Chapter 18
Economic Efficiency

The benchmark for any notion of optimal policy, be it optimal monetary policy or optimal fiscal policy, is the economically efficient outcome. Once we know what the efficient outcome is for any economy, we can ask "how good" the optimal policy is (note that optimal policy need not achieve economic efficiency – we will have much more to say about this later). In a representative agent context, there is one essential condition describing economic efficiency: social marginal rates of substitution are equated to their respective social marginal rates of transformation.[147] We already know what a marginal rate of substitution (MRS) is: it is a measure of the maximal willingness of a consumer to trade **consumption** of one good for **consumption** of one more unit of another good. Mathematically, the MRS is the ratio of marginal utilities of two distinct goods.[148] The MRS is an aspect of the demand side of the economy. The marginal rate of transformation (MRT) is an analogous concept from the production side (firm side) of the economy: it measures how much **production** of one good must be given up for **production** of one more unit of another good. Very simply put, the economy is said to be operating efficiently if and only if the consumers' MRS between any (and *all*) pairs of goods is equal to the MRT between those goods. MRS is a statement about consumers' preferences: indeed, because it is the ratio of marginal utilities between a pair of goods, clearly it is related to consumer preferences (utility). MRT is a statement about the production technology of the economy.

To illustrate further the notion of economic efficiency, we proceed in two simple steps. First, we use the simple one-period consumption-leisure model to understand economic efficiency in a static (non-dynamic) setting. Then, we use the simple two-period consumption-savings model to understand the dynamic analog. Before proceeding in these two steps, we introduce a device that is useful for determining efficient allocations.

The Social Planner

It is quite easy to characterize, in terms of the solution to an optimization problem, economically-efficient outcomes. To do so, we introduce the concept of a **Social**

[147] We qualify this statement with "in a representative agent context" because if we consider heterogeneous agents (both heterogeneous consumers as well as heterogenous firms), there are two additional conditions that are components of the definition of economic efficiency: marginal rates of substitution between any two goods are equated across all consumers; and marginal rates of transformations between any two inputs are equated across all firms. Clearly (and trivially), with a representative (single) consumer, marginal rates of substitution are equated across "all" consumers, and with a representative (single) firm, marginal rates of transformation are equated across "all" firms.
[148] Review these basic ideas if you need to.

Planner. The Social Planner is an all-knowing "individual" or "institution" that is able to perfectly control and allocate the resources of an economy.[149] The Social Planner is a "dictator," but a benevolent one. The Social Planner is able to simply take, by order or decree, any resources he needs from any parties he sees fit. The Social Planner does not need to resort to "taxes" in order to achieve this. Thus, we should not think of the Social Planner as a "government" in the sense we typically have in mind in capitalist societies. Rather, the Social Planner is able to directly command how production in the economy occurs and how the fruits of that production get distributed to consumers. Most importantly, the Social Planner does **not** have any need for markets or prices (or, as alluded to already, taxes). The Social Planner does not care if "markets" (and hence market prices) exist – because he *directly and independently* chooses what happens, markets are irrelevant for the Social Planner.[150]

Economic Efficiency in the Static Consumption-Leisure Model

So what does the Social Planner do? Consider the one-period consumption-leisure model we have studied. The representative consumer has preferences over consumption and leisure, described by the utility function $u(c,1-n)$, where, as usual, c denotes consumption, and $1-n$ denotes leisure (with n being labor and the total time available is normalized to one; i.e., if l denotes leisure $n+l=1$ here).

Suppose the production technology is simple: linear in labor. One unit of labor always yields one unit of output, with no diminishing marginal product in labor. We have used this simple production technology for illustrative purposes before. The **resource constraint** of the economy is thus given by

$$c=n. \tag{1.61}$$

The Social Planner understands the economy's resource frontier – it embodies the production technology of the economy. The Social Planner understands consumers' utility functions – it embodies consumers' preferences. The optimization problem of the benevolent Social Planner is to *maximize (one-period) consumer utility with respect to the resource frontier*. Pay close attention to the economic content of this maximization statement: there is an institution in the economy (the Social Planner) that is choosing how to satisfy demand in the economy (which consumers' utility functions give rise to) with *direct* regard for the supply constraints (which the resource constraint/production function gives rise to) of the economy. This is something that does not occur in modern capitalist economies: consumers maximize their utility *with regard to market prices and their budget constraints,* and, separately, firms maximize their profits *with regard to market prices and their production technologies.* Note the intermediary in market-based

[149] The Social Planner is also often referred to as a "central planner."
[150] This will not be true of another type of planner – the Ramsey Planner – that we encounter soon.

transactions: market prices mediate the exchange between suppliers and demanders. In contrast, the Social Planner ignores markets, examines consumers' preferences, examines firm's production technologies, and simply *commands* both consumers and firms to do what he decides.

The formal maximization, then, that the Social Planner performs is to choose c and n to maximize $u(c,1-n)$ subject to the resource constraint (1.61). Clearly, we can simplify the problem by simply inserting (1.61) into the utility function, avoiding the need to set up a Lagrangian.

Doing so, the representative consumer's one-period utility function is simply $u(c,1-c)$. The Social Planner chooses c to maximize this; the first-order condition with respect to c (which by now should be trivial to compute) is simply

$$u_1(c,1-c)+u_2(c,1-c)(-1)=0. \tag{1.62}$$

Note the "-1" term in the second term on the left-hand-side, which arises from using the chain rule to differentiate with respect to c. Clearly, (1.62) states that if the Social Planner gets to choose c (and hence n) for this economy, we would choose it in such a way that

$$\frac{u_1(c,1-c)}{u_2(c,1-c)}=1. \tag{1.63}$$

We have seen this condition before: it was part of the characterization of optimal monetary policy that we studied. With our brief review of the notion of economic efficiency and the idea of the Social Planner, we can now understand a bit better the (deep) idea behind this condition.

The left-hand-side of (1.63) is, as we know by now, the MRS between consumption and leisure (because, as usual, $u_1(.)$ is the marginal utility of consumption and $u_2(.)$ is the marginal utility of leisure). What we did not emphasize before was the right-hand-side of (1.63): in the simple economy we are studying here, it is the *economy's MRT between consumption and leisure.*

To understand why "1" is the economy's MRT between consumption and leisure in this example, return to the resource frontier shown in (1.61). If as a whole the economy takes – "produces" – one *less* unit of leisure, clearly it works one *more* unit of time. But the resource constraint tells us that that means there is one *more* unit of consumption produced in the economy. Hence, in order for the economy to produce one more unit of consumption, it must "produce" one less unit of leisure – but this means, by the definition of MRT, that the MRT between consumption and leisure is one.

Condition (1.63) thus states that if the Social Planner makes choices for the economy, he makes sure that the MRS between consumption and leisure is equated to the MRT between consumption and leisure – but this means, by the definition of economic efficiency, that the Social Planner's choice is economically efficient.[151] Condition (1.63) characterizes economic efficiency along the consumption-leisure dimension. The importance of this efficiency condition is evident when studying optimal monetary policy (as we have already seen) and when studying optimal labor taxation (which we will soon see).

Economic Efficiency in the Two-Period Consumption-Savings Model

We just studied economic efficiency along the (intratemporal) consumption-leisure margin. Let's turn to economic efficiency along the (intertemporal) consumption-savings margin. To think about this issue, let's return to our simple two-period model, before we started to think about monetary issues.

The lifetime (here, of course, lifetime means only two periods) utility function of the representative consumer is

$$v(c_1)+\beta v(c_2), \tag{1.64}$$

where $v(.)$ is the single-period utility function and $\beta\in(0,1)$ is the subjective discount factor.[152] Note that we are assuming additive-separability across time – this, of course, is nothing new; we have been doing this all along in our infinite-period models whenever we write $u(c_t)+\beta u(c_{t+1})+\beta^2 u(c_{t+2})+\beta^3 u(c_{t+3})+\dots$.

The production technology of the economy is the following. In each period, there is a *diminishing-returns* production technology that transforms capital into output, all of which is consumed. In period 1, the production function is $f(k_1)$, and in period 2 the production function is $f(k_2)$. The notational convention we are adopting is that k_1 is the capital used *in production in period 1* and which was *decided upon in period 0*; likewise, k_2 is the capital used *in production in period 2* and which was *decided upon in period 1.* Thus, even the Social Planner has to respect that fact that machines "take time" to build – the Social Planner cannot create machines by magic. If the Social Planner

[151] Return now to our study of optimal monetary policy. Was the benevolent central bank, through its control of the (steady-state) rate of money growth, able to achieve economic efficiency?

[152] We use the notation $v(.)$ only to avoid confusion with the notation $u(.)$ we used in the previous section. Of course, as always, the names of utility functions don't matter: we could just as well call the function $Bob(c)$ or anything else we care to.

wants there to be a certain quantity of machines (capital) available for use in period 2, he must plan for that in period 1 by appropriately choosing how many machines he wants available the next period.

To describe the resource frontier of the economy, we actually need to proceed period by period. In period 1, the resource frontier is

$$c_1 + k_2 - (1-\delta)k_1 = f(k_1), \tag{1.65}$$

and in period 2 the resource frontier is

$$c_2 + k_3 + (1-\delta)k_2 = f(k_2). \tag{1.66}$$

The term $\delta \in (0,1)$ is the depreciation rate of capital (that portion of the machines that get "used up" or "worn out" by the act of production), hence the term $k_2 - (1-\delta)k_1$ is gross investment in period 1, and $k_3 - (1-\delta)k_2$ is gross investment in period 2.

As in our earlier study of the two-period model, there is no reason ever for the economy to have positive assets left over at the end of period 2 (and it is not feasible to have negative assets left over at the end of period 2), which immediately tells us that $k_3 = 0$. By our "time to build" assumption regarding capital, at the beginning of period 1 (which is where our analysis is focused), it is too late for the Social Planner to alter k_1; remaining to be chosen, though, is k_2.

The Social Planner's objective is to maximize consumers' lifetime utility subject to the pair of resource constraints (1.65) and (1.66), which will yield the economically-efficient outcome in this two-period model. To proceed, let's first formulate the sequential, two-period Lagrangian for the Social Planner's problem,

$$\begin{aligned}
&v(c_1) + \beta v(c_2) \\
&+\lambda_1 \left[f(k_1) - c_1 - (k_2 - (1-\delta)k_1) \right] \\
&+\beta\lambda_2 \left[f(k_2) - c_2 - (k_3 - (1-\delta)k_2) \right]
\end{aligned}$$

where, as usual λ_1 is the multiplier on the period-1 resource constraint and λ_2 is the multiplier on the period-2 resource constraint. The maximization here is from the perspective of the very beginning of period 1, and the objects of choice are c_1, c_2, and k_2; the three first-order conditions are, respectively,

$$v'(c_1) - \lambda_1 = 0, \tag{1.67}$$

$$\beta v'(c_2) - \beta\lambda_2 = 0, \tag{1.68}$$

and

$$-\lambda_1 + \beta\lambda_2\left[f'(k_2) + 1 - \delta\right] = 0. \tag{1.69}$$

To emphasize again, by the time the beginning of period 1 arrives, k_1 has already been chosen, so there can be no first-order condition surrounding k_1; and $k_3 = 0$ simply must be, so we have no need for a first-order condition surrounding k_3.

Conditions (1.67), (1.68), and (1.69) describe the economically-efficient outcome in this two-period economy. We can rearrange these three conditions to emphasize what is happening along the consumption-savings (intertemporal) margin. Condition (1.67) tells us $\lambda_1 = v'(c_1)$; condition (1.68) tells us $\lambda_2 = v'(c_2)$. If we insert both of these in condition (1.69) and rearrange a bit, we have

$$\frac{v'(c_1)}{\beta v'(c_2)} = f'(k_2) + 1 - \delta. \tag{1.70}$$

The left-hand-side of (1.70) is nothing but the consumer's MRS between period-1 consumption and period-2 consumption (the β appears because it accounts for the consumer's impatience between the two-periods).

The right-hand-side of condition (1.70) is the economy's MRT between period-1 output and period-2 output. How can we understand why the term $f'(k_2) + 1 - \delta$ is the MRT? Recall that the MRT measures how much production of one good must be foregone in order to have production of one more unit of another good. In our simple two-period economy, there are only two goods being produced: period-1 consumption and period-2 consumption. Conduct the following thought experiment. Suppose the economy wants to forgo one unit of period-1 consumption – i.e., the economy needs to produce one less unit of period-1 output. Foregoing one unit of period-1 consumption means the economy (controlled, as it is here, by the Social Planner) can have one more unit of capital available for production in period 2. We can see this fact by examining the period-1 resource constraint, (1.65): recalling that k_1 is fixed, a one-unit reduction in c_1 means a one-unit rise in k_2. In yet other words, forgoing one unit of consumption in a given period means one extra unit of *savings* (which is what, at the economy-wide level, capital accumulation is) available for the next period. Next, we must consider what that one extra unit of k_2 implies for production in period 2. First off, a fraction δ of the one extra unit of k_2 will disappear in the form of depreciation. Thus, of the one extra unit of k_2, only $(1-\delta)$ of it will remain. However, the extra unit of k_2 also means a little extra production can take place in period 2. How much extra production is measured by the marginal product $f'(k_2)$ -- after all, the marginal product is *defined* as the extra production that results from a one-unit increase in input. So, on net, the total extra resources that are *available* to the economy *in period 2* as a result of the economy forgoing one unit of consumption in period 1 is $f'(k_2)+1-\delta$, and this extra $f'(k_2)+1-\delta$ is available for period-2 consumption.

Thus, the economy's MRT between period-1 consumption and period-2 consumption is $f'(k_2)+1-\delta$. Condition (1.70) tells us, as we know by now it should, that if the Social Planner makes choices for the two-period economy, he makes sure that the MRS between period-1 consumption and period-2 consumption is equated to the MRT between period-1 consumption and period-2 consumption. Condition (1.70) characterizes economic efficiency along the consumption-savings dimension. The importance of this efficiency condition is evident when studying the optimal tax rate on savings. An interesting point to note is that optimal *monetary* policy has nothing to do with the consumption-savings efficiency condition; even though *money* and *interest rates* are commonly thought of as being "intertemporal" features of the economy (that is, money and interest rates "link" together different periods), the basic consideration of optimal monetary policy only has to do with efficiency along the consumption-leisure (an *atemporal*, aka *intratemporal*) dimension, an idea that we saw in our first look at optimal monetary policy.