiments is thus not to preserve empirical relevance of the exogenous-policy model, but rather to shed light on the quantitative importance of these two structural features in determining the dynamics of optimal taxes.

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<sup>33</sup>We emphasize that this is an efficiency-based motivation for tax volatility, unlike the results in Aiyagari, Marcat, Marimon, and Sargent (2002), Schmitt-Grohe and Uribe (2005), or Chugh (2006), in which the inability (or undesirability) of the government to make debt repayments fully state contingent leads to large fluctuations in tax rates in order to meet budget shocks; such a channel does not exist in our model because government debt payments are fully state contingent.

<sup>34</sup>Although not formalized by Arseneau and Chugh (2008) as we have done here, the result that possibly-time-varying policy can achieve efficiency in a search-and-bargaining economy is the idea of the “dynamic bargaining power effect” in their monetary policy study.

Raising worker bargaining power to its Hosios value by itself or reducing unemployment transfers to zero by itself leads to an order-of-magnitude reduction in tax variability. What raising  $\eta$  and lowering  $\chi$  have in common is that each shifts surplus-sharing through Nash bargaining towards efficient surplus-sharing: the former because of standard Hosios reasons, the latter because, given the primitives of the model,  $\chi$  has no role in determining efficient allocations because it represents neither preferences nor technology. Indeed,  $\eta$  also has no role in determining efficient allocations because bargaining is *only* a feature of decentralization.

The first three rows of Table 5 show that it is really the combination of large unemployment transfers and very low worker bargaining power that is important in driving the very large tax volatility in the baseline model. If both structural parameters are simultaneously set to their efficient values (the fourth row of Table 5), then both static and intertemporal wedges are completely stabilized across time. The mapping from wedges to taxes in this case is easy. Comparing (29) with (26) shows that intertemporal efficiency is achieved with  $\tau_t^s = 0 \forall t$ . In turn, condition (28) shows that static wedge smoothing implies labor tax smoothing.<sup>35</sup> Moreover, the dynamics of *all Ramsey allocations are identical* regardless of the  $(\eta, \chi)$  pair in the decentralized economy: Tables 6, 7, and 8, which appear in Appendix F, show this.

## 7.5 Wage Dynamics: The Importance of Wage Rigidity

The reinstatement of tax smoothing conditional on the efficient surplus split demonstrates that it is not matching frictions per se that cause tax volatility. Moreover, it also demonstrates that there is nothing special about Nash bargaining in generating optimal tax volatility. Rather, tax volatility is due to inefficient wage flexibility, which is formalized in our Nash bargaining setup by low worker bargaining power and high unemployment transfers. These parameters are not chosen purposefully to generate tax volatility, but rather because they are one way to generate sufficiently rigid real (pre-tax) wages that: 1) for the exogenous-policy equilibrium, go in the right direction in explaining empirically-plausible wage volatility in the data (compare Table 1 and Table 2); and 2) for the normative Ramsey results, create the inefficient sharing of match rents between workers and firms. More broadly, any wage-setting protocol that delivers rigid wages requires tax variability as part of the optimal policy.<sup>36</sup>

As noted above, Tables 6, 7, and 8 in Appendix F show that the fluctuations implemented by the Ramsey government are completely independent of the  $(\eta, \chi)$  pair in the decentralized economy.

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<sup>35</sup>It is interesting that this one-to-one mapping between perfect wedge smoothing and perfect tax smoothing corresponds to Werning's (2007) proof for frictionless labor markets for the case of the isoelastic labor subutility function (15), despite the fact that here labor markets are subject to random matching.

<sup>36</sup>We have confirmed this for the case of the Ramsey equilibrium in which  $w_t = \bar{w} \forall t$ , in which  $\bar{w}$  is a constant real wage that lies inside the equilibrium bargaining set.

The tables also show that wage dynamics *do* depend on  $(\eta, \chi)$ , however. The Ramsey government cares about the dynamics of allocations and uses whatever *after-tax wage* dynamics are required to achieve them, including tax smoothing if that achieves the objective. Thus, all Ramsey equilibria achieve the same welfare because welfare depends only on allocations.

## 7.6 Long-Run Ramsey Distortions

Our main focus has been on the cyclical behavior of taxes in achieving efficient *fluctuations*. This does not mean that Ramsey equilibria achieve the efficient *level* of activity. Figure 3 plots a few indicators of the long-run inefficiency of Ramsey equilibria, which is unavoidable because the Ramsey government must generate revenue using proportional — distortionary — taxes.

Consistent with the preceding analysis, the upper left and upper middle panels of Figure 3 show that long-run inefficiencies are loaded entirely on the static margin. This amounts to a distortion in the long-run participation rate: the upper right panel shows that participation is about 3 percent lower than its efficient level. In the exogenous-policy economy, both long-run static and intertemporal distortions vary incredibly as  $\eta$  varies.<sup>37</sup>

Long-run Ramsey inefficiency is independent of the parameters  $\eta$  and  $\chi$ . Figure 3 demonstrates this by varying  $\eta$  and holding fixed all other parameters, including  $\chi$ , from the exogenous-policy model.<sup>38</sup> Just as with the business-cycle results, Ramsey allocations are independent of  $\eta$  and  $\chi$ . The decentralization of the Ramsey allocation, however, depends on how wages are determined, which is the reason optimal long-run tax rates, shown in the lower panel of Figure 3, do vary with  $\eta$ . Such a result — allocations being invariant to a structural parameter, even though taxes do vary with that parameter — does not have a counterpart in baseline Ramsey models, and arises due to the degree of freedom regarding price-setting that is inherent in a matching model, one of the main points made by Hall (2005).

## 7.7 Elasticity of Labor Force Participation

An independent point of interest is to what extent the elasticity of participation is important for the results. As discussed in the introduction, although there has been growing interest in modeling the participation margin in matching frameworks, most models continue to assume fixed participation. Our model captures inelastic participation by setting  $\iota = 0$  in the labor subutility function (15).

The upper left panel of Figure 4 shows that labor tax volatility, as measured by our main metric of volatility relative to GDP, is higher for  $\iota = 0$  than the baseline  $\iota = 0.18$ . By the metric of absolute

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<sup>37</sup>The magnitudes of distortions are so incomparable to that of the Ramsey allocations that they are plotted on the right-hand axes of the upper left and upper middle panels.

<sup>38</sup>For brevity, we do not present the experiment of varying  $\chi$  while holding fixed all other parameters, including  $\eta$ . The main message of this mirror-image experiment is the same.

volatility (the upper right panel), tax volatility is instead lower with inelastic participation, but would still be characterized as “tax volatility” by the benchmarks of the Ramsey literature.<sup>39</sup> Thus, while the elasticity of participation does matter for precise quantitative policy predictions, it does not alter the main result of optimal tax volatility.

## 7.8 Optimal Taxation Issues: Completeness of Tax System and Persistence

An important issue in models of optimal taxation is whether or not the available tax instruments constitute a *complete* tax system. The tax system is complete in our model. Establishing this is important for two reasons. First, at a technical level, proving completeness reaffirms that the Ramsey problem as formulated in Section 4 is indeed correct. As shown by Chari and Kehoe (1999, p. 1680), Correia (1996), Armenter (2008), and many others, incompleteness of the tax system requires imposing additional constraints that reflect the incompleteness. Second, it is well-understood in Ramsey theory that incomplete tax systems can lead to a wide range of “unnatural” policy prescriptions in which the use of some instruments (in either the short run or the long run) proxy for other, perhaps more natural, instruments. Demonstrating completeness therefore establishes that none of our results is due to any policy instrument serving as imperfect proxies for other, unavailable, instruments.

As Chari and Kehoe (1999, pp. 1679-1680) explain, an incomplete tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that, in the decentralized economy, uniquely creates a wedge between MRS of those goods and the corresponding MRT. Based on the model-appropriate concepts of MRTs and wedges developed in Sections 5 and 6, it is trivial to show that the pair of instruments  $(\tau_t^n, \tau_t^s)$  constitutes a complete tax system.

The argument is as follows: Proposition 1 proved that there are two margins of adjustment in the economy. Completeness thus requires two policy instruments whose *joint* setting induces a unique wedge in each of the two margins. The two instruments  $\tau_t^n$  and  $\tau_t^s$  do exactly this. Even though *both* instruments appear in *both* the static wedge (28) *and* the intertemporal wedge (29), they appear in different relation to each other in the two different wedges. A policy pair  $(\tau_t^n, \tau_t^s)$  thus determines each wedge uniquely.

A consequence of completeness of the tax system is that the introduction of any additional tax instruments into the environment necessarily implies indeterminacy of the decentralization of Ramsey allocations. Some of the resulting new policy decentralizations would feature constant labor income tax rates along the business cycle. If one were to prefer this way of “restoring the

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<sup>39</sup>The difference between relative and absolute volatilities is due to the fact that the long-run labor tax rate rises as  $\iota$  rises, as the bottom row of Figure 4 shows.

optimality of tax-smoothing,” it must be driven by considerations outside the scope of the model. The model does not provide any basis for preferring one decentralization over another, which is a well-understood point in Ramsey models. Hence, loading redundant policy instruments onto the static and intertemporal wedges would be an uninteresting way of restoring labor tax smoothing.

Finally, we have noted that optimal tax dynamics in the model have nothing to do with completeness or incompleteness of government debt markets, as Aiyagari, Marcet, Sargent, and Sepphala (2002) showed can be important. They showed that tax rates inherit the serial correlation of the underlying shocks to the economy with complete markets, but display near-unit-root behavior with incomplete markets. Our simulations (the main results presented in Table 4 and the robustness results presented in Tables 6, 7, and 8 in Appendix F) show that the serial correlation of taxes depends on the  $(\eta, \chi)$  pair in the decentralized economy. This result is not one that can be or should be compared to the Aiyagari et al types of results because the *fluctuations* of taxes in our model are due to pure efficiency concerns, not to government financing issues. It is only the *level* of taxes in our model that is driven by government financing needs.<sup>40</sup>

## 8 Conclusion

The main results are easy to summarize. We started with a simple DSGE labor search-and-matching model, calibrated so that it generates reasonable business-cycle fluctuations of several key labor-market outcomes. When the exogenous government is replaced by a Ramsey government, labor-market fluctuations look very different. Optimal policy sharply dampens the volatility of labor-market outcomes, and the Beveridge curve disappears. These Ramsey fluctuations are *efficient*, and implementing them requires volatility of labor income tax rates, contrary to the basic tax-smoothing insights of the Ramsey literature. Nonetheless, wedges, or distortions, in the economy are smoothed over the business cycle. Thus, not only is the connection between wedge-smoothing and tax-smoothing much more tenuous in a matching model than in a Walrsian-based model, for empirically-relevant parameters the connection is non-existent.

We make the connection (or lack thereof) between wedges and taxes in a matching model clear by developing static and intertemporal dimensions of efficiency that are analogs to their counterparts in RBC models. This notion of the “labor wedge” is structural with respect to matching models and thus is the welfare-relevant measure of wedges for normative studies. We advocate that this independent contribution of our work should be applied by other researchers studying policy issues

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<sup>40</sup>An impulse response experiment, which is available upon request from the authors, shows this especially clearly: with zero persistence of exogenous shocks, the optimal tax rate jumps on impact, but then returns almost completely to its long-run value one period later and has returned to its long-run value within just a few periods. Thus, taxes do *not* display near-unit-root behavior, which is the hallmark of optimal tax dynamics with incomplete markets.

(whether fiscal or monetary) in quantitative matching models because it brings intuitive clarity to the forces shaping optimal policy and is easy to communicate to a wide audience.

Not only is our concept of labor wedges applicable to theoretical policy analysis, it also can provide a new basis for the “labor-wedge accounting” measurements that have become common since Chari, Kehoe, and McGrattan (2007) and Shimer (2009). A virtue of our view of efficiency and wedges in this positive regard is that it nests the standard RBC-based view on which existing empirical studies are based. As such, it may provide a basis for explaining, among other empirical applications, the contribution of matching frictions to standard measures of labor wedges.

Important to emphasize is that neither matching frictions nor Nash bargaining is necessary to generate the main optimal tax volatility result. The result is driven by inefficiencies in the wage-setting process. Fundamentally, what is required to generate these inefficiencies is a friction that creates a surplus — matching frictions in our model — that is then split in an inefficient way — real wage rigidity in our model — regardless of whether that inefficiency stems from Nash bargaining with a particular calibration or otherwise. In this regard, our results are similar to the monetary policy results of Blanchard and Gali (2010) and Ravenna and Walsh (2011). A broader read of our results is that any wage-determination mechanism, such as a perfectly rigid wage as in Hall (2005), that generates empirically-relevant labor market volatility but implies inefficient surplus-sharing will cause optimal taxes to be very volatile. It has become widely-understood that the wage-determination mechanism in a matching model is critically important for business-cycle fluctuations conditional on exogenous fiscal policy; our results show that it is as important for the dynamics of optimal fiscal policy.

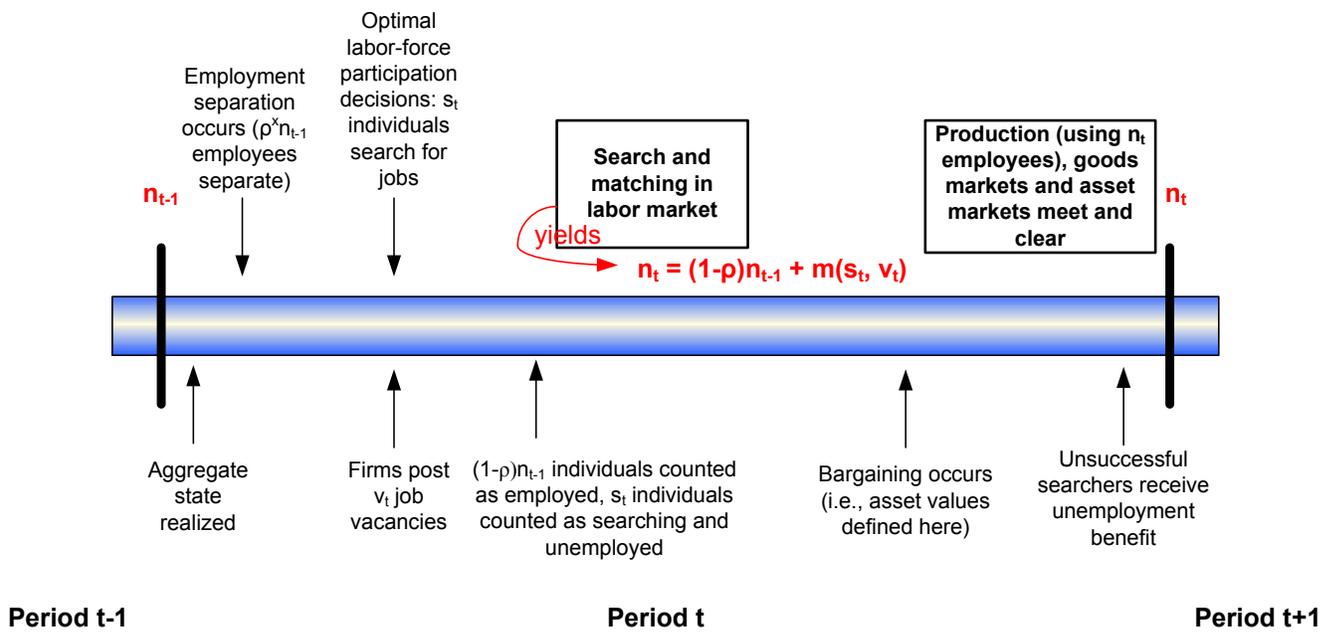


Figure 1: Timing of events.

|                             | $y_t$        | $s_t$ | $v_t$ | $\theta_t$ | $lfp_t$ | $n_t$ | $w_t$ | $\tau_t^n$ |
|-----------------------------|--------------|-------|-------|------------|---------|-------|-------|------------|
| Relative standard deviation | 1            | 5.15  | 6.30  | 11.28      | 0.20    | 0.60  | 0.52  | 1.92       |
| Autocorrelation             | 0.87         | 0.91  | 0.91  | 0.91       | 0.68    | 0.94  | 0.91  | 0.66       |
| Correlation with $y$        | 1            | -0.86 | 0.91  | 0.90       | 0.39    | 0.78  | 0.56  | 0.20       |
| Correlation ( $s_t, v_t$ )  | <b>-0.89</b> |       |       |            |         |       |       |            |
| Long-run $lfp$              | <b>0.74</b>  |       |       |            |         |       |       |            |

Table 1: Cyclical dynamics of U.S. labor markets. Quarterly business-cycle statistics (1964:1-2005:1) for  $y$ ,  $s$ ,  $v$ ,  $\theta$ ,  $n$ , and  $w$  taken from Gertler and Trigari (2009, Table 2). Quarterly business cycle statistics for  $lfp$  taken from Veracierto (2008, Table 2).

| Parameter Value                              | Description/Notes                                                        |
|----------------------------------------------|--------------------------------------------------------------------------|
| <u>Preferences</u>                           |                                                                          |
| $\beta = 0.99$                               | Households' quarterly subjective discount factor                         |
| $\iota = 0.18$                               | Elasticity of participation with respect to real wage                    |
| $\kappa = 7$                                 | Labor subutility parameter                                               |
| <u>Labor Markets</u>                         |                                                                          |
| $\chi = 0.76$                                | Unemployment benefits                                                    |
| $\eta = 0.05$                                | Nash bargaining power of workers                                         |
| $\xi = 0.40$                                 | Elasticity of aggregate matches with respect to search unemployment      |
| $\psi = 0.77$                                | Matching function calibrating parameter                                  |
| $\rho = 0.10$                                | Quarterly job separation rate                                            |
| $\gamma = 0.27$                              | Fixed cost of posting vacancies                                          |
| <u>Exogenous Government Spending Process</u> |                                                                          |
| $\bar{g} = 0.11$                             | Long-run level of government spending (excluding unemployment transfers) |
| $\rho_g = 0.97$                              | Quarterly persistence of log $g$ process                                 |
| $\sigma_g = 0.027$                           | Standard deviation of log TFP shock                                      |
| <u>Exogenous TFP Process</u>                 |                                                                          |
| $\rho_z = 0.95$                              | Quarterly persistence of log TFP process                                 |
| $\sigma_z = 0.006$                           | Standard deviation of log TFP shock                                      |
| <u>Exogenous Tax Policy</u>                  |                                                                          |
| $\bar{\tau}^n = 0.20$                        | Long-run labor-income tax rate                                           |
| $\rho_{\tau^n} = 0.66$                       | Quarterly persistence of log tax process                                 |
| $\sigma_{\tau^n} = 0.02$                     | Standard deviation of log tax shock                                      |

Table 2: Baseline calibration of exogenous-policy model.

|                             | $y_t$        | $s_t$ | $v_t$ | $\theta_t$ | $lfp_t$ | $n_t$ | $w_t$ | $\tau_t^n$ |
|-----------------------------|--------------|-------|-------|------------|---------|-------|-------|------------|
| <u>All shocks</u>           |              |       |       |            |         |       |       |            |
| Relative standard deviation | 1            | 5.36  | 6.97  | 10.88      | 0.20    | 0.79  | 0.28  | 1.37       |
| Autocorrelation             | 0.91         | 0.91  | 0.58  | 0.84       | 0.93    | 0.90  | 0.68  | 0.66       |
| Correlation with $y$        | 1            | -0.90 | 0.80  | 0.97       | -0.75   | 0.88  | -0.15 | -0.44      |
| Correlation ( $s_t, v_t$ )  | <b>-0.55</b> |       |       |            |         |       |       |            |
| Long-run $lfp$              | <b>0.74</b>  |       |       |            |         |       |       |            |
| <u>TFP shocks</u>           |              |       |       |            |         |       |       |            |
| Relative standard deviation | 1            | 5.08  | 5.72  | 10.06      | 0.17    | 0.72  | 0.09  | 0          |
| Autocorrelation             | 0.94         | 0.96  | 0.74  | 0.92       | 0.96    | 0.94  | 0.91  | —          |
| Correlation with $y$        | 1            | -0.95 | 0.87  | 0.99       | -0.99   | 0.94  | 0.99  | —          |
| Correlation ( $s_t, v_t$ )  | <b>-0.74</b> |       |       |            |         |       |       |            |
| Long-run $lfp$              | <b>0.74</b>  |       |       |            |         |       |       |            |

Table 3: Baseline model under exogenous policy. Top panel: shocks to TFP, government purchases, and labor-income tax rate. Bottom panel: shocks only to TFP.

|                        | $y$          | $s$   | $v$   | $\theta$ | $lfp$ | $n$  | $w$   | $\tau^n$ | $\tau^s$ |
|------------------------|--------------|-------|-------|----------|-------|------|-------|----------|----------|
| Volatility             | 1            | 1.39  | 1.25  | 1.10     | 0.13  | 0.14 | 0.50  | 5.58     | 16.23    |
| Autocorrelation        | 0.89         | -0.33 | -0.48 | 0.89     | 0.92  | 0.58 | -0.44 | 0.67     | 0.89     |
| Correlation with $y$   | 1            | -0.38 | 0.44  | 0.97     | -0.08 | 0.10 | -0.33 | -0.86    | -0.98    |
| Correlation ( $s, v$ ) | <b>0.66</b>  |       |       |          |       |      |       |          |          |
| Long-run $\tau^n$      | <b>10.5%</b> |       |       |          |       |      |       |          |          |
| Long-run $\tau^s$      | <b>2.2%</b>  |       |       |          |       |      |       |          |          |
| Long-run $lfp$         | <b>0.74</b>  |       |       |          |       |      |       |          |          |

Table 4: Optimal policy. Volatilities are relative to the volatility of GDP, except  $\tau^s$ , which is the level of volatility around the long-run  $\tau^s$ . Shocks are to TFP and government purchases.

| Parameter Set          | SD(%) of static wedge |             | SD(%) of intertemporal wedge |             | Optimal tax dynamics |                    |
|------------------------|-----------------------|-------------|------------------------------|-------------|----------------------|--------------------|
|                        | Exog. policy          | Opt. policy | Exog. policy                 | Opt. policy | Vol. of $\tau_t^n$   | Vol. of $\tau_t^s$ |
| Baseline               | 22.9                  | 0.08        | 12.3                         | 0           | 5.57                 | 16.2               |
| $\eta = \xi$           | 20.6                  | 0.04        | 8.3                          | 0           | 0.40                 | 1.5                |
| $\chi = 0$             | 0.66                  | 0.24        | 0.12                         | 0           | 0.46                 | 3.5                |
| $\eta = \xi, \chi = 0$ | 0.66                  | 0           | 0.63                         | 0           | 0                    | 0                  |

Table 5: Volatility of static and intertemporal wedges in exogenous-policy equilibria and Ramsey equilibria, and volatility of taxes in Ramsey equilibria. Volatility of labor income tax reported as coefficient of variation relative to that of GDP, and volatility of vacancy subsidy reported as absolute level of around the long-run subsidy. Shocks are to TFP and government purchases.

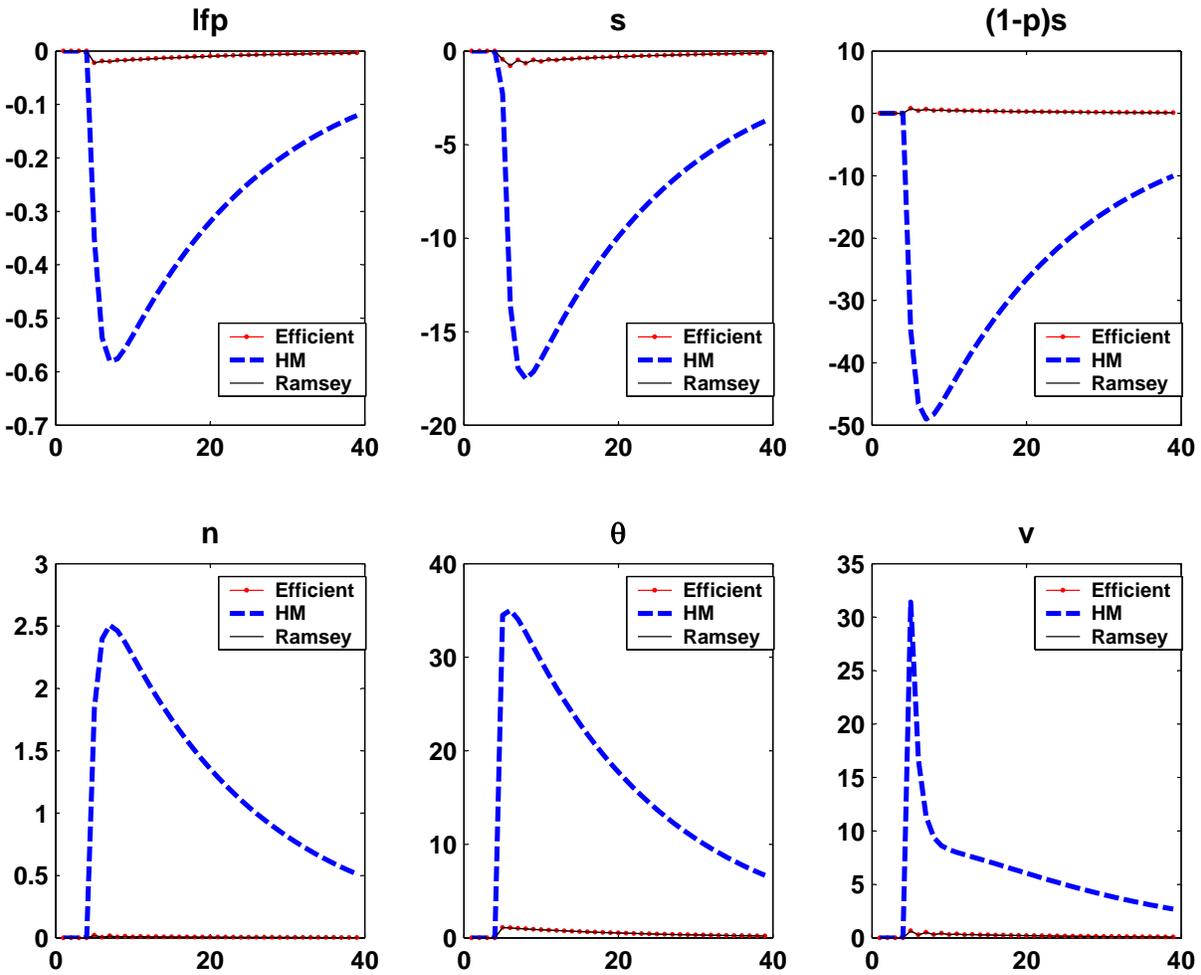


Figure 2: Labor-market responses to one-percent positive shock to  $z_t$  in three different equilibria: efficient equilibrium, baseline exogenous-policy model with HM calibration, and Ramsey equilibrium. Vertical axes plot percentage deviation from respective long-run equilibrium. (Note: The efficient responses and Ramsey responses are identical in each panel.)

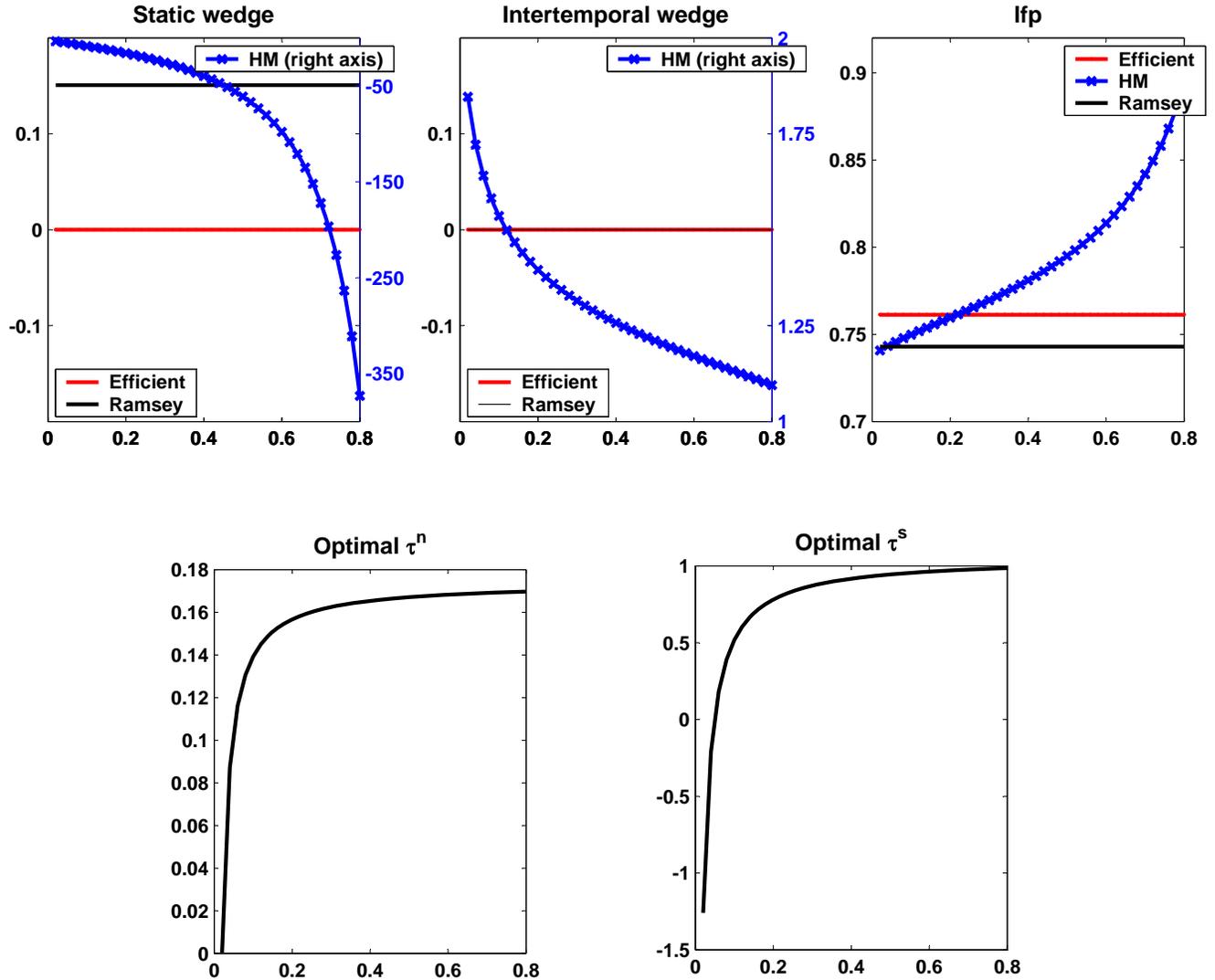


Figure 3: Long-run effects of bargaining power  $\eta$  on efficient allocations, Ramsey allocations, and allocations in baseline exogenous-policy model. Bottom row plots optimal labor tax rate and vacancy subsidy rate. Top left panel and top middle panel plot exogenous-policy model on right axis. Both the static and intertemporal wedges are plotted as  $(1 - MRS/MRT)$ . Horizontal axis in each panel plots bargaining power over the range  $\eta \in (0, 0.8)$ .

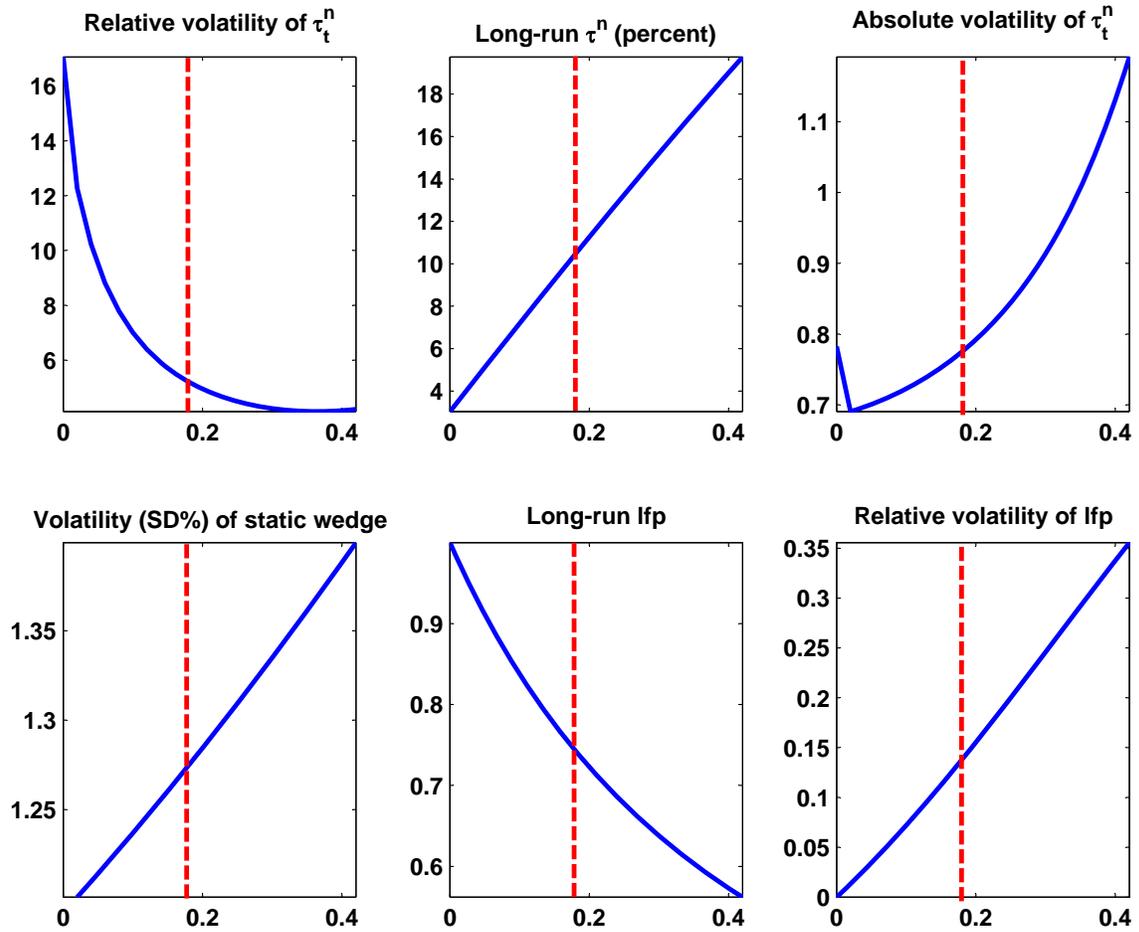


Figure 4: Short-run and long-run features of Ramsey equilibrium for various value of elasticity of labor-force participation,  $\nu$ . Baseline value  $\nu = 0.18$  shown as vertical dashed line.

## A Household Optimization

Using the definition  $ue_t = (1 - p_t)s_t$ , which follows from the assumed timing of events within any period  $t$ , the household optimization problem is: choose state-contingent processes for  $\{c_t\}$ ,  $\{b_t\}$ ,  $\{s_t\}$ , and  $\{n_t^h\}$  to maximize

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) - h((1 - p_t)s_t + n_t^h) \right] \quad (32)$$

subject to sequences of flow budget constraints

$$c_t + b_t = (1 - \tau_t^n)w_t n_t^h + (1 - p_t)s_t \chi + R_t b_{t-1} + (1 - \tau^d)d_t \quad (33)$$

and perceived laws of motion for the employment stock

$$n_t^h = (1 - \rho)n_{t-1}^h + s_t p_t. \quad (34)$$

Denote by  $\{\lambda_t\}$  and  $\{\mu_t^h\}$  the sequences of Lagrange multipliers on the sequences of these constraints, respectively. The first-order conditions with respect to  $c_t$ ,  $b_t$ ,  $s_t$ , and  $n_t^h$  are, respectively,

$$u'(c_t) - \lambda_t = 0, \quad (35)$$

$$-\lambda_t + \beta R_t E_t \lambda_{t+1} = 0, \quad (36)$$

$$-(1 - p_t)h'((1 - p_t)s_t + n_t^h) + \lambda_t(1 - p_t)\chi + \mu_t^h p_t = 0, \quad (37)$$

and

$$-\mu_t^h + \lambda_t(1 - \tau_t^n)w_t - h'((1 - p_t)s_t + n_t^h) + \beta(1 - \rho)E_t \mu_{t+1}^h = 0. \quad (38)$$

Conditions (35) and (36) clearly imply a standard bond-Euler equation, which is expression (4) in the main text. With first-order conditions now computed, switch to the notation  $lfp_t = ue_t + n_t^h = (1 - p_t)s_t + n_t^h$ , which follows from the accounting identities of the model.

To obtain the labor-force-participation (LFP) condition, first solve (37) for the multiplier  $\mu_t^h$ :

$$\mu_t^h = \left( \frac{1 - p_t}{p_t} \right) [h'(lfp_t) - \lambda_t \chi]. \quad (39)$$

Substituting this along with  $\lambda_t = u'(c_t)$  into the FOC on  $n_t^h$  gives

$$\left( \frac{1 - p_t}{p_t} \right) [h'(lfp_t) - u'(c_t)\chi] = u'(c_t)(1 - \tau_t^n)w_t - h'(lfp_t) + \beta(1 - \rho)E_t \mu_{t+1}^h. \quad (40)$$

Solving this expression for the marginal rate of substitution  $h'(lfp_t)/u'(c_t)$  gives

$$\frac{h'(lfp_t)}{u'(c_t)} = p_t \left[ (1 - \tau_t^n)w_t + (1 - \rho)E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\mu_{t+1}^h}{u'(c_{t+1})} \right\} \right] + (1 - p_t)\chi, \quad (41)$$

which is the LFP condition (5) in the main text (given the equilibrium expression for  $\mu_t^h$ , updated to period  $t + 1$ , shown above and the definition of the household stochastic discount factor  $\Xi_{t+1|t} \equiv \beta u'(c_{t+1})/u'(c_t)$ ). As shown below, the continuation term  $(1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\mu_{t+1}^h}{u'(c_{t+1})} \right\}$  is the envelope condition of the period- $t + 1$  household optimization problem, discounted so that it is expressed in terms of period- $t$  goods.

An alternative representation of the LFP condition, however, is more immediately useful for the Nash bargaining problem in Appendix B. To obtain this alternative representation, divide both sides by  $p_t$ , and subtract  $\chi/p_t$  from both sides, which gives

$$\frac{h'(lfp_t) - u'(c_t)\chi}{p_t u'(c_t)} = (1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\mu_{t+1}^h}{u'(c_{t+1})} \right\}. \quad (42)$$

Next, use the period- $t + 1$  version of (39) to substitute for the  $\mu_{t+1}^h$  on the right-hand side, which gives

$$\frac{h'(lfp_t) - u'(c_t)\chi}{p_t u'(c_t)} = (1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) \frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{p_{t+1} u'(c_{t+1})} \right\}. \quad (43)$$

This representation of the LFP condition is recursive in the term  $\frac{h'(ue_t + n_t^h) - u'(c_t)\chi}{p_t u'(c_t)}$ , an observation that is useful for the setup of the Nash bargaining problem in Appendix B.

Also for use in Appendix B, define the value function associated with the household problem by  $\mathbf{V}(n_{t-1}^h)$ . The associated envelope condition is thus

$$\mathbf{V}'(n_{t-1}^h) = (1 - \rho)\mu_t^h \quad (44)$$

$$= (1 - \rho) \left( \frac{1 - p_t}{p_t} \right) [h'(lfp_t) - u'(c_t)\chi], \quad (45)$$

where the second line follows from (39). Finally, for use in Appendix B, the period  $t + 1$  envelope condition can be expressed in discounted terms as

$$\begin{aligned} \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\mathbf{V}'(n_t^h)}{u'(c_{t+1})} &= (1 - \rho) \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\mu_{t+1}^h}{u'(c_{t+1})} \\ &= (1 - \rho) \frac{\beta u'(c_{t+1})}{u'(c_t)} (1 - p_{t+1}) \left( \frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{p_{t+1} u'(c_{t+1})} \right) \end{aligned} \quad (46)$$

## B Nash Bargaining

This section presents the details of the derivation of the Nash wage equation. From the analysis of the household problem in Appendix A, the conditions that are needed here are the LFP condition (43) and the envelope condition (46).

The other primitives that must be defined are the values to the household of the marginal employed individual and the marginal unemployed individual *at the time bargaining occurs*. As Figure 1 shows, these values are properly defined in the “second subperiod” of period  $t$ , immediately after labor matching has taken place and thus each individual’s measured labor market status for period  $t$  is known. In contrast, household-level decisions (in particular, the participation decision of how many individuals to send to search for jobs) occurs in the “first subperiod” of period  $t$ , *before* matching has taken place. The temporal separation of events in the model requires that we construct the bargaining-relevant value equations by simply accounting for the payoffs (viewed from the perspective of the household) that accrue to an employed individual and to unemployed (unsuccessful search) individual; denote these values, respectively, by  $\mathbf{W}_t$  and  $\mathbf{U}_t$ .

An individual who is employed (whether newly employed or not) following labor matching in period  $t$  has value (measured in terms of goods) to the household

$$\mathbf{W}_t = (1 - \tau_t^n)w_t + E_t \left\{ \Xi_{t+1|t} \frac{\mathbf{V}'(n_t^h)}{u'(c_t)} \right\}. \quad (47)$$

The payoffs are an immediate after-tax wage and the marginal value to the household of entering period  $t+1$  with another pre-existing employment relationship, which is measured by the household-level envelope condition.

An individual who unsuccessfully searched for a job in period  $t$  is classified as unemployed and has value (measured in terms of goods) to the household

$$\mathbf{U}_t = \chi. \quad (48)$$

There is zero continuation payoff to the household of an unemployed individual because the household re-optimizes participation at the start of period  $t+1$ , and  $ue_t$  is *not* a state variable for the household at the start of period  $t+1$ .

The surplus of employment is thus

$$\mathbf{W}_t - \mathbf{U}_t = (1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\mathbf{V}'(n_t^h)}{u'(c_{t+1})} \right\} \quad (49)$$

$$= (1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) \left( \frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{p_{t+1}u'(c_{t+1})} \right) \right\}, \quad (50)$$

in which the second line makes use of (46). The  $1 - p_{t+1}$  term appears because of the “instantaneous hiring” timing of the model, in which an individual that is newly unemployed can immediately

(i.e., within the same time period) search, find a job, and begin working again. Comparing (52) to condition (43), it is clear that

$$\mathbf{W}_t - \mathbf{U}_t = \frac{h'(lf p_t) - u'(c_t)\chi}{p_t u'(c_t)}. \quad (51)$$

The surplus earned by the household of successfully completing wage negotiations can finally be expressed as

$$\mathbf{W}_t - \mathbf{U}_t = (1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t}(1 - p_{t+1})(\mathbf{W}_{t+1} - \mathbf{U}_{t+1}) \right\}. \quad (52)$$

An analogous derivation on the firm side leads to an expression for the surplus to a firm of the marginal worker:

$$\mathbf{J}_t = z_t - w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t}\mathbf{J}_{t+1} \right\}. \quad (53)$$

For use below, note that  $\mathbf{J}_t = \frac{\gamma(1-\tau_t^s)}{q_t}$ .

In generalized Nash bargaining, the parties choose  $w_t$  to maximize

$$(\mathbf{W}_t - \mathbf{U}_t)^\eta \mathbf{J}_t^{1-\eta}. \quad (54)$$

The solution to this problem gives the time- $t$  generalized Nash sharing rule,  $\frac{\mathbf{W}_t - \mathbf{U}_t}{1 - \tau_t^n} = \frac{\eta}{1 - \eta} \mathbf{J}_t$ .

Now proceed to derive an explicit expression for  $w_t$ . Inserting the expression for  $\mathbf{W}_t - \mathbf{U}_t$  into the Nash sharing rule,

$$-\frac{\chi}{1 - \tau_t^n} + w_t + \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t}(1 - p_{t+1})(\mathbf{W}_{t+1} - \mathbf{U}_{t+1}) \right\} = \frac{\eta}{1 - \eta} \mathbf{J}_t, \quad (55)$$

and then using the time- $t + 1$  Nash sharing rule,

$$-\frac{\chi}{1 - \tau_t^n} + w_t + \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t}(1 - p_{t+1})(1 - \tau_{t+1}^n) \frac{\eta}{1 - \eta} \mathbf{J}_{t+1} \right\} = \frac{\eta}{1 - \eta} \mathbf{J}_t. \quad (56)$$

Make the substitution  $\mathbf{J}_t = \frac{\gamma(1-\tau_t^s)}{q_t}$ , and similarly for  $\mathbf{J}_{t+1}$ , which yields

$$-\frac{\chi}{1 - \tau_t^n} + w_t + \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t}(1 - p_{t+1})(1 - \tau_{t+1}^n) \frac{\eta}{1 - \eta} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} = \frac{\eta}{1 - \eta} \frac{\gamma(1 - \tau_t^s)}{q_t}. \quad (57)$$

Next, use the job-creation condition  $\frac{\gamma(1-\tau_t^s)}{q_t} = z_t - w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1-\tau_{t+1}^s)}{q_{t+1}} \right\}$  to substitute on the right-hand-side, which gives

$$-\frac{\chi}{1 - \tau_t^n} + w_t + \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t}(1 - p_{t+1})(1 - \tau_{t+1}^n) \frac{\eta}{1 - \eta} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} = \frac{\eta}{1 - \eta} \left[ z_t - w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} \right]. \quad (58)$$

Grouping terms involving  $w_t$ ,

$$w_t \left[ 1 + \frac{\eta}{1 - \eta} \right] = \frac{\eta}{1 - \eta} z_t + \frac{\chi}{1 - \tau_t^n} - \frac{\eta}{1 - \eta} \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t}(1 - p_{t+1})(1 - \tau_{t+1}^n) \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} + \frac{\eta}{1 - \eta} (1 - \rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} \quad (59)$$

Finally, multiplying by  $1 - \eta$  gives the wage equation

$$\begin{aligned}
w_t &= \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau_t^n} \\
&\quad - \eta \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) (1 - \tau_{t+1}^n) \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} + \eta(1 - \rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} \\
&= \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau_t^n} + \eta(1 - \rho) E_t \left\{ \Xi_{t+1|t} \left[ 1 - (1 - p_{t+1}) \frac{1 - \tau_{t+1}^n}{1 - \tau_t^n} \right] \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}, \quad (60)
\end{aligned}$$

which is expression (10) in the main text.

If tax rates never fluctuated, so that  $\tau_t^n = \tau^n$  and  $\tau_t^s = \tau^s \forall t$ , this simplifies to

$$w_t = \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau^n} + \eta(1 - \tau^s)(1 - \rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma p_{t+1}}{q_{t+1}} \right\}, \quad (61)$$

which, because  $\theta_{t+1} = p_{t+1}/q_{t+1}$  due to constant-returns matching, is

$$w_t = \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau^n} + \eta(1 - \tau^s)(1 - \rho) E_t \left\{ \Xi_{t+1|t} \gamma \theta_{t+1} \right\}. \quad (62)$$

Furthermore, if we were considering just the steady state, the wage equation becomes

$$w = \eta [z + \beta(1 - \tau^s)(1 - \rho)\gamma\theta] + (1 - \eta) \frac{\chi}{1 - \tau^n}. \quad (63)$$

## C Efficient Allocations

A social planner in this economy optimally allocates the measure one of individuals in the representative household to leisure, search, and employment. There are several representations of the planning problem available: suppose that  $c_t$ ,  $v_t$ ,  $lfp_t$ , and  $n_t$  are the formal objects of choice. Given the accounting identities of the model, search can thus be expressed  $s_t = lfp_t + (1 - \rho)n_{t-1}$ .

The social planning problem is thus

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - h(lfp_t)] \quad (64)$$

subject to the sequence of goods-market resource constraints

$$c_t + g_t + \gamma v_t = z_t n_t \quad (65)$$

and laws of motion for the employment stock

$$n_t = (1 - \rho)n_{t-1} + m(lfp_t - (1 - \rho)n_{t-1}, v_t). \quad (66)$$

Denote by  $\lambda_t^1$  and  $\lambda_t^2$  the Lagrange multipliers on these two constraints, respectively. The first-order conditions with respect to  $c_t$ ,  $v_t$ ,  $lfp_t$ , and  $n_t$  are thus

$$u'(c_t) - \lambda_t^1 = 0, \quad (67)$$

$$-\lambda_t^1 \gamma + \lambda_t^2 m_v(s_t, v_t) = 0, \quad (68)$$

$$-h'(lfp_t) + \lambda_t^2 m_s(s_t, v_t) = 0, \quad (69)$$

and

$$\lambda_t^1 z_t - \lambda_t^2 + (1 - \rho)\beta E_t \left\{ \lambda_{t+1}^2 (1 - m_s(s_{t+1}, v_{t+1})) \right\} = 0. \quad (70)$$

### C.1 Static Efficiency

First, work just with the static conditions (67), (68), and (69). Eliminating  $\lambda_t^2$  between conditions (68) and (69) gives

$$\frac{h'(lfp_t)}{u'(c_t)} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}. \quad (71)$$

For Cobb-Douglas matching and its associated marginals,<sup>41</sup> static efficiency is characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \gamma \theta_t \frac{\xi}{1 - \xi}. \quad (72)$$

---

<sup>41</sup>Cobb-Douglas matching has the properties:

1.  $m(s_t, v_t) = s_t^\xi v_t^{1-\xi}$
2.  $m_s(s_t, v_t) = \xi s_t^{\xi-1} v_t^{1-\xi} = \xi \theta_t^{1-\xi}$
3.  $m_v(s_t, v_t) = (1 - \xi) s_t^\xi v_t^{-\xi} = (1 - \xi) \theta_t^{-\xi}$

Because its derivation relies only on the static first-order conditions (67), (68), and (69), we interpret (71) (or (72)) as the model's static efficiency condition.

## C.2 Intertemporal Efficiency

Using conditions (67) and (68) to eliminate the multipliers from (70) gives

$$\frac{\gamma}{m_v(s_t, v_t)} = z_t + (1 - \rho)E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{\gamma}{m_v(s_{t+1}, v_{t+1})} (1 - m_s(s_{t+1}, v_{t+1})) \right\}. \quad (73)$$

Condition (73) is one representation of efficiency along the intertemporal margin. Instead, using conditions (67) and (69) to eliminate the multipliers from (70) gives

$$\frac{h'(lfp_t)}{u'(c_t)} \frac{1}{m_s(s_t, v_t)} = z_t + (1 - \rho)E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \frac{h'(lfp_{t+1})}{u'(c_{t+1})} \frac{1}{m_s(s_{t+1}, v_{t+1})} (1 - m_s(s_{t+1}, v_{t+1})) \right\}. \quad (74)$$

Condition (74) is a second representation of efficiency along the intertemporal margin.

These two representations of intertemporal efficiency, (73) and (74), are equivalent as long as condition (71) holds, which it does at the efficient allocation. That is, substituting condition (71) into either condition (73) or (74) yields identical representations for intertemporal efficiency. Hence, given that static efficiency is characterized by (71), intertemporal efficiency is equivalently characterized by either (73) or (74). We proceed by considering (73) as characterizing intertemporal efficiency, which is condition (22) in the main text.

## C.3 MRS-MRT Representation of Efficiency

The efficiency conditions (71) and (73) can be described in terms of appropriately-defined concepts of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). Defining MRS and MRT in a model-appropriate way allows us to describe efficiency in terms of the basic principle that efficient allocations are characterized by  $MRS = MRT$  conditions along all optimization margins.

Consider the static efficiency condition (71). The left-hand side is clearly the within-period MRS between consumption and participation (search) in any period  $t$ . We claim that the right-hand side is the corresponding MRT between consumption and participation. Rather than take the efficiency condition (71) as *prima facie* evidence that the right-hand side *must be* the static MRT, however, this MRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization).

First, though, we define MRS and MRT relevant for intertemporal efficiency. To do so, we first restrict attention to the non-stochastic case because it makes clearer the separation of components of preferences from components of technology (due to endogenous covariance terms implied by the

$E_t(\cdot)$  operator). The non-stochastic intertemporal efficiency condition can be expressed as

$$\frac{u'(c_t)}{\beta u'(c_{t+1})} = \frac{(1 - \rho) \left( \frac{\gamma}{m_v(s_{t+1}, v_{t+1})} \right) (1 - m_s(s_{t+1}, v_{t+1}))}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}. \quad (75)$$

The left-hand side of (75) is clearly the intertemporal MRS (hereafter abbreviated IMRS) between  $c_t$  and  $c_{t+1}$ . We claim that the right-hand side is the corresponding intertemporal MRT (hereafter abbreviated IMRT).

Applying this definition to the fully stochastic condition (73), we can thus express intertemporal efficiency as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[ \frac{\frac{(1-\rho)\gamma(1-m_s(s_{t+1}, v_{t+1}))}{m_v(s_{t+1}, v_{t+1})}}{\frac{\gamma}{m_v(s_t, v_t)} - z_t} \right] \right\} = E_t \left\{ \frac{IMRT_{c_t, c_{t+1}}}{IMRS_{c_t, c_{t+1}}} \right\}. \quad (76)$$

Rather than take the efficiency condition (75) as *prima facie* evidence that the right-hand side *must be* the IMRT, however, the IMRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization), to which we now turn.

#### C.4 Proof of Proposition 1: Transformation Frontier and Derivation of MRTs

Based only on the primitives of the environment — that is, independent of the context of any optimization — we now prove that the right-hand sides of (71) and (75) are, respectively, the model-appropriate concepts of the static MRT and deterministic IMRT. Doing so thus proves Proposition 1 in the main text. To conserve on notation, suppose in what follows that government spending is always zero ( $g_t = 0 \forall t$ ), which has no bearing on any of the arguments or conclusions.

Consider the period- $t$  goods resource constraint and law of motion for employment:  $c_t + \gamma v_t = z_t n_t$  and  $n_t = (1 - \rho)n_{t-1} + m(s_t, v_t)$ . Solving the former for  $v_t$ , and substituting in the latter gives

$$n_t - (1 - \rho)n_{t-1} - m\left(s_t, \frac{z_t n_t - c_t}{\gamma}\right) = 0. \quad (77)$$

Next, use the accounting identity  $lfp_t = (1 - \rho)n_{t-1} + s_t$  to substitute for  $s_t$  on the right-hand side, and define

$$\Upsilon(c_t, lfp_t, n_t; \cdot) \equiv n_t - (1 - \rho)n_{t-1} - m\left(lfp_t - (1 - \rho)n_{t-1}, \frac{z_t n_t - c_t}{\gamma}\right) = 0 \quad (78)$$

as the period- $t$  transformation frontier. The function  $\Upsilon(\cdot)$  is a more general notion of a transformation, or resource, frontier than either the goods resource constraint or the law of motion for employment alone because  $\Upsilon(\cdot)$  *jointly* describes the *two* technologies in the economy: the technology that creates employment matches and, conditional on employment matches, the technology that creates output. The dependence of  $\Upsilon(\cdot)$  on (among other arguments)  $c_t$  and  $lfp_t$  is highlighted because the period- $t$  utility function is defined over  $c_t$  and  $lfp_t$ .

By the implicit function theorem, the static MRT between consumption and participation (search) is thus

$$-\frac{\Upsilon_{lfp_t}}{\Upsilon_{c_t}} = \gamma \frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}, \quad (79)$$

which formalizes, independent of the social planning problem, the notion of the static MRT on the right-hand side of the efficiency condition (71) and presented in Proposition 1.

For use in deriving the IMRT below, note that the implicit function theorem also allows us to compute

$$\frac{\partial n_t}{\partial c_t} = -\frac{\Upsilon_{c_t}}{\Upsilon_{n_t}} = -\frac{m_v(s_t, v_t)}{\gamma - m_v(s_t, v_t)z_t}, \quad (80)$$

which gives the marginal effect on period- $t$  employment of a change in period- $t$  consumption. This effect has intertemporal consequences because  $n_t$  is the stock of employment entering period  $t + 1$ ; because (78) cannot be solved explicitly for  $n_t$ , the effect must be accounted for implicitly.

Next, define the transformation frontier that links period  $t$  and period- $t + 1$

$$G(c_{t+1}, n_{t+1}, c_t, n_t; \cdot) = n_{t+1} - (1 - \rho)n_t - m\left(lfp_{t+1} - (1 - \rho)n_t, \frac{z_{t+1}n_{t+1} - c_{t+1}}{\gamma}\right) = 0. \quad (81)$$

The function  $G(\cdot)$  in form is the same as the function  $\Upsilon(\cdot)$ , but, for the purpose at hand, it is useful to view it as a generalization of  $\Upsilon(\cdot)$  in that  $G(\cdot)$  is explicitly viewed as a function of period  $t$  and period  $t + 1$  allocations.<sup>42</sup> The two-period transformation frontier  $G(\cdot)$  has partials with respect to  $c_{t+1}$  and  $c_t$

$$G_{c_{t+1}} = \frac{m_v(s_{t+1}, v_{t+1})}{\gamma} \quad (82)$$

and

$$\begin{aligned} G_{c_t} &= -(1 - \rho)\frac{\partial n_t}{\partial c_t} + (1 - \rho)m_s(s_{t+1}, v_{t+1})\frac{\partial n_t}{\partial c_t} \\ &= (1 - \rho)\left(\frac{m_v(s_t, v_t)}{\gamma - m_v(s_t, v_t)z_t}\right) - (1 - \rho)\left(\frac{m_v(s_t, v_t)}{\gamma - m_v(s_t, v_t)z_t}\right)m_s(s_{t+1}, v_{t+1}) \\ &= (1 - \rho)\left(\frac{m_v(s_t, v_t)}{\gamma - m_v(s_t, v_t)z_t}\right)(1 - m_s(s_{t+1}, v_{t+1})); \end{aligned}$$

the second line follows from substituting (80).

By the implicit function theorem, the IMRT between  $c_t$  and  $c_{t+1}$  is thus

$$\begin{aligned} \frac{G_{c_t}}{G_{c_{t+1}}} &= \frac{(1 - \rho)\left(\frac{m_v(s_t, v_t)}{\gamma - m_v(s_t, v_t)z_t}\right)(1 - m_s(s_{t+1}, v_{t+1}))}{\frac{m_v(s_{t+1}, v_{t+1})}{\gamma}} \\ &= \frac{(1 - \rho)\left(\frac{\gamma}{m_v(s_{t+1}, v_{t+1})}\right)(1 - m_s(s_{t+1}, v_{t+1}))}{\frac{\gamma}{m_v(s_t, v_t)} - z_t}, \quad (83) \end{aligned}$$

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<sup>42</sup>Rather than as a function of only period- $t$  allocations, as we viewed  $\Upsilon(\cdot)$ . Note also that, as must be the case, we could use  $G(\cdot)$ , rather than  $\Upsilon(\cdot)$ , to define the within-period MRT between consumption and participation. By the implicit function theorem, the within-period MRT (for period  $t + 1$ ) is  $-\frac{G_{lfp_{t+1}}}{G_{c_{t+1}}} = \gamma \frac{m_s(s_{t+1}, v_{t+1})}{m_v(s_{t+1}, v_{t+1})}$ , obviously identical to the static MRT derived above.

which formalizes, independent of the social planning problem, the notion of the IMRT on the right-hand side of the (deterministic) efficiency condition (75) and presented in Proposition 1.

With the static MRT and IMRT defined from the primitives of the environment, the efficiency conditions (71) and (75) are indeed interpretable as appropriately-defined  $MRS = MRT$  conditions.

## D Equilibrium Wedges

To present the algebra behind the wedges defined in Section 6, the following equilibrium conditions are needed: the vacancy-creation condition

$$\frac{\gamma(1 - \tau_t^s)}{q_t} = z_t - w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}, \quad (84)$$

the Nash wage outcome

$$w_t = \eta z_t + (1 - \eta) \frac{\chi}{1 - \tau_t^n} - \eta \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) (1 - \tau_{t+1}^n) \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} + \eta (1 - \rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}, \quad (85)$$

and the household's LFP condition

$$\frac{h'(lfp_t) - u'(c_t)\chi}{p_t u'(c_t)} = (1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) \frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{p_{t+1} u'(c_{t+1})} \right\}. \quad (86)$$

### D.1 Definition of Static Wedge

To obtain the decentralized economy's static wedge, divide the LFP condition (86) by the vacancy-creation condition (84), which gives

$$\frac{q_t [h'(lfp_t) - u'(c_t)\chi]}{\gamma(1 - \tau_t^s) p_t u'(c_t)} = \frac{(1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) \frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{p_{t+1} u'(c_{t+1})} \right\}}{z_t - w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}}. \quad (87)$$

Using the result that  $q(\theta_t)/p(\theta_t) = \theta_t^{-1}$  (which follows from constant-returns matching), the left-hand side can be rearranged as

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\gamma(1 - \tau_t^s) \theta_t u'(c_t)} = \frac{(1 - \tau_t^n)w_t - \chi + (1 - \rho)E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1}) \frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{p_{t+1} u'(c_{t+1})} \right\}}{z_t - w_t + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}}. \quad (88)$$

Note from the derivations in Appendix A and in Appendix B that the numerator on the right-hand side of this last expression is  $(\mathbf{W}_t - \mathbf{U}_t)$ , the surplus to the household of having the marginal member enter into an employment relationship.<sup>43</sup> Also note that the denominator on the right-hand side of the previous expression is  $\mathbf{J}_t$ , the surplus to the firm of entering into an employment relationship with one additional worker.

Thus, the last expression is simply

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\gamma(1 - \tau_t^s) \theta_t u'(c_t)} = \frac{\mathbf{W}_t - \mathbf{U}_t}{\mathbf{J}_t}. \quad (89)$$

<sup>43</sup>Specifically, to see this, use condition (39), the envelope condition (44), and condition (52), which defines the surplus to a worker/household  $\mathbf{W}_t - \mathbf{U}_t$ .

Substituting in this expression the private economy's Nash-bargaining outcome  $\frac{\mathbf{W}_t - \mathbf{U}_t}{1 - \tau_t^n} = \frac{\eta}{1 - \eta} \mathbf{J}_t$ , we have

$$\frac{h'(lfp_t) - u'(c_t)\chi}{\gamma(1 - \tau_t^s)\theta_t u'(c_t)} = \frac{\eta}{1 - \eta}(1 - \tau_t^n). \quad (90)$$

Rearranging, we have that in the decentralized Nash-bargaining economy with taxes and unemployment benefits,

$$\frac{h'(lfp_t) - u'(c_t)\chi}{u'(c_t)} = (1 - \tau_t^n)(1 - \tau_t^s)\gamma\theta_t \frac{\eta}{1 - \eta}, \quad (91)$$

in which, recall,  $\eta \in (0, 1)$  is the Nash bargaining power of households. To represent this in terms of a wedge in the static efficiency condition (72), we have

$$\frac{h'(lfp_t)}{u'(c_t)} = \gamma\theta_t \frac{\xi}{1 - \xi} \left[ \frac{\chi(1 - \xi)}{\gamma\xi\theta_t} + (1 - \tau_t^n)(1 - \tau_t^s) \frac{\eta(1 - \xi)}{\xi(1 - \eta)} \right], \quad (92)$$

with the term in square brackets the wedge between MRS and MRT. This is condition (28) in the main text.

As discussed following the definition of the static wedge, comparing (92) with the static efficiency condition (72) shows that four conditions are sufficient for the decentralized economy to achieve static efficiency: the decentralized economy features  $\eta = \xi$ ; the unemployment transfer is  $\chi = 0$ ; proportional labor income taxation is  $\tau_t^n = 0$ ; and the proportional vacancy subsidy is  $\tau_t^s = 0$ . These conditions are not necessary, however, because for any arbitrary ( $\eta \neq \xi, \chi \neq 0$ ), an appropriate setting for policy  $(\tau_t^n, \tau_t^s)$  achieves efficiency.

## D.2 Definition of Intertemporal Wedge

To obtain the decentralized economy's intertemporal wedge, first substitute the Nash wage outcome (85) into the vacancy-creation condition (84):

$$\begin{aligned} \frac{\gamma(1 - \tau_t^s)}{q_t} &= (1 - \eta)z_t - (1 - \eta) \frac{\chi}{1 - \tau_t^n} \\ &+ (1 - \eta)(1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\} + \eta \frac{1 - \rho}{1 - \tau_t^n} E_t \left\{ \Xi_{t+1|t} (1 - p_{t+1})(1 - \tau_{t+1}^n) \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \right\}. \end{aligned} \quad (93)$$

Combining terms inside the  $E_t(\cdot)$  operator,

$$\frac{\gamma(1 - \tau_t^s)}{q_t} = (1 - \eta)z_t - (1 - \eta) \frac{\chi}{1 - \tau_t^n} + (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{q_{t+1}} \left[ 1 - \eta + \frac{\eta(1 - \tau_{t+1}^n)}{1 - \tau_t^n} (1 - p_{t+1}) \right] \right\}. \quad (94)$$

Next, substitute the relationships implied by Cobb-Douglas matching between matching probabilities and the marginal products of the matching function,<sup>44</sup> which gives

$$\begin{aligned} \frac{\gamma(1 - \xi)(1 - \tau_t^s)}{m_v(s_t, v_t)} &= (1 - \eta)z_t - (1 - \eta) \frac{\chi}{1 - \tau_t^n} \\ &+ (1 - \rho)E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \xi)(1 - \tau_{t+1}^s)}{m_v(s_{t+1}, v_{t+1})} \left[ 1 - \eta + \frac{\eta(1 - \tau_{t+1}^n)}{1 - \tau_t^n} \left( 1 - \frac{m_s(s_{t+1}, v_{t+1})}{\xi} \right) \right] \right\}. \end{aligned} \quad (95)$$

<sup>44</sup>Given  $m(s, v) = s^\xi v^{1-\xi}$ , these relationships are  $q = m_v(s, v)/(1 - \xi)$  and  $p = m_s(s, v)/\xi$ .

Finally, divide by  $1 - \xi$  to get

$$\begin{aligned} \frac{\gamma(1 - \tau_t^s)}{m_v(s_t, v_t)} &= \left( \frac{1 - \eta}{1 - \xi} \right) z_t - \left( \frac{1 - \eta}{1 - \xi} \right) \frac{\chi}{1 - \tau_t^n} \\ &+ (1 - \rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma(1 - \tau_{t+1}^s)}{m_v(s_{t+1}, v_{t+1})} \left[ 1 - \eta + \frac{\eta(1 - \tau_{t+1}^n)}{1 - \tau_t^n} \left( 1 - \frac{m_s(s_{t+1}, v_{t+1})}{\xi} \right) \right] \right\}, \end{aligned} \quad (96)$$

which is a representation of the decentralized economy's intertemporal equilibrium condition. This condition is to be compared with the intertemporal efficiency condition (73), which is repeated here for convenience:

$$\frac{\gamma}{m_v(s_t, v_t)} = z_t + (1 - \rho) E_t \left\{ \Xi_{t+1|t} \left[ \frac{\gamma}{m_v(s_{t+1}, v_{t+1})} (1 - m_s(s_{t+1}, v_{t+1})) \right] \right\}. \quad (97)$$

Comparing (96) with (97), it is clear that the sufficient conditions for intertemporal efficiency in the decentralized equilibrium are the same as those that achieve static efficiency:  $\eta = \xi$ ;  $\chi = 0$ ;  $\tau_t^n = 0 \forall t$ ; and  $\tau_t^s = 0 \forall t$ .

We showed above that (97) can be expressed as

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[ \frac{\frac{(1 - \rho)\gamma(1 - m_s(s_{t+1}, v_{t+1}))}{m_v(s_{t+1}, v_{t+1})}}{\frac{\gamma}{m_v(s_t, v_t)} - z_t} \right] \right\} = E_t \left\{ \frac{IMRT_{c_t, c_{t+1}}}{IMRS_{c_t, c_{t+1}}} \right\}. \quad (98)$$

Writing (96) in a similar form, we have

$$1 = E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[ \frac{(1 - \rho) \frac{\gamma(1 - \tau_{t+1}^s)}{m_v(s_{t+1}, v_{t+1})} \left[ 1 - \eta + \frac{\eta(1 - \tau_{t+1}^n)}{1 - \tau_t^n} \left( 1 - \frac{m_s(s_{t+1}, v_{t+1})}{\xi} \right) \right]}{\frac{\gamma(1 - \tau_t^s)}{m_v(s_t, v_t)} - \left( \frac{1 - \eta}{1 - \xi} \right) \left( z_t - \frac{\chi}{1 - \tau_t^n} \right)} \right] \right\}. \quad (99)$$

Comparison of (99) with (98) implicitly defines the intertemporal wedge.

## E Derivation of Implementability Constraint

The derivation of the implementability constraint follows that laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). For notational convenience, in what follows we use the definition  $ue_t = (1 - p_t)s_t$  where possible and also use  $lfp_t = ue_t + n_t$  to conserve on notation. In deriving the implementability constraint, the household's perceived law of motion for employment,

$$n_t^h = (1 - \rho)n_{t-1}^h + p_t s_t, \quad (100)$$

and the labor-force-participation condition

$$\begin{aligned} \left( \frac{1 - p_t}{p_t} \right) [h'(lfp_t) - u'(c_t)\chi] &= u'(c_t)(1 - \tau_t^n)w_t - h'(lfp_t) \\ &+ \beta(1 - \rho)E_t \left\{ \left( \frac{1 - p_{t+1}}{p_{t+1}} \right) [h'(lfp_{t+1}) - u'(c_{t+1})\chi] \right\}, \end{aligned} \quad (101)$$

will be useful.

The rest of the analysis applies to equilibrium conditions, hence we drop  $h$  superscripts on variables. Start with the household flow budget constraint in equilibrium

$$c_t + b_t = (1 - \tau_t^n)w_t n_t + (1 - p_t)s_t \chi + R_t b_{t-1} + (1 - \tau^d)d_t. \quad (102)$$

Multiply by  $\beta^t u'(c_t)$  and sum over dates and states starting from  $t = 0$ ,

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) c_t + E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) b_t &= E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - \tau_t^n) w_t n_t \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - p_t) s_t \chi + E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) R_t b_{t-1} + E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - \tau^d) d_t. \end{aligned}$$

Use the household's Euler equation,  $u'(c_t) = E_t [\beta u'(c_{t+1}) R_{t+1}]$ , to substitute for the  $u'(c_t)$  in the term on the left-hand-side involving  $b_t$ ,

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) c_t + E_0 \sum_{t=0}^{\infty} \beta^{t+1} u'(c_{t+1}) R_{t+1} b_t &= E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - \tau_t^n) w_t n_t \\ &+ E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - p_t) s_t \chi + E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) R_t b_{t-1} + E_0 \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - \tau^d) d_t. \end{aligned}$$

From here on, we suppress the  $E_0$  operator to further conserve on notation.

Canceling terms in the second summation on the left-hand-side with the third summation on the right-hand-side leaves only time-zero bond wealth,

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - \tau_t^n) w_t n_t + \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - p_t) s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t) (1 - \tau^d) d_t + u'(c_0) R_0 b_{-1}. \quad (103)$$

Next, use (101) to substitute for the sequence of terms  $u'(c_t)(1 - \tau_t^n)w_t$  in the first summation on the right-hand-side, which gives

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t \left( \frac{1-p_t}{p_t} \right) [h'_t - u'(c_t)\chi] n_t + \sum_{t=0}^{\infty} \beta^t h'_t n_t \quad (104)$$

$$- (1-\rho) \sum_{t=0}^{\infty} \beta^{t+1} \left( \frac{1-p_{t+1}}{p_{t+1}} \right) [h'_{t+1} - u'(c_{t+1})\chi] n_t + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-p_t)s_t \chi \quad (105)$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1}.$$

To further conserve on notation, we now use  $h'_t$  to stand for  $h'(lf p_t)$ .

Next, use  $n_t = (1-\rho)n_{t-1} + p_t s_t$  to substitute for the sequence of  $n_t$  terms that appears in the first summation on the right-hand-side, which gives

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = (1-\rho) \sum_{t=0}^{\infty} \beta^t \left( \frac{1-p_t}{p_t} \right) [h'_t - u'(c_t)\chi] n_{t-1} \quad (106)$$

$$+ \sum_{t=0}^{\infty} \beta^t \left( \frac{1-p_t}{p_t} \right) [h'_t - u'(c_t)\chi] p_t s_t + \sum_{t=0}^{\infty} \beta^t h'_t n_t \quad (107)$$

$$- (1-\rho) \sum_{t=0}^{\infty} \beta^{t+1} \left( \frac{1-p_{t+1}}{p_{t+1}} \right) [h'_{t+1} - u'(c_{t+1})\chi] n_t + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-p_t)s_t \chi \quad (108)$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1}. \quad (109)$$

The first summation on the right-hand-side cancels with the fourth summation on the right-hand-side, leaving only the time-zero term:

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t \left( \frac{1-p_t}{p_t} \right) [h'_t - u'(c_t)\chi] p_t s_t + \sum_{t=0}^{\infty} \beta^t h'_t n_t \quad (110)$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-p_t)s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} + (1-\rho) \left( \frac{1-p_0}{p_0} \right) [h'_0 - u'(c_0)\chi] n_{-1}.$$

Expanding and rearranging the first summation on the right-hand-side,

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t (1-p_t) h'_t s_t + \sum_{t=0}^{\infty} \beta^t h'_t n_t - \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-p_t)s_t \chi \quad (111)$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-p_t)s_t \chi + \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} + (1-\rho) \left( \frac{1-p_0}{p_0} \right) [h'_0 - u'(c_0)\chi] n_{-1}.$$

Canceling the summations involving  $(1-p_t)s_t \chi$ ,

$$\sum_{t=0}^{\infty} \beta^t u'(c_t) c_t = \sum_{t=0}^{\infty} \beta^t h'_t (1-p_t) s_t + \sum_{t=0}^{\infty} \beta^t h'_t n_t \quad (112)$$

$$+ \sum_{t=0}^{\infty} \beta^t u'(c_t)(1-\tau^d)d_t + u'(c_0)R_0 b_{-1} + (1-\rho) \left( \frac{1-p_0}{p_0} \right) [h'_0 - u'(c_0)\chi] n_{-1}.$$

Finally, collecting terms, using the identity  $lfp_t = (1-p_t)s_t + n_t$ , and re-introducing the conditional expectation  $E_0$ , we have the present-value implementability constraint

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u'(c_t)c_t - h'(lfp_t)lfp_t - u'(c_t)(1 - \tau^d)d_t \right\} = A_0, \quad (113)$$

where

$$A_0 \equiv u'(c_0)R_0b_{-1} + (1 - \rho) \left( \frac{1 - p_0}{p_0} \right) [h'(lfp_0) - u'(c_0)\chi] n_{-1}. \quad (114)$$

|                            | $y_t$        | $s_t$ | $v_t$ | $\theta_t$ | $lfp_t$ | $n_t$ | $w_t$ | $\tau_t^n$ | $\tau^s$ |
|----------------------------|--------------|-------|-------|------------|---------|-------|-------|------------|----------|
| Volatility                 | 1            | 1.39  | 1.25  | 1.10       | 0.13    | 0.14  | 0.88  | 0.40       | 1.51     |
| Autocorrelation            | 0.89         | -0.33 | -0.48 | 0.89       | 0.92    | 0.58  | 0.89  | 0.44       | 0.89     |
| Correlation with $y$       | 1            | -0.38 | 0.44  | 0.97       | -0.08   | 0.10  | 0.99  | -0.79      | -0.98    |
| Correlation ( $s_t, v_t$ ) | <b>0.66</b>  |       |       |            |         |       |       |            |          |
| Long-run $lfp$             | <b>0.74</b>  |       |       |            |         |       |       |            |          |
| Long-run $\tau^n$          | <b>16.5%</b> |       |       |            |         |       |       |            |          |
| Long-run $\tau^s$          | <b>91.7%</b> |       |       |            |         |       |       |            |          |

Table 6: Optimal policy with Hosios condition ( $\eta = \xi$ ). Shocks are to TFP and government purchases. Volatilities are relative to the volatility of GDP, except  $\tau^s$ , which is the level of volatility around the long-run  $\tau^s$ .

## F Restoring the Optimality of Tax Smoothing

Tables 6, 7, and 8 present the Ramsey dynamics of policies and allocations for the cases of, respectively: the baseline parameter set except with the Hosios condition ( $\eta = \xi$ ) in place; the baseline parameter set except with zero unemployment transfers ( $\chi = 0$ ); and the baseline parameter set except with both the Hosios condition in place and zero unemployment transfers. As comparison of these tables with the main Ramsey results presented in Table 4 shows, optimal policy achieves exactly the same dynamics of quantities no matter the  $(\eta, \chi)$  pair in the decentralized economy.

|                                 | $y_t$         | $s_t$ | $v_t$ | $\theta_t$ | $lfp_t$ | $n_t$ | $w_t$ | $\tau_t^n$ | $\tau^s$ |
|---------------------------------|---------------|-------|-------|------------|---------|-------|-------|------------|----------|
| Volatility                      | 1             | 1.39  | 1.25  | 1.10       | 0.13    | 0.14  | 1.39  | 0.45       | 3.45     |
| Autocorrelation                 | 0.89          | -0.33 | -0.48 | 0.89       | 0.92    | 0.58  | 0.89  | 0.91       | 0.91     |
| Correlation with $y$            | 1             | -0.38 | 0.44  | 0.97       | -0.08   | 0.10  | 0.97  | 0.93       | -0.93    |
| Correlation ( $s_t, v_t$ )      | <b>0.66</b>   |       |       |            |         |       |       |            |          |
| Long-run $lfp$                  | <b>0.74</b>   |       |       |            |         |       |       |            |          |
| Long-run $\tau^n$               | <b>-60.4%</b> |       |       |            |         |       |       |            |          |
| Long-run $\tau^s$               | <b>-57.3%</b> |       |       |            |         |       |       |            |          |
| Correlation ( $w_t, \tau_t^n$ ) | 0.97          |       |       |            |         |       |       |            |          |

Table 7: Optimal policy with zero unemployment benefits ( $\chi = 0$ ). Shocks are to TFP and government purchases. Volatilities are relative to the volatility of GDP, except  $\tau^s$ , which is the level of volatility around the long-run  $\tau^s$ .

|                                 | $y_t$        | $s_t$ | $v_t$ | $\theta_t$ | $lfp_t$ | $n_t$ | $w_t$ | $\tau_t^n$ | $\tau^s$ |
|---------------------------------|--------------|-------|-------|------------|---------|-------|-------|------------|----------|
| Volatility                      | 1            | 1.39  | 1.25  | 1.10       | 0.13    | 0.14  | 1.05  | 0          | 0        |
| Autocorrelation                 | 0.89         | -0.33 | -0.48 | 0.89       | 0.92    | 0.58  | 0.89  | —          | —        |
| Correlation with $y$            | 1            | -0.38 | 0.44  | 0.97       | -0.08   | 0.10  | 0.98  | —          | —        |
| Correlation ( $s_t, v_t$ )      | <b>0.66</b>  |       |       |            |         |       |       |            |          |
| Long-run $lfp$                  | <b>0.74</b>  |       |       |            |         |       |       |            |          |
| Long-run $\tau^n$               | <b>14.7%</b> |       |       |            |         |       |       |            |          |
| Long-run $\tau^s$               | <b>0</b>     |       |       |            |         |       |       |            |          |
| Correlation ( $w_t, \tau_t^n$ ) | —            |       |       |            |         |       |       |            |          |

Table 8: Optimal policy with zero unemployment benefits ( $\chi = 0$ ) and Hosios condition ( $\eta = \xi$ ). Shocks are to TFP and government purchases. Volatilities are relative to the volatility of GDP, except  $\tau^s$ , which is the level of volatility around the long-run  $\tau^s$ .

|                      | $y$  | $s$          | $v$   | $\theta$ | $lfp$ | $n$   | $w$   | $\tau^n$ | $\tau^s$ |
|----------------------|------|--------------|-------|----------|-------|-------|-------|----------|----------|
| Volatility           | 1    | 1.85         | 1.56  | 1.38     | 0.23  | 0.23  | 1.86  | 8.42     | 10.35    |
| Autocorrelation      | 0.85 | -0.15        | -0.51 | 0.92     | 0.94  | 0.72  | 0.18  | 0.25     | 0.71     |
| Correlation with $y$ | 1    | -0.40        | 0.36  | 0.95     | -0.30 | -0.16 | -0.06 | -0.30    | -0.80    |
| Long-run $\tau^n$    |      | <b>11.6%</b> |       |          |       |       |       |          |          |
| Long-run $\tau^s$    |      | <b>10.5%</b> |       |          |       |       |       |          |          |
| Long-run $lfp$       |      | <b>0.74</b>  |       |          |       |       |       |          |          |

Table 9: Optimal policy with zero dividend-income taxation. Volatilities are relative to the volatility of GDP, except  $\tau^s$ , which is the level of volatility around the long-run  $\tau^s$ . Shocks are to TFP and government purchases.

## G Zero Taxation of Dividend Income

The main result of tax volatility is not sensitive to the assumption of zero dividend taxation. As noted in Section 2.2, it is well-understood in Ramsey models that the unavailability of profit taxes can affect policy prescriptions. This is because, following production, profit flows (which is what households' dividend income reflects) represent an inelastic source of revenue which the government would like to tax heavily. Because dividend receipts are inelastic with respect to household decisions, their inclusion or absence has nothing to do with our proof in Section 7 about the completeness of tax instruments in our model. A 100-percent profit/dividend tax is trivially optimal because it does not distort any margins, and this is the case on which we have focused.

To demonstrate the robustness of the main results to the absence of a profit tax, Table 9 reports simulation-based results under the polar opposite assumption of zero dividend taxation,  $\tau^d = 0$  for the baseline exogenous-policy calibration. As comparison of the results in Table 9 with those reported in Table 4 shows, the cyclical properties of optimal policy in both the pseudo-LFP model and the LFP model are virtually identical under zero or full taxation of dividend income. Table 9 tabulates results for the case of zero government-provided unemployment benefits. For the case of positive government-provided unemployment benefits, we find results very nearly the same as those reported in Table 9; for brevity, though, we do not report these results.

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