Chapter 16
Fiscal Theory of Exchange Rates

We now turn to the subject of international monetary economics. Specifically, we will consider the interaction of a fixed exchange rate system with fiscal policy. Exchange rate management is typically thought to be in the domain of monetary, not fiscal, policy. However, we will learn that fiscal policy considerations also impact exchange rates. We will build a small theoretical model that allows us to study this interaction. Our theoretical model will consist of four building blocks:

1. Money demand function
2. Purchasing power parity (PPP)
3. Interest parity condition
4. Government budget constraint

Before we describe these four building blocks, we first discuss the timing of the model. Specifically, rather than a two-period economy we have considered in much of our study, we will consider an infinite-period economy. Then, after laying out the four building blocks, we consider the workings of the model, paying close attention to the influence of fiscal policy on nominal exchange rates.

Infinite-Period Economy

By now you are comfortable with the idea of the two-period economy we used in studying the representative consumer's consumption-savings decision. The two-period economy served our purposes in that task, but turns out to be insufficient in our present study of the interaction between fiscal policy and exchange rates. Thus, we now generalize our model economy to allow for an infinite number of time periods. The reasons why we need an arbitrarily large number of time periods will become clearer as we proceed.

In the two-period economy, the "names" of each of the two periods was fairly natural – we named them period 1 and period 2. We could analogously name the time periods in our present infinite-period economy as period 1, period 2, period 3, period 4, period 5, etc, without end. However, again as will become clearer below, the specific "name" of a given time period will have no relevance – all that will matter is how far (in time) a given time period is from any other given time period and whether it comes before or after it. For example, period 2 is two time periods earlier than period 4. But period 11 is also two time periods earlier than period 13. And period 134 is two time periods earlier than time period 136. Because all we will need to care about is how time periods relate to each other, rather than any absolute sense of time, we will name the time periods in a more general fashion, specifically by calling them t, $t+1$, $t+2$, $t+3$, $t+4$, … With this notation, t can take on any value: we could have $t=0$, in which case $t+1=1$ and $t+2=2$. Or we could have $t=11$, in which case $t+1=12$ and $t+2=13$. And so forth. With this notation, obviously period t is two time periods earlier than period $t+2$, and that is as specific as we will need to be.

Money Demand Function

Consumers are assumed to need money in order to purchase their consumption in every period. That is, all consumption purchases require the use of cash, which implies that "checks" drawn against bank deposits do not exist in this economy. This need for cash gives rise to a money demand function which has the usual properties we have associated with a money demand function. Specifically, we will denote the **nominal money demand function** as

$$M_t = P_t \cdot \phi\left(c_t, i_t\right), \tag{14}$$

where P_t denotes, as usual, the nominal price level (also the nominal price of consumption) in period t, the function $\phi\left(\bullet\right)$ is the real money demand function, c_t denotes consumption in period t, and i_t denotes the nominal interest rate on bank deposits held by consumers between period t and $t+1$. The reason that $\phi\left(\bullet\right)$ is real money demand is that if we divide both sides of expression (14) by the price level P_t, we have real money demand M_t / P_t equals $\phi(c_t, i_t)$.

In general, consumption may be different in each period, which is why we subscript consumption by t. However, to focus our attention on the important issues, we will assume for simplicity that $c_t = \bar{c}$ in every period, where $\bar{c}$ is simply some constant amount of consumption. All this says is that in period t consumption equals $\bar{c}$, in period $t+1$ consumption equals the same value $\bar{c}$, in period $t+2$ consumption equals the same value $\bar{c}$, etc.[119]

[119] In terms of the two-period consumption-savings model, you should be able to convince yourself that it is possible for an individual's optimal choice (i.e., the tangency between an indifference curve and the lifetime budget constraint) to occur at a point such that consumption is equal in the two periods. Our assumption here that consumption is constant in each of the infinite number of periods is simply the infinite-period analog of that outcome.

Purchasing Power Parity (PPP)

We will follow a long-standing tradition in international economics and denote foreign country variables using a superscripted asterisk. Domestic country variables are thus denoted without an asterisk. With this convention, we define P_t^* as the foreign country price level in period t, P_t as the domestic country price level in period t, and E_t as the nominal exchange rate between the currencies of the two countries.

It is very important to be clear about the units associated with these variables, especially those of the nominal exchange rate. The units associated with the foreign price level P_t^* is foreign currency per one basket of foreign goods. The units associated with the domestic price level P_t is domestic currency per one basket of domestic goods. The units associated with the nominal exchange rate is domestic currency per unit of foreign currency. For example, if we call the U.S. the domestic country and Japan the foreign country, then we have P_t is the dollar price of one basket of U.S. goods (i.e., think of the CPI basket here), P_t^* is the yen price of one basket of Japanese goods, and E_t is the number of U.S. dollars needed to buy one Japanese yen.[120] It is of course possible to define E_t in the inverse way, as the number of Japanese yen needed to buy one U.S. dollar. Thus, our convention here is simply a matter of preference. However, it is crucial that once we adopt a convention (and our convention is domestic currency needed to buy one unit of foreign currency) we be consistent throughout our analysis.

With this notation clear, we can proceed to describe the second fundamental building block of our model, purchasing power parity, abbreviated PPP. Simply put, the concept of PPP states that when prices of goods in any two different countries are converted into the same currency units (using the nominal exchange rate), they are the same. Consider a simple example: suppose in the U.S. in 2002 a McDonald's Big Mac costs \$2.00 and in Japan in 2002 a McDonalds's Big Mac[121] costs 270 yen. Further suppose the exchange rate between the U.S. dollar and the Japanese yen in 2002 was 135 yen per one dollar (alternatively, 1/135 = 0.007 dollars per yen). If we convert the yen price of a Big Mac in Japan into dollar terms, we find that the dollar price of a Big Mac in Japan was

$$\left(\frac{270\,\text{yen}}{\text{Big Mac}}\right)\left(\frac{1\,\text{dollar}}{135\,\text{yen}}\right)=\frac{2\,\text{dollars}}{\text{Big Mac}},$$

[120] In early April 2003, it cost 120 Japanese yen to purchase one U.S. dollar. Thus, in terms of our notation, where E_t measures how many U.S. dollars are needed to buy one Japanese yen, we would have $E_t = 1/120 = 0.008$ dollars per yen.

[121] Let's ignore any regional variation (i.e., Japanese hamburgers typically have more or less ketchup than a U.S. hamburger) and suppose the Japanese Big Mac is exactly the same product as the U.S. Big Mac.

which is exactly the same as the dollar price of a Big Mac in the U.S. The notion of **purchasing power parity (PPP)** is the macroeconomic analog of the idea this example illustrates. PPP states that identical baskets of goods in different countries have the same price level when converted into a common currency. To extend our example, if the average basket of goods consumed by U.S. consumers is the same as the average basket of goods consumed by Japanese consumers, then PPP dictates that the price levels of the two economies are the same once converted into a common currency.[122]

Recall our notation: P_t^* denotes the foreign country price level, P_t denotes the domestic country price level, and E_t denotes the nominal exchange rate in units of domestic currency per one unit of foreign currency. Using this notation, PPP is the condition[123]

$$P_t = E_t P_t^* . \tag{15}$$

Expression (15) is basically the algebraic definition of PPP. We will assume that PPP always holds in our model economy.

An important question that should naturally arise is: does PPP hold in reality? The answer, as the answer to most questions about the validity of assumptions in economics, is not exactly. In fact, PPP does not hold between any two given countries at every point in time (i.e., every year). However, data suggest that PPP does seem to hold in the long run. That is, if we take averages of price levels and exchange rates over several or many years, the above condition (15) is often met. PPP is thus a long-run phenomenon. We are making the stronger assumption that PPP is also a short-run phenomenon because we will assume that it holds at every time period t.

Finally, we will make one further auxiliary assumption associated with PPP. This second assumption is simply to make our subsequent mathematical analysis simpler, and none of the general economic results we will derive depend on it. We will assume that the foreign country price level is constant and equal to one in every period. That is, $P_t^* = 1$ in every period. Imposing this assumption on the PPP condition above gives us

$$P_t = E_t , \tag{16}$$

which states that the domestic price level equals the nominal exchange rate in every period t. Note that if the domestic price level rises for any reason, it must be

[122] The assumption that U.S. consumers and Japanese consumers consume exactly the same basket of goods is obviously a simplification. For example, it's likely that a good such as sushi is a more important component of the Japanese basket of goods than of the U.S. basket of goods. But to the extent that Japanese consumers and U.S. consumers generally consume the same types of things (i.e., food, TV's, cars, etc.), it's perhaps not such a bad approximation to reality.

[123] Convince yourself that this expression is essentially what we used in our Big Mac example.

accompanied by a rise in E_t. A rise in E_t means that the domestic currency becomes weaker relative to the foreign currency because it now takes more units of domestic currency to purchase one unit of the foreign currency. Such a weakening of one currency versus another is called a **depreciation.** Thus, again referring to expression (16), we can conclude that domestic price inflation implies, and is implied by, depreciation of the exchange rate. Similarly, nominal price deflation (a fall in the domestic price level) implies, and is implied by, **appreciation** of the exchange rate, which is a strengthening of the domestic currency versus the foreign currency.

Finally, notice that using condition (16) in the money demand function (14), we can write the money demand function as

$$\frac{M_t}{E_t} = \phi\left(c_t, i_t\right). \tag{17}$$

Interest Parity Condition

The third building block of our model is the interest parity condition. To build this element of the model, we must introduce a concept known as **arbitrage.** When investors are faced with the option of investing in the nominal assets of different countries, not surprisingly, all other things equal, they will invest in those country's assets that yield the highest returns. Specifically, they will invest in those country's assets that yield the highest real, as opposed to nominal, returns, even though assets typically have only nominal returns associated with them. As a result of the highly competitive nature of global financial markets,[124] the real returns of different countries' assets are equalized.[125] **This does not mean, however, that real interest rates across countries are equalized.** In fact, the condition that describes this equalization of real returns is more subtle, as we now discuss.

Different countries' assets are denominated in different currencies. U.S. assets are denominated in dollars and Japanese assets are denominated in yen. Because of the different currency units associated with different countries' assets, comparing their relative attractiveness requires **converting their returns into the same currency units.** For example, a U.S. investor presumably cares about the total dollar return on his investment regardless of whether he invests in the U.S. or Japan.

[124] In our study of New Keynesian Economics, we made the point that perhaps product markets are imperfectly competitive. The arena in which perfect competition **does** seem to be a very good approximation is financial markets, and this extends to international financial markets. Financial markets are in general highly competitive.

[125] Recall from basic microeconomics that in perfect competition, firms do not set prices, but rather simply take the market price as given. Because of this, the price of each firm's output is the same.

Denote by i_t the domestic (in this example, the U.S.) nominal interest rate, i_t^* the foreign (in this example, Japan) nominal interest rate, r_t^* the foreign real interest rate, and π_t^* the foreign inflation rate. If a U.S. investor invests $1 in a U.S. bond, after the bond matures (at time $t+1$ say) he clearly will get back $1+i_t$ dollars. Consider his thought process about investing $1 in a Japanese bond, however. First, he would have to convert that $1 into yen at time t, which he can do at the exchange rate E_t dollars per yen. After the currency exchange he has $1/E_t$ yen which he can invest in the Japanese asset. If he holds the Japanese bond until maturity (also in period $t+1$), he will earn interest so that at maturity he will have $\left(1+i_t^*\right)\left(1/E_t\right)$ yen. Because he lives in the U.S., however, he needs to convert these yen back into dollars, which at time $t+1$ he can do at the exchange rate E_{t+1} dollars per yen. Completing this final transaction would leave him with a total of $\left(1+i_t^*\right)\left(E_{t+1}/E_t\right)$ dollars. **However –** the exchange rate in the future is obviously not known. All the investor can base his decision on is what he expects the future exchange rate to be. Denote this expectation of the future exchange rate by E_{t+1}^e. He thus expects that by investing $1 in a Japanese bond he will get back a total of $\left(1+i_t\right)\left(E_{t+1}^e/E_t\right)$ in the future.

If arbitrage holds, as is generally the case in global financial markets, then the expected returns of investing in the U.S. should be the same as investing in Japan. From the above analysis, this implies

$$1+i_t=\left(1+i_t^*\right)\frac{E_{t+1}^e}{E_t}, \tag{18}$$

which is known as the **interest parity condition.** Because of our earlier assumption that the foreign price level is always $P_t^*=1$, foreign inflation is always $\pi_t^*=0$. This implies, by the Fisher Equation, that the foreign nominal interest rate equals the foreign real interest rate in every period of the economy, $i_t^*=r_t^*$. Substituting this result into the interest parity condition allows us to rewrite it as

$$1+i_t=\left(1+r_t^*\right)\frac{E_{t+1}^e}{E_t}. \tag{19}$$

Again, you are probably questioning how accurate interest rate parity is in reality. Because of the highly-competitive nature of global financial markets, it actually is a very good approximation. Most deviations which do occur from interest rate parity[126] last for only a short time. So we will take in our model that interest rate parity holds all the time.

Government Budget Constraint

Finally, we describe the most important building block of our model, the government budget constraint. This budget constraint is not a lifetime budget constraint, but rather a period-by-period budget constraint.

In each period t, the government has three sources of income: nominal tax revenues T_t, money creation $M_t - M_{t-1}$, and interest earnings on foreign reserves. Foreign reserves are foreign countries' assets that a central bank holds for the purposes of official international financial transactions. Foreign reserves is a stock variable, similar to the net assets of the representative consumer that we encountered in our study of the consumption-savings model. Denote by B_t^G the foreign reserves held by the central bank at the end of period t. Foreign reserves are usually not held in the form of hard currency but rather in the form of government bonds. For example, Argentina's central bank's reserves of dollars are not held as U.S. currency but instead as U.S. bonds. Thus, foreign reserves pay interest. We will denominate interest earnings on foreign reserves in terms of domestic currency. To do this, we need three pieces: the nominal exchange rate, the foreign interest rate, and foreign reserve holdings at the end of the previous period. In period t, interest earnings on foreign reserves are then given by $E_t i_t^* B_{t-1}^G$. Again because of our assumption of zero foreign inflation, we can replace i_t^* by r_t^* in this last expression. Thus we have algebraic expressions for the three sources of government revenue in each period t. The two expenditure items for the government are nominal government purchases G_t and additions (or subtractions) to its holdings of foreign reserves, which is represented by $E_t\left(B_t^G - B_{t-1}^G\right)$. We can finally write the government budget constraint,

$$E_t\left(B_t^G - B_{t-1}^G\right) + G_t = T_t + \left(M_t - M_{t-1}\right) + E_t r_t^* B_{t-1}^G \tag{20}$$

After a couple of algebraic manipulations, we can rewrite this expression in the following useful way,

$$B_t^G - B_{t-1}^G = \frac{M_t - M_{t-1}}{P_t} - \left[\frac{G_t}{P_t} - \frac{T_t}{P_t} - r_t^* B_{t-1}^G\right]. \tag{21}$$

[126] Deviations occur because of activities such as currency trading and bond trading by investors and financial institutions.

In this manipulation, we have used the fact that the price level equals the nominal exchange rate. The left hand side of expression (21) is the change in foreign reserve holdings during period t. The first term on the right hand side of expression (21) is real government seignorage revenue, which is the government's revenue from money creation (in real terms because it is divided by the nominal price level). The second term on the right hand side of (21) is the difference between government expenditure and income from the collection of taxes and the receipt of interest payments on foreign reserve holdings. This term is called the **real secondary deficit**, and we will denote it by DEF_t,

$$DEF_t = \frac{G_t}{P_t} - \frac{T_t}{P_t} - r_t^* B_{t-1}^G . \tag{22}$$

Using this definition and the fact that $P_t = E_t$ allows us to write expression (21) as

$$B_t^G - B_{t-1}^G = \frac{M_t - M_{t-1}}{E_t} - DEF_t . \tag{23}$$

This expression makes clear that a fiscal deficit $\left(DEF_t > 0\right)$ must be associated with money creation $\left(M_t - M_{t-1} > 0\right)$ or with a decline in the government's foreign reserves $\left(B_t^G - B_{t-1}^G < 0\right)$, or both. Expression (23) will be the workhorse of our analysis of fixed exchange rate systems, to which we now turn.

Analyzing a Fixed Exchange Rate System

We now use the model we have just built to analyze the interaction of a fixed exchange rate system with fiscal policy.[127] To further focus our attention on the most important issues, we add one more assumption, that the foreign real interest rate is constant in every time period. Algebraically, $r_t^* = \bar{r}^*$, where $\bar{r}^*$ is simply some constant.

Suppose a country is currently maintaining a fixed nominal exchange rate vis-à-vis a foreign country, which means that E_t is a constant. Even more specifically, the public expects the exchange rate to always be constant. Let E with no subscript denote this constant value of the nominal exchange rate. Because the public expects the exchange rate to continue to be pegged, $E_{t+1}^e = E_t = E$, meaning that the interest parity condition

[127] The analysis of a floating exchange rate system using this model is more complicated, and we leave this topic for a more advanced course in International Economics.

(19) yields $i_t = \bar{r}^*$. In words, if a fixed exchange rate system is in place and is expected to remain in place, the domestic nominal interest rate equals the constant foreign real interest rate.[128]

Next, insert this constant value for the domestic nominal interest rate into the money demand function. With a constant nominal interest rate and consumption constant at $c_t = \bar{c}$ every period, real money demand M_t / P_t must be a constant every period. But the domestic price level P_t is itself a constant during the fixed exchange rate system because $P_t = E_t$. This implies that nominal money M_t must also be constant – that is, $M_t = M_{t-1}$ always. With constant nominal money in the economy, the government budget constraint (23) shows that seignorage revenue is zero. This is the first important result we have derived in this model: **under a fixed exchange rate system, the government earns zero seignorage revenue because the money supply of the economy must be constant.** The government is unable to print money under a fixed exchange rate system. Recall from our study of domestic macroeconomics that increasing the money supply leads to a short-term boost to GDP but leads in the long-run to inflation. Under a fixed exchange rate system, the central bank loses this channel of boosting GDP. But the potential benefit is that by tying their hands, the central bank avoids creating inflation in the economy.

Now, because seignorage revenue is zero, the government budget constraint becomes

$$B_t^G - B_{t-1}^G = -DEF_t \,. \tag{24}$$

This expression shows that under a fixed exchange rate system, a fiscal deficit necessarily implies a loss of foreign reserves. As a simple example, suppose a country has a fixed exchange rate system in place and the government has $20 billion of U.S. foreign reserves at the end of period $t-1$ (the year 1999, say). During period t, the exchange rate peg continues in force, meaning seignorage revenue is zero. The government simultaneously runs a real secondary deficit, so that $DEF > 0$ in period t. By expression (24), this necessarily means that foreign reserves at the end of period t are smaller than foreign reserves at the end of period $t-1$. Essentially, the government had to use some of its stock of foreign reserves to pay for its deficit.

The Collapse of a Fixed Exchange Rate System

The natural lower limit on foreign reserves is zero. When a country runs out of foreign reserves ($B^G = 0$), that's it.[129] Thus, if a country is running a fixed exchange rate system

[128] More generally, it equals the foreign **nominal** interest rate. But we have $i_t = r_t$ always here in our model.

simultaneous with a real secondary deficit, eventually foreign reserves will be completely drained. Once the country runs out of foreign reserves, it has two options. One option is to reverse its secondary deficit and preserve its fixed exchange rate. The other option, if it doesn't have the political will to reverse the deficit, is to abandon the fixed exchange rate by beginning to print money. We briefly analyze these two alternatives using the model we have laid out.

When foreign reserves run out (and they cannot go negative), no further depletion of foreign reserves can occur. Suppose at the end of time $t-1$, foreign reserves have been depleted so $B_{t-1}^{G}=0$. The government must decide how to manage its finances in period t. The government budget constraint (23) must hold in period t (as it must in every period!) – that is, the government must somehow make it hold. If the government wishes to maintain the fixed exchange rate in period t, that automatically means the money supply will not change, so that $M_t = M_{t-1}$. Seignorage revenue is zero. With no more foreign reserves to use, expression (23) shows that DEF cannot be strictly negative in period t. If the government somehow balances its budget so that $DEF=0$, foreign reserves continue to be zero but at least the situation is sustainable. Alternatively, the government may somehow find the political will to turn the deficit into a surplus, $DEF>0$ in period t, which would mean that foreign reserves would once again begin to accumulate.

If the government is unable to reverse the deficit, however, the only available recourse is to devalue the currency (that is, purposely weaken the domestic currency versus the foreign currency). The devaluation may be either anticipated or unanticipated by the public. Suppose first that the devaluation is unanticipated, meaning that $E_t^e = E_{t-1}$, but the actual exchange rate turns out to be $E_t > E_{t-1}$. By instituting a one-time surprise devaluation, the central bank is able to raise seignorage revenue in period t. To see this, first note that a devaluation in period t necessarily means $P_t > P_{t-1}$. If the public expects there to never be a devaluation again, then $c_t = \overline{c}$ continues to hold and $i_t = \overline{r}^*$ continues to hold (the latter follows from the interest parity condition). Thus, real money demand remains unchanged in period t and the entire rise of the price level must be met with increased nominal money in period t. The fact that $M_t > M_{t-1}$ means seignorage revenue becomes strictly positive in period t. The amount of seignorage revenue needed is at least that required to pay off the deficit in period t, because the left hand side of expression (23) cannot be negative in period t.

[129] Actually, this is not technically true. It is possible for a country to have "negative" foreign reserves, through arrangements known as central banks' special drawing rights. Essentially, such arrangements allow central banks to borrow foreign reserves from each other. We leave this more technical aspect to a more advanced course on international economics and simply suppose that the lower limit on foreign reserves is zero.

On the other hand, the devaluation could have been anticipated by the public. An anticipated devaluation manifests itself **in period** $t-1$ as $E_t^e > E_{t-1}$. By the interest parity condition, the anticipated devaluation means that the domestic nominal interest rate **in period** $t-1$ rises compared to its usual level. The money demand function then shows us that real money demand **in period** $t-1$ falls relative to that in period $t-2$. The price level in period $t-1$ has not yet risen, which necessarily implies that nominal money in period $t-1$ is **smaller** than nominal money in period $t-2$. With $M_{t-1} < M_{t-2}$, seignorage revenue is **negative** in the period preceding the fall of the nominal exchange rate peg. Inspecting the government budget constraint shows that the negative seignorage revenue coupled with the fiscal deficit means foreign reserves drain even more quickly in period $t-1$ than if the devaluation were not anticipated. This situation is termed a **balance of payments crisis.** What is going on is that domestic residents, fearing an impending devaluation of their currency, rush to the central bank to exchange their domestic currency for foreign currency. When they do so, the exchange rate is still pegged. In order to honor the commitment to the fixed exchange rate, the central bank must cash in some of its stockpile of foreign reserves to give its residents foreign currency. This cashing in of foreign reserves represents a second drain in addition to the fiscal deficit. The end result is that the fixed exchange rate must be abandoned even sooner than period $t+1$ (call it "period t and a half" if you like).

We consider below how to model a balance of payments crisis in more detail. First, however, we consider the equilibrium of the model we have set up under a floating exchange rate system.

Equilibrium Under a Floating Exchange Rate Regime[130]

Under a floating exchange rate regime, the nominal exchange rate is market-determined (that is, set by the forces of supply and demand). In order to consider nominal exchange rate determination in such an environment, we must take a stand on what type of monetary policy the central bank follows. For simplicity, and because it serves to illustrate the main issues, we will assume the central bank simply determines how much money is in circulation each period – that is, we will assume the central bank follows a money growth rule.

Specifically, suppose the central bank expands the money supply at the constant rate μ between any two consecutive periods, so that

$$M_{t+1} = (1+\mu)M_t;$$

[130] This and the subsequent sections are adapted from *International Macroeconomics* by Stephanie Schmitt-Grohe and Martin Uribe.

thus, for example, if the central bank expands the money supply by five percent between any two periods t and $t+1$, we would have $\mu = 0.05$. Our goal is to determine how the nominal exchange rate, the price level, real balances, and the domestic nominal interest rate evolve over time when the central bank is following this money supply rule. To do this, we will guess, and then verify, that in equilibrium the nominal exchange rate depreciates at the rate μ. Thus, we guess that the nominal exchange rate evolves over time according to

$$E_{t+1} = (1+\mu)E_t .$$

Because in our model PPP holds and the foreign price level equals one in every period, we know that $P_t = E_t$ in every period; this means that the domestic price level evolves over time according to

$$P_{t+1} = (1+\mu)P_t .$$

This expression says that, given our guess for how the nominal exchange rate moves over time, the rate of inflation equals the rate of money growth chosen by the central bank. There is a lot of empirical evidence across countries that shows that on average a country's inflation rate is very closely related to the money growth rate; furthermore, evidence also shows that, for currencies that have floating exchange rates, the rate of depreciation of the nominal exchange rate is also very closely related to the money growth rate.[131] Our model formalizes these relationships.

Next, to determine the domestic nominal interest rate i_t, use the interest parity condition, to determine

$$1+i_t = (1+r^*)\frac{E^e_{t+1}}{E_t} = (1+r^*)\frac{(1+\mu)E_t}{E_t} = (1+r^*)(1+\mu) .$$

We can solve this expression for the domestic nominal interest rate,

$$i_t = (1+r^*)(1+\mu) - 1 ,$$

which shows that i_t depends on μ. Let's compactly denote this dependence by writing $i_t = i(\mu)$, where the functional notation $i(\mu)$ on the right-hand-side abstractly captures the relationship. If $\mu = 0$, clearly $i_t = r^*$. If $\mu > 0$, the domestic nominal interest rate

[131] The observation that inflation and money growth are highly correlated is the foundation of the *quantity theory of money,* a theory of how monetary policy affects inflation that owes much of its original articulation to the late Nobel prize-winning economist Milton Friedman.

exceeds the foreign interest rate. The economic intuition behind this is that expansion of the domestic money supply makes it less valuable; the resulting depreciation of the domestic currency requires that bonds denominated in domestic currency must carry a higher interest return in order to induce investors to purchase it. Note that the function $i(\mu)$ is increasing in μ; the higher is the domestic money growth rate, the higher is the domestic nominal interest rate. Mathematically, $i'(\mu) > 0$.

Substituting the relationship $i_t = i(\mu)$ into the money demand function yields

$$\frac{M_t}{E_t} = \phi(\bar{c}, i(\mu)).$$

Consumption $\bar{c}$ is as before constant over time; with a time-invariant money growth rate μ, the nominal interest rate is also constant over time. Hence, the right-hand-side of the previous expression is constant over time. For the money market to be in equilibrium, the left-hand-side of the previous expression must therefore also be constant over time. We already know – because of the assumed money growth rule – that M grows at the rate μ every period. The only way the left-hand-side can be constant is for E to grow at the rate μ every period, as well. This is indeed true under our initial guess that $E_{t+1} = (1+\mu)E_t$. Thus, we have verified our original guess and have determined how the nominal exchange rate, the domestic price level, and the domestic nominal interest rate all evolve over time if the central bank is following a constant money growth rule with a flexible nominal exchange rate.

Balance of Payments (BOP) Crises

A balance of payments (BOP) crisis is a situation in which the government is unable or unwilling to meets its international financial obligations. These difficulties may manifest themselves in a variety of ways, such as the failure to honor the domestic and/or foreign public debt or the suspension of currency convertibility.

Often, the root cause of a BOP crisis is an unsustainable mix of fiscal policy and monetary policy. A classic example of such an unsustainable policy mix is a situation in which the government pegs the nominal exchange rate at a level stronger than under the floating rate and the government simultaneously runs a fiscal deficit. As we saw earlier, under a fixed exchange rate system, the government must finance any fiscal deficit by running down its foreign reserves because it cannot change the nominal money supply – that is, when the nominal exchange rate is pegged, $\mu = 0$. As we alluded to above, however, in the days or weeks immediately **before** a peg collapses, the equilibrium money supply **shrinks** because holders of domestic currency, fearing the coming devaluation of their nominal assets, rush to rid themselves of their domestic currency holdings. In this rest of this section, we will study in detail the most popular model used to study the dynamics of a collapse of a fixed exchange rate system.

Consider a country that is running a constant fiscal deficit $DEF > 0$ every period. Also suppose that the government has fixed the nominal exchange rate at E units of domestic currency per unit of foreign currency. The government has positive foreign reserves, but its foreign reserves can never go below zero. Based on our earlier discussion, it is thus clear that as long as the fixed exchange rate is in place, the fiscal deficit causes a continuous drain on foreign reserves, which at some point will be completely depleted. Put differently, if the fiscal deficit is no eliminated, at some point the government will be forced to abandon the currency peg and start printing money in order to cover the deficit.

Let T denote the period in which, as a result of having run out of foreign reserves, the government abandons the peg and begins printing money to pay for its fiscal deficit. The dynamics of the currency crisis can be characterized by three distinct phases:

1. Pre-collapse phase: during this phase, which lasts until (and including) period $T-2$, the currency peg is in place
2. BOP crisis: takes place in period $T-1$ and is the period in which the domestic central bank faces a run against the domestic currency, resulting in massive losses of foreign reserves
3. Post-collapse phase: the period from T onwards; in this phase, the nominal exchange rate floats freely and the central bank expands the money supply at a rate consistent with paying for the fiscal deficit.

Pre-Collapse Phase

From some point in the past through period $T-2$, the nominal exchange rate is pegged, so the variables of interest behave just as described earlier. To recap, the nominal exchange rate is constant and equal to E, that is, $E_t = E$ for $t = ..., T-4, T-3, T-2$. By PPP and our assumption $P_t^* = 1$, the domestic price level is also constant over time and equal to E ($P_t = E$ for $t = ..., T-4, T-3, T-2$). Because the exchange rate is fixed, the devaluation rate $(E_t - E_{t-1})/E_{t-1}$ is equal to zero. The nominal interest rate i_t, which by the interest parity condition satisfies $1+i_t = (1+r^*)E_{t+1}^e / E_t$, is equal to r^*. Note in particular that the nominal interest rate in period $T-2$ is equal to r^* because the fixed exchange rate is still in place in period $T-1$ -- thus, $i_t = r^*$ for $t = ..., T-4, T-3, T-2$.

Also as discussed earlier, by pegging the nominal exchange rate, the government relinquishes the ability to change the nominal money supply. Also as before, the fact that seignorage revenue equals zero under the peg means that the dynamics of foreign reserves are governed by

$$B_t^G - B_{t-1}^G = -DEF$$

for $t = ..., T-4, T-3, T-2$. The central bank loses DEF units of foreign reserves every period during the pre-collapse phase. The continuous loss of foreign reserves in combination with the zero lower bound on the central bank's foreign reserve holdings makes it clear that a currency peg is unsustainable in the long run in the presence of persistent fiscal deficits.

Post-Collapse Phase

At the beginning of period T, the government has zero foreign reserves ($B_T^G = 0$). Given that B^G cannot go below zero and that government cannot (or does not) eliminate the fiscal deficit, it follows than in period T the monetary authority is forced to abandon the currency peg and to print money in order to finance the fiscal deficit. Thus, in the post-collapse phase, the government lets the nominal exchange rate float. Consequently, the behavior of all variables is as we discussed above when we studied equilibrium under a floating exchange rate. In particular, the central bank must choose a money growth rate μ in order to generate enough seignorage revenue to finance the fiscal deficit, implying, just as above, that the nominal exchange rate depreciates each period at the rate μ, the domestic price level grows each period at the rate μ (that is, the domestic inflation rate is μ), and the domestic nominal interest is higher than the foreign interest rate r^* by an amount that depends on μ.

Let's compare the economy's dynamics pre- and post-crisis. The first thing to note is that with the demise of the fixed exchange rate, price stability disappears as inflation sets in. In the pre-collapse phase, the rate of money growth, the rate of devaluation, and the rate of inflation are all zero. In contrast, in the post-collapse phase, these variables are all constant at the positive rate μ. Second, the sources used to finance the government's fiscal deficit are very different in the two phases. In the pre-crisis phase, the deficit is financed entirely with foreign reserves. As a result, foreign reserves display a steady decline during the pre-collapse phase. On the other hand, in the post-collapse phase the fiscal deficit is financed through seignorage income and foreign reserves are constant (and equal to zero in our example). Finally, in the post-collapse phase, real money balances are lower than in the pre-collapse phase because the domestic nominal interest rate is higher ($i_t > r^*$ during the post-collapse phase, while $i_t = r^*$ during the pre-collapse phase).

BOP Crisis: Period $T-1$

In period $T-1$, the fixed exchange rate has not yet collapsed. Thus, the nominal exchange rate and the domestic price level are still equal to their values during the pre-collapse phase: $E_{T-1} = E$ and $P_{T-1} = E$. However, the important difference is that the domestic nominal interest rate is no longer equal to r^*. In period $T-1$, the public expects a depreciation of the domestic currency to occur in period T.

Supposing markets' expectations are rational (that is, they will, on average, be correct), the expected rate of depreciation between period $T-1$ and period T is μ: the same μ we have already been considering. That is,

$$\frac{E_T^e - E_{T-1}}{E_{T-1}} = \mu .$$

Therefore, the nominal interest rate in period $T-1$ jumps up to its post-crisis level, $i_{T-1} = (1+r^*)(1+\mu) - 1$ **even though the depreciation hasn't yet happened.**

As a result of the increase in the nominal interest rate, real balances fall in period $T-1$ to their post-collapse level; that is,

$$\frac{M_{T-1}}{E} = \phi(\bar{c}, i(\mu)) .$$

Because the nominal exchange rate in period $T-1$ is still E, the fall in **real** money balances in period $T-1$ (the right-hand-side of the last expression) must be brought about entirely through a fall in **nominal** money balances. In period $T-1$, fearing the imminent collapse of the domestic currency, the public runs to the central bank to exchange domestic currency for foreign currency. In period $T-1$, the government still honors its commitment to exchange currency at the rate E, so it must dip into its foreign reserves (which, after all, represent claims to **foreign** currency) in order to do so. Thus, in period $T-1$, foreign reserves at the central bank fall by **more than** DEF **.**

To see this more formally, the government budget constraint in period $T-1$ tells us

$$\begin{aligned} B_{T-1}^G - B_{T-2}^G &= \frac{M_{T-1} - M_{T-2}}{E} - DEF \\ &= \phi(\bar{c}, i(\mu)) - \phi(\bar{c}, r^*) - DEF \\ &< -DEF \end{aligned}$$

The second equality follows from the fact that $M_{T-1}/E = \phi(\bar{c}, i(\mu))$ and $M_{T-2}/E = \phi(\bar{c}, r^*)$. The inequality follows from the fact that $i(\mu) = (1+r^*)(1+\mu) - 1 > r^*$ and the fact that the money demand function is decreasing in the nominal interest rate: the higher is the nominal interest rate, the lower is the value of $\phi(.)$ (for a given value of $\bar{c}$).

The government budget in period $T-1$ formalizes why the demise of currency pegs is typically preceded by a speculative run against the domestic currency and large losses of foreign reserves by the central bank: even though the exchange rate is still fixed in period $T-1$, the nominal interest rate rises in anticipation of a devaluation in period T, which in turn causes a contraction of the demand for real money balances in period $T-1$. Because in period $T-1$ the domestic currency is still convertible at the fixed rate E, the central bank must absorb the entire decline in the real demand for money by surrendering foreign reserves, which accelerates the onset of the crisis.