Searching for Wages in an Estimated Labor Matching Model*

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Abstract

We estimate a real business cycle economy with search frictions in the labor market in which the latent wage follows a non-structural ARMA process. The estimated model does an excellent job matching a broad set of quantity data and wage indicators. Under the estimated process, wages respond immediately to shocks but converge slowly to their long-run levels, inducing substantial variation in labor's share of surplus. These results are not consistent with either a rigid real wage or flexible Nash bargaining. Despite inducing a strong endogenous response of wages, neutral shocks to productivity account for the vast majority of aggregate fluctuations in the economy, including labor market variables.

Keywords: Search and Matching, Wages, Unemployment, Business Cycles

JEL Classification: E32, E24

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1 Introduction

Over the past decade, researchers have debated the theoretical and empirical merits of alternative structural models of wage determination in labor search and matching models. This paper steps back from that debate, initiated by Shimer (2005) and Hall (2005), and instead asks: What properties of a generic wage-determination process are needed to match the historical comovements of output, consumption, investment, unemployment, labor-force participation, and job creation in the US? To answer this question, we estimate a search and matching model that is structural in every way except for the determination of wages, which we link to the fundamental shocks in the economy via a non-structural ARMA process. Because we do not take an a priori stance on how wages are determined, our specification nests a variety of possible structural models of wage determination. Estimation of the model delivers a striking result: The estimated wage process is not consistent with either a rigid real wage or Nash bargaining, yet productivity shocks account for the vast majority of aggregate fluctuations in the economy, including labor market variables and observed wage measures.

Figure 1 depicts the response of our estimated wage process to a permanent neutral productivity shock, which our estimation determines is the principal driver of business cycles.[1] The impact response of the wage to a productivity shock is nearly as large as under Nash bargaining, inconsistent with most models of real wage rigidity. However, while the Nash wage continues to rise in the periods just after the shock, the estimated wage approaches its new long-run level only gradually, with full adjustment taking several years. This combination of a strong impact response, but slow convergence—accommodated by our agnostic approach to modeling wages—is the key feature that allows productivity shocks to simultaneously match the empirical properties of real quantities, labor market variables, and observed wage measures. Our results thus provide new guidance in the ongoing search for structural foundations of wage determination and offer strong support for technology as an important driver of business cycle fluctuations.

We estimate a real dynamic stochastic general equilibrium (DSGE) model, endowed with external habit formation, adjustment costs in investment, variable capital utilization, and search and matching frictions in the labor market. More or less complicated versions of this basic model underlie the vast majority of the recent literature on structural estimation of

[1] In addition to permanent total factor productivity, our model allows for fundamental shocks to investment-specific technology, labor supply, government spending, labor tax rates, and a non-fundamental shock directly impacting only wages.

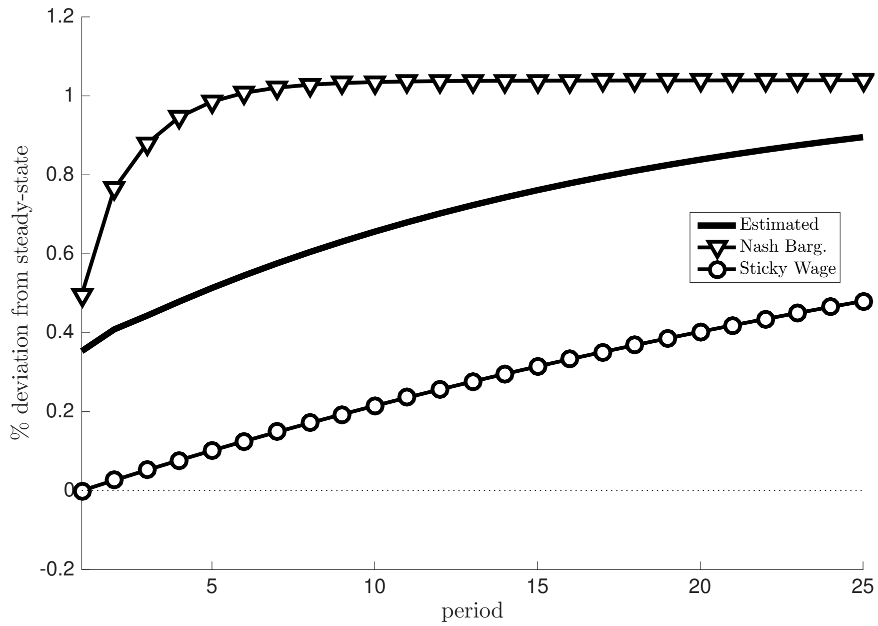


Figure 1: Impulse responses of the real wage to a permanent neutral productivity shock.

DSGE models. Critically, however, we do not commit to any particular wage-determination mechanism. We are able to do this because, as observed by Mortensen (1992), Hall (2005), Rogerson and Shimer (2011) and others, wages can coherently be treated as parametric so long as they remain within the bargaining set. Our contribution is to treat the wage as a time-varying parameter endowed with a flexible ARMA specification that, given a rich-enough lag structure, is sufficient to describe the dynamics of wages under nearly all assumptions the literature has made regarding wage determination.

We estimate the model on quantity data and a set of seven wage indicators (which are assumed to be measured with error) and study the degree to which the model can reconcile both wage and quantity fluctuations. We find that it does quite well: Unconditional moments for quantities are closer to the data than typically found in other estimated medium-scale DSGE models, and the implications for the latent wage are realistic. As a basis for comparison, we also estimate equivalent models with (i) Nash bargaining and (ii) a slowly-adjusting real wage following Hall (2005), and show that the baseline model is preferred by the data.

Having shown that the estimated model performs quite well in terms of fit, we show that the estimated ARMA process governing the real wage exhibits several distinctive features. First, the share of joint match surplus allocated to workers varies significantly over the business cycle, rising substantially during recessions. This type of variation is explicitly precluded by constant-share Nash bargaining. Second, the estimated wage process is remarkably flexible in certain respects. In particular, wages respond strongly on impact to permanent productivity shocks, but adjust slowly to their long-run level. Responses to other shocks, including investment-specific technology, labor supply, and government spending also exhibit strong impact effects followed by gradual reversion to long-run levels. There is little in our estimation to suggest that the data call for hump-shaped responses to shocks.[2]

Given the apparent flexibility of wages, our model has a surprising implication regarding the sources of business cycles: the vast majority of variation in both standard aggregate quantities and labor market variables is driven by productivity shocks. In particular, roughly 70 percent of the variation in output, consumption, unemployment, and vacancies is accounted for by permanent neutral productivity shocks. This result stands in sharp contrast with other estimated labor search and matching models, which find a much smaller role for productivity shocks.[3] Importantly, we arrive at this result despite estimating a high impact elasticity of wages to productivity shocks and a relatively low value of non-market activity.[4]

The estimated process for wages is tied to seven wage indicator series via a factor structure similar to that of Boivin and Giannoni (2006): real compensation per worker, real compensation per hour, real weekly compensation, two Employment Cost Index measures (one that includes benefits and one that excludes benefits) and two quality-adjusted wage series from Haefke et al. (2013a) (one for all workers and one for new hires only). Because the correspondence of the model wage concept with the data is imperfect and because we have little reason to prefer any single measure, we let each series load differently on the underlying model wage and assume that each is measured with error. We tie the model closely to the wage data, however, by imposing tight priors that each wage series does in fact contain substantial information regarding the wage in the model. Since the wage series have very different dynamics, no single factor can capture them all. We find that the model prefers, by a substantial margin, the compensation per worker measure of wages. While they

[2]Whether stickiness is real or nominal is not especially important, since nominal wage stickiness matters only to the extent that it induces a sluggishness in real wages. Both types of frictions are potentially encompassed by our ARMA specification of the wage process.

[3]Gertler et al. (2008) find that productivity shocks explain roughly one third of the variation in the growth rate of output, and less than half of labor market outcomes. Lubik (2009) finds that productivity shocks are critical for explaining output, but do not explain any of the variation in unemployment and vacancies.

[4]Hall (2005) generates volatility in unemployment by dispensing with Nash bargaining and simply assuming wages are inelastic with respect to productivity. Hagedorn and Manovskii (2008) achieve a similar effect under Nash bargaining by assuming that workers have low bargaining power and that their outside option is very close to the bargained wage. We estimate a replacement rate of roughly 25 percent.

have a short history and the magnitude of their fluctuation is muted, the Employment Cost Index measures also do a good job capturing the cyclical patterns of our estimated wage process.

This paper contributes to a growing empirical literature on equilibrium search and matching models of the labor market based on the seminal work of Diamond (1982), Mortensen (1982), and Pissarides (1985). Early contributions include Mortensen and Pissarides (1994), Merz (1995), and Andolfatto (1996). Following the Shimer (2005) critique, the literature has modeled wage determination in many different ways. A few prominent examples include Hall (2005), Farmer and Hollenhorst (2006), Hall and Milgrom (2008), Gertler et al. (2008), and Kennan (2010). Like us, Michaillat (2012) treat wages as parametric, in his case by assuming a log-linear relationship between wages and current productivity. Our approach here builds on this work, treating the wage as a *time-varying* parameter that responds dynamically — and potentially with a lag — to several fundamental shocks. Motivated by the results of Haefke et al. (2013b), Michaillat (2012) calibrates a relatively high elasticity of the real wage with respect to productivity shocks and finds that cyclical job rationing leads to substantial fluctuations in unemployment. Our estimate of a large impact response of wages to technology is consistent with his calibration. Our estimated model, however, does not rely on job rationing to reconcile relatively flexible wages with large unemployment fluctuations. Moreover, our flexible parameterization of wages encompasses many possible wage-determination mechanisms, and is able to capture the dynamics of aggregate wage series while matching a broad range of business cycle variables.

Methodologically, our approach belongs to a shorter literature that integrates non-structural components within otherwise structural models. Structural models with non-structural blocks have been estimated by Sargent (1989), Altug (1989), Ireland (2004), and Boivin and Giannoni (2006). The advantage of these approaches is that they allow the researcher to avoid making strong structural assumptions when the *a priori* grounds for such assumptions are unclear. Of course, care must be taken if such models are to be used to analyze changes in policy, and we do not attempt to do so here. Nevertheless, we view the methodology as a valuable strategy for identifying patterns in the data that candidate models should match. Also related is the literature initiated by McGrattan (1994), Hall (1996), and Chari et al. (2007), which uses dynamic models as measurement tools to infer the non-structural "wedges" driving the economy. Cheremukhin and Restrepo-Echavarria (2014) use a labor search and matching model with (variable-share) Nash bargaining in order to examine the sources of the RBC labor wedge.

The remainder of the paper is organized as follows: Section 2 develops the model, Section 3 outlines our estimation strategy, and Section 4 describes the results and their implications for modeling the real wage. Section 5 concludes.

2 The Model

The economy consists of a representative household and a representative firm who each trade in markets for consumption, labor, and capital. Consumption and capital markets are competitive, while transactions in labor markets are subject to search and matching frictions in the spirit of Mortensen and Pissarides (1994).

2.1 Households

The representative household consists of a continuum of ex-ante identical members each of whose unit time endowment can be allocated to working, searching for work, or leisure.[5] The household derives utility at time t from consumption and leisure (non-participation) according to the period utility function $u(c_t, f_t; C_{t-1}, \iota_t)$, where c_t is household consumption, f_t is the household's labor force participation, C_{t-1} is lagged aggregate consumption capturing the habit stock in the economy, and ι_t is a preference shock shifting the marginal disutility of labor.[6] Each period, the household dedicates a portion s_t of its members to search for a match in the labor market. Searching members match with probability p_t, which the household takes as given. Moreover, newly-created matches become productive within the period, so that the total labor force participation of the representative household is given by

$$f_t = n_t + (1 - p_t)\, s_t, \tag{1}$$

where f_t denotes the measure of household members in the labor force, n_t denotes the measure of currently matched workers, and $(1 - p_t)\, s_t$ denotes the measure of household workers who

[5]Consistent with the labor search literature incorporating a participation margin, we interpret non-participation in the labor force as leisure in the representative household's optimization problem.

[6]As is common in the literature, we assume external habits. Since both aggregate consumption C_{t-1} and ι_t are exogenous to the representative household, we suppress the dependence of $u(\cdot)$ on C_{t-1} and ι_t in subsequent expressions. Henceforth, we denote by capitalized letters aggregate counterparts to household- and firm-specific variables.

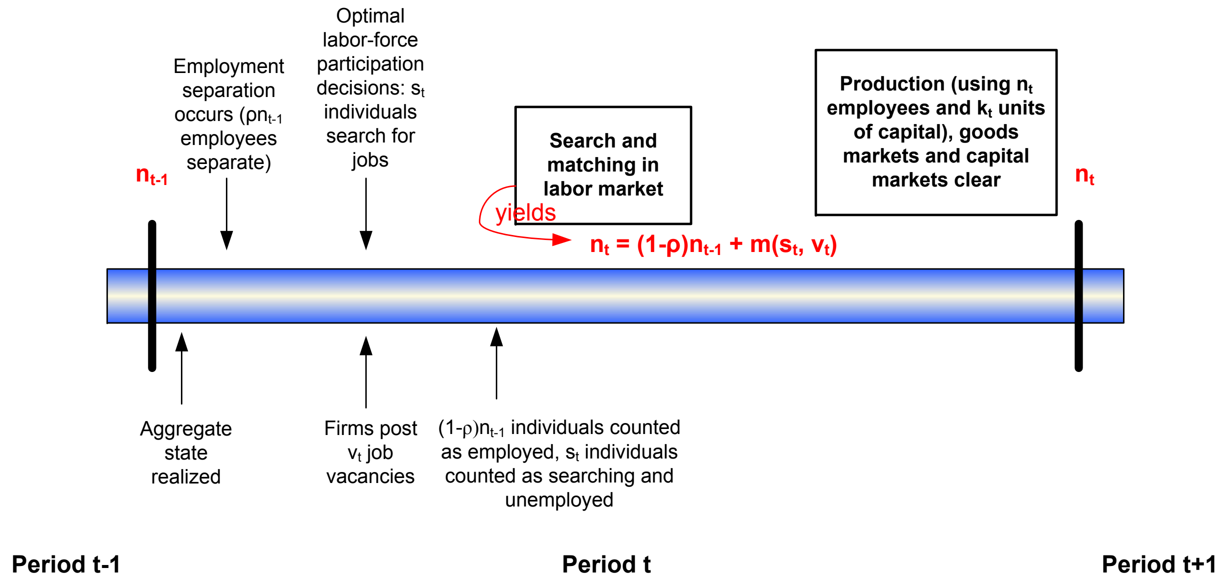


Figure 2: Timing of events in labor markets.

search but fail to find a match in period t.[7] Each period, previously productive matches dissolve with exogenous probability λ, so that the law of motion for matched workers faced by the household is

$$n_t = (1-\lambda)n_{t-1} + p_t s_t. \tag{2}$$

The unemployment rate in the economy is therefore given by

$$ur_t = \frac{(1-p_t)s_t}{n_t + (1-p_t)s_t}. \tag{3}$$

Figure 2 displays the timing of events in labor markets.

In addition to choosing its consumption and labor force participation, each period the household must also choose a level of capacity utilization for the current capital stock and a new level of investment subject to an investment adjustment cost as in Christiano et al. (2005). The law of motion for the stock of capital is given by

$$k_{t+1} = (1-\delta(u_t))k_t + i_t, \tag{4}$$

[7] This timing convention is consistent with the evidence on labor market flows at quarterly frequency. See Davis et al. (2006).

where $\delta(u_t)$ is the depreciation rate of the capital stock, which is an increasing and convex function of capacity utilization, denoted u_t.

The household budget constraint is given by

$$c_t + i_t Z_t \left(1 + \Phi\left(\frac{i_t}{i_{t-1}}\right)\right) + \tau_t = R_t u_t k_t + (1 - \tau_t^n) W_t n_t + (1 - p_t) s_t \kappa_t + d_t. \tag{5}$$

In equation (5), households take the rental rate of capital, the wage rate of labor, the relative price of investment in consumption units, and benefits paid to unemployed workers (R_t, W_t, Z_t and κ_t respectively), as given. They also receive d_t, lump-sum dividends from firms to the household, pay τ_t^n, an exogenous distortionary tax on labor income, and pay τ_t, a lump-sum tax used to finance any exogenous stream of government expenditure and unemployment benefits that remain unfunded after labor income tax revenue is collected. In order to maintain balanced growth, we assume that the benefit paid to unemployed workers gradually adjusts towards current wages,

$$\kappa_t = (\kappa W_{t-1})^{\phi_\kappa} \kappa_{t-1}^{1-\phi_\kappa}. \tag{6}$$

Substituting the expression for f_t in equation (1) into the utility function, the representative household's problem may be expressed as

$$\max_{c_t, s_t, i_t, u_t, n_t, k_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t + (1 - p_t) s_t) \tag{7}$$

subject to (2), (4), (5) and (6).

The first-order conditions for investment, i_t, and capital next period, k_{t+1}, are given by

$$\mu_t^K = u_{c,t} Z_t \left[1 + \Phi\left(\frac{i_t}{i_{t-1}}\right) + \left(\frac{i_t}{i_{t-1}}\right) \Phi'\left(\frac{i_t}{i_{t-1}}\right)\right] - E_t \left\{\beta u_{c,t+1} Z_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 \Phi'\left(\frac{i_{t+1}}{i_t}\right)\right\} \tag{8}$$

$$\mu_t^K = E_t \left\{\beta \left[\left(1 - \delta(u_{t+1})\right) \mu_{t+1}^K + u_{c,t+1} R_{t+1} u_{t+1}\right]\right\} \tag{9}$$

where μ_t^K denotes the Lagrange multiplier on the law of motion for capital in (4). Equation (8) states that the utility gain from supplying an additional unit of capital must equal the marginal utility of current consumption, taking into consideration the costs associated with changing the level of investment. Equation (9) states that the utility gained from taking an additional unit of capital into the next period is the sum of its after-depreciation continuation value and the utility gained from consumption of future rental income. Observe that, absent investment adjustment costs and a utilization margin, (8) implies that $\mu_t^K = u_{c,t}Z_t \; \forall t$, and so from (9) we recover the familiar real business cycle Euler equation.

Optimality of capacity utilization requires

$$u_{c,t}R_t = \delta^{'}(u_t)\mu_t^K. \tag{10}$$

Finally, after some simplification, the household's labor force participation condition may be expressed as

$$-\frac{u_{f,t}}{u_{c,t}} = p_t\left[(1-\tau_t^n)W_t + (1-\lambda)E_t\left\{\Omega_{t,t+1}\left(\frac{1-p_{t+1}}{p_{t+1}}\right)\left(-\frac{u_{f,t+1}}{u_{c,t+1}} - \kappa_{t+1}\right)\right\}\right] + (1-p_t)\kappa_t, \tag{11}$$

where $\Omega_{t,t+1} \equiv \beta\frac{u_{c,t+1}}{u_{c,t}}$. This condition states that the utility gained from keeping one more person out of the labor force (i.e. the marginal utility of leisure) is, at the optimum, equated to the utility gained from sending one more person to search for a job. The utility from joining the labor force to search, in turn, depends on the utility derived from consuming out of the earned wage in the same period, plus the asset value of the job, both discounted according to the exogenous job-finding probability. Note that as the matching probability and separation rate tend to unity, equation (11) becomes the RBC labor supply condition.

2.2 Firms

The representative firm chooses labor, capital, and vacancy postings so as to maximize the present value of real dividends, discounted according to the consumer's stochastic discount factor. The firm produces output with a production function of the form,

$$y_t = F(u_t k_t, X_t n_t), \tag{12}$$

where X_t is a non-stationary, labor-augmenting technology shock, with long-run growth rate γ_x.

The law of motion of employed labor from the firm's perspective is given by

$$n_t = (1-\lambda)n_{t-1} + q_t v_t \tag{13}$$

where v_t denotes vacancies posted in the labor market, while q_t denotes the probability of a vacancy posting returning a match. Firm profits are given by the expression

$$d_t = A_t F(u_t k_t, X_t n_t) - W_t n_t - R_t u_t k_t - a_n v_t. \tag{14}$$

where a_n governs the cost of posting a vacancy.

The problem of the firm may therefore be expressed as

$$\max_{v_t, n_t, k_t} E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} d_t \tag{15}$$

subject to (13) and (14). The first order condition for capital is given by

$$A_t F_{k,t} = u_t R_t. \tag{16}$$

The first-order condition for vacancies is given by $\mu_t^N = a_n / q_t$ and the first-order condition for labor is given by

$$\mu_t^N = A_t F_{n,t} - W_t + (1 - \lambda) E_t \left\{ \Omega_{t,t+1} \mu_{t+1}^N \right\}. \tag{17}$$

Equation (17) constitutes the vacancy posting condition, which states that the firm optimally posts vacancies until the marginal cost of doing so is equalized with the marginal product of labor net of the wage bill, plus the continuation value of a match. Note that as a_n tends to zero and the separation rate tends to unity, the vacancy posting condition approaches the standard RBC labor demand condition.

2.3 Government

The government runs a balanced budget, financing an exogenous stream of aggregate purchases G_t through a combination of distortionary labor taxes τ_t^n and lump-sum tax revenues τ_t net of unemployment benefit transfers to households $(1 - p_t) s_t \kappa_t$. The government's resource constraint is thus given by

$$G_t = \tau_t + \tau_t^n W_t n_t - (1 - p_t) s_t \kappa_t. \tag{18}$$

We follow Schmitt-Grohé and Uribe (2012) in assuming that government spending consists of a trend and stationary component

$$G_t = G_t^T G_t^S. \tag{19}$$

The trend component gradually adjusts to restore the long-run share of government spending in the economy,

$$G_t^T = G_{t-1}^T \left(\frac{G_{t-1}^T}{\bar{g}} \right)^{\phi_{g,y}}, \tag{20}$$

while transient changes in government expenditure follow an AR(1) process, specified below.

2.4 Wages

In order to close the model, we must make an assumption regarding wage determination in the economy. Rather than take a particular stand on the microeconomic foundations of wage determination in frictional markets, our baseline specification builds on the observation of Hall (2005) that prices can be treated as parameters in markets characterized by search and matching frictions. However, instead of taking the level of wages as fixed, as do both Hall (2005) and Michaillat and Saez (2013), we treat the *statistical process* by which wages evolve in the economy as parametric, and it is this process that we seek to estimate.

The starting point of our exercise, therefore, is a representation of *equilibrium* wages as

$$\log (W_t) = \beta^W(L)\epsilon_t. \tag{21}$$

Here, $\beta^W(L)$ is an infinite-lag polynomial and ϵ_t potentially includes all shocks affecting the economy. Equation (21) is generic, but in our implementation we restrict the wage process in two ways. First, we assume that equilibrium wages are cointegrated with the stochastic trends in the economy. This assumption is necessary in order to ensure the economy has a balanced growth path, and is consistent with all of the wage-determination mechanisms that we are aware of in the literature. Second, we need to assume that, when making their vacancy posting decisions, firms take as given the eventual wage a hired worker earns. With single-worker firms, or constant returns to scale production, there is very little loss of generality with this assumption.[8] If we could estimate the infinity of coefficients implied in $\beta^W(L)$,

[8] With multiple worker firms and decreasing returns, firms may internalize the effect on surplus, and therefore wages, created by hiring additional workers. In some contexts this leads to an inefficiency in which

we would therefore directly nest the vast majority of wage-determination mechanisms that appear in the literature. Of course, in practice, such an estimation is infeasible. In Section 3, we discuss the parametric restrictions we place on equation (21) in order to implement our empirical strategy.

For comparison, we contrast our baseline specification of wages with two alternatives. The first alternative we consider is the model of a "wage norm", or fixed real wage, as specified by Hall (2005). Hall (2005) shows that a deterministic and commonly known trend can easily be incorporated into the norm without changing the implications. With our stochastic trend, the case is not quite so straightforward, since the wage norm needs to adjust to surprise shocks at some horizon in order to stay within the bargaining set. Accordingly, we assume that wages adjust slowly to permanent shocks according to the equation

$$W_t^{Hall} = c\left(X_t Z_t^{\frac{\alpha}{1-\alpha}}\right)^{\phi_h} \left(W_{t-1}^{Hall}\right)^{1-\phi_h} W_{w,t}, \tag{22}$$

where the constant c depends on the steady-state labor share of the economy, ϕ captures the rate of adjustment, or error-correction, of the wage norm, and $W_{w,t}$ is a stationary function of an exogenous shock to wages so that the structural wage model need not hold exactly in all periods.[9] In the rigid-wage version of our economy, we assume $\phi_h = 0.025$, or an error-correction of 2.5 percent-per-quarter.

We also consider a version of the economy where the wage is determined by Nash bargaining. Following a standard derivation (reproduced in Appendix C) the Nash-bargained wage is given by

$$W_t^{NB} = \left\{(1-\eta)\kappa_t + \eta\left[A_t F_{n,t} + (1-\lambda)E_t\left\{\Omega_{t,t+1} p_{t+1} \mu_{t+1}^N\right\}\right]\right\} W_{w,t}. \tag{23}$$

This version of the model explicitly allows for a calibration of low worker bargaining power and high outside options, which Hagedorn and Manovskii (2008) argue can address the challenge to Nash bargaining posed by Shimer (2005). As in equation (22), we allow for independent shocks to the wage via $W_{w,t}$, which generate exogenous fluctatuations in the split of surplus between workers and firms.

firms hire excess labor in order to suppress marginal products and therefore all wages.

[9]The introduction of this error term allows for the Hall (and Nash) models to be compared directly with our reduced-form wage model, which also contains direct shocks to the wage. We elaborate on this point below.

2.5 Equilibrium

The number of matches formed in the labor market is governed by a constant returns matching function $m(V_t, S_t)$.S_t represents the aggregate mass of job-seekers and V_t represents the aggregate mass of vacancies posted by firms. The probabilities of forming a match in the labor market, which both the consumer and firm take as given, are respectively

$$p_t = \frac{m(V_t, S_t)}{S_t} \tag{24}$$

$$q_t = \frac{m(V_t, S_t)}{V_t}. \tag{25}$$

Substituting out the matching probability definitions in (24) and (25), equilibrium is described by a set of allocations $\{C_t, S_t, I_t, N_t, V_t, K_{t+1}\}$, the capital rental rate R_t, the capital utilization rate u_t, and a Lagrange multiplier μ_t^K that satisfy consumer optimality conditions (8), (9), (10) and (11), firm optimality conditions (16) and (17), the aggregate laws of motion for labor and capital

$$N_t = (1-\lambda)N_{t-1} + m(V_t, S_t) \tag{26}$$

$$K_{t+1} = (1-\delta(u_t))K_t + I_t \tag{27}$$

and the aggregate resource constraint

$$C_t + I_t Z_t \left(1 + \Phi\left(\frac{I_t}{I_{t-1}}\right)\right) + G_t = A_t F(u_t K_t, X_t N_t) - a_n V_t \tag{28}$$

taking as given the exogenous processes for X_t, Z_t, ι_t, G_t, τ_t^n, as well as the parametric wage process described in equation (21).

3 Empirical Strategy

3.1 Functional Forms

The representative household derives utility at time t from the utility function

$$u(c_t, f_t) = \frac{\left[(c_t - hC_{t-1}) \cdot \nu(f_t; \iota_t)\right]^{1-\sigma} - 1}{1-\sigma} \tag{29}$$

where h represents the degree of external habit formation with respect to last period's aggregate consumption C_{t-1}, and

$$\nu(f_t;\iota_t) = \psi\frac{(1-f_t)^{1-\iota_t}-1}{1-\iota_t} \tag{30}$$

where ι_t controls the curvature of the disutility of labor and is allowed to vary stochastically.[10] Output is produced using capital and labor according to a standard Cobb-Douglas production function

$$F(u_tk_t, X_tn_t) = (u_tk_t)^\alpha(X_tn_t)^{1-\alpha}. \tag{31}$$

The investment adjustment cost is given by

$$\Phi\left(\frac{i_t}{i_{t-1}}\right) = \frac{\phi}{2}\left(\frac{i_t}{i_{t-1}} - \gamma_x\gamma_z^{\frac{1}{\alpha-1}}\right)^2 \tag{32}$$

so that in the steady state $\Phi(\gamma_x\gamma_z^{\frac{1}{\alpha-1}}) = \Phi^{'}(\gamma_x\gamma_z^{\frac{1}{\alpha-1}}) = 0$, and $\Phi^{''}(\gamma_x\gamma_z^{\frac{1}{\alpha-1}}) = \phi > 0$.[11] The depreciation rate depends on utilization according to

$$\delta(u_t) = \delta_0 + \delta_1(u_t-\bar{u}) + \frac{\delta_2}{2}(u_t-\bar{u})^2 \tag{33}$$

so that $\delta(\bar{u}) = \delta_0$, $\delta^{'}(\bar{u}) = \delta_1 > 0$ and $\delta^{''}(\bar{u}) = \delta_2 > 0$. Finally, we assume a standard Cobb-Douglas matching function

$$m(V_t, S_t) = \chi S_t^\epsilon V_t^{1-\epsilon}, \tag{34}$$

confirming ex post that matching probabilities remain between zero and one.

[10]In the log-linearized model used in estimation, shocking ι is isomorphic to shocking ψ, which is used to calibrate the steady state.

[11]Our investment adjustment cost function is a special case of the form specified in Christiano et al. (2005).

There are five fundamental exogenous processes hitting the economy:

$$\log(\gamma_{x,t}/\gamma_x) = \rho_x \log(\gamma_{x,t-1}/\gamma_x) + \epsilon_t^x \tag{35}$$

$$\log(\gamma_{z,t}/\gamma_z) = \rho_z \log(\gamma_{z,t-1}/\gamma_x) + \epsilon_t^z \tag{36}$$

$$\log(\iota_t/\bar{\iota}) = \rho_\iota \log(\iota_{t-1}/\bar{\iota}) + \epsilon_t^\iota \tag{37}$$

$$\log(G_t^S) = \rho_g \log(G_{t-1}^S) + \epsilon_t^g \tag{38}$$

$$\log(\tau_t^n) = \rho_{\tau^n} \log(\tau_{t-1}^n) + \epsilon_t^{\tau^n}. \tag{39}$$

In the above $\gamma_{x,t} \equiv X_t/X_{t-1}$ and $\gamma_{z,t} \equiv Z_t/Z_{t-1}$ describe the growth rates of the two stochastic trends in the economy. In preliminary estimations, we consistently find ρ_x and ρ_z close to zero, and therefore fix these equal to zero in our baseline estimation. The preferences and technology described here are consistent with balanced growth. Since we will linearize the economy around a steady state, we must first stationarize the first-order conditions of the economy. The details of these steps are given in Appendix A.

3.2 A Process for Wages

Because it is not feasible to estimate the infinity of coefficients of the MA representation of the wage in equation (21), we seek a more compact representation that spans, as much as possible, the same set of possible dynamics for wages. A large variety of representations is feasible, and the most appropriate one is not a priori obvious. In order to maximize the transparency of the estimated parameters, we proceed to directly parameterize the impulse response of wages to each shock. Formally, we assume the log of the wage evolves as the sum of six independent components

$$w_t = \bar{w} + w_{x,t} + w_{z,t} + w_{\iota,t} + w_{g,t} + w_{\tau^n,t} + w_{w,t}. \tag{40}$$

Each term in the sum evolves according to a univariate ARMA(p,q) process, with a form that depends on the specification of the underlying shock. For transitory shocks $j \in \{\iota, g, \tau^n, w\}$, we assume a level-stationary process,

$$w_{j,t} = \rho^j(L) w_{j,t-1} + \psi^j(L)\epsilon_{j,t}. \tag{41}$$

For the non-stationary shocks, $j \in \{x, z\}$, we assume growth-stationary processes

$$\Delta \hat{w}_{x,t} = \rho^x(L)\Delta \hat{w}_{x,t-1} + \psi^x(L)\epsilon_{x,t} - \phi_x\left(w_{x,t-1} - \log(X_{t-1})\right) \tag{42}$$

$$\Delta \hat{w}_{z,t} = \rho^z(L)\Delta \hat{w}_{z,t-1} + \psi^z(L)\epsilon_{z,t} - \phi_z\left(w_{z,t-1} - \frac{\alpha}{\alpha-1}\log(Z_{t-1})\right) \tag{43}$$

where $\Delta \hat{w}_{x,t} \equiv \Delta w_{x,t} - \log(\gamma_x)$, $\Delta \hat{w}_{z,t} \equiv \Delta w_{z,t} - \frac{\alpha}{\alpha-1}\log(\gamma_z)$ and the terms multiplied by the $\phi_j \in (0, 1]$ represent error-correction terms that ensure the level of the wage converges to a constant ratio with the other trending variables in the economy.

Note that (40) contains an additional term, $w_{w,t}$, that follows the same ARMA process as the other terms but does not correspond to one of the fundamental shocks in the economy. The presence of this term is important for the interpretation of our results. If this term plays a large role in driving wages, we would conclude that the data demand large disturbances to the wage-determination arrangement itself, and that causality in the economy runs from wages to other labor market variables and aggregate quantities. Conversely, if our estimates indicate that wages largely respond to the other fundamental shocks hitting the economy, then we can conclude that the data are consistent with a strong endogenous response of wages to economic conditions, with causality running from those fundamental shocks to wages. As noted above, we also include this term in the rigid-wage and Nash wage processes, thus allowing for temporary deviations from those wage-determination arrangements in our estimated versions of those economies.

We also estimate the model using our two alternative specifications of the wage. Rearrangement of equation (22) shows that the slow-adjustment specification is actually nested in our ARMA specification when $\phi_x = \phi_z = \phi_h$ and the remaining ARMA parameters are set to zero. The Nash-wage specification is not explicitly nested in our ARMA process, and so requires specification of an additional parameter, η, that is not present in either of our other estimations. In order to ensure that our model comparison is not driven by prior choices on non-nested parameters, we use the least-restrictive priors that prove feasible for parameters related to wage determination. See Appendix D.2 for details on the prior choices.

We have also considered another natural candidate for the (stationarized) log-wage, namely

$$\tilde{w}_t = \Psi x_t, \tag{44}$$

where x_t represents the state vector, which includes both exogenous and endogenous variables. The advantage of this approach is that the endogenous states contain a summary of the full history of past shocks, and in a linear combination that is by construction relevant to the economy. This approach also directly nests many wage-determination mechanisms, such as Nash bargaining, that do not themselves introduce additional states into the system.

This approach has drawbacks, however. First, it is hard to formulate priors on the parameter vector Ψ and, in practice, the endogeneity of some elements of x_t makes it especially difficult to constrain the Bayesian sampler (or any other search algorithm) to regions in which equilibrium both exists and is unique. The endogenous elements of x_t also mean that it is quite difficult to characterize the restrictions implied by the representation in equation (44) on possible dynamics for wages. These challenges notwithstanding, we have explored representation (44) using a reliable global optimizer and found that it offers no advantage in terms of model fit relative to the representation we adopt above.[12]

3.3 Data

We estimate the model using quarterly data from 1972Q1 to 2013Q4, with an additional six years of data used to initialize the Kalman filter. We classify the data used into two groups. The first set of observations, $\hat{Y}_{1,t}$, consists of real quantity indicators for which we have a single measure. For these variables, the link between the model and data is uncontroversial, and measurement errors are generally considered modest. Accordingly, with the exception of output, we assume the data are a perfect measure of the corresponding model concept. The vector $\hat{Y}_{1,t}$ consists of log-changes in real per-capita output, consumption, investment, and government expenditure, as well as the unemployment rate, per-capita employment, and labor tax rates. The measurement equation linking observables with model analogues is

$$\hat{Y}_{1,t} = \begin{bmatrix} \Delta \log y_t \\ \Delta \log c_t \\ \Delta \log i_t \\ \Delta \log g_t \\ \Delta \log ur_t \\ \Delta \log n_t \\ \Delta \log \tau_t^n \end{bmatrix} + \begin{bmatrix} \epsilon_{y,t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{45}$$

[12]The above discussion of determinacy contrasts with the common finding that *exogenously* specifying a price — e.g. the interest rate in New Keynesian models — often leads to indeterminacy of equilibrium. In the labor search and matching model, the endogenous adjustment of market tightness clears markets, thereby pinning down equilibrium.

Table 1: Summary Moments - Data

	Δy	Δc	Δi	Δg	Δur	Δn
$\sigma(X)$	0.82	0.53	2.15	0.92	5.06	0.48
$\sigma(X)/\sigma(\Delta Y)$	1.00	0.64	2.62	1.12	6.15	0.58
$\rho(X_t, X_{t-1})$	0.35	0.46	0.47	0.17	0.66	0.62
$\rho(X, \Delta Y)$	1.00	0.62	0.74	0.23	-0.68	0.62

The covariance properties of these series are summarized in Table 1.

The inclusion of the measurement error in the output equation is necessary because the resource constraint of the economy implies a fixed relation between output, consumption, investment, government expenditure and vacancy postings. Since our model counterfactually assumes balanced trade, the data do not exactly respect this relation. Nevertheless, this measurement error is estimated to be quite small.

To these "basic" data, we then append an additional set of variables $\hat{Y}_{2,t}$, for which the link to the model concepts, as well as the quality of measurement, is far less clear. In this vector, we include two measures of vacancy postings as well as seven diverse measures of the aggregate wage. The first measure of vacancies comes from the BLS JOLTS survey, and is first available in 2001Q1. The second vacancy measure is given by the "Help-wanted" advertising index produced by the Conference Board through 2009. This series demonstrates a secular downward trend associated with the popularization of the internet and web-based job search, so we therefore drop all observations from 2000Q1 onward. Summary statistic for these series are included in Table 2. Given the rather different inputs into these two series, we assume each represents a potentially rescaled version of the model concept, v_t, measured with error.

The choice of an appropriate empirical counterpart for the model wage is not obvious for at least two reasons. First, the failure of standard measures of the wage to account for unobserved worker heterogeneity can impart a countercyclical compositional bias upon real wages, an issue noted by Alan Stockman and subsequently studied by Solon et al. (1994). In recessions, unemployment should occur disproportionately among the low-skilled who earn low wages. Average measured wages thus exhibit a countercyclical bias due to the changing composition of the workforce, a fact that cannot be accounted for without comprehensive individual-level data. Second, much recent literature has emphasized a distinction between the wage of new hires in the economy and the wage of ongoing matches. In many search and

Table 2: Moments - Vacancy-posting Measures (Estimated Model)

	JOLTS	**Help-Wanted**	**Model**
$\sigma(X)$	6.27	10.17	4.78
$\sigma(X)/\sigma(\Delta Y)$	7.63	12.38	5.37
$\rho(X_t, X_{t-1})$	0.49	0.68	0.59
$\rho(X, \Delta Y)$	0.38	0.78	0.78
Signal Power	0.58	0.91	1.00

matching contexts it is the wage of new hires which plays the primary allocative role in the economy.[13]

For the reasons cited above, we incorporate seven different real wage series in our estimation. The properties of these data are provided in Table 3. The first three series are somewhat standard measures of compensation. The first is total real compensation per worker. Given the lack of an explicit hours margin in our model, and the growing importance of non-wage compensation in the US economy, we view this as conceptually the closest match to the real wage in our model economy. We add to this an index of compensation per hour and real weekly compensation. These two measures are both somewhat more volatile than our preferred measure of the wage. To address concerns about composition bias, we include four additional series: two composition-adjusted series of the Employment Cost Index (ECI), and two quality-adjusted series from Haefke et al. (2013a). The Employment Cost Index, compiled by the BLS, seeks to follow wages holding constant the sectoral distribution of labor in the economy. We consider the standard ECI and the ECI adjusted to include benefits. The quality-adjusted series, constructed by Haefke et al. (2013a), seek to adjust for individual-level characteristics to better account for the cyclical variation in the quality of workers. We consider their standard quality-adjusted series for all workers, as well as their quality-adjusted series for new hires only. Several of the aforementioned series do not exist for the full sample period, and so the wage data constitute an unbalanced panel. As with the vacancy data, we assume each series represents a potentially rescaled version of the model concept W_t measured with error.

[13]For some recent theory and empirical work on the issue see Pissarides (2009) and Kudlyak (2014), among others.

Table 3: Moments - Wage Measures (Estimated Model)

	cmp/wrkr	cmp/hr	wkly cmp	ECI	ECI+ben	HSV	HSV-new	modl
$\sigma(X)$	0.61	0.79	1.11	0.29	0.29	1.06	5.25	0.58
$\sigma(X)/\sigma(\Delta Y)$	0.74	0.96	1.35	0.35	0.35	1.29	6.39	0.65
$\rho(X_t, X_{t-1})$	-0.01	-0.18	0.25	0.29	0.11	-0.12	-0.19	0.26
$\rho(X, \Delta Y)$	0.33	-0.01	0.03	0.11	0.19	0.03	-0.02	0.49
Signal Power	0.28	0.16	0.13	0.39	0.41	0.13	0.01	1.00

In summary, the vector $\hat{Y}_{2,t}$ is related to the model according to the measurement equation

$$\hat{Y}_{2,t} = \begin{bmatrix} \gamma_{v,1}\Delta\log v_t \\ \gamma_{v,2}\Delta\log v_t \\ \gamma_{w,1}\Delta\log w_t \\ \gamma_{w,2}\Delta\log w_t \\ \gamma_{w,3}\Delta\log w_t \\ \gamma_{w,4}\Delta\log w_t \\ \gamma_{w,5}\Delta\log w_t \\ \gamma_{w,6}\Delta\log w_t \\ \gamma_{w,7}\Delta\log w_t \end{bmatrix} + \begin{bmatrix} \epsilon_{v_1,t} \\ \epsilon_{v_2,t} \\ \epsilon_{w_1,t} \\ \epsilon_{w_2,t} \\ \epsilon_{w_3,t} \\ \epsilon_{w_4,t} \\ \epsilon_{w_5,t} \\ \epsilon_{w_6,t} \\ \epsilon_{w_7,t} \end{bmatrix}. \tag{46}$$

This approach to measurement is similar to the integration of DSGE and factor-based methods proposed by Boivin and Giannoni (2006). Further details on the data construction are in Appendix D.2.

3.4 Identification

We log-linearize the model economy around the non-stochastic steady state and estimate it using Bayesian methods. To do this, we first calibrate a subset of parameters according to the values listed in Table 4. These parameters are either set to standard values, or to match long run relationships found in the data. In particular, the compensation-to-output share $\mu_{w/y}$ is used to pin down the long-run wage in the economy, F pins down the scaling parameter of the disutility of labor ψ, the long-run employment rate, $\bar{ur}$, determines the separation parameter λ, and the long-run firm matching probability q pins down the vacancy cost a_n, which is exactly neutral to dynamics in the economy.

Model dynamics are driven by shocks to five fundamental exogenous AR(1) processes:

Table 4: Calibrated Parameters

Parameter	Concept	Value
β	Discount factor	0.992
ι	Curvature of $v(\cdot)$	5.000
σ	Risk aversion	2.500
$\mu_{w/y}$	Compensation share of output	0.700
α	Capital share of output	0.250
q	Labor matching prob., firm side	0.500
$\bar{ur}$	Steady-state unempl. rate	0.062
F	Steady-state LFP rate	0.603
δ_0	Average depreciation rate	0.025
δ_2	Curvature of depreciation rate	0.005
$\bar{g}/\bar{y}$	Government share of GDP	0.205
$\bar{\tau}^n$	Steady-state labor tax rate	0.224
γ_x	Long-run TFP growth	1.002
γ_z	Long-run RPI growth	0.996

the growth rate of total factor productivity ($\gamma_{x,t}$), the growth rate of the relative price of investment ($\gamma_{z,t}$), labor supply (ι_t), government spending (G_t^S), and the labor tax rate (τ_t^n). We proceed to estimate the parameters governing these fundamental exogenous processes, the preference and technology parameters,

$$\Theta_1 = \{h, \phi, \epsilon, p, \kappa, \phi_\kappa, \phi_{g,y}\}\,, \tag{47}$$

and the parameters describing the ARMA wage processes.

In order to implement the procedure, we need to select the order of the ARMA processes underlying wages in the economy. For this we fix ex ante an ARMA(2,1) to describe the wage response to each shock. We have experimented with adding subsequent autoregressive and moving average terms, but find very little improvement of fit. In principle, one could estimate the ARMA orders of each process within the Bayesian context following the strategy of Meyer-Gohde and Neuhoff (2014). Given the complication involved and the modest increase in fit that we find for models with higher order ARMAs, this step does not seem warranted.

We impose priors that enforce two additional identification assumptions on the estimated parameters of the AR process. First, we impose the natural restriction that the impact response of the wage to an innovation in permanent productivity is non-negative. Second, we impose the restriction that the first autoregressive coefficient of the polynomial $\Gamma_j(L)$

is non-negative for stationary shocks $j \in \{\iota, g, \tau^n, w\}$. This latter restriction prevents wage responses from exhibiting periodic "zig-zags", a feature we believe is unlikely to be implied by structural models of wage determination. These restrictions do not come close to binding at the posterior mode, but rather they help to prevent the MCMC algorithm from engaging in occasional diversions into regions of the state space that we believe are reasonably excluded on a priori grounds. See appendix D.2 for a detailed description of our choices for prior distributions.

Given the relatively large number of parameters required to parameterize the exogenous wage process, it is natural to wonder whether indeed the data contain sufficient information to identify the relevant parameters in the context of the model. We therefore test local identification using the method proposed by Iskrev (2010), which examines whether a set of model moments displays linearly independent variation for a marginal adjustment of parameters.[14] The model passes this test. We also check global identification within the posterior distribution using a numerical optimizer that employs a genetic algorithm to discover a broad range of initial values, then completes the search for each starting value with a hill-climbing procedure. We consistently find that the optimizer delivers the same posterior mode regardless of starting point.

4 Results

Table 5 reports estimates of deep parameters of our model economy. The estimated values of these parameters are generally consistent with values found elsewhere in the literature. In particular, we estimate reasonable values for the extent of habit formation h, investment adjustment costs ϕ, and the household's matching probability p (a quarterly matching probability of 0.96 is consistent with average unemployment duration of just over three months). Notably, we estimate a relatively low value for the replacement rate of benefits κ. The latter suggests that our estimated model does not require a Hagedorn and Manovskii (2008)-style calibration to match the data. Table 5 also reports estimated autoregressive coefficients and standard deviations of innovations for the fundamental driving processes in our model. Our estimates of these parameters are broadly in line with those found in the DSGE estimation literature.[15]

[14] Because it relies only on the model moments, the outcome of this test is independent of our prior choice.

[15] See, e.g., Christiano et al. (2005), Smets and Wouters (2007), Gertler et al. (2008).

Table 5: Posterior Estimates of Model Parameters, Part I.

Parameter	Concept	Median	5%	95%
h	Habit formation	0.77	0.73	0.80
ϕ	Investment adj. cost	2.91	2.23	3.86
ϵ	S share in matching fcn.	0.05	0.04	0.06
p	Labor matching prob., consumer side	0.96	0.95	0.97
κ	UI benefit replacement rate	0.24	0.07	0.53
ϕ_κ	UI benefit adjustment rate	0.49	0.17	0.83
$\phi_{g,y}$	Reversion to long-run government share	0.11	0.08	0.16
ρ_ι	AR(1) coeff. on labor supply	0.99	0.99	1.00
ρ_g	AR(1) coeff. on government exp.	0.95	0.92	0.97
ρ_τ	AR(1) coeff. on labor tax	0.98	0.93	1.00
σ^x	100 $\times$ std. dev. of ϵ^x	1.05	0.94	1.19
σ^z	100 $\times$ std. dev. of ϵ^z	2.16	1.96	2.41
σ^ι	100 $\times$ std. dev. of ϵ^ι	0.45	0.41	0.49
σ^g	100 $\times$ std. dev. of ϵ^g	0.93	0.85	1.02
σ^{τ^n}	100 $\times$ std. dev. of ϵ^{τ^n}	2.57	2.34	2.83

Table 6 reports the estimated reduced-form parameters governing the evolution of the real wage. Interpreting these values in isolation is somewhat difficult. However, it is worth observing that only contemporaneous shocks have statistically significant effects on the wage process: All impact coefficients are statistically significant, whereas none of the MA(1) coefficients are. Furthermore, with the exception of TFP and the relative price of investment, which are estimated to be pure random walks, the autoregressive elements of the wage process for each of the remaining shocks only have significant first-order components. Our decision to allow each component of the wage process to evolve according to an ARMA(2,1) therefore seems to be allowing the model more than enough flexibility to match the data.

Because we use a reduced-form model of wages, we need to check if the estimated model ever implies negative surplus for firms or households. In simulations, the estimated wage never violates the participation constraint of households (i.e. the wage never falls below the reservation wage) or firms.

4.1 Model Fit

The model does an exceptionally good job at matching moments from the data. Table 7 reports unconditional moments from the model for the corresponding series from Table 1.

Table 6: Posterior Estimates of Model Parameters, Part II.

Parameter	Concept	Median	5%	95%
ψ_0^x	Impact - TFP	0.35	0.29	0.42
ψ_0^z	Impact - RPI	-0.13	-0.17	-0.08
ψ_0^ι	Impact - labor supply	0.19	0.14	0.23
ψ_0^g	Impact - government spending	0.21	0.17	0.25
$\psi_0^{\tau^n}$	Impact - labor tax	0.02	0.00	0.05
ψ_0^w	Impact - wage (exog. component)	0.22	0.19	0.25
ψ_1^x/ψ_0^x	MA(1) - TFP	0.03	-0.09	0.15
ψ_1^z/ψ_0^z	MA(1) - RPI	0.13	-0.03	0.28
$\psi_1^\iota/\psi_0^\iota$	MA(1) - labor supply	0.03	-0.11	0.16
ψ_1^g/ψ_0^g	MA(1) - government spending	0.08	-0.04	0.21
$\psi_1^{\tau^n}/\psi_0^{\tau^n}$	MA(1) - labor tax	-0.02	-0.18	0.14
ψ_1^w/ψ_0^w	MA(1) - wage (exog. component)	0.05	-0.07	0.17
ρ_0^x	AR(1) - TFP	0.01	-0.12	0.14
ρ_0^z	AR(1) - RPI	-0.11	-0.27	0.07
ρ_0^ι	AR(1) - labor supply	0.79	0.60	0.89
ρ_0^g	AR(1) - government spending	0.93	0.88	0.97
$\rho_0^{\tau^n}$	AR(1) - labor tax	0.47	0.15	0.84
ρ_0^w	AR(1) - wage (exog. component)	0.95	0.89	0.98
ρ_1^x	AR(2) - TFP	-0.01	-0.09	0.06
ρ_1^z	AR(2) - RPI	-0.11	-0.22	0.00
ρ_1^ι	AR(2) - labor supply	0.01	-0.12	0.13
ρ_1^g	AR(2) - government spending	-0.07	-0.20	0.06
$\rho_1^{\tau^n}$	AR(2) - labor tax	0.02	-0.14	0.18
ρ_1^w	AR(2) - wage (exog. component)	-0.02	-0.16	0.11
ϕ_x	Error correction - TFP	0.06	0.03	0.10
ϕ_z	Error correction - RPI	0.27	0.18	0.35

Note: For consistency of units, MA(0) terms (impact effects) are percent changes for a one-standard deviation shock and all MA(1) parameters are normalized in terms of their MA(0) counterpart.

Table 7: Summary Moments (Estimated Model)

	Δy	Δc	Δi	Δg	Δur	Δn
$\sigma(X)$	0.89	0.61	2.18	0.99	5.71	0.51
$\sigma(X)/\sigma(\Delta Y)$	1.00	0.69	2.44	1.12	6.42	0.57
$\rho(X_t, X_{t-1})$	0.64	0.54	0.42	0.06	0.56	0.34
$\rho(X, \Delta Y)$	1.00	0.84	0.89	0.15	-0.78	0.65

Second moments, autocorrelations, and correlations with output growth all mirror the data very closely. The covariogram in Figure 3 shows that the model fits the data very well at both short and medium horizons, with the data lying in between the Bayesian 95 percent confidence intervals at the vast majority of points. In fact, the alignment of the covariogram with the data between the 1-quarter and 8-quarter horizons is comparable to that which would be achieved with an unrestricted VAR of four lags; even more lags would be needed to replicate the model's performance at matching longer-horizon covariances.

Much of the debate concerning the textbook search and matching model's inability to generate adequate volatility in unemployment from productivity shocks focuses on the behavior of vacancies. Absent stochastic variation in the separation rate, search and matching models require volatility in job creation in order to drive volatility in unemployment. Models featuring Nash bargaining fail precisely because the wage is too elastic with respect to productivity; employers stand to gain little from posting vacancies when increased productivity is matched by commensurate increases in real wages. Models featuring a fixed real wage such as Hall (2005), on the other hand, generate excessive volatility in vacancy creation. Table 2 indicates that our model does not suffer from these issues, and instead matches the cross correlation of vacancies quite well.[16]

[16] These results are borne out in our own estimation of models featuring Nash bargaining and a rigid real wage. See Tables 15 and 19, respectively, in Appendix D.4 for more details.

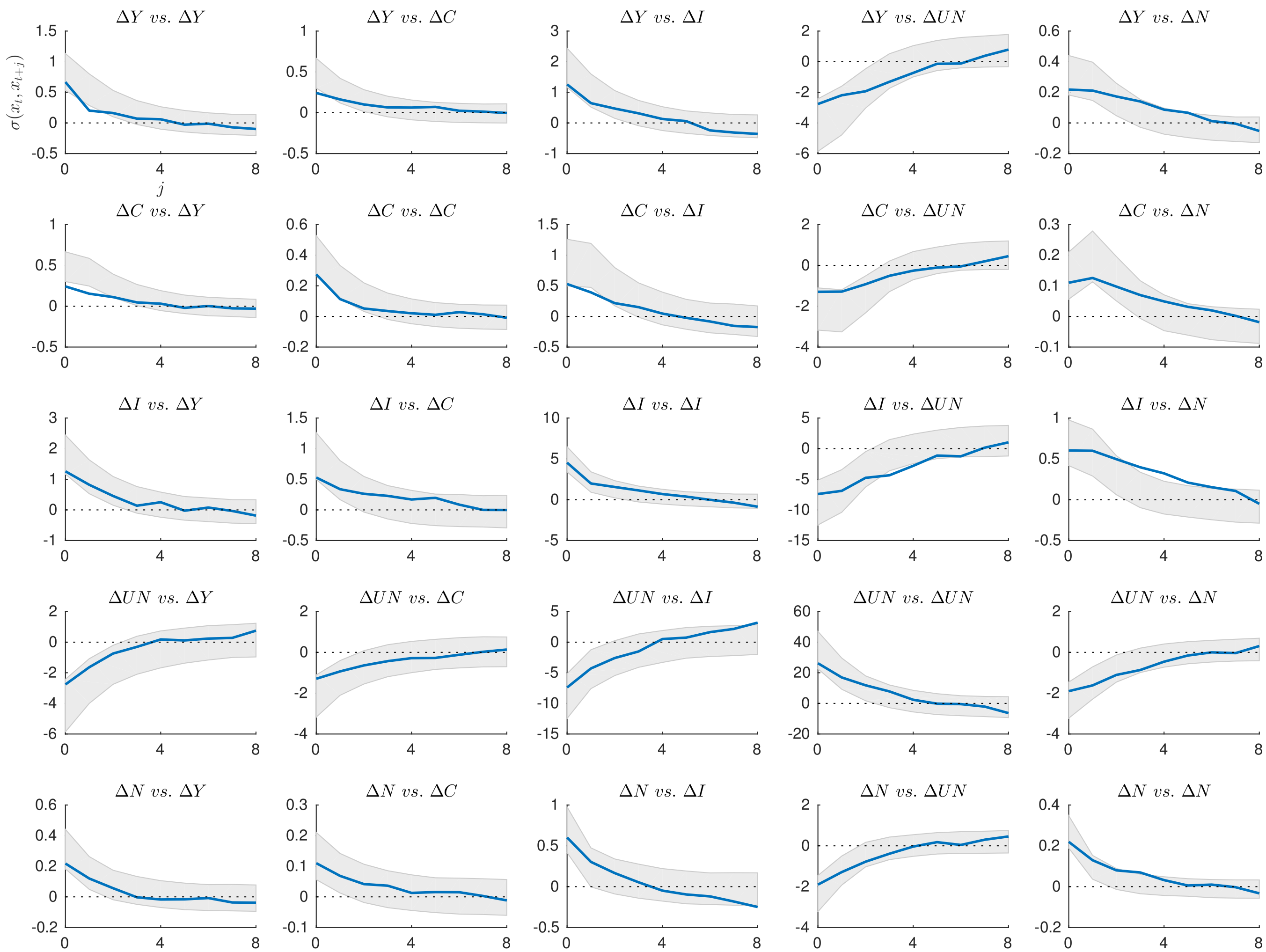


Figure 3: Covariogram comparing estimated model and actual data.

Table 3 compares the moments of the model-implied wage with the set of seven empirical wage measures. The final row provides a summary measure of the precision of the information contained in each wage series.[17] These moments suggest that the compensation per worker measure of the wage, as well as both ECI measures, are the most consistent with observed quantities, given the model. The standard deviation, autocorrelation, and correlation with output growth for compensation per worker are all quite close to the model-based wage. The signal power in the final row suggests that both ECI measures also conform closely to the model wage.

Figure 4 delves a bit deeper into the wage measurement relationships by comparing the compensation per worker measure of cumulative wage growth with the model-implied wage over the same period. Inspection of the figure confirms the results in Table 3: Compensation per worker tracks the model-implied wage quite closely over the sample period. The figure indicates that, while the series can differ in non-trivial ways over short horizons, the match between the two series improves over longer time periods.

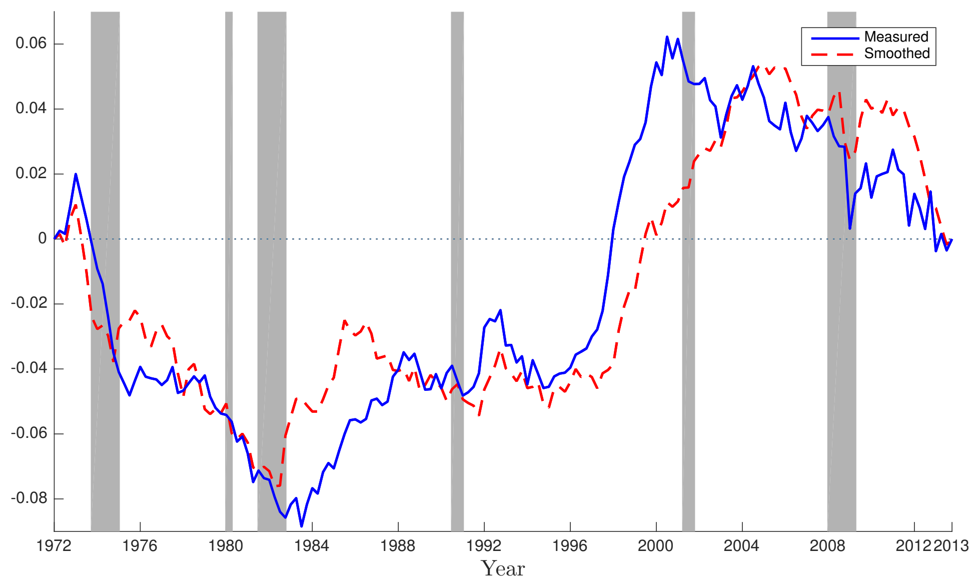


Figure 4: Compensation per worker and the smoothed model wage.

[17] We define the signal power of a particular wage measure i as $\frac{Var(\gamma_{w,i}\Delta \log w_t)}{Var(\Delta \log \hat{w}_{i,t})}$ where $\hat{w}_{i,t}$ denotes the observation on empirical wage series i at time t.

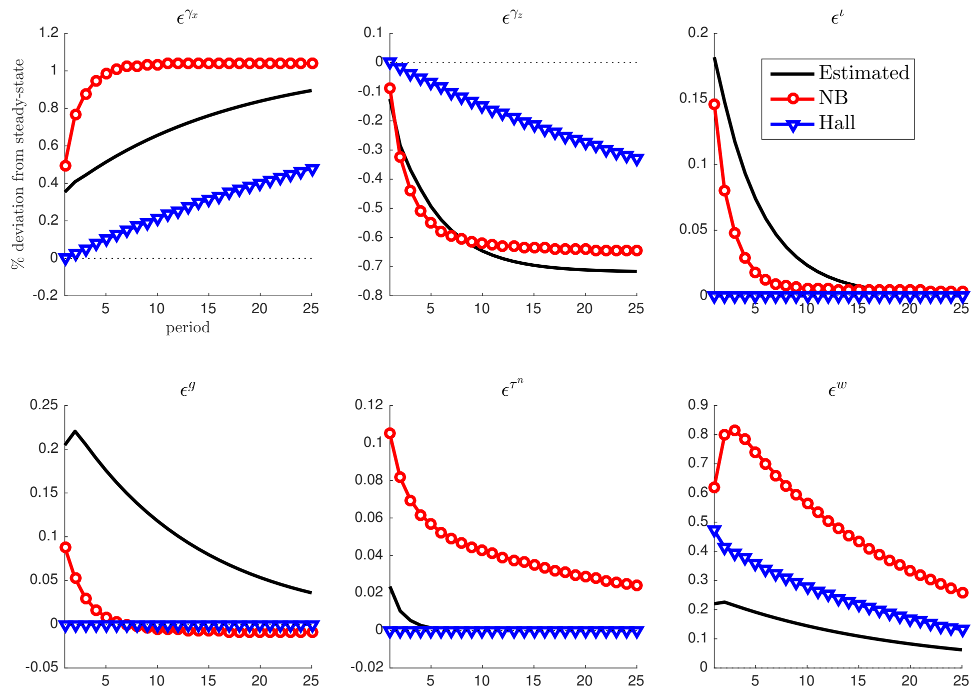
4.2 Impulse Responses

Figure 5 documents impulse responses of the real wage and labor's share of match surplus to each of the six shocks in the model (five fundamental shocks plus the residual wage shock) for three cases: our estimated model, Nash bargaining, and our implementation of the Hall (2005) rigid-wage model.[18] By construction, the wage in the rigid-wage model only responds to growth rate shocks and the residual shock to wages. Furthermore, as expected, the share of match surplus accruing to labor in the Nash bargaining model only responds to the residual shock to wages, as this may be thought of as the wedge between the model wage and the structural Nash wage which implies constant surplus shares.

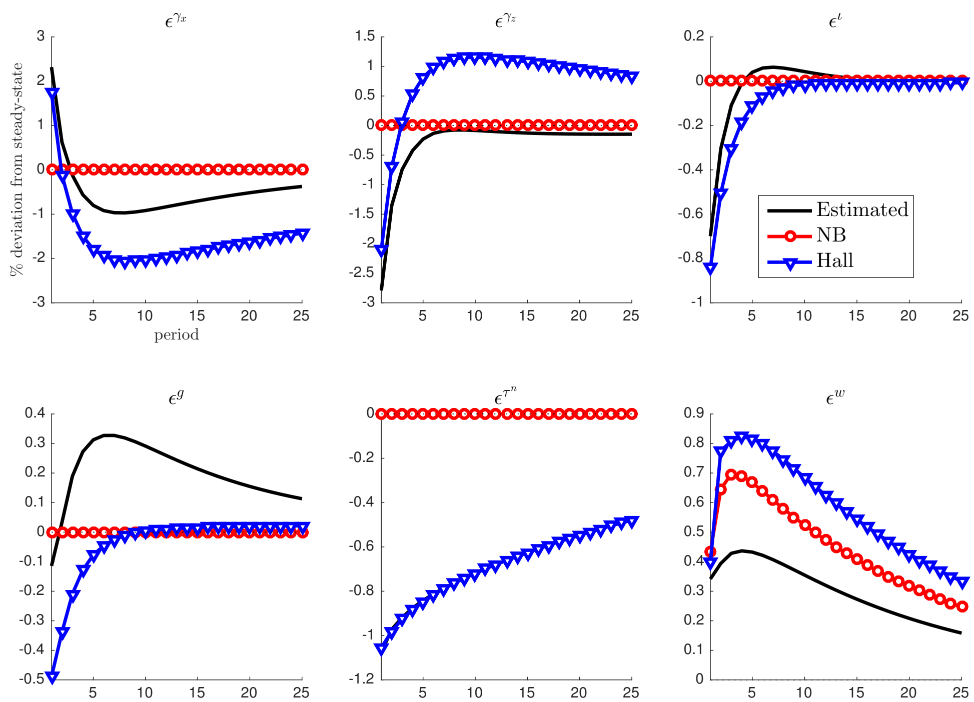
The responses of the estimated wage process differ significantly from both the rigid-wage model and the Nash bargaining model. The estimated wage responds to permanent shocks on impact—more rapidly than the rigid-wage model—but is slightly muted relative to the Nash bargaining wage response in the case of permanent technology shocks. Furthermore, the estimated wage responds to investment-specific technology shocks and labor supply shocks in a manner that is broadly consistent with the Nash bargaining responses, but deviates from the Nash bargaining responses markedly for the government spending, labor tax, and residual wage shocks. Indeed, there is no consistent similarity between the wage responses in the estimated model, the rigid-wage model, and the Nash bargaining model. Moreover, there is no evidence in our estimated process for wages that the data call for hump-shaped responses of the real wage. Instead, in response to both permanent and temporary shocks, the pattern is that of a substantial impact response of the wage followed by gradual adjustment to its long-run level.

The impulse responses of labor's share of match surplus demonstrate, in part, how the estimated model achieves realistic volatility in labor market variables without requiring a fixed real wage. Because of dynamic decision making on the part of firms and households, the surplus shares depicted depend on the full path of future wages. Thus, despite the fact that the two processes indicate very different impact effects for wages, both the rigid and estimated wage processes result in a strong initial increase in the household's share of surplus. The estimated wage process is therefore simultaneously capturing *both* the impact effects of real wages and the fluctuations in labor's surplus share needed to deliver fluctuations in

[18] In computing impulse responses, we fix fundamental parameters at their values estimated in the baseline economy. Thus, differences in impulse responses in these figures are driven only by different assumptions regarding wage determination.



(a) Wage Level



(b) Surplus Share

Figure 5: Impulse responses to each of the six shocks.

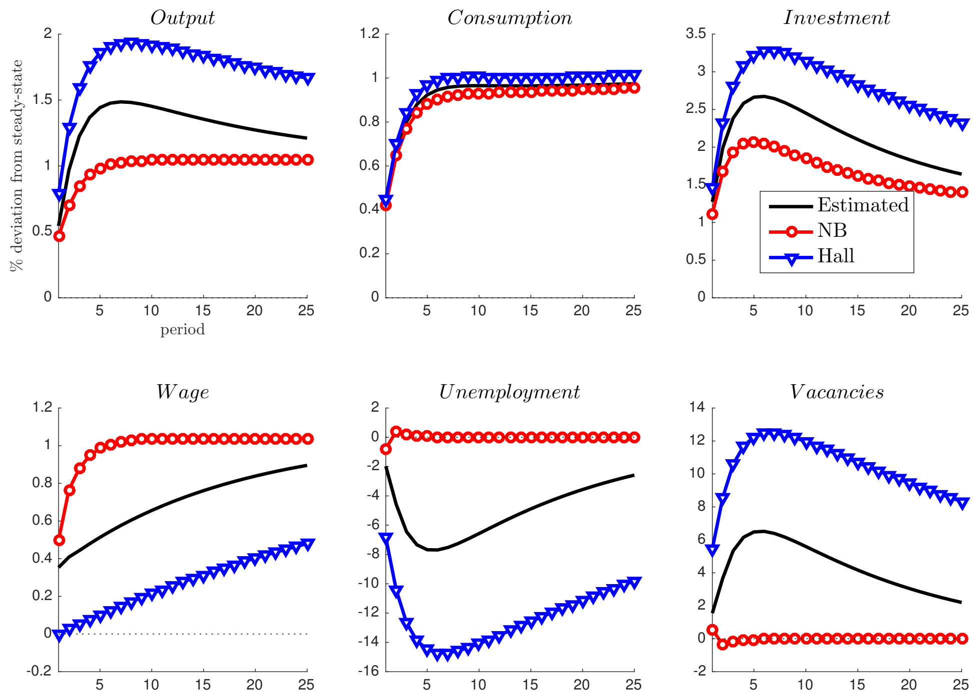


Figure 6: Impulse responses to permanent neutral productivity shock.

vacancies and unemployment. This is possible because of the sluggish adjustment of wages (relative to Nash bargaining) following the flexible initial response. Over time, the rigid-wage model demonstrates a counterfactually large share of surplus accruing to firms, but the qualitative response of the surplus share continues to resemble that of the rigid-wage model. In short, while wages appear in many cases to roughly approximate the flexible Nash benchmark, the implications for the division of surplus more closely resemble those of the rigid-wage model.

Figure 6 documents impulse responses of output, consumption, investment, the real wage, unemployment and vacancies to permanent neutral technology, the key shock according to our estimated model. As with the wage and surplus share responses discussed above, the agnostic specification of wages in our estimated model implies markedly different dynamics for each of these variables than those implied by the Nash bargaining or rigid-wage models. The response of the estimated model to technology shocks lies squarely between the rigid-wage and Nash responses. On impact, output, consumption and investment respond similarly to technology shocks in each of the three models, but responses subsequently fan out, with Nash bargaining delivering relatively muted responses (due to its relatively flexible wage) and

Table 8: Variance Decomposition of Observables: Business Cycle Frequency

	σ^x	σ^z	σ^ι	σ^g	σ^{τ^n}	σ^w
y	77.0	15.0	4.7	1.3	0.1	1.9
c	72.1	18.0	8.2	1.6	0.0	0.1
i	61.0	30.4	3.5	4.1	0.5	0.5
ur	69.1	3.2	1.9	10.4	0.1	15.4
n	50.7	4.0	27.6	6.5	0.5	10.7
vn	68.4	3.2	2.9	10.2	0.0	15.2
w	34.4	37.6	5.7	11.2	0.0	11.1

Note: Unconditional variance decomposition of fluctuations with duration between 6 and 32 quarters, using the Baxter and King (1999) approximate band-pass filter.

the rigid-wage model delivering relatively volatile responses. Similar dynamics are evident in the responses of unemployment and vacancies, albeit with more divergent responses on impact.

4.3 Variance Decomposition

Table 8 provides the unconditional variance decomposition for our estimated model. An overwhelming majority of the variation in observable macroeconomic aggregates is attributable to technology shocks. Indeed, roughly 70 percent of the variation in output, consumption, unemployment and vacancies owes to technology shocks under our estimated model. Moreover, technology shocks account for over half of the variation in investment and employment. Interestingly, and despite their importance for every macroeconomic quantity variable included in estimation, technology shocks account for only one third of total variation in the model wage. Although decidedly second-order, shocks to the growth rate of the relative price of investment also account for a non-trivial share of variation in observables, most notably investment (30 percent) and the real wage (37 percent).

Because our model wage process depends on the five structural shocks as well as a reduced-form wage shock, Table 8 also allows us to assess the extent to which the "fundamental" components of our wage process are sufficient for accounting for variation in observables. The sixth column of Table 8 suggests that less than 15 percent of variation in the wage is accounted for by our wage shock, with the balance accounted for by the five remaining structural shocks. More generally, the importance of the residual wage shock is decidedly limited, as it explains no more than 15 percent of variation in any other observable.

Table 9: Model Comparison

Model	$P(\mathcal{M})$	Bayes Factor
Baseline	7471.5	exp(0)
Nash Bargaining	7339.0	exp(132.5)
Fixed Wages	7318.2	exp(153.3)

Moreover, any limited relevance is confined to the labor market, as wage shocks explain only two percent of variation in output, and less than one percent of variation in consumption and investment.

4.4 Model Comparison

We have suggested that our baseline model provides a substantial improvement in fit relative to prominent structural models of wage determination. To make this comparison rigorous, we estimate otherwise identical models with (i) a flexible Nash-bargained wage and (ii) a rigid wage that adjusts only slowly in response to permanent shocks. For both of the alternative models, we allow for exogenous ARMA(2,1) deviations from the underlying model of wage determination, exactly analogous to the reduced-form wage shock in our baseline model. We then perform a Bayesian model comparison, computing marginal likelihoods for the three models we wish to consider. An important feature of this statistic is that it penalizes free parameters, so that (nested) models with relatively more degrees of freedom are not always preferred.[19] Table 9 shows that, while the Nash bargaining model outperforms the sticky real wage model, both are clearly dominated by our reduced-form specification for the wage, with a Bayes factor on the order of $exp(140)$.

The reasons for the failure of these two models are rather different, and in ways that corroborate previous observations about these two wage-determination mechanisms. In particular, the model estimated with Nash bargaining is unable to generate sufficiently large variation in vacancies, on account of an excessively volatile real wage. Interestingly, the data do not seek a calibration that is consistent with Hagedorn and Manovskii's (2008) suggestion of a low bargaining power combined with a high replacement rate of benefits (median estimates are $\eta = 0.41$ and $\kappa = 0.46$, respectively). Conversely, the tables in Appendix D.4

[19]Since our estimation relies very little any of the MA(1) or AR(2) terms we have included in the baseline model of the wage, our comparison actually implicitly penalizes it.

show that the rigid-wage economy delivers counterfactually high volatility of unemployment and, to a lesser extent, output growth.

4.5 Discussion

We believe our results deliver several important insights that can guide future researchers considering alternative models of wage determination. First, our results — in particular the comparison to the alternative models — provide strong evidence that the *endogenous* response of wages to fundamental shocks is crucial for our model to match the data. If causality in the economy was passing from exogenous shocks in wage determination to the broader economy, then the alternative models could easily account for such fluctuations through the exogenous component of the wage process. Instead, the large role of fundamental shocks (and relatively small role of exogenous wage shocks) in the baseline economy supports the conclusion that wages are an endogenous outcome. The orthogonal wage shocks in our model may then be thought of as deviations from an underlying wage-determination paradigm through which fundamental shocks drive wages. Unable to account for the endogenous pattern of wage fluctuations, the alternative models rely far more heavily on the exogenous component of wages to best-fit the data, and do so without approaching the fit provided by the baseline model.

Second, we have shown that the data are consistent with relatively flexible real wages and demand an important role for productivity shocks in driving both real quantities and labor market variables. In Appendix D.4, we provide variance decompositions for the estimated models featuring Nash bargaining and a rigid real wage, respectively. Interestingly, the data do not demand a dominant role for productivity shocks in either case. In the case of Nash bargaining, productivity shocks account for non-labor market variables—output, consumption, and investment—but fail to account for any significant variation in labor market quantities. This is consistent with the critique of Shimer (2005). Perhaps more surprisingly, in the case of the rigid wage model à la Hall (2005), productivity shocks play a substantial but smaller role than in the baseline economy. By construction, productivity shocks can account for very little of the variation in the real wage, leading the model to depend largely on residual wage shocks to explain wage fluctuations, and thus to a strongly counterfactual countercyclical real wage.

By contrast, when the model is estimated with our more general wage specification, the data demand a *central* role for productivity shocks. Our wage specification allows for

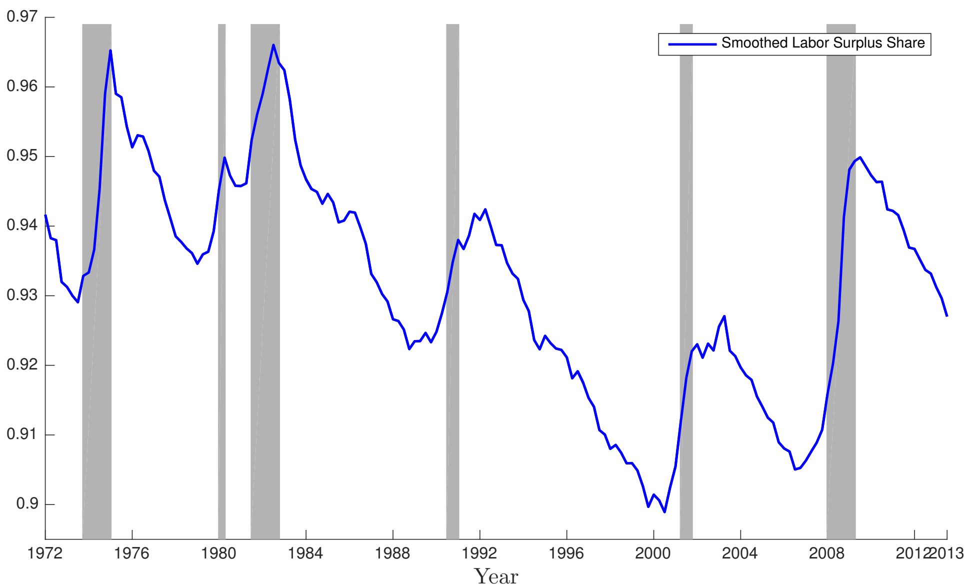


Figure 7: Labor's share of surplus.

this precisely because it can accommodate wage fluctuations that are significantly correlated both with productivity and with the endogenous changes in surplus share that cause vacancy postings to fluctuate. This contrast highlights a more general point that has guided our modeling approach: Results concerning the sources of business cycle fluctuations are predicated on the structural assumptions embodied in the model being estimated. By avoiding structural assumptions on wage determination, we have avoided making an a priori commitment that would otherwise largely determine our conclusion regarding the importance of different shocks.

Finally, our results provide a set of qualitative facts that can help in evaluating which models of wage determination are likely to be consistent with aggregate data. Our results indicate that successful wage-determination mechanisms will allow substantial flexibility of wages on impact, with wages generally moving *in the direction* of a Nash-bargained wage. However, the magnitude of the initial response is typically different from that implied by Nash bargaining, and mean reversion is slower than under Nash bargaining. Moreover, as Figure 7 demonstrates, our estimated wage process implies that labor's share of surplus should rise during recessions, a result that is consistent with the hypothesis that firms provide some degree of business cycle insurance to their workers.[20] A model that captures these three features — particularly in response to productivity shocks — is very likely to match the

[20]Cheremukhin and Restrepo-Echavarria (2014) directly model an exogenous shock to bargaining power and report a similar finding.

data.

5 Conclusion

The search and matching framework has become a popular modeling device in mainstream macroeconomics. But no consensus has been reached on the nature of wage determination in such markets. We draw upon search theory for what we consider to be critical components for realistically representing market interactions–costly search, nontrivial job-finding rates, equilibrium unemployment–while dispensing with a priori wage-determination mechanisms. Instead, we model wages as evolving according to an ARMA process, thus approximately nesting most structural wage-determination mechanisms used in the literature. We can do this because the search and matching framework per se requires nothing of wages except that they remain within the bargaining set. The flexibility of our approach allows us to study the empirical properties of the search and matching framework constraining ourselves to a particular theory of wages. The wage process that we estimate sheds light on the properties that a data-consistent theory of the wage must have.

Our model is able to match the data remarkably well. We match key business cycle frequency moments of output, consumption, investment, employment and the unemployment rate with a high level of accuracy relative to existing DSGE estimation literature. The dynamics of our estimated model are driven primarily by permanent shocks to productivity while our estimated process for the wage suggests that real wages can be relatively flexible, while still generating empirically realistic cyclical fluctuations. This result stands in marked contrast with a large literature seeking amplification by way of nominal rigidities or other partial adjustment mechanisms.

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Appendices

A Stationary Representation

The model described in the body of the text is trend stationary with respect to labor-augmenting technological progress, X_t, and the relative price of investment, Z_t. Denoting by tildes the stationary counterparts to non-stationary variables, we can rewrite the model in terms of only stationary variables as follows:

$$\tilde{Y}_t = A_t \left(u_t \tilde{K}_t\right)^{\alpha} \left(\gamma_{x,t} N_t\right)^{1-\alpha} \tag{48}$$

$$F_t = N_t + (1 - p_t) S_t \tag{49}$$

$$N_t = (1 - \lambda) N_{t-1} + M_t \tag{50}$$

$$\tilde{K}_{t+1} = \gamma_{x,t}^{-1} \gamma_{z,t}^{\frac{-1}{\alpha-1}} \left[(1 - \delta(u_t)) \tilde{K}_t + \tilde{I}_t\right] \tag{51}$$

$$\tilde{Y}_t = \tilde{C}_t + \tilde{I}_t \left[1 + \Phi\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \gamma_{x,t-1} \gamma_{z,t-1}^{\frac{1}{\alpha-1}}\right)\right] + a_n V_t \tag{52}$$

$$\tilde{D}_t = \tilde{Y}_t - \tilde{W}_t N_t - \tilde{R}_t u_t \tilde{K}_t - a_n V_t \tag{53}$$

$$\tilde{R}_t = \frac{\tilde{\mu}_t^K}{\tilde{\mu}_t} \delta'(u_t) \tag{54}$$

$$\frac{\tilde{\mu}_t^K}{\tilde{\mu}_t} = E_t \left\{\Omega_{t,t+1} \gamma_{z,t} \left[u_{t+1} \tilde{R}_{t+1} + (1 - \delta(u_{t+1})) \frac{\tilde{\mu}_{t+1}^K}{\tilde{\mu}_{t+1}}\right]\right\} \tag{55}$$

$$\begin{aligned} \frac{\tilde{\mu}_t^K}{\tilde{\mu}_t} &= \gamma_{z,t} \left[1 + \Phi\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \gamma_{x,t-1} \gamma_{z,t-1}^{\frac{1}{\alpha-1}}\right) + \Phi'\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \gamma_{x,t-1} \gamma_{z,t-1}^{\frac{1}{\alpha-1}}\right) \left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}} \gamma_{x,t-1} \gamma_{z,t-1}^{\frac{1}{\alpha-1}}\right)\right] \\ &\quad - E_t \left\{\Omega_{t,t+1} \left[\gamma_{z,t+1} \Phi'\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} \gamma_{x,t} \gamma_{z,t}^{\frac{1}{\alpha-1}}\right) \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t} \gamma_{x,t} \gamma_{z,t}^{\frac{1}{\alpha-1}}\right)^2\right]\right\} \end{aligned} \tag{56}$$

$$\tilde{\mu}_t^N = \frac{a_n}{q_t} \tag{57}$$

$$\tilde{\mu}_t^N = (1 - \alpha) A_t \left(\frac{u_t \tilde{K}_t}{\gamma_{x,t} N_t}\right)^{\alpha} \gamma_{x,t} - \tilde{W}_t + (1 - \lambda) E_t \left\{\Omega_{t,t+1} \gamma_{x,t} \gamma_{z,t}^{\frac{\alpha}{\alpha-1}} \tilde{\mu}_{t+1}^N\right\} \tag{58}$$

$$\tilde{R}_t = \alpha A_t \left(\frac{u_t \tilde{K}_t}{\gamma_{x,t} N_t}\right)^{\alpha-1} \tag{59}$$

$$-\frac{u_{f,t}}{u_{\tilde{c},t}} = p_t \left[\tilde{W}_t + (1-\lambda) E_t \left\{ \Omega_{t,t+1} (\gamma_{x,t} \gamma_{z,t}^{\frac{\alpha}{\alpha-1}})^{1-\sigma} \left(\frac{1-p_{t+1}}{p_{t+1}} \right) \left(-\frac{u_{f,t+1}}{u_{\tilde{c},t+1}} - \tilde{\kappa}_{t+1} \right) \right\} \right] + (1-p_t)\tilde{\kappa}_t \tag{60}$$

where

$$\Omega_{t,t+1} \equiv \beta \frac{\tilde{\mu}_{t+1}}{\tilde{\mu}_t} \left(\gamma_{x,t} \gamma_{z,t}^{\frac{\alpha}{\alpha-1}} \right)^{-\sigma} \tag{61}$$

$$\tilde{\mu}_t = \frac{\mu_t}{\left(X_{t-1} Z_{t-1}^{\frac{\alpha}{\alpha-1}} \right)^{-\sigma}} \tag{62}$$

$$\tilde{\mu}_t^K = \frac{\mu_t^K}{\left(X_{t-1} Z_{t-1}^{\frac{\alpha}{\alpha-1}} \right)^{-\sigma} Z_{t-1}} \tag{63}$$

$$\tilde{\mu}_t^N = \frac{\mu_t^N}{X_{t-1} Z_{t-1}^{\frac{\alpha}{\alpha-1}}} \tag{64}$$

and variables are detrended according to $\tilde{\Delta}_t \equiv \frac{\Delta_t}{X_{t-1} Z_{t-1}^{\frac{\alpha}{\alpha-1}}}$ for $\Delta_t \in \{Y_t, C_t, D_t, W_t\}$, $\tilde{\Delta}_t \equiv \frac{\Delta_t}{X_{t-1} Z_{t-1}^{\frac{1}{\alpha-1}}}$ for $\Delta_t \in \{K_t, I_t\}$, and $\tilde{R}_t \equiv \frac{R_t}{Z_t}$.

B Steady State and Calibration

We use the restrictions imposed by the deterministic steady-state of the model, together with long-run values for $\bar{p}$, $\bar{q}$, $\bar{N}$, $\bar{un}$, $\bar{\phi}^N$, $\bar{u}$ taken from the data, to analytically solve for values for all remaining endogenous variables, as well as χ, λ, ψ, a_n, δ_1 and W. In what follows, we make reference only to detrended variables, though we neglect tildes for ease of notation.

The first-order conditions for investment and utilization, evaluated at the deterministic steady state, imply

$$\mu^K/\mu = \gamma_z \tag{65}$$

$$R = (\mu^K/\mu)\delta_1. \tag{66}$$

Substituting these expressions into the household's first-order condition for period-ahead capital and solving for δ_1, we obtain

$$\delta_1 = \left[(\Omega\gamma_z)^{-1} - 1 + \delta_0 \right] / \bar{u} \tag{67}$$

from which we recover

$$R = \gamma_z \delta_1. \tag{68}$$

From the firm's first-order condition for capital, we obtain

$$K = \left[\alpha A \bar{u}^{\alpha-1} (\gamma_x \bar{N})^{1-\alpha} r^{-1}\right]^{\frac{1}{1-\alpha}} \tag{69}$$

from which, in turn, we can solve for output

$$Y = A(uK)^{\alpha} (\gamma_x \bar{N})^{1-\alpha}. \tag{70}$$

The definition of the unemployment rate, together with its long-run value and that of total employment, can be used to solve for steady-state labor force participation

$$F = \frac{\bar{N}}{1 - \bar{un}}. \tag{71}$$

The definition of the labor force, in turn, allows us to solve for the mass of searching workers,

$$S = \frac{F - \bar{N}}{1 - \bar{p}}. \tag{72}$$

The law of motion for employment (from the perspective of the household and the firm, respectively) yield

$$\lambda = \frac{\bar{p} S}{\bar{N}} \tag{73}$$

$$V = \frac{\lambda \bar{N}}{\bar{q}}. \tag{74}$$

From the definition of the matching function, we obtain

$$\theta = \frac{\bar{p}}{\bar{q}} \tag{75}$$

which in turn allows us to solve for χ,

$$\chi = \bar{p} \theta^{-\epsilon}. \tag{76}$$

Next, from the definition of labor's share, together with its steady state value $\bar{\phi}^N$, we can solve for the real wage,

$$W = \bar{\phi}^N \bar{N}^{-\alpha} A(\bar{u}K)^{\alpha} \gamma_x^{1-\alpha}. \tag{77}$$

With the steady-state real wage in hand, we use the vacancy posting condition to solve for the vacancy posting cost

$$a_n = \frac{\bar{q}}{1-(1-\lambda)\Omega\gamma_x\gamma_z^{\frac{\alpha}{\alpha-1}}}\left[(1-\alpha)A\left(\frac{\bar{u}K}{\bar{N}\gamma_x}\right)^{\alpha} - W\right] \tag{78}$$

where $\Omega = \beta\left(\gamma_x\gamma_z^{\frac{\alpha}{\alpha-1}}\right)^{-\sigma}$. The law of motion for capital and the aggregate resource constraint imply, respectively,

$$I = K\left[\frac{1-(1-\delta_0)\gamma_x^{-1}\gamma_z^{\frac{-1}{\alpha-1}}}{\gamma_x^{-1}\gamma_z^{\frac{-1}{\alpha-1}}}\right] \tag{79}$$

and

$$C = Y - G - a_n V - \gamma_z I. \tag{80}$$

Finally, making use of the labor force participation condition, we obtain

$$\psi = \frac{W\left[\bar{p} + (1-\bar{p})\kappa - \kappa(1-\lambda)\beta(1-\bar{p})\left(\gamma_x\gamma_z^{\frac{\alpha}{\alpha-1}}\right)^{1-\sigma}\right]}{\left(1-(1-\lambda)\beta(1-\bar{p})\left(\gamma_x\gamma_z^{\frac{\alpha}{\alpha-1}}\right)^{1-\sigma}\right)\left((1-F)^{-\iota}C(1-h(\gamma_x\gamma_z^{\frac{\alpha}{\alpha-1}})^{-1})\right)}. \tag{81}$$

C Match Surplus and Nash Bargaining

In order to compare the dynamics of our estimated model with those implied by Nash bargaining, we first use the expressions for household and firm match surpluses to derive the wage associated with Nash bargaining in our model. This also yields expressions for household and firm match surplus shares, which we discuss in the body of the text.

The wage that results from Nash bargaining between firms and workers is given by

$$W_t^{NB} = \arg\max_{W_t}[\mathbf{W_t}(W_t) - \mathbf{U_t}]^{\eta}[\mathbf{J_t}(W_t) - \mathbf{V_t}]^{1-\eta} \tag{82}$$

where $\mathbf{W_t}$ denotes the value of a match for the household, $\mathbf{U_t}$ denotes the value of unemployment for the household, $\mathbf{J_t}$ denotes the value of a match for the firm, and $\mathbf{V_t}$ denotes the value of a vacancy for the firm. Free-entry of firms implies that $\mathbf{V_t} = 0$, and our specification of unemployment benefits, combined with the existence of a search margin for households, implies that $\mathbf{U_t} = \kappa_t$. Thus, the standard Nash sharing rule reduces to

$$\mathbf{W_t} - \mathbf{U_t} = \left(\frac{\eta}{1-\eta}\right)\mathbf{J_t}. \tag{83}$$

The household match surplus (in units of consumption) may be expressed as the sum of the wage payment earned in the period of the match (due to our timing assumption) and the continuation value of the match, less the lump-sum transfer to the unemployed,

$$\mathbf{W_t} - \mathbf{U_t} = W_t(1-\tau_t^n) - \kappa_t + (1-\lambda)E_t\left\{(1-p_{t+1})\Omega_{t,t+1}(\mathbf{W_{t+1}} - \mathbf{U_{t+1}})\right\}. \tag{84}$$

The value of a match to the firm (again, in units of consumption) is given by the current marginal product of the match net of the wage bill plus the continuation value,

$$\mathbf{J_t} = A_t F_{n,t} - W_t + (1-\lambda)E_t\left\{\Omega_{t,t+1}\mathbf{J_{t+1}}\right\}. \tag{85}$$

Note that the firm's first-order conditions imply

$$\mathbf{J_t} = \mu_t^N. \tag{86}$$

To solve for the wage associated with Nash bargaining, begin by substituting the expressions for $\mathbf{W_t}$ and $\mathbf{U_t}$ into the Nash sharing rule,

$$W_t^{NB}(1-\tau_t^n) - \kappa_t + (1-\lambda)E_t\left\{(1-p_{t+1})\Omega_{t,t+1}(\mathbf{W_{t+1}} - \mathbf{U_{t+1}})\right\} = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{87}$$

Iterating the sharing rule forward and substituting in for $\mathbf{W_{t+1}} - \mathbf{U_{t+1}}$,

$$W_t^{NB}(1-\tau_t^n) - \kappa_t + (1-\lambda)E_t\left\{(1-p_{t+1})\Omega_{t,t+1}\left(\frac{\eta}{1-\eta}\right)\mathbf{J_{t+1}}\right\} = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{88}$$

Replacing $\mathbf{J_t}$ with the firm's first-order condition for labor and then $\mathbf{J_{t+1}}$ with the time $t+1$

Lagrange multiplier,

$$W_t^{NB}(1-\tau_t^n) - \kappa_t + (1-\lambda)E_t\left\{(1-p_{t+1})\Omega_{t,t+1}\left(\frac{\eta}{1-\eta}\right)\mu_{t+1}^N\right\}$$
$$= \frac{\eta}{1-\eta}\left(A_tF_{n,t} - W_t^{NB} + (1-\lambda)E_t\left\{\Omega_{t,t+1}\mu_{t+1}^N\right\}\right). \quad (89)$$

Solving for W_t^{NB}, we obtain

$$W_t^{NB} = \frac{1-\eta}{\eta+(1-\eta)(1-\tau_t^n)}\kappa_t + \frac{\eta}{\eta+(1-\eta)(1-\tau_t^n)}\left[A_tF_{n,t} + (1-\lambda)E_t\left\{\Omega_{t,t+1}p_{t+1}\mu_{t+1}^N\right\}\right]. \quad (90)$$

From here, dividing by $X_{t-1}Z_{t-1}^{\frac{\alpha}{\alpha-1}}$ yields the stationary representation,

$$\tilde{W}_t^{NB} = \frac{1-\eta}{\eta+(1-\eta)(1-\tau_t^n)}\tilde{\kappa}_t + \frac{\eta}{\eta+(1-\eta)(1-\tau_t^n)}\left[A_t\tilde{F}_{n,t} + (1-\lambda)\gamma_{x,t}\gamma_{z,t}^{\frac{\alpha}{\alpha-1}}E_t\left\{\Omega_{t,t+1}p_{t+1}\tilde{\mu}_{t+1}^N\right\}\right]. \quad (91)$$

Then, for a given wage-determination mechanism, the standard expression for the household's share of match surplus is given by

$$SS^H = \frac{\mathbf{W}-\mathbf{U}}{\mathbf{W}-\mathbf{U}+\mathbf{J}-\mathbf{V}} \quad (92)$$

and the expression for the firm's share of match surplus is

$$SS^F = \frac{\mathbf{J}-\mathbf{V}}{\mathbf{W}-\mathbf{U}+\mathbf{J}-\mathbf{V}} = 1 - SS^H. \quad (93)$$

D Estimation Details

D.1 Data Construction

Nominal quantity variables are converted to their real per capita counterparts by dividing by the seasonally-adjusted chain-weighted GDP deflator and the civilian non-institutional population. Consumption is measured as the sum of personal consumption expenditures on non-durables and services, while investment is measured as the sum of personal consumption expenditure on durables and total private fixed investment. Unemployment is measured as the number of unemployed persons as a percentage of the civilian non-institutional labor force. The labor tax series is constructed by Arseneau and Chugh (2012), and ends in 2009Q4.

D.2 Priors

The priors used in the baseline estimation are summarized in the tables below.

Table 10: Prior Distributions - I

Parameter	**Dist.**	**Mean**	**Std. Dev.**	**Low**	**High**
h	beta	0.60	0.20	0.10	1.00
ϕ	gaminv	4.00	2.00	0.01	75.00
ϵ	beta	0.50	0.20	0.01	1.00
p	beta	0.50	0.20	0.01	1.00
κ	beta	0.50	0.20	0.01	1.00
ϕ_κ	beta	0.50	0.20	0.01	1.00
$\phi_{g,y}$	beta	0.30	0.20	0.01	1.00
ρ_ι	beta	0.60	0.20	0.00	1.00
ρ_g	beta	0.60	0.20	0.00	1.00
ρ_τ	beta	0.60	0.20	0.00	1.00
σ^x	gaminv	0.02	0.10	0.00	0.30
σ^z	gaminv	0.02	0.10	0.00	0.30
σ^ι	gaminv	0.02	0.10	0.00	0.30
σ^g	gaminv	0.02	0.10	0.00	0.30
σ^{τ^n}	gaminv	0.02	0.10	0.00	0.30

Table 11: Prior Distributions - II

Parameter	**Dist.**	**Mean**	**Std. Dev.**	**Low**	**High**
ψ_0^x	gaminv	1.00	1.00	0.00	5.00
ψ_0^z	norm	0.00	1.00	-5.00	5.00
ψ_0^ι	norm	0.00	1.00	-5.00	5.00
ψ_0^g	norm	0.00	1.00	-5.00	5.00
$\psi_0^{\tau^n}$	norm	0.00	1.00	-5.00	5.00
ψ_0^w	gaminv	1.00	1.00	0.00	5.00
ψ_1^x/ψ_0^x	norm	0.00	0.10	-0.99	0.99
ψ_1^z/ψ_0^z	norm	0.00	0.10	-0.99	0.99
$\psi_1^\iota/\psi_0^\iota$	norm	0.00	0.10	-0.99	0.99
ψ_1^g/ψ_0^g	norm	0.00	0.10	-0.99	0.99
$\psi_1^{\tau^n}/\psi_0^{\tau^n}$	norm	0.00	0.10	-0.99	0.99
ψ_1^w/ψ_0^w	norm	0.00	0.10	-0.99	0.99
ρ_0^x	norm	0.00	0.10	-0.99	0.99
ρ_0^z	norm	0.00	0.10	-0.99	0.99
ρ_0^ι	beta	0.50	0.20	0.00	1.00
ρ_0^g	beta	0.50	0.20	0.00	1.00
$\rho_0^{\tau^n}$	beta	0.50	0.20	0.00	1.00
ρ_0^w	beta	0.50	0.20	0.00	1.00
ρ_1^x	norm	0.00	0.10	-0.99	0.99
ρ_1^z	norm	0.00	0.10	-0.99	0.99
ρ_1^ι	norm	0.00	0.10	-0.99	0.99
ρ_1^g	norm	0.00	0.10	-0.99	0.99
$\rho_1^{\tau^n}$	norm	0.00	0.10	-0.99	0.99
ρ_1^w	norm	0.00	0.10	-0.99	0.99
ϕ_x	beta	0.50	0.20	0.00	1.00
ϕ_z	beta	0.50	0.20	0.00	1.00

Table 12: Prior Distributions - III

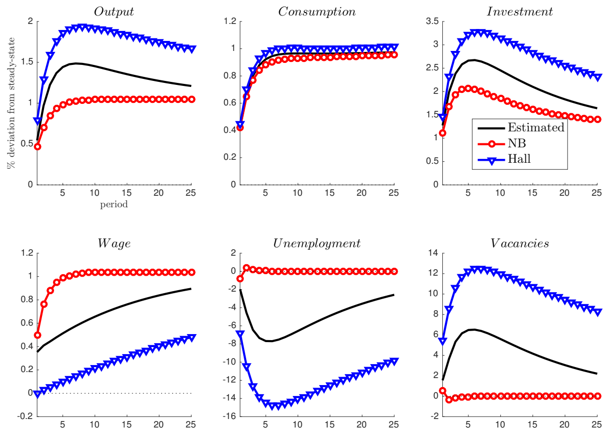
Parameter	**Dist.**	**Mean**	**Std. Dev.**	**Low**	**High**
$\gamma_{w,1}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{w,2}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{w,3}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{w,4}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{w,5}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{w,6}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{w,7}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{v,1}$	gaminv	1.00	0.20	0.00	5.00
$\gamma_{v,2}$	gaminv	1.00	0.20	0.00	5.00
σ_y	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,1}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,2}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,3}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,4}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,5}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,6}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{w,7}$	gaminv	0.00	0.03	0.00	0.20
$\sigma_{v,1}$	gaminv	0.01	0.03	0.00	2.00
$\sigma_{v,2}$	gaminv	0.01	0.03	0.00	2.00

D.3 Estimated Model

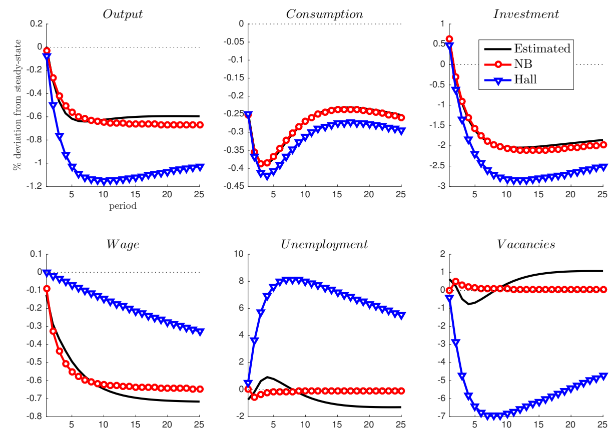
We compute the posterior parameter distribution using the MCMC algorithm with 12 chains of 1.5 million draws each and dropping the first 25% of each chain. This appendix summarizes the estimated parameters for the measurement equations (45)-(46), and provides impulses responses for the main quantities and labor market variables for each of the model's six shocks. The final figure also plots prior and posterior distributions for the estimated parameters.

Table 13: Posterior Estimates of Model Parameters, Part III.

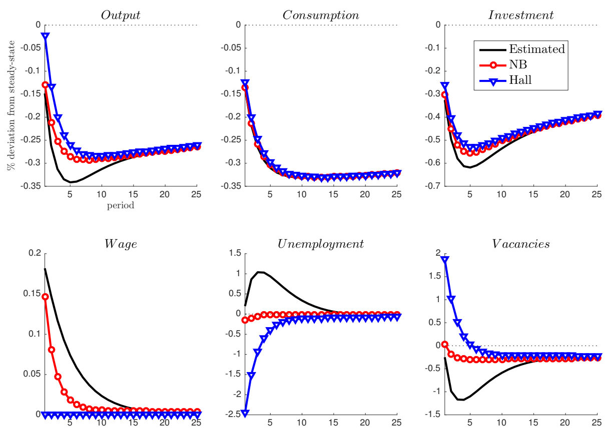
Parameter	**Concept**	**Median**	**5%**	**95%**
$\gamma_{w,1}$	Wage loading - Total comp. per worker	0.62	0.53	0.73
$\gamma_{w,2}$	Wage loading - Comp. per hour index	0.60	0.50	0.72
$\gamma_{w,3}$	Wage loading - Real weekly comp.	0.74	0.60	0.93
$\gamma_{w,4}$	Wage loading - ECI (wages)	0.41	0.35	0.46
$\gamma_{w,5}$	Wage loading - ECI (wages and benefits)	0.42	0.37	0.49
$\gamma_{w,6}$	Wage loading - Quality-adj. wage (HSvR)	0.73	0.58	0.92
$\gamma_{w,7}$	Wage loading - Quality-adj. wage for new workers (HSvR)	0.93	0.70	1.28
$\gamma_{v,1}$	Vacancy loading - JOLTS survey	1.01	0.82	1.25
$\gamma_{v,2}$	Vacancy loading - Help-wanted	2.23	1.89	2.65
σ_{y}	Meas. error - GDP	0.00	0.00	0.01
$\sigma_{w,1}$	Measurement error - Total comp. per worker	0.01	0.01	0.01
$\sigma_{w,2}$	Measurement error - Comp. per hour index	0.01	0.01	0.01
$\sigma_{w,3}$	Measurement error - Real weekly comp.	0.01	0.01	0.01
$\sigma_{w,4}$	Measurement error - ECI (wages)	0.00	0.00	0.00
$\sigma_{w,5}$	Measurement error - ECI (wages and benefits)	0.00	0.00	0.00
$\sigma_{w,6}$	Measurement error - Quality-adj. wage (HSvR)	0.01	0.01	0.01
$\sigma_{w,7}$	Measurement error - Quality-adj. wage for new workers (HSvR)	0.05	0.05	0.06
$\sigma_{v,1}$	Measurement error - JOLTS survey	0.04	0.04	0.05
$\sigma_{v,2}$	Measurement error - Help-wanted	0.03	0.03	0.04



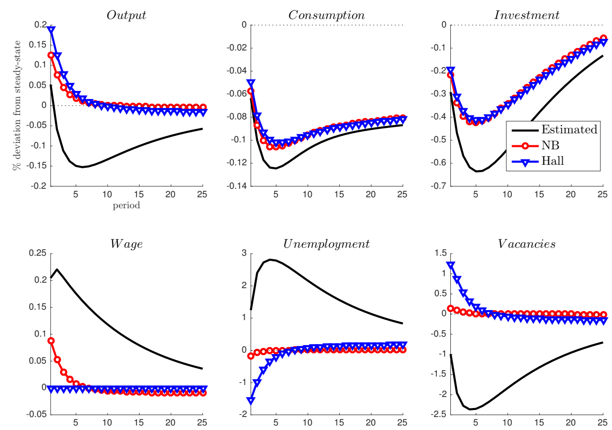
(a) Total factor productivity



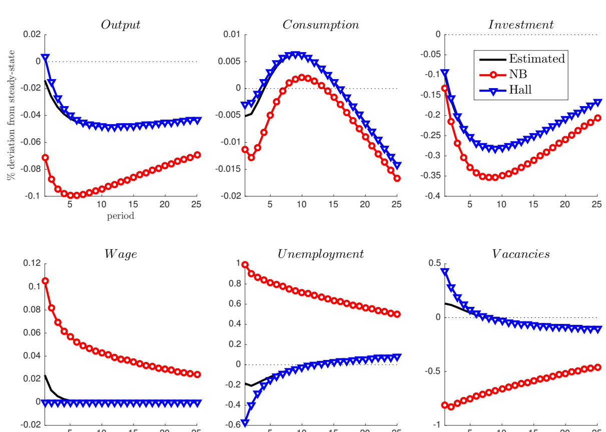
(b) Relative price of investment



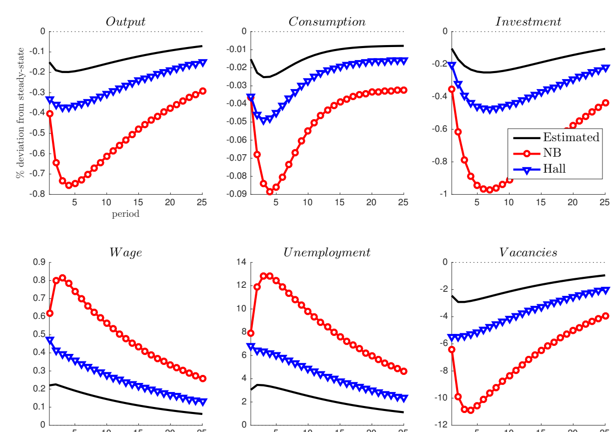
(c) Labor supply



(d) Government spending



(e) Labor tax



(f) Wage

Figure 8: Quantity impulse responses to six exogenous shocks.

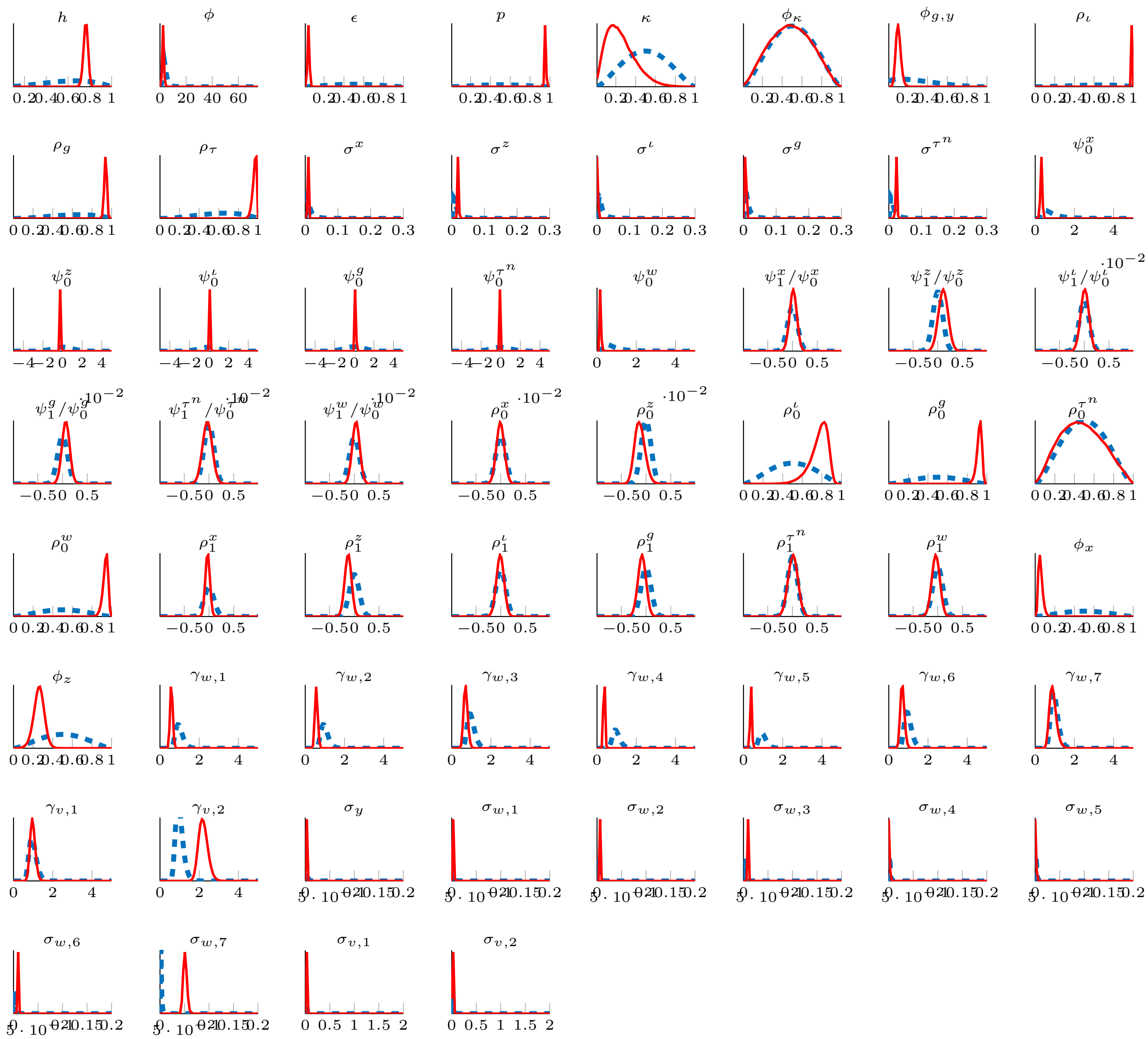


Figure 9: Prior (dashed) and posterior (solid) distributions for estimated parameter values.

D.4 Alternative Models

This appendix summarizes the implications of the estimated versions of the Nash-bargaining and the rigid-wage economies.

Table 14: Summary Moments (Nash Bargaining)

	Δy	Δc	Δi	Δg	Δur	Δn
$\sigma(X)$	0.75	0.62	2.03	0.96	5.32	0.45
$\sigma(X)/\sigma(\Delta Y)$	1.00	0.82	2.72	1.29	7.11	0.60
$\rho(X_t, X_{t-1})$	0.51	0.46	0.41	0.05	0.39	0.28
$\rho(X, \Delta Y)$	1.00	0.78	0.82	0.20	-0.40	0.40

Table 15: Moments - Vacancy-Posting Measures (Nash Bargaining)

	JOLTS	**Help-Wanted**	**Model**
$\sigma(X)$	6.27	10.17	5.18
$\sigma(X)/\sigma(\Delta Y)$	7.63	12.38	6.92
$\rho(X_t, X_{t-1})$	0.49	0.68	0.40
$\rho(X, \Delta Y)$	0.38	0.78	0.41

Table 16: Moments - Wage Measures (Nash Bargaining)

	cmp/wrkr	**cmp/hr**	**wkly cmp**	**ECI**	**ECI+ben**	**HSV**	**HSV-new**	**modl**
$\sigma(X)$	0.61	0.79	1.11	0.29	0.29	1.06	5.25	0.79
$\sigma(X)/\sigma(\Delta Y)$	0.74	0.96	1.35	0.35	0.35	1.29	6.39	1.06
$\rho(X_t, X_{t-1})$	-0.01	-0.18	0.25	0.29	0.11	-0.12	-0.19	0.44
$\rho(X, \Delta Y)$	0.33	-0.01	0.03	0.11	0.19	0.03	-0.02	0.54

Table 17: Nash Model, Variance Decomposition of Observables: Business Cycle Frequency

	σ^x	σ^z	σ^ι	σ^g	σ^{τ^n}	σ^w
y	54.6	23.5	5.2	0.5	0.7	15.6
c	69.8	20.0	8.7	1.4	0.0	0.1
i	47.0	43.8	3.3	2.6	1.2	2.1
ur	0.1	0.1	0.0	0.0	1.0	98.8
n	0.8	1.4	24.7	0.2	3.8	69.1
vn	0.1	0.1	0.1	0.0	1.2	98.5
w	55.9	21.7	0.5	0.2	0.4	21.3

Note: Unconditional variance decomposition of fluctuations with duration between 6 and 32 quarters, using the Baxter and King (1999) approximate band-pass filter.

Table 18: Summary Moments (Hall)

	Δy	Δc	Δi	Δg	Δur	Δn
$\sigma(X)$	1.17	0.57	2.10	0.97	9.03	0.60
$\sigma(X)/\sigma(\Delta Y)$	1.00	0.49	1.79	0.82	7.70	0.51
$\rho(X_t, X_{t-1})$	0.48	0.46	0.39	0.06	0.21	0.26
$\rho(X, \Delta Y)$	1.00	0.71	0.82	0.21	-0.90	0.85

Table 19: Moments - Vacancy-Posting Measures (Hall)

	JOLTS	**Help-Wanted**	**Model**
$\sigma(X)$	6.27	10.17	12.38
$\sigma(X)/\sigma(\Delta Y)$	7.63	12.38	10.54
$\rho(X_t, X_{t-1})$	0.49	0.68	0.25
$\rho(X, \Delta Y)$	0.38	0.78	0.90

Table 20: Moments - Wage Measures (Hall)

	cmp/wrkr	**cmp/hr**	**wkly cmp**	**ECI**	**ECI+ben**	**HSV**	**HSV-new**	**modl**
$\sigma(X)$	0.61	0.79	1.11	0.29	0.29	1.06	5.25	0.50
$\sigma(X)/\sigma(\Delta Y)$	0.74	0.96	1.35	0.35	0.35	1.29	6.39	0.43
$\rho(X_t, X_{t-1})$	-0.01	-0.18	0.25	0.29	0.11	-0.12	-0.19	-0.00
$\rho(X, \Delta Y)$	0.33	-0.01	0.03	0.11	0.19	0.03	-0.02	-0.24

Table 21: Hall Model, Variance Decomposition of Observables: Business Cycle Frequency

	σ^x	σ^z	σ^ι	σ^g	σ^{τ^n}	σ^w
y	67.3	27.0	2.2	0.6	0.1	2.9
c	66.3	20.5	11.1	1.8	0.0	0.2
i	48.5	44.4	3.5	2.5	0.9	0.1
ur	59.5	21.7	1.3	0.5	0.1	16.8
n	49.6	23.4	11.4	0.9	0.4	14.2
vn	59.4	21.9	1.3	0.6	0.1	16.8
w	6.4	3.5	0.0	0.0	0.0	90.1

Note: Unconditional variance decomposition of fluctuations with duration between 6 and 32 quarters, using the Baxter and King (1999) approximate band-pass filter.