Chapter 20
Optimal Fiscal and Monetary Policy

We now proceed to study jointly optimal monetary and fiscal policy. The motivation behind this topic stems directly from observations regarding the consolidated government budget constraint. Specifically, a broad lesson emerging from our study of fiscal-monetary interactions is that *money creation* and thus inflation potentially helps the *fiscal authority* to pay for its government spending. Alternatively, a broad interpretation we made when we studied optimal monetary policy earlier was that steady-state inflation (more precisely, any steady-state deviation from the Friedman Rule) acted as a *tax* on consumers. At that stage, we did not note that a deviation from the Friedman Rule, acting as a "tax," potentially *raised revenue* for the government; now, with our notion of a consolidated budget constraint, we are in a position to understand this latter idea.

Here, the question that we take up is: if *both* monetary *and* fiscal policy are conducted optimally, what is the *optimal steady-state mix* of labor taxes and inflation needed to finance some fixed amount of government spending? Our approach to answering this question will hew very closely to the methods of analysis we have already developed in our separate looks at optimal monetary policy (without regard for fiscal policy) and optimal fiscal policy (without regard for monetary policy).

The model we use to answer this question mostly combines elements we have already seen. To overview the key elements of the model we will use to try to think about our main question, our model will:

- Feature an infinite number of periods
- Model money using the money-in-the-utility function (MIU) approach
- Feature labor income taxes as the only direct fiscal instrument (i.e., no consumption taxes and no taxes on savings)
- Feature a consolidated government budget constraint
- Feature a simple linear-in-labor production technology
- Focus on the steady state

Because by now most of these model elements are familiar to us, we will not spend much time developing the details of the basic model; rather we will spend most of our time analyzing the optimal policy problem and its solution.

Firms

The way in which we model firms is as we have often done: the representative firm simply hires labor each period in perfectly-competitive labor markets and sells its output. The production technology we assume here is also as simple as possible, linear in labor: $y_t = f(n_t) = n_t$. Firms' profits in period t (in nominal terms) are thus simply $P_t y_t - W_t n_t$, where the notation is standard: P_t is the nominal price of goods, W_t is the nominal wage, and n_t is the quantity of labor. When the firm is maximizing profits, we assume it takes as given both the nominal price P and the nominal wage W.[162] Substituting the linear production technology into the profit function and optimizing with respect to n_t (the only thing the firm decides here is how many units of labor to hire on a period-by-period basis) yields the firm first-order condition $P_t - W_t = 0$. If we define, as usual, the real wage as $w_t = W_t / P_t$, the result of firm profit-maximization is

$$w_t = 1 \tag{1.85}$$

Condition (1.25) is one of the equilibrium conditions of the simple model we are developing, and is the only one that arises from the firm (supply) side of the model.

Consumers

As mentioned above, we will model consumers using our money-in-the-utility function (MIU) specification. The representative consumer begins period t with nominal money holdings M_{t-1}, nominal bond holdings B_{t-1}, and stock (a real asset) holdings a_{t-1}. The period-t budget constraint of the consumer is

$$P_t c_t + P_t^b B_t + M_t + S_t a_t = (1-t_t)W_t n_t + M_{t-1} + B_{t-1} + (S_t + D_t)a_{t-1}, \tag{1.86}$$

where the notation again is as in the MIU model presented earlier: S_t is the nominal price of a unit of stock, D_t is the nominal dividend paid by each unit of stock, and P_t^b is the nominal price of a one-period nominal bond with face-value \$1. Because we continue to assume that all bonds are one-period bonds and the face value of each bond is $FV = 1$, we have that $P_t^b = \frac{1}{1+i_t}$ (which you should recall), where i_t is the net nominal interest rate on a nominal bond held from period t to period t+1. Note the term

[162] Nothing more than our usual assumption of price-taking behavior; here, price-taking describes the firm's behavior in both output markets and input markets.

$(1-t_t)W_t n_t$ in the budget constraint: it represents total after-tax labor income in period t. The consumer takes both the wage W_t and the tax rate t_t as given.[163]

Note the *absence* in the consumer budget constraint of the lump-sum amount of transfer from the government which was present in our earlier study of optimal monetary policy. This is one subtle but crucial difference in the model we are using here to study *jointly optimal* fiscal and monetary policy.

The present value of lifetime utility of the consumer is, as expected, given by

$$u\left(c_t, \frac{M_t}{P_t}, 1-n_t\right) + \beta u\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}, 1-n_{t+1}\right) + \beta^2 u\left(c_{t+2}, \frac{M_{t+2}}{P_{t+2}}, 1-n_{t+2}\right) + \dots, \quad (1.87)$$

in which each period's utility depends on consumption c, real money balances M/P, and leisure $1-n$ and, also as is standard by now, future utility is discounted by the factor β.

Setting up a sequential Lagrangian (with λ_t the multiplier on the consumer's time-t budget constraint),

$$\begin{aligned}
&u\left(c_t, \frac{M_t}{P_t}, 1-n_t\right) + \beta u\left(c_{t+1}, \frac{M_{t+1}}{P_{t+1}}, 1-n_{t+1}\right) + \beta^2 u\left(c_{t+2}, \frac{M_{t+2}}{P_{t+2}}, 1-n_{t+2}\right) + \dots \\
&+\lambda_t\left[(1-t_t)W_t n_t + M_{t-1} + B_{t-1} + (S_t + D_t)a_{t-1} - P_t c_t - P_t^b B_t - M_t - S_t a_t\right] \\
&+\beta\lambda_{t+1}\left[(1-t_{t+1})W_{t+1} n_{t+1} + M_t + B_t + (S_{t+1} + D_{t+1})a_t - P_{t+1} c_{t+1} - P_{t+1}^b B_{t+1} - M_{t+1} - S_{t+1} a_{t+1}\right] \\
&+\dots.
\end{aligned} \quad (1.88)$$

In period t, the consumer chooses $(c_t, n_t, M_t, B_t, a_t)$. Proceeding mechanically, the first-order-conditions with respect to each of these five choice variables, respectively, are:

$$u_1\left(c_t, \frac{M_t}{P_t}, 1-n_t\right) - \lambda_t P_t = 0 \quad (1.89)$$

$$-u_3\left(c_t, \frac{M_t}{P_t}, 1-n_t\right) + \lambda_t (1-t_t)W_t = 0 \quad (1.90)$$

[163] As before, we, "the modeler," know from the firm optimality condition (1.25) that it will (in equilibrium) be the case that $W_t = P_t$; however, the consumer need not "understand" this; all the consumer does is take *whatever* W_t is as given.

$$\frac{u_2\left(c_t, \frac{M_t}{P_t}, 1-n_t\right)}{P_t} - \lambda_t + \beta\lambda_{t+1} = 0 \qquad (1.91)$$

$$-\lambda_t P_t^b + \beta\lambda_{t+1} = 0 \qquad (1.92)$$

$$-\lambda_t S_t + \beta\lambda_{t+1}(S_{t+1} + D_{t+1}) = 0 \qquad (1.93)$$

Conditions (1.29) through (1.33) describe how consumers make optimal choices; as such, they represent equilibrium conditions. As usual, though, it is instructive to not work with these raw first-order conditions directly, but instead combine them into interpretable expressions of the form "MRS equals a price ratio" which are the cornerstone of consumer theory. From here on, to save on notation, we will adopt the following convention regarding arguments of functions. Rather than write, for example, $u_1\left(c_t, \frac{M_t}{P_t}, 1-n_t\right)$ to stand for the marginal utility of consumption in period t, we will simply write u_{1t}, and it will be understood that the second subscript "t" indicates that it is time-t arguments (specifically, c_t, M_t / P_t, and $1-n_t$) that are inside the function. Thus, u_{2t} stands for the marginal utility of real money balances in period t, u_{3t} stands for the marginal utility of leisure in period t, u_{1t+1} stands for the marginal utility of consumption in period t+1, u_{2t+1} stands for the marginal utility of real money balances in period t+1, and so on.

With this notational convention, condition (1.89) implies that $\lambda_t = \frac{u_{1t}}{P_t}$. Inserting this in condition (1.90) and rearranging, we have

$$\frac{u_{3t}}{u_{1t}} = (1-t_t)w_t, \qquad (1.94)$$

where, as usual, $w_t = W_t / P_t$ stands for the *real* wage in period t. We have seen condition (1.94) countless times by now: it is simply the consumer's consumption-leisure optimality condition, stating that the MRS between consumption and leisure (the left-hand-side) equals the after-tax real wage. Condition (1.94) is an equilibrium condition of the model, and it describes how consumers make optimal consumption-leisure tradeoffs.

Next, condition (1.92) tells us $\beta\lambda_{t+1} = \lambda_t P_t^b$. Using this fact in condition (1.91), we can write $\frac{u_{2t}}{P_t} - \lambda_t + \lambda_t P_t^b = 0$, or, equivalently. Next, recall that with bonds that always have a face value of one, $P_t^b = \frac{1}{1+i_t}$, meaning we can write the previous expression as $\frac{u_{2t}}{P_t} = \lambda_t \left[1 - \frac{1}{1+i_t}\right]$, or, simplifying, $\frac{u_{2t}}{P_t} = \lambda_t \left[\frac{i_t}{1+i_t}\right]$. Recalling that $\lambda_t = \frac{u_{1t}}{P_t}$, we can therefore write

$$\frac{u_{2t}}{u_{1t}} = \frac{i_t}{1+i_t}, \tag{1.95}$$

which states that when consumers are making optimal choices, the MRS between consumption and money (the left hand side) depends on the nominal interest rate.[164] Condition (1.95) is the *consumption-money optimality condition* of this model, in analogy with the consumption-leisure optimality condition, and is an equilibrium condition of the model.

Finally, the first-order condition on stock, equation (1.93), can be manipulated (along with condition (1.89) and the time-t+1 version of condition (1.89)) to yield a consumption-savings optimality condition,

$$\frac{u_{1t}}{u_{1t+1}} = \beta(1+r_t). \tag{1.96}$$

To recall details, refer back to the analysis of the consumer's optimization problem in Chapter 17.

Resource Constraint

As always, the resource constraint describes all of the different uses of total output (GDP) of the economy. In the model here, output is produced by the linear-in-labor production technology and, as in the model we used to study just fiscal policy, there are *two* uses for output: private consumption (by consumers) and *public consumption* (i.e., government expenditures). Hence the resource constraint in any arbitrary period t is

[164] Don't be misled by the notation: here, u_2 stands for the marginal utility *of real money balances* because real money balances is the second argument of the utility function. In much of what we've done before, the second argument of the utility function was leisure, meaning that in previous models u_2 stood for the marginal *of leisure;* in the model we are studying here, the marginal utility of leisure is u_3 because leisure is the *third* argument of the utility function. This is simply a notational choice, however; we could have just as readily chosen to make leisure the second argument and real money balances the third argument.

$$c_t + govt_t = n_t\,. \tag{1.97}$$

Government

The government is a consolidated fiscal-monetary authority, as in our study of fiscal-monetary interactions. The period-t budget constraint of the consolidated government is

$$t_t \cdot W_t \cdot n_t + P_t^b B_t + M_t - M_{t-1} = P_t \cdot govt_t + B_{t-1}\,, \tag{1.98}$$

which is an adaptation of the consolidated period-t government budget constraint we encountered in Chapter 15; the only difference is that rather than regular tax revenue being specified arbitrarily as T_t, here we have $t_t \cdot W_t \cdot n_t$. The consolidated government budget constraint (GBC) has the same interpretation as in Chapter 15: the GBC states that government spending on goods and services as well as repayments of maturing government debt (the right-hand-side of expression (1.98)) can be covered by three sources (the left-hand-side of expression (1.98)): labor income tax revenue, issuance/sales of new government bonds, and money creation. Compare the GBC (1.98) with the government budget constraints that underpinned our analysis of optimal monetary policy in Chapter 17 and optimal fiscal policy in Chapter 19: $M_t - M_{t-1} = \tau_t$ and $t_t \cdot W_t \cdot n_t = P_t \cdot govt_t$, respectively.[165]

Equilibrium and Steady-State Equilibrium

The next step, as usual, is to describe the private-sector equilibrium. Because the general idea is the same as in our earlier (separate) studies of optimal monetary policy and optimal fiscal policy, we do not discuss this in detail here. Rather, we simply proceed to list the equilibrium conditions and then condense things down to a small set of steady-state equilibrium conditions.

The firm optimality condition, expression, (1.85), is the only equilibrium condition arising from the supply side of the model. On the demand side of the model, expressions (1.94), (1.95), and (1.96) describe, respectively, the representative consumer's optimal consumption-leisure tradeoff, optimal consumption-money tradeoff, and optimal consumption-savings tradeoff. As such, all three are also equilibrium conditions of our model.

In principle, the resource constraint is an equilibrium condition of the model, as well. But, as we were able to do in our study of optimal fiscal policy, we can use the

[165] Of course, in our previous study of just optimal fiscal policy, we did not put explicit time subscripts on things nor did we formulate the analysis in nominal terms, but the two modifications are straightforward.

consumer's budget constraint, given by expression (1.86), in place of the resource constraint. Hence, expression (1.86) is the final condition describing the private-sector equilibrium.

We are concerned with steady-states, so we must impose steady-state on all of the equilibrium conditions. At this stage, imposing steady-state should be a relatively straightforward exercise. Let's analyze in some detail, though, the steady-state version of the consumer budget constraint.

For reasons that will become a bit more clear when we formulate the optimal policy problem below, let's assume that $B = 0$ always. Also, it turns out that for our purpose (studying optimal fiscal and monetary policy) the steady-state quantity of stock the consumer has is irrelevant, thus let's also assume (without further proof of its irrelevance) that $a = 0$ always.[166] With these simplifying assumptions, we can write (1.86) in real terms (i.e., dividing through by P_t) as

$$c_t + \frac{M_t}{P_t} = (1 - t_t) w_t n_t + \frac{M_{t-1}}{P_t}, \tag{1.99}$$

or, putting both terms involving money on the same side of the equation,

$$c_t = (1 - t_t) w_t n_t + \frac{M_{t-1}}{P_t} - \frac{M_t}{P_t}. \tag{1.100}$$

Defining $m_t \equiv M_t / P_t$ as real money balances, and using the manipulation $\frac{M_{t-1}}{P_t} = \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} = \frac{m_{t-1}}{1+\pi_t}$, we have

$$c_t = (1 - t_t) w_t n_t + \frac{m_{t-1}}{1+\pi_t} - m_t. \tag{1.101}$$

Imposing steady-state,

$$c = (1 - t) wn + m\left[\frac{1}{1+\pi} - 1\right]. \tag{1.102}$$

[166] Note that we are making these assertions *after* we have already obtained the consumer's FOCs. If we had made these assumptions *before* computing FOCs, the structure of the entire model would be drastically different; as it stands, it is relatively innocuous, but for reasons that we leave for a more advanced course in macroeconomic theory.

Next, we know that in steady state, the inflation rate equals the money growth rate; if it did not, then real money balances could not be constant in the steady state.[167] Re-adopting our notation from before, let g be the steady-state growth rate of the nominal money supply. Then,

$$c = (1-t)wn - m\left[\frac{g}{1+g}\right]; \tag{1.103}$$

notice the appearance of the minus sign on the right-hand-side. Substituting $w=1$, we have that the consumer's choice of consumption depends on his choice of labor supply and real money holdings,

$$c = (1-t)n - m\left[\frac{g}{1+g}\right]. \tag{1.104}$$

As we did in our analysis in purely optimal fiscal policy, we next substitute this expression for steady-state equilibrium consumption into the remaining private-sector equilibrium conditions, the (steady-state version of the) consumption-leisure optimality condition (1.94) and the (steady-state version of the) consumption-money optimality condition (1.95). Note that we do **not** need to make this substitution into the (steady-state version of the) consumption-savings optimality condition because if we impose steady state on equation (1.96), we find, as always, that $\frac{1}{\beta} = 1+r$. Of course, by the exact Fisher equation and the fact that $\pi = g$ in steady state, this can in turn be expressed as $\frac{1}{\beta} = \frac{1+i}{1+\pi} = \frac{1+i}{1+g}$, which reveals that in steady-state equilibrium,

$$1+i = \frac{1+g}{\beta}, \tag{1.105}$$

which of course was also true in our discussion of purely optimal monetary policy.

Making the substitution for c in the consumption-leisure and consumption-money optimality conditions thus give us

[167] In other words, having already asserted that **real** money balances become constant in the steady state, it must be, by the definition of real money balances, that M/P is constant. The only way for M/P to be constant is for the numerator and the denominator to both be changing at the same exact rate. This is nothing more than our usual monetarist/quantity-theoretic notion that in the long run (i.e., in steady state), the money growth rate is equal to the inflation rate.

$$\frac{u_3\left((1-t)n-m\left[\frac{g}{1+g}\right],m,1-n\right)}{u_1\left((1-t)n-m\left[\frac{g}{1+g}\right],m,1-n\right)}=1-t \quad (1.106)$$

and

$$\frac{u_2\left((1-t)n-m\left[\frac{g}{1+g}\right],m,1-n\right)}{u_1\left((1-t)n-m\left[\frac{g}{1+g}\right],m,1-n\right)}=\frac{1+g-\beta}{1+g}. \quad (1.107)$$

In writing these two expressions, we have re-introduced the arguments to the marginal utility functions and also used the relationship in (1.105) to eliminate the nominal interest rate.

Conditions (1.106) and (1.107) condense the entire description of the private-sector equilibrium of the economy down to two conditions. Jointly, these two conditions should be thought of as defining *a pair of functions* $n(t,g)$ and $m(t,g)$.[168]

Formulation of Optimal Policy Problem

Our objective is to study *jointly-optimal steady-state* fiscal and monetary policy. The policy problem is to choose a t and a g that maximizes the representative consumer's utility taking into account the function $n(t,g)$, the function $m(t,g)$, and the government budget constraint. Because we are only concerned with steady-state policy, to move towards this goal, let's first rearrange the government budget constraint (1.98) and turn it into a steady-state expression. First, recognize as usual that $P_t^b=\frac{1}{1+i_t}$ and divide through by P_t to put everything in real terms:

[168] You should think of this just as the equilibrium "reaction function" $c(g)$ in our consideration of purely optimal monetary policy and $n(t)$ in our consideration of purely optimal fiscal policy. The technical difference here is that the government has *two* policy instruments (the labor tax rate and the money growth rate) and there are *two* steady-state equilibrium objects to be determined. However, for a wide class of utility functions used in quantitative macroeconomic models, it can be shown (in a more advanced treatment of monetary theory) that labor will depend only on the labor tax rate and money balances will depend only on the money growth rate. Thus, we simply assert in the rest of what we do that this is true.

$$t_t w_t n_t + \frac{1}{1+i_t}\frac{B_t}{P_t} + \frac{M_t - M_{t-1}}{P_t} = govt_t + \frac{B_{t-1}}{P_t}. \qquad (1.108)$$

On the right-hand-side, notice, as is always the case in a consolidated fiscal-monetary budget constraint, the appearance of seignorage revenues, $sr_t = \frac{M_t - M_{t-1}}{P_t}$. As above and in Chapter 15, define $b_t \equiv B_t / P_t$ as the real amount of government debt outstanding at the end of period t. Also, break up the seignorage revenue term as $\frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}$. Recalling that $m_t \equiv M_t / P_t$ is real money balances and recalling that $P_{t-1} / P_t = 1/(1+\pi_t)$, we can rearrange (1.108) further to get

$$t_t w_t n_t + \frac{1}{1+i_t} b_t + m_t - \frac{m_{t-1}}{1+\pi_t} = govt_t + \frac{B_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t}, \qquad (1.109)$$

or, a little more compactly,

$$t_t w_t n_t + \frac{1}{1+i_t} b_t + m_t - \frac{m_{t-1}}{1+\pi_t} = govt_t + \frac{b_{t-1}}{1+\pi_t} \qquad (1.110)$$

where, in the last step, we used the manipulation $\frac{B_{t-1}}{P_t} = \frac{B_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t} = \frac{b_{t-1}}{1+\pi_t}$. Our next step is to impose steady state on expression (1.110); doing so and combining terms,

$$twn + b\left[\frac{1}{1+i} - \frac{1}{1+\pi}\right] + m\left[1 - \frac{1}{1+\pi}\right] = govt. \qquad (1.111)$$

Because $\pi = g$ in steady-state, we can write the previous expression as

$$twn + b\left[\frac{1}{1+i} - \frac{1}{1+g}\right] + m\left[1 - \frac{1}{1+g}\right] = govt. \qquad (1.112)$$

We can condense this expression even further. The Fisher equation tells us $1+i = (1+r)(1+\pi)$, which in turn can be expressed as $1+i = (1+r)(1+g)$. Next, we know from our consumption-savings optimality condition (equation (1.96)) that in steady state, $1+r = 1/\beta$. Thus, in steady-state, $1+i = \frac{1+g}{\beta}$, which we saw in expression (1.105). Inserting all of this on the left-hand-side of (1.112), we have

$$twn + b\left[\frac{\beta - 1}{1+g}\right] + m\left[\frac{g}{1+g}\right] = govt. \quad (1.113)$$

After several manipulations and rearrangements, we have arrived at a very useful intermediate form of the steady-state equilibrium version of the government budget constraint.[169] Expression (1.113) shows that government spending must be financed in the long run (i.e., in the steady state) by a combination of labor income taxes (the first term on the left-hand-side), seignorage revenue (the third term on the left-hand-side), and **deflation of government debt** (the second term on the left-hand-side).

This last "revenue source," deflation of government debt, can be thought of as a steady-state version of the ideas of the fiscal theory of the price level and the fiscal theory of inflation that we studied earlier. In that analysis, recall that the two ideas were distinct, and the distinction between them lay in *when* the inflation wrought by an active fiscal policy was going to occur: the fiscal theory of the price level stated that it would occur *now,* while the fiscal theory of inflation stated that it would occur *at some time in the current period or future periods, or perhaps spread out over multiple periods.* In steady state, however, which is what we are focused on here, the very notions of "now" and "later" disappear: in steady-state, time "disappears," thus "now" and "later" are blurred. Hence, in steady-state we cannot distinguish between the fiscal theory of the price level and the fiscal theory of inflation; the two roll into what we are here calling *deflation of government debt.*[170]

It turns out that for the purpose at hand (studying the optimal steady-state *mix* of money growth/seignorage and labor taxes) the deflation of government debt channel is not important.[171] Thus, from now on, we will assume $b = 0$ (i.e., the government has no debt obligations), which also justifies why we assumed above that $B = 0$ when we were describing the private-sector equilibrium. The GBC can thus now be written as

$$twn + m\left[\frac{g}{1+g}\right] = govt. \quad (1.114)$$

Recall our mode of analysis of optimal policy problems: at the stage of determining the optimal policy, the government (in this case, the consolidated fiscal-monetary government) *takes into account all equilibrium conditions,* including functions that describe how the private sector responds to *any arbitrary* policy that it sets. Thus, there

[169] Note that in our study of the FTPL and the FTI, we were not focused on the *steady-state* version of the government budget constraint; there, we were explicitly concerned with the *dynamics* (of inflation and seignorage revenue) implied by the intertemporal government budget constraint.

[170] In yet other words, the fiscal theory of the price level and the fiscal theory of inflation are inherently *dynamic* concepts.

[171] We leave the precise reasons behind the *steady-state* irrelevance of the debt-deflation mechanism for a more advanced course in monetary theory.

are three more things to do with (1.114): insert the equilibrium steady-state real wage rate $w=1$ (recall equilibrium condition (1.85)), insert the function $n(t,g)$, and insert the function $m(t,g)$. Making these insertions,

$$t \cdot n(t,g) + m(t,g)\left[\frac{g}{1+g}\right] = govt\,. \tag{1.115}$$

The government's policy problem thus boils down to the government choosing t and g to satisfy its budget constraint (1.115). The reason that the optimal policy problem boils down to just the government budget constraint is just as it was in our study of optimal fiscal policy: the functions $n(t,g)$ and $m(t,g)$ already capture how the private sector responds to a given policy the government chooses.

There are in principle an infinite number of combinations of (t,g) that satisfy (1.115). In Chapter 19, when we arrived at the analogous place in the analysis, what we had was one equation (the government budget constraint) in one unknown (the tax rate); here we have one equation in *two* unknowns. Clearly, if we knew either t or g, then we would know the other as well – that is, if we somehow pick either t or g, then equation (1.115) would again reduce to one equation in one unknown.

In order to pin down one of the policies, let's proceeding to compute the first-order conditions of (1.115) with respect to t and g; using the product and quotient rules, they are, respectively,

$$n(t,g) + t\frac{\partial n}{\partial t} + \frac{\partial m}{\partial t}\left[\frac{g}{1+g}\right] = 0 \tag{1.116}$$

and

$$t\frac{\partial n}{\partial g} + \frac{\partial m}{\partial g}\left[\frac{g}{1+g}\right] + \frac{m(t,g)}{(1+g)^2} = 0\,. \tag{1.117}$$

Conditions (1.116) and (1.117) define *either* the optimal labor tax rate *or* the optimal growth rate of the money supply; they do *not* define both. We will make this point more clear through an example in the next section, but when free to "choose" two variables (here, policy variables) to satisfy one equation, one is of course not really free to "choose" both of them. As was the case in Chapter 19, we cannot make any more progress *actually* computing the optimal values of t and g without making some assumptions about the utility function.[172] This is the task we take up in the next section. In the next section, we first assume a conventional form for the utility function, make some progress towards analyzing the jointly-optimal policy, and draw on lesions we have learned previously to draw some general conclusions.

[172] Note that in the analysis of *only* optimal monetary policy, we *were* able to completely solve for optimal monetary policy (in isolation from fiscal policy) *without* making any assumptions about the utility function. Things are different in the analysis of *only* optimal fiscal policy and the joint analysis because of the presence of the government budget constraint – that is, the presence of a *financing* concern (i.e., how should the government raise revenue?) makes things much more complicated, and the level of generality of proofs/results that we can obtain is not as high as it was in in the case of *only* optimal monetary policy.

A Workhorse Utility Function

A utility function that is a staple in modern macroeconomic models and one that we have had many occasions to work with already is the additively-separable function that is log in consumption and money balances and linear in leisure. For the rest of our analysis, we thus assume that the utility function is

$$u(c,m,1-n)=\log c+\log m+\log(1-n), \tag{1.118}$$

which means the marginal utility functions are $u_1=1/c$, $u_2=1/m$, and $u_3=1/(1-n)$. Before we can use equations (1.116) and (1.117) to figure out what *either* the optimal tax rate *or* the optimal money growth rates is given this utility function, we must first determine what the functions $n(t,g)$ and $m(t,g)$ are for this utility function (because we need these functions for use in expressions (1.116) and (1.117)).

In order to determine the functions $n(t,g)$ and $m(t,g)$, recall that we must use conditions (1.106) and (1.107). Using the marginal utility functions associated with our assumed utility function in these two conditions, respectively, we have

$$\frac{(1-t)n-m\left[\frac{g}{1+g}\right]}{1-n}=1-t \tag{1.119}$$

and

$$\frac{(1-t)n-m\left[\frac{g}{1+g}\right]}{m}=\frac{1+g-\beta}{1+g}. \tag{1.120}$$

The task is to solve equations (1.119) and (1.120) for n and m. There are obviously a number of ways one can attack this problem since all it requires is some brute-force (though tedious…) algebra. Let's first solve (1.120) for m. After a couple of steps of algebra and rearrangement, we have

$$m=\frac{(1-t)n(1+g)}{1+2g-\beta}. \tag{1.121}$$

Next, take this expression for m and insert it in equation (1.119); doing so, we have

$$\frac{(1-t)n}{1-n}-\left[\frac{g}{1+g}\right]\left[\frac{(1-t)(1+g)}{1+2g-\beta}\right]\left[\frac{n}{1-n}\right]=1-t\,. \tag{1.122}$$

Canceling some terms gives us

$$\frac{n}{1-n} - \frac{n}{1-n}\left[\frac{g}{1+2g-\beta}\right] = 1 \qquad (1.123)$$

or even more compactly,

$$\frac{n}{1-n}\left[\frac{1+g-\beta}{1+2g-\beta}\right] = 1. \qquad (1.124)$$

Solving this for n, we find

$$n(g) = \frac{1+2g-\beta}{2+3g-2\beta}, \qquad (1.125)$$

which shows that n is a function of g **but not a function of t.** This is not a general statement, of course, but rather simply a property of the utility function we are using here; nonetheless, it is an interesting property to note.[173]

Next, we need the function $m(t,g)$. To compute it, insert (1.125) into (1.121); doing so gives us

$$m(t,g) = \frac{(1-t)(1+g)}{2+3g-2\beta}. \qquad (1.126)$$

If neither expression (1.125) nor expression (1.126) strikes you as particularly informative – don't worry, they really are not. They are intermediates, though, required to take our next step, which is to insert these functions *and their partial derivatives* into expressions (1.116) and (1.117). Omitting all the algebraic steps in computing the appropriate partials and doing the necessary substitutions, etc, making these insertions leaves us with the following two expressions that determine *either* the optimal tax rate *or* the optimal money growth rate*:*

$$\frac{1+g-\beta}{2+3g-2\beta} = 0 \qquad (1.127)$$

and

$$\frac{(2-t)(1-\beta)}{(2(\beta-1)-3g)^2} = 0. \qquad (1.128)$$

[173] With log utility, optimal labor supply is not a function of the labor tax rate.

Assuming $\beta < 1$, equation (1.128) can only be satisfied if $t = 2$, i.e., a labor tax rate of 200 percent! Clearly, this makes no economic sense because, as we saw in Chapter 19, if the labor tax rate were even just 100 percent, nobody would ever work. This is the sense, then in which we meant above that the FOCs of the optimal policy problem here determine *either* the optimal money growth rate *or* the optimal labor tax rate.[174]

Equation (1.127), on the other hand, immediately tells us $g = \beta - 1$ is the optimal money growth rate. We've of course seen this policy prescription before – it's simply the Friedman Rule. Thus, in the context of the *joint* conduct of fiscal and monetary policy, the optimal *monetary* component of policy is to implement the Friedman Rule, which recall, means that there should deflation of prices and, equivalently, a zero nominal interest rate.

The last step is then to solve for the labor tax rate. Inserting the Friedman Rule $g = \beta - 1$ in the government budget constraint (1.115) tells us that the labor tax rate must satisfy

$$t \cdot n(t, g) + m(t, \beta - 1)\left[\frac{\beta - 1}{\beta}\right] = govt\ . \tag{1.129}$$

[174] A useful analogy is to think in terms of the solution of a general quadratic equation. The quadratic formula typically returns *two* solutions, but in practice, only one of them makes "sense" (i.e., in applications in economics, physics, engineering, etc.)