Department of Economics Boston College

Economics 861.01
Advanced Macroeconomics I (aka Monetary Theory I)
Project 1
Professor Sanjay Chugh
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Objective

As a building block to working with some set(s) of general equilibrium and/or partial equilibrium models (both in this class and, should your research interests eventually take you in that direction, your own continued research), you will compute a **first-order approximation** to the decisions rules of a DSGE/RBC economy. You will use the Schmitt-Grohe and Uribe (2004 *Journal of Economic Dynamics and Control*) algorithm.

Because the primary methodological objective here is **to learn how to implement** such solutions yourself, you are **not** permitted to use off-the-shelf programs provided by Schmitt-Grohe and Uribe or others or packaged programs such as Dynare.

A good, complete submission (excluding code) should be relatively brief; but a few brief notes are:

1. **I'd be surprised if the entire submission (written well, not cramped together, double-spaced, and so on) were LESS than five pages long.**
2. **Perhaps the best papers (written well, not cramped together, double-spaced, and so on) are around 10 or so pages long.**
3. **A very good submission could be longer than 10 pages, but it has to clearly justify WHY the submission is long.**

Regardless of submission length, your first FIVE sentences, AT MOST, should contain a BRIEF summary of the findings (and I am the one who decides whether or not it is "brief"). This should be the ABSTRACT of your text.

This abstract should include (very importantly!) CLEAR, GOOD economic INTUITION, and (if it helps you explain something important) either one or at most two important numerical results. (Note: is "intuition" simply a verbal description of the numerical results?). Further details about "what to submit" appear at the end of this document. Note that, despite a very long project description, "what to submit" is actually a fairly SHORT amount of results from everybody; and then a section, to be determined by each person individually, that might further expand on the results.

The Problem

For the Social Planning problem of a DSGE/RBC economy with long-run growth, construct a linear approximation of the model's (dynamic) decision rules around the **deterministic steady state.** Specifically, in terms of the notation of Schmitt-Grohe and Uribe (2004), you must solve (using Matlab or another program such as Fortran, etc) the system of equations

$$f_{y'} \cdot g_x \cdot h_x + f_y \cdot g_x + f_{x'} \cdot h_x + f_x = 0$$

for the **matrices** g_x and h_x . **Keep in mind that each term of this expression is evaluated at the deterministic steady state of the model.**

The Social Planning problem for the **transformed DSGE/RBC economy with possibly time-varying long-run growth** is: maximize the representative household's lifetime expected utility

$$E_0 \sum_{t=0}^{\infty} b^t X_t^{1-\sigma} u(c_t, n_t)$$

subject to the sequence of resource constraints

$$c_t + \gamma_t k_{t+1} - (1-\delta)k_t = z_t f(k_t, n_t),$$

taking as given the initial capital stock k_0, the exogenous law of motion governing (high-frequency) productivity fluctuations z_t,

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}^z,$$

and the exogenous law of motion governing (low-frequency) trend growth fluctuations,

$$\ln \gamma_{t+1} = (1-\rho_\gamma)\ln \bar{\gamma} + \rho_\gamma \ln \gamma_t + \varepsilon_{t+1}^\gamma,$$

which in turn governs the evolution of the deterministic component of productivity

$$X_{t+1} = \gamma_t X_t.$$

The stationary transformation of the (stochastically) growing economy is taken with respect to the "very-long-run growth rate" $\bar{\gamma}$.

The household's subjective discount factor between period t and t+1 is β_t, which in turn is given by $\beta_t \equiv b^t \gamma_t^{1-\sigma}$. The parameter b is the (constant) subjective discount factor in the underlying growing economy, and the utility parameter σ is introduced below.

The deterministic component of productivity grows at a stochastic (gross) growth rate γ_t between period *t* and period *t*+1. The growth rate γ_t is revealed to agents in the economy at the same time as z_t is revealed, which is at the very start of period *t*. Thus, agents are (somewhat) informed (although not perfectly informed) about how outcomes (realized prices and quantities) will fluctuate between period *t* and period *t*+1, which is the essence of deterministic growth.

The steady state level of TFP is $\bar{z} = 1$, with innovations to productivity ε_t^z : iid $N(0, \sigma_z^2)$. The steady state level of long run productivity **growth** (in gross terms) is $\bar{\gamma}$ (i.e., the "very-long-run growth rate"), with innovations to long-run growth ε_t^γ : iid $N(0, \sigma_\gamma^2)$.

Functional forms for period-t utility and production are

$$u(c_t, n_t) = \frac{\left[c_t(1-n_t)^\psi\right]^{1-\sigma} - 1}{1-\sigma}$$

and

$$f(k_t, n_t) = k_t^{\alpha} n_t^{1-\alpha}.$$

Finally, government absorption and taxes are always zero.

Parameterization AND Possible Sets of Experiments

This section describes **ALL OF THE POSSIBLE** sets of experiments that you **COULD** perform as you investigate the performance of the model. As noted above, you will report in a **BRIEF** section **SOME** results; and in a **subsequent** section (which cannot have length zero), you should report some **OTHER** set(s) of results that in principle help you learn more about the model. These sections are described further below.

Quantitatively analyze this DSGE/RBC economy with TFP and long-run growth shocks for the three parameter sets shown below:

	Parameter Set A (Baseline)	**Parameter Set B (Constant Growth)**	**Parameter Set C (Variable Growth)**
$\bar{\gamma}$	1	$1.03^{1/4}$	$1.03^{1/4}$
b	0.99	0.99	0.99
δ	0.02	0.02	0.02
σ	1.5	1.5	1.5
α	0.36	0.36	0.36
ψ	**???**	**???**	**???**
ρ_z	0.95	0.95	0.95
σ_z	0.006	0.006	0.006
ρ_γ	---	---	**???**
σ_γ	---	---	**???**

Note the "**???**" listed for the parameter ψ in all three parameter sets. In each case, you must calibrate the value of ψ that makes the steady state fraction of time spent in employment $= 0.3$ of the total (unit) time endowment.

Also note the "**???**" listed in Parameter Set C for the persistence and standard deviation of shocks to the long-run growth rate. To my knowledge, no empirical evidence seems to exist in the literature for a "long-run-growth-shock process" for the U.S. economy. In the context of small open economy analysis, Aguiar and Gopinath (2007 *Journal of Political Economy*) estimate such a process for several (small open) countries. It would be of independent interest to estimate such a process for the U.S. economy, and then those estimates could be used as input to the model you are studying here. This empirical exercise is left outside the scope of this project (but you are free and very welcome to undertake it, because it seems it would be an independent contribution).

If you are not conducting experiments to uncover the values of these two parameters, **when allowing for shocks to the long-run growth rate, experiment with a couple of values of ρ_γ and σ_γ.** Specifically, use the pair $\rho_\gamma \in \{0.01, 0.5\}$ AND the pair $\sigma_\gamma \in \{0.001, 0.01\}$. (Hence, there are **four** pairs of ρ_γ and σ_γ values to try.)

Simulations

Having computed the matrices g_x and h_x, the next step is to conduct simulations of your model(s). In order to generate simulations, recall that the first-order approximations are given by

$$y_t = g(x_t, \sigma) \approx g(\bar{x}, 0) + g_x \cdot (x_t - \bar{x})$$
$$x_{t+1} = h(x_t, \sigma) \approx h(\bar{x}, 0) + h_x \cdot (x_t - \bar{x}) + \eta \sigma \varepsilon_{t+1}$$

in which it is easiest to set the perturbation parameter $\sigma = 1$, in which case the matrix η must contain the standard deviations of the model's exogenous state variables. You will be provided with sequences of shocks for the vector process $\{z_t, \gamma_t\}$ which will be the forcing process for your time-series simulations. Specifically, you will be provided with 200 sequences each of length 200 periods (quarters). These shocks are drawn from an *iid* $N(0, 1)$ distribution, which, when pre-multiplied with the appropriate row of the matrix η yields an *iid* $N(0, \sigma_i^2)$ sequence, $i \in \{z, \gamma\}$.[1]

Using **both HP-filtered cyclical components and (separately) band-pass-filtered cyclical components of your simulated time series** (specifically, the net percentage deviation of each simulated series from its respective trend), calculate, for each time series of interest in a given simulation, standard deviations, first-order serial correlations, and contemporaneous correlation of each variable with GDP.[2,3] Then, compute and report the means and standard deviations of these means, the means and standard deviations of these standard deviations, and the means and standard deviations of these correlations across all simulations. These sets of second-moment statistics (along with the steady state values of the endogenous variables you decide are interesting/relevant to analyze) are what you should report as your simulation-based results (in some appropriate and informative combination of tables and/or graphs and/or text).

[1] The shocks were generated using Matlab's built-in `randn` function. For this project, use the provided sequence of shocks for your simulations (for the sake of some comparability). In subsequent projects of your own, you can use the `randn` function to generate your own random numbers.

[2] Note that some series (such as GDP) may have to be constructed residually if you do not include them as part of your state or costate vectors.

[3] You will be provided with two Matlab files that implement the HP filter and one Matlab file that implements the band-pass filter.

Analysis/Discussion

Given the novelty of growth shocks in a closed economy model, parts of your paper should be structured around it (i.e., your experiments, your analysis, including any appropriate steady state analysis you decide to do, your discussions, etc.).

There are also other "standard" experiments you can conduct, namely simulations in the face of only shocks to z_t, omitting growth altogether (i.e., Parameter Set A). To the extent possible, compare your results on this dimension with appropriate empirical data (either collected and summarized yourself or referencing existing studies – for example, King and Rebelo (1999), Cooley and Hansen (1995), or some other existing and credible study).

All your experimental analysis and presentation of results is best made through some informative collection of tables and/or graphs and/or text. The exact data to which you compare your model is left up to you – this should also be reflected in how you motivate your paper in the introduction and abstract.

The discussion of results is in many ways the most important part of your paper. Here, you should provide interesting and relevant analysis from the (informative) experiments you run, describing the successes as well as shortcomings of your model. Describe the intuition/economic mechanism for any major successes; discuss the intuition/economic mechanism for any important shortcomings. Your discussion need not describe every nitty-gritty detail of the results you obtain (and should certainly NOT be just a verbal description of what a reader could find in tables, etc.), but should provide a fair and scientific view of your results and how they do or do not shed light on the study's basic hypotheses and goals.

An issue/question that you **must** analyze is (and this is repeated below): Analytical and/or quantitative exploration of the Frisch elasticity of labor supply in the model.

(Some) Computational/Programming Guidance

Using Matlab's `fsolve` function to solve for the matrices g_x and h_x is once again the key computational step, as in Project 0.

In order to conduct simulations using the sequences of shocks with which you will be provided, you must essentially proceed "iteratively" through each simulation. To do so, begin with k_0 (which is simply the deterministic steady state value $\bar{k}$) and the "first realization" of the shocks to z and γ (that is, the first (period-zero) shocks to log z and log γ) and compute the period-zero equilibrium outcome using

$$y_0 = g(\bar{x},0) + g_x \cdot (x_0 - \bar{x})$$
$$x_1 = h(\bar{x},0) + h_x \cdot (x_0 - \bar{x}) + \eta\sigma\varepsilon_1$$

Once you have the period-zero equilibrium outcome of the model in hand, compute the period-one equilibrium outcome of the model using

$$y_1 = g(\bar{x},0) + g_x \cdot (x_1 - \bar{x})$$
$$x_2 = h(\bar{x},0) + h_x \cdot (x_1 - \bar{x}) + \eta\sigma\varepsilon_2$$

Continue this way through all periods of the simulation, and then repeat this for each of the simulations. In conducting these simulations, you can and should try to cleverly arrange matrices and vectors in a way that takes advantage of Matlab's comparative advantage (compared to other software programs) in performing matrix manipulations. Be careful about issues such as matrix conformability, in particular with your g_x and h_x matrices.

A "sensibility check" you may want to try on your programs is to check the convergence (to the deterministic steady state) implied by your computed g_x and h_x matrices. To check this, begin with some arbitrary k_0 (say, perhaps 2% above or below the steady state $\bar{k}$) and construct a vector of zeros for the sequence of TFP shocks and deterministic productivity shocks. Iteratively apply your approximated decision rules (as described above) to construct a time-series simulation of the model – the difference, of course, is that this will be a *deterministic simulation* because each period the TFP shock and the trend shock is by assumption zero. If you have computed the correct g_x and h_x, your model variables should clearly converge to their deterministic steady state counterparts.

If you do not find convergence to the deterministic steady state (and you are convinced you are conducting the simulations correctly), there likely is an error in your computed g_x and/or h_x matrix. One "simple" error is that you have found the explosive root of the system – in that case, you should begin again in your computations.

What To Submit

Your submission should be a stand-alone, complete paper – i.e., one should be able to read it independent of knowing what the "description" of "Project 1" was. As before, your submission **must be typed, not hand-written.**

The sections that EVERYONE must do are the following, for the following parameterizations: Parameterization A; Parameterization C with $\rho_{\gamma} = 0.01$ AND $\sigma_{\gamma} = 0.01$. THAT IS, there are only TWO total parameter sets to approximate and solve here.

Abstract

Section 1. Introduction (NOTE the details that I present on the NEXT page regarding WHAT should be in the introduction)

Section 2. Model

- **(…fill in details…note: do you need to include all the DETAILS of the model in this section? OR should you include it in the Appendix?)**
- **Last section: General equilibrium definition**

Section 3. Parameterization

- **Your calibration strategy for the parameter psi**
- **Your general calibration strategy for other parameters**

Section 4. Numerical Results

- **BRIEF note(s) about HOW you approximate the model (note: do you have to present all the DETAILS about the SGU algorithm?)**
- **(BUT DO PRESENT the numerical results for the g_x and h_x matrices for the cases being considered – which are as described above)**
- **Steady state results**
- **Impulse responses to z(t) shocks**
- **Impulse responses gamma(t) shocks in the gamma = 1 CASE**
- **Economic intuition behind these impulse responses (note: should these be very long? And a second note: are these explanations just a VERBAL DESCRIPTION of the results one can see in the tables?)**
- **Basic "conclusions," summarizing your results thus far, their potential importance, and suggestions, if any, for future work**

Section(s) that FILL IN some of the rest of the issues

Section 5 – XX: (You determine how to set this up)

As for Project 0, attach a print-out of your code to your submission.

<u>**Advice on Writing**</u>

In addition to "replicating" some "standard" results in basic RBC analysis, your analysis is also novel in that no(?) work has been done studying the effects of **shocks to long-run growth in a closed-economy DSGE/RBC model.** As such, you should center the writing and presentation of your paper around this issue since it makes a novel contribution to the literature. The paper must also include the "standard" analysis of the cyclical fluctuations induced in the model by shocks to the usual Solow productivity measure.

Because what you will be submitting is a complete research paper, some advice regarding "how to" write good research papers:

1. **Get to the main point(s) quickly.** Different researchers have different writing styles, and you will develop your own as you gain experience, but here are concrete guidelines for you to follow and then later develop from: **by the end of the first two paragraphs of the introduction of your paper,** the following points should be made very clear:
   a. Big-picture motivation(s).
   b. The precise question(s) you ask in the project
   c. A very brief(!) description of the method(s) you use to address your precise question(s)
   d. The main result(s) your analysis yields
   e. Big-picture conclusion(s) you can draw from your analysis and results
   f. Where your work lies in the context of a larger literature (or even what literature it lies in), along with the **marginal contribution** of your paper.

2. **There is no need for a history lesson in the introduction.** Closely related to the "get to the main point(s) quickly" theme is the fact that, although many things may be new to you as you are learning various techniques and literatures, there is little place in a research paper (and certainly not in the introduction!) for a long-winded discussion of how and why you or someone might come to ask a particular research question(s).

3. **There is no need to report any and every single experiment that you conducted.** In the course of any research project, there are **MANY** ideas one explores. In working with theoretical frameworks, there are **MANY** experiments one conducts in order to learn how the model behaves and what sorts of positive and/or normative predictions it generates. Not every experiment directly yields useful results or insights – **hence there is no need to report the results of every single permutation of experiments you ran.**

4. **This isn't a creative writing exercise.** Which returns back to point #1 above – get to the main points without excessive or flowery language and without repeating the same thing(s) over and over.