Monopolistically Competitive Search Equilibrium[*]

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Abstract

This paper introduces a monopolistically-competitive recruiting (intermediated) market in a standard (non-intermediated) search and matching model to explore the implications of intermediated labor markets, whose importance in new job creation is rising. We analytically show that: (1) the surplus to recruiters from successful monopolistic intermediation appears *directly and additively* in the surplus-sharing condition between newly matched workers and firms; (2) the surplus that accrues to monopolistic recruiters arises due to aggregate increasing returns in matching; (3) deviations from efficient wage setting (Nash-Hosios) in non-intermediated random-search markets spill over into recruiter creation and matching via intermediated markets, but deviations from efficient matching aggregation in recruiting markets have *no* impact on non-intermediated markets; and (4) in general equilibrium, the aggregate increasing returns in matching expands the aggregate resource frontier. We quantitatively show how the implications of wage distortions in non-intermediated markets on aggregate unemployment and labor force participation depend on the existence of intermediated labor markets.

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1 Introduction

Labor market intermediaries play an important role in helping firms meet their employment needs and job seekers find employment opportunities. The services and reach of these intermediaries has grown over time, especially with a dramatic expansion of e-recruiting firms and their services since the middle of the 1990s (Nakamura et al., 2009; Bagues and Sylos Labini, 2009).[1] Despite the rising prominence of intermediated labor markets and the latter's potentially important consequences for aggregate outcomes, the structure of intermediated job matching markets has received little, but recently growing, attention in the macro-labor literature.

This paper introduces a monopolistically-competitive recruiting sector with endogenous entry of recruiters alongside a standard (random-) search and matching model of labor markets. The core of our model builds on the work of Moen (1997) and Shimer (1996), who are the first to characterize competitive search equilibrium, and on the monopolistically-competitive endogenous entry model of Bilbiie, Ghironi, and Melitz (2012).[2] Using our framework, we address several important questions: 1) What is the nature of the monopolistically-competitive surplus sharing condition? 2) What are the implications for aggregate labor-market matching? 3) What, if any, are the spillover effects on new job creation via recruiting markets from distortions in random-search matching (as well as spillover effects operating in the opposite direction)? 4) What are the general equilibrium consequences of monopolistically-competitive intermediation? 5) What are the implications of monopolistically-competitive intermediation for unemployment, participation, and labor market conditions?

We obtain several main results that answer these questions. First, we analytically characterize the surplus sharing condition between labor suppliers and labor demanders that are matched via intermediation. This surplus sharing condition contains an *additive* positive surplus received by monopolistic recruiters for successful intermediation. This central result arises because of imperfect competition in the intermediated labor market and is not one that could be attributed to, say,

[1]Examples of labor market intermediaries (LMIs, for short) include employment agencies and recruiting and staffing firms and job search engines and services. For an comprehensive summary of online job services, their proliferation, and their importance, see Nakamura et al. (2009). Among other things, labor market intermediaries build resume databases, provide services that centralize job applications, provide customized matching services for firms, and advertise open employment positions. Well-known providers of online (for-profit) e-recruiting services include Monster.com (one of the largest e-recruiting firms in the U.S. that started in 1995), Indeed.com, and Career-Buider.Similar services operate in the non-profit realm as well (for example, America's Job Bank). For the benefits of e-recruiting (which include the reduction of variable recruiting costs and processing costs, among others), see Nakamura et al. (2009). Survey evidence from the Society for Human Resource Management for 2007 suggests that more than 40 percent of new hires in both the public and private sectors originated from e-recruiting (Nakamura et al., 2009). Using data from iLogos Research, Nakamura et al. (2009) document a sharp expansion in the corporate website employment sections in global 500 companies: while in 1998 these sections represented 29 percent of corporate website use, these sections expanded to 94 percent of website use in the 2000s. For related work on e-recruiting and the labor market, see Autor, Katz, and Krueger (1998), Kuhn (2003), Kuhn and Skuterud (2004), and Stevenson (2008), among others. See the Appendix for evidence on the growth of staffing firms as measured by the evolution of employment in these firms.

[2]The application of Bilbiie, Ghironi, and Melitz (2012) is to product markets.

proportional taxes on labor income, consumption purchases, or on goods-producing firms' profits. This central result is novel compared to the existing literature on labor-market intermediation and stands in contrast to existing work on the distortionary effects of fiscal policy on labor markets. To illustrate the economic rationale behind the novel surplus sharing rule, consider the (qualitative) surplus sharing condition

Profit of recruiter + (1-Share) x Surplus of new employee = (Share) x Surplus of new employer,

in which the "Share" term is a scalar between zero and one that measures the percentage of the total surplus from a matched employer-employee pair. The presence of the additive term in the surplus sharing condition is, to the best of our knowledge, a novel contribution to the literature on labor market intermediation.

As the surplus sharing expression shows, the shares of the total surplus received by the worker and the employer sum to one. This observation leads to the second analytical result, which is that the presence of additional resources above and beyond those received by the worker and the firm are due to the *aggregate* increasing returns to scale (IRTS) in matching that arises in our model. This pair of analytical results arise in the partial equilibrium model of labor markets; moving to the general equilibrium environment leads to another, third, analytical result, which is that the aggregate resource frontier *expands* due to the aggregate increasing returns in new job creation. While the idea of increasing returns to scale amid differentiated goods production traces back to at least Romer (1987), its application to intermediated labor markets is new relative to existing literature on labor-market intermediation.[3]

The fourth analytical result shows that deviations from efficient wage setting in non-intermediated random-search matching spill over into and distort the development of new recruiters and hence matching via intermediated recruiting markets. However, distortions that emerge from inefficient aggregation in monopolistic recruiting markets have *no* impact on non-intermediated markets. Spillovers are thus asymmetric in nature and only occur in one direction. Intuitively, this result arises from the *differential* degree of (in)efficiencies between non-intermediated markets and (monopolistically-competitive) intermediated markets *coupled* with endogenous recruiting firm entry. More precisely, if workers' Nash bargaining power in the determination of wages in non-intermediated job creation is inefficiently *low*, then, in intermediated recruiting markets, the number of recruiting firms, labor-market tightness, and vacancies directed to recruiting markets are all *increasing* in worker bargaining power, whereas for inefficiently *high* bargaining power, all three of these variables are *decreasing* in bargaining power.

The quantitative results in the general equilibrium model verify the analytics regarding spillover

[3]Masters (2007) highlights the fact that matching technologies with increasing returns imply that intermediaries can bring about welfare gains. However, his work does not directly focus on intermediation in labor markets.

effects, and the model's cyclical results are consistent with broad business-cycle patterns: conditional on TFP shocks, the model delivers procyclical labor force participation, procyclical consumption, and procyclical physical capital investment, along with countercyclical unemployment. The quantitative results also show that steady-state reductions in the flow cost of a vacancy in intermediated labor markets lead to a small increase in steady state unemployment and a decrease in participation and output. Conversely, commensurate reductions in the flow cost of posting a vacancy in non-intermediated labor markets lead to a significant reduction in unemployment and an expansion in output and consumption. This asymmetry in the steady-state consequences of differences in flow vacancy creation costs across labor markets translates into non-negligible implications for cyclical unemployment dynamics. In particular, relative to the baseline economy, an economy with lower flow vacancy posting costs in non-intermediated (intermediated) labor markets exhibits sharper (virtually identical) fluctuations in unemployment amid TFP shocks. Therefore, our findings suggest that, in an environment where intermediated and non-intermediated labor markets coexist, not all vacancies are created equal and differential changes in low vacancy costs across labor markets can have important implications for unemployment volatility.

The rest of the paper is organized as follows. Section 2 describes the structure of recruiting markets and the surplus-sharing function that arises as recruiters intermediate frictional labor demand and labor supply. Section 3 then embeds the recruiting sector in a general equilibrium framework. Section 4 analytically characterizes the asymmetric spillover of inefficiencies across labor markets; more specifically, the general equilibrium inefficiencies that arise in otherwise efficient intermediated (recruiting) markets as a result of distortionary wage-setting in non-intermediated markets, and the absence of distortions in efficient non-intermediated markets, *even under inefficient aggregation in intermediated markets.* Section 5 contains quantitative results from the general equilibrium model. Section 6 briefly places our main results within the context of existing work on intermediation and matching frictions, and Section 7 concludes. Many of the algebraic derivations are provided in a detailed set of Appendices.

2 Recruiters — Partial Equilibrium

We begin with a partial equilibrium model of the imperfectly competitive recruiting sector with endogenous entry.

2.1 Recruiting Market j

There is a continuum $[0, 1]$ of perfectly-competitive recruiting markets. As shown in Figure 1, in each recruiting market $j \in [0, 1]$, perfectly-competitive recruiting agencies purchase differentiated

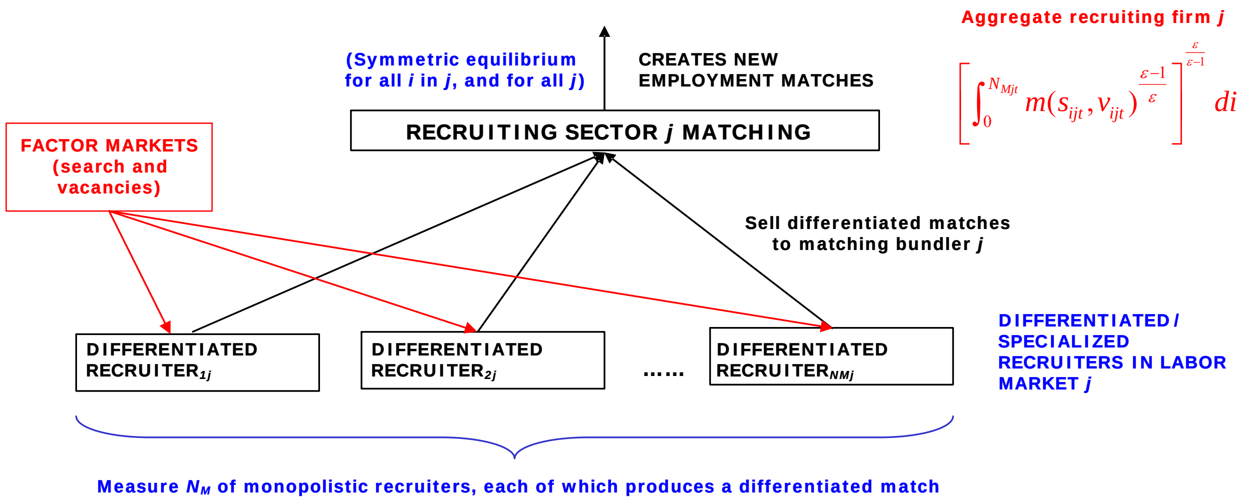

Figure 1: **Structure of Matching Markets.** Differentiated recruiting agencies produce specialized matches in their particular submarkets, which are then aggregated by perfectly-competitive recruiting agencies in labor market j. In each labor market j, there are N_M differentiated recruiting agencies. The matching aggregator displayed (as but one example) is the Dixit-Stiglitz technology, in which the parameter ε measures the elasticity of substitution between any pair of differentiated matches.

submarket ij matches and aggregate them using a technological aggregator. As shown in Figure 1, in each recruiting market $j \in [0, 1]$, perfectly-competitive recruiting agencies purchase differentiated submarket ij matches and aggregate them using a technological aggregator. Table 1 shows the several matching aggregators considered in the theoretical and quantitative analysis, and, for reference, Table 2 provides definitions of notation used in the partial equilibrium analysis.

The representative labor-market j recruiting agency is modeled as being a "large" recruiting agency that develops "many" differentiated recruiting agencies. The labor-market j recruiting agency is "large" in the sense that it produces multiple recruiting agencies, but the assumption of a continuum of recruiting firms ensures that each is small relative to the overall labor market, and hence does not internalize the effects of its decisions on the outcomes in matching-market j. Thus, we are assuming that recruiting agency j's decisions regarding the development of new differentiated matching agencies do not internalize the fact that by creating new differentiated matching agencies the profits of *any* existing agencies within the firm are adversely affected (which is dubbed the "profit destruction externality"). This can be rationalized by assuming that new differentiated matching agencies are developed by independent recruiting line managers who communicate little with each other or are even encouraged to compete with each other.[4]

This rationale allows us to independently characterize the entry of new recruiters in labor market

[4]This assumption is standard in the Bilbiie, Ghironi, and Melitz (2012) class of models on which our recruiting sector builds.

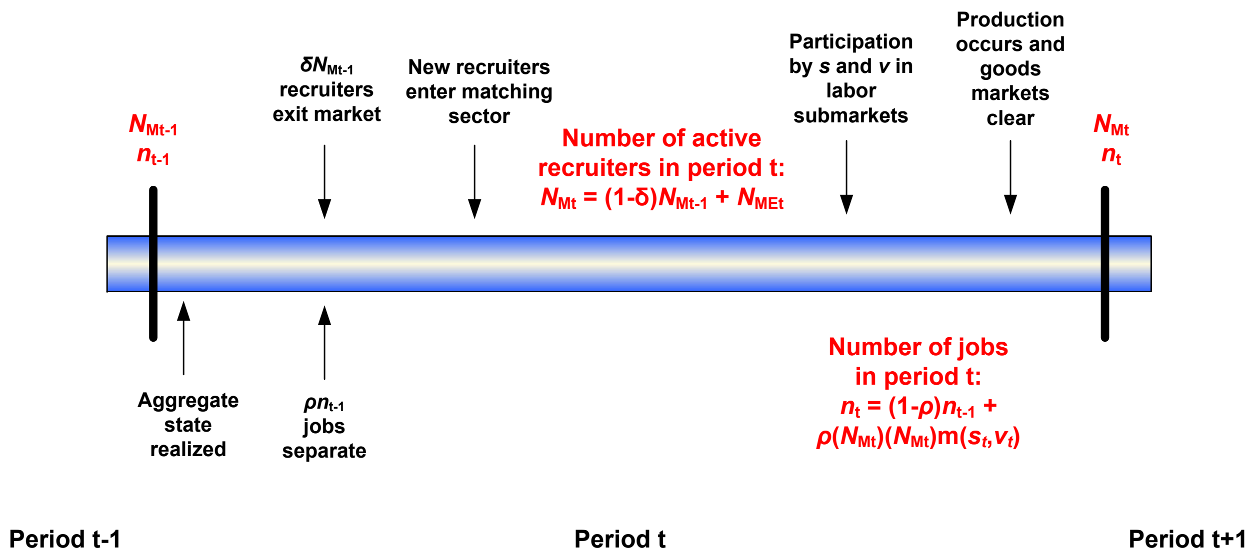

Figure 2: **Ordering of events in intermediated labor markets.** Newly-developed monopolistic recruiting agencies begin operations in period t, and newly-created job matches in period t begin producing goods in period t. The $\rho(N_{Mt})N_{Mt}$ term measures the increasing returns to scale in matching.

j and the demand for each differentiated recruiter i's match $m(s_{ijt}, v_{ijt})$ in labor market j, to which we now turn.

Entry of New Recruiters.

Temporarily define aggregate profits received by the representative recruiter in labor market j as $\Pi(N_{Mjt}) \equiv \int_0^{N_{Mjt}} (\rho_{ijt} - mc_{jt}) \cdot m(s_{ijt}, v_{ijt})\, di$. Expressed in real terms (that is, in units of consumption goods), the intertemporal profit function of the representative recruiter in labor market j is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(\Pi(N_{Mjt}) - \Gamma_{Mt} N_{MEjt} \right), \tag{1}$$

in which $\Xi_{t|0}$ is the period-zero discount factor of the ultimate owners of the recruiting firm.[5] Entry of a new recruiter in period t entails a sunk cost Γ_{Mt}, which is identical across all potential entrants.

The total number of new recruiters in labor market j is N_{MEjt}. The law of motion for the total number of monopolistic recruiters in labor market j is

$$N_{Mjt} = (1-\omega)N_{Mjt-1} + N_{MEjt}, \tag{2}$$

which is a constraint on recruiter j's optimization problem. Given this constraint, recruiter j maximizes its intertemporal profit function (1) by choosing N_{Mjt} and N_{MEjt}. The first-order

[5] As will be clear in the general equilibrium model in Section 3, the ultimate owner of recruiting firms and hence any flow profits they earn is the representative household.

Dixit-Stiglitz	Benassy	Translog
$\mu(N_M) = \mu = \frac{\varepsilon}{\varepsilon-1}$	$\mu(N_M) = \mu = \frac{\varepsilon}{\varepsilon-1}$	$\mu(N_M) = 1 + \frac{1}{\sigma N_M}$
$\rho(N_M) = N_M^{\mu-1} = N_M^{\frac{1}{\varepsilon-1}}$	$\rho(N_M) = N_M^{\varphi}$	$\rho(N_M) = \exp\left(-\frac{1}{2}\frac{\tilde{N}_M - N_M}{\sigma \tilde{N} N_M}\right)$
$\epsilon(N_M) = \mu - 1$	$\epsilon(N_M) = \varphi$	$\epsilon(N_M) = \frac{1}{2\sigma N_M} = \frac{1}{2}(\mu(N_M) - 1)$

Table 1: **Matching aggregators.** The markup, relative price of symmetric good, and aggregate increasing returns as functions of the number of recruiters for the Dixit-Stiglitz, Benassy, and translog variety aggregators. The Benassy aggregator nests the Dixit-Stiglitz aggregator if $\varphi = \frac{\varepsilon}{\varepsilon-1} - 1$, in which φ characterizes, in terms of elasticity, the welfare benefits of increasing returns. $\tilde{N}_M$ denotes the mass of potential submarket recruiters for the translog aggregator.

conditions with respect to N_{Mjt} and N_{MEjt} yield the matching-market j free-entry condition

$$\Gamma_{Mt} = \Pi'(N_{Mjt}) + (1-\omega)E_t\left\{\Xi_{t+1|t}\Gamma_{Mt+1}\right\}. \tag{3}$$

Intuitively, the free-entry condition equates the marginal cost of entering submarket j to the expected marginal benefit, which, in turn, depends on the flow of marginal profits $\Pi'(N_{Mjt})$ and, conditional on the Poisson exit rate ω, the continuation term. This expression can be thought of as pinning down the endogenous measure of newly-entered monopolistic recruiters N_{MEjt}.

Demand Function for $m(s_{ijt}, v_{ijt})$.

Next, we characterize the representative labor-market j recruiter's demand for submarket ij new job matches $m(s_{ijt}, v_{ijt})$. For ease of exposition, we assume that the recruiting-market j aggregator is of Dixit-Stiglitz form

$$\left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \tag{4}$$

(in which the parameter ε measures the elasticity of substitution between any pair of specialized matches), but the results also hold for other aggregators.[6]

Characterization of demand functions for submarket ij new job matches requires a reformulation of the profit function stated in (1), the rationale for which is, as described above, the "autonomous" recruiting line managers within the "large" recruiting agency j. The reformulated profit function is the "static" profit function

$$\left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di - \int_0^{N_{Mjt}} \rho_{ijt} \cdot m(s_{ijt}, v_{ijt})\ di. \tag{5}$$

[6]Such as the Benassy aggregator and the translog aggregator.

Variable Name	Definitions/Notes
N_{Mjt}	Stock of recruiting agencies in submarket ij
N_{MEjt}	New recruiting agencies in submarket ij
ρ_{ijt}	Relative price of recruiter ij
w_{ijt}	Wage for newly-hired employees in submarket ij
v_{ijt}	Vacancies directed to submarket ij
s_{ijt}	Active job search directed to submarket ij
θ_{ijt}	Labor-market tightness ($\equiv v_{ijt}/s_{ijt}$) in submarket ij
$k^f(\theta_{ijt})$	Probability of job filling in submarket ij
$k^h(\theta_{ijt})$	Probability of job finding in submarket ij
$\mathbf{W}(w_{ijt}, \theta_{ijt})$	Value of active job search participating in submarket ij that successfully finds an employer
$\mathbf{U}_t$	Value of active job search in submarket ij that fails to find a job
$\mathbf{J}(w_{ijt}, \theta_{ijt})$	Value of job vacancy in submarket ij that successfully finds an employee
Γ_{Mt}	Exogenous cost of developing a specialized recruiting agency and entering the recruiting market
ω	Exogenous Poisson exit rate of recruiting agencies

Table 2: **Notation.** Partial equilibrium model of labor market.

Optimization yields the demand functions

$$m(s_{ijt}, v_{ijt}) = \rho_{ijt}^{-\varepsilon} \cdot \left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di \tag{6}$$

for each underlying differentiated matching firm ij. Rewriting the demand function to isolate ρ_{ijt} gives

$$\rho_{ijt} = m(s_{ijt}, v_{ijt})^{-\frac{1}{\varepsilon}} \cdot \left[\left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di\right]^{\frac{1}{\varepsilon}}. \tag{7}$$

2.2 Monopolistically-Competitive Surplus Sharing

We now turn to the optimization problem of a differentiated recruiter i in labor market j which in turn leads to the novel surplus-sharing condition that arises in monopolistically-competitive recruiting markets.

Profit Maximization.

As standard in monopolistically competitive models, a differentiated firm (in our application, a differentiated recruiting agency) maximizes profits by choosing its price based on its demand function. Because the matching function $m(s_{ijt}, v_{ijt})$ is constant-returns-to-scale, it is sufficient to describe its cost-per-match in terms of the marginal cost mc_{jt}, which is independent across submarkets.[7] Recruiting agency ij's period-t profits are thus given by

$$\rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt}). \tag{8}$$

Continuing with the Dixit-Stiglitz matching aggregator shown in (4), substitution of the Dixit-Stiglitz demand function (6) allows us to rewrite recruiting agency ij's period-t profits as

$$\left(\rho_{ijt}^{1-\varepsilon} - \rho_{ijt}^{-\varepsilon} \cdot mc_{jt}\right) \cdot \left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} di. \tag{9}$$

The first-order condition of (9) with respect to ρ_{ijt} yields the Dixit-Stiglitz pricing condition

$$\rho_{ijt} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) mc_{jt}, \tag{10}$$

in which $\mu_t = \frac{\varepsilon}{\varepsilon-1}$ is the constant gross markup that emerges from the Dixit-Stiglitz aggregator.[8]

[7] In equilibrium, the factor prices will be equated to the marginal cost of creating a new job match, which is $mc_{jt} = p_{sjt}/m_s(\cdot) = p_{v_{jt}}/m_v(\cdot)$.

[8] More generally (referring to Table 1), the pricing condition can be expressed as $\rho(N_{Mjt}) = \mu(N_{Mjt}) \cdot mc(N_{Mjt})$.

Monopolistic Surplus Sharing.

In terms of the ordering of events (refer to Figure 2), recruiter ij has already maximized profits (and thus minimized costs) before the posting phase (w_{ijt}, θ_{ijt}) that attract both suppliers and demanders to submarket ij. Due to the ordering of events, the posting phase only requires use of recruiter ij's *marginal profit*. More precisely, define the value function associated with the recruiter ij problem as

$$\mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot) = \max \left\{ \rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m(s_{ijt}, v_{ijt}) \right\}, \tag{11}$$

which implies there are two associated envelope conditions. The envelope condition with respect to s_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot)}{\partial s_{ijt}} &= \rho_{ijt} \cdot m_s(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_s(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot \xi \cdot k^h(\theta_{ijt}), \end{aligned} \tag{12}$$

in which the second line follows from the properties of the Cobb-Douglas matching function $m(s, v) = s^{\xi} v^{1-\xi}$ and $k^h(\theta_{ijt})$ denotes the probability that an active job searcher in submarket ij successfully gains employment. Analogously, the envelope condition with respect to v_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot)}{\partial v_{ijt}} &= \rho_{ijt} \cdot m_v(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_v(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot (1 - \xi) \cdot k^f(\theta_{ijt}), \end{aligned} \tag{13}$$

in which the second line follows from the properties of the Cobb-Douglas matching function $m(s, v) = s^{\xi} v^{1-\xi}$ and $k^f(\theta_{ijt})$ denotes the probability that a job opening in submarket ij is successfully filled.

Similar to Moen (1997), recruiter ij has to incentivize both labor suppliers and labor demanders to participate in submarket ij. The incentive mechanism for recruiter ij is to take as constraints the participation conditions of labor suppliers and labor demanders. We detail the foundations of the participation constraints in Section 3; for now, though, we simply take them as given. Referring to the definitions in Table 2, the participation constraint of a labor supplier is

$$k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}, \theta_{ijt}) + \left(1 - k^h(\theta_{ijt})\right) \mathbf{U} = \mathbf{X}^H \tag{14}$$

and the participation constraint of a labor demander is

$$k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}) = \mathbf{X}^F. \tag{15}$$

Expression (14) states that the value of a labor supplier that directs search towards submarket ij must be the same as the value $\mathbf{X}^H$ of directing search to any other submarket. Analogously, expression (15) states that the value of a labor demander that directs its job openings towards submarket ij must be the same as the value $\mathbf{X}^H$ of directing its job openings to any other submarket.

Regardless of whether the envelope condition (12) or (13) is used, the following surplus-sharing rule (the proof for which appears in Appendix A) arises.

Proposition 1. ***Monopolistic Surplus Sharing.*** *The surplus-sharing rule between labor suppliers and labor demanders that meet via monopolistically-competitive labor-market intermediation is*

$$\xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_{jt}) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}, \theta_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}), \tag{16}$$

in which $\xi \in (0,1)$ *denotes the elasticity of total matches with respect to the measure of active job searchers.*

Proof. See Appendix A. □

Intuitively, there are *three* parties involved in the surplus, and each of the three parties earns a positive share of the overall surplus. The three parties are the newly-employed worker (whose value is $\mathbf{W}(\cdot) - \mathbf{U}$), the newly-filled job opening (the value of which is $\mathbf{J}(\cdot)$), and the monopolistic recruiter ij (whose value is $\mathbf{V}_{Mij}(\cdot)$) that matches the other two parties. Observation of the monopolistic surplus sharing rule shows that the percentage of the total surplus received by workers $(1-\xi)$ and the percentage of the total surplus received by goods-producing firms (ξ) sum to 100%. This observation naturally leads to the question of the source of the *extra* resources needed to provide monopolists the *positive economic profit* $\rho_{ijt} - mc_{jt}$.

2.3 Increasing Aggregate Returns in Matching

The ultimate source is the *increasing returns* that arise in the *aggregate match*. More precisely, substitute the labor market-j matching aggregator (4) into the profit function of the representative labor-market-j recruiter (1).[9] Impose symmetric equilibrium first across all submarkets i in a given labor market j, and then impose symmetric equilibrium across all labor markets j. The aggregate match that arises in labor market j is

$$N_{Mjt}^{\frac{\varepsilon}{\varepsilon-1}} \cdot m(s_{jt}, v_{jt}), \tag{17}$$

in which the $N_{Mjt}^{\frac{\varepsilon}{\varepsilon-1}}$ term represents the aggregate increasing returns.

[9] Use of the Dixit-Stiglitz aggregator in (4) is sufficient to make the point, but the result holds for other aggregators such as the Benassy aggregator and the translog aggregator.

To understand the aggregate increasing returns in matching, examine the recruiting aggregator. Continuing to use the Dixit-Stiglitz aggregator (4) for the sake of simplicity, the *perfectly-competitive* aggregate recruiter j constructs, in decentralized labor-market j, new job matches via the technology stated in (4). However, if its matching technology were the more general

$$\tilde{m}\left(N_{Mjt}, m(s_{jt}, v_{jt})\right) = \left[\int_0^{N_{Mjt}} m(s_{ijt}, v_{ijt})^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

there are constant returns to scale for the intermediate ij matches in producing the final labor-market match j for a given measure of differentiated recruiters N_{Mjt}. However, there are increasing returns to scale once N_{Mjt} is treated as an input argument to production of market-j matches, which implies that operating this $\tilde{m}(.)$ technology in the *perfectly-competitive* labor market j is infeasible. To see the increasing returns more clearly, imposing symmetry across i yields

$$\begin{aligned}
\tilde{m}\left(N_{Mjt}, m(s_{jt}, v_{jt})\right) &= \left[m(s_{jt}, v_{jt})^{\frac{\varepsilon-1}{\varepsilon}} \int_0^{N_{Mjt}} 1\, di\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[m(s_{jt}, v_{jt})^{\frac{\varepsilon-1}{\varepsilon}} \cdot N_{Mjt}\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= m(s_{jt}, v_{jt}) \cdot N_{Mjt}^{\frac{\varepsilon}{\varepsilon-1}} \\
&= m(s_{jt}, v_{jt}) \cdot \rho(N_{Mjt}) \cdot N_{Mjt}, \qquad (18)
\end{aligned}$$

from which it is clear that, given $\varepsilon < \infty$, aggregate increasing returns arises in matching. Based on this Dixit-Stiglitz example, Table 1 informs us that the increasing returns term is $N_{Mjt}^{\frac{\varepsilon}{\varepsilon-1}} = \rho(N_{Mjt}) \cdot N_{Mjt}$. But aggregate increasing returns in matching is not restricted to this particular function. Thus, the general formulation of the increasing returns component is $\rho(N_{Mt})N_{Mt}$, which takes into account other aggregators (Table 1 shows two other commonly-used aggregators).

Aggregate increasing returns in production is a well-known idea starting from at least Romer (1987). However, to the best of our knowledge, the aggregate increasing returns in production model has not been applied to recruiting markets in the way that our model does.

Monopolistic Wages.

To better understand the implicit monopolistic wage in Proposition 1, we first need to characterize the foundations of the value expressions $\mathbf{W}(w_{ijt}, \theta_{ijt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{ijt}, \theta_{ijt})$ and hence the participation constraints. Section 3 provides these foundations, from which wages arising from monopolistic intermediation can be expressed in closed form.

Variable Name	Definitions/Notes
v_{Nt}	Vacancies posted in non-intermediated matching market (i.e., random search)
s_{Nt}	Active job search in non-intermediated matching market (i.e., random search)
$\gamma(v_{ijt})$	Vacancy posting cost function for submarket ij
$\gamma_N(v_{Nt})$	Vacancy posting cost function for non-intermediated matching market
w_{Nt}	Wage for employees hired in non-intermediated matching market
θ_{Nt}	Labor-market tightness ($\equiv v_{Nt}/s_{Nt}$) in non-intermediated matching market
$k^f(\theta_{Nt})$	Probability of v_{Nt} matching in non-intermediated matching market
$k^h(\theta_{Nt})$	Probability of s_{Nt} matching in non-intermediated matching market
$p_{v_{jt}}$	Income per job vacancy v_{ijt} posted in labor market j
$p_{s_{jt}}$	Income per unit search s_{ijt} directed towards labor market j
k_t	Physical capital
r_t	Real interest rate
χ	Government-provided unemployment benefits
Π^M_{jt}	Period-t flow profits, recruiting firms in labor market j
Π^F_t	Period-t flow profits, goods-producing firms

Table 3: **Notation.** General equilibrium model.

3 General Equilibrium

We now place the partial equilibrium recruiting model into a general equilibrium framework. The general equilibrium framework characterizes the foundations of the directed search constraints faced by monopolistic recruiters. The general equilibrium framework also relaxes the assumption that recruiting is the only channel by which new job matches are created by introducing a second process for new job creation, which is the well-known Pissarides (1985) non-intermediated *random search* matching process that has become common in macroeconomic models that use the labor search and matching structure. For reference, Table 3 provides definitions of notation for the general equilibrium model.

3.1 Households

There is a continuum $[0, 1]$ of identical households. In each household, there is a continuum $[0, 1]$ of family members. In period t, each family member in the representative household has a labor-market status of employed, unemployed and actively seeking a job, or being outside the labor force. Regardless of which labor-market status a particular family member is in, each family member receives the same exact amount of consumption c_t due to full risk-sharing within each household (see Andolfatto (1996) for formal details).

The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h \left(n_t + \underbrace{(1 - k^h_{Nt}) \cdot s_{Nt}}_{=ue^N_t} + \int_0^1 \left(\int_0^{N_{Mjt}} \underbrace{(1 - k^h_{ijt}) \cdot s_{ijt}}_{=ue_{ijt}} \, di \right) dj \right) \right] \tag{19}$$

subject to the budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta) k_t + w_t(1 - \rho) n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k^h_{ijt} \cdot s_{ijt} \, di \, dj + \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt} \, di \, dj \\ &+ (1 - k^h_{Nt}) \cdot s_{Nt} \chi + \int_0^1 \int_0^{N_{Mjt}} \left(1 - k^h_{ijt}\right) \cdot s_{ijt} \chi \, di \, dj + \int_0^1 \Pi^M_{jt} \, dj + \Pi^F_t, \end{aligned} \tag{20}$$

in which Π^M_{jt} is the period-t flow profits from labor market j that the household receives lump-sum, and Π^F_t is the period-t flow profits from the goods-producing firms that the household receives lump-sum. The representative household also faces the period-t perceived law of motion of employment, which is

$$n_t = (1 - \rho) n_{t-1} + k^h_{Nt} \cdot s_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k^h_{ijt} \cdot s_{ijt} \, di \, dj. \tag{21}$$

The optimality conditions (the details of which are provided in Appendix C) that emerge are the standard Euler expression for the supply of physical capital

$$1 = E_t \left\{ \Xi_{t+1|t} \left(1 + r_{t+1} - \delta\right) \right\}, \tag{22}$$

in which $\Xi_{t+1|t} \equiv \beta u'(c_{t+1})/u'(c_t)$ denotes the stochastic discount factor, and a set of labor-force participation conditions

$$\frac{h'(lfp_t)}{u'(c_t)} = (1-k^h_{Nt})\underbrace{\chi}_{\equiv\mathbf{U}} \tag{23}$$
$$+ \quad k^h_{Nt}\underbrace{\left[w_{Nt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)\right\}\right]}_{\equiv\mathbf{W}(w_{Nt},\theta_{Nt})}$$

and

$$\frac{h'(lfp_t)}{u'(c_t)} = p_{s_{jt}} + (1-k^h_{ijt})\underbrace{\chi}_{\equiv\mathbf{U}} \tag{24}$$
$$+ \quad k^h_{ijt}\underbrace{\left[w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})}-\chi\right)-\frac{p_{s_{jt+1}}}{k^h_{jt+1}}\right]\right\}\right]}_{\equiv\mathbf{W}(w_{ijt},\theta_{ijt})} \quad \forall ij.$$

The participation function (23) characterizes endogenous, but *random*, job search in the non-intermediated labor market, whereas the set of participation functions (24) characterize endogenous *directed* job search towards intermediated labor submarket ij. Given the household-level envelope conditions, around the optimum, active job search in all submarkets must yield the same value $k^h(\theta_{ijt})\cdot\mathbf{W}(w_{ijt},\theta_{ijt}) + \left(1-k^h(\theta_{ijt})\right)\cdot\mathbf{U}(\cdot) = k^h(\theta_{kjt})\cdot\mathbf{W}(w_{kjt},\theta_{kjt}) + \left(1-k^h(\theta_{kjt})\right)\cdot\mathbf{U}(\cdot), \forall i \neq k.$

3.2 Firms

There is a continuum $[0,1]$ of identical goods-producing firms. The representative goods-producing firm's lifetime profit function is

$$E_0\sum_{t=0}^{\infty}\Xi_{t|0}\left\{\ z_t f(k_t,n_t) - r_t k_t - \gamma_N(v_{Nt})\ \right\}$$
$$-\quad E_0\sum_{t=0}^{\infty}\Xi_{t|0}\left\{\int_0^1\int_0^{N_{Mjt}}\gamma(v_{ijt})\ di\ dj - \int_0^1\int_0^{N_{Mjt}}p_{v_{jt}}\cdot v_{ijt}\ di\ dj\right\}$$
$$-\quad E_0\sum_{t=0}^{\infty}\Xi_{t|0}\left\{w_t\cdot(1-\rho)n_{t-1} + w_{Nt}\cdot k^f_{Nt}\cdot v_{Nt} + \int_0^1\int_0^{N_{Mjt}} w_{ijt}\cdot k^f_{ijt}\cdot v_{ijt}\ di\ dj\right\} \tag{25}$$

subject to the period-t perceived law of motion of employment

$$n_t = (1-\rho)n_{t-1} + k^f_{Nt}\cdot v_{Nt} + \int_0^1\int_0^{N_{Mjt}} k^f_{ijt}\cdot v_{ijt}\ di\ dj. \tag{26}$$

Profit-maximization (see Appendix B for the formal analysis) leads to the set of job-creation conditions

$$\gamma'_N(v_{Nt}) = k^f_{Nt} \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma'_N(v_{Nt+1})}{k^f_{Nt+1}} \right\} \right)}_{\equiv \mathbf{J}(w_{Nt}, \theta_{Nt})}, \tag{27}$$

and

$$\gamma'(v_{ijt}) = p_{v_{jt}} + k^f_{ijt} \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f_{jt+1}} \right) \right\} \right)}_{\equiv \mathbf{J}(w_{ijt}, \theta_{ijt})} \ \forall ij. \tag{28}$$

The job-creation condition (27) characterizes endogenous, but *random*, vacancy postings in the non-intermediated labor market, whereas the set of job-creation condition (28) characterize endogenous *directed* vacancy postings in intermediated labor submarkets ij. Around the optimum, the firm is indifferent between directing new job vacancies to intermediated submarket i or intermediated submarket k, $k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}) = k^f(\theta_{kjt}) \cdot \mathbf{J}(w_{kjt}, \theta_{kjt})$, $\forall i \neq k$.

3.3 Wage Determination

Wages in Intermediated Labor Market (Directed Search).

With the foundations of the value expressions $\mathbf{W}(w_{ijt}, \theta_{ijt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{ijt}, \theta_{ijt})$ for wage determination now in place, we can express the wage implicit in the monopolistically-competitive surplus sharing condition in Proposition 1 in explicit form. Substitution of the (symmetric equilibrium) value expressions $\mathbf{W}(w_t, \theta_t)$, $\mathbf{U}_t$, and $\mathbf{J}(w_t, \theta_t)$ into (16) yields the (symmetric equilibrium) explicit-form wage

$$\begin{aligned} w_t &= \xi z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho) E_t \left\{ \Xi_{t+1|t} \gamma'(v_{t+1}) \cdot \theta_{t+1} \right\} \\ &- \ \xi(1-\xi) \left[\left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) - (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\rho(N_{Mt+1}) - \frac{\rho(N_{Mt+1})}{\mu(N_{Mt+1})} \right) \right\} \right], \end{aligned} \tag{29}$$

the algebraic details for which appear in Appendix D. If the recruiting market were perfectly competitive ala Moen (1997), then $\rho(N_{Mt}) = \frac{\rho(N_{Mt})}{\mu(N_{Mt})} (= mc(N_{Mt})) \ \forall ij$ and $p_{v_t} = m_v(\cdot)$, in which case the real wage is characterized by the completely-standard first line of (29). However, if the recruiting market is monopolistically competitive, then it is not only the period-t profits accruing to the recruiting sector that affect the period-t wage, *period-*$(t+1)$ *profits also affect the period-t wage*, despite the fact that monopolistic recruiters only make static decisions. The reason that recruiters' period-$t+1$ rents affect the period-t wage is the long-lasting nature of employment relationships.

Nash-Bargained Wages in Non-Intermediated Labor Market (Random Search).

We assume that the wage model in the non-intermediated labor market is generalized Nash bargaining. Without going into details (which can easily be found in a textbook such as Pissarides (2000, Chapter 1)), the Nash surplus-sharing condition is

$$\mathbf{W}(w_{Nt}, \theta_{Nt}) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right) \mathbf{J}(w_{Nt}, \theta_{Nt}), \tag{30}$$

in which $\eta \in (0, 1)$ denotes the potential new employee's generalized Nash bargaining power. Substitution of the value expressions $\mathbf{W}(w_{Nt}, \theta_{Nt})$, $\mathbf{U}_t$, and $\mathbf{J}(w_{Nt}, \theta_{Nt})$ yields the explicit-form wage

$$w_{Nt} = \eta \cdot z_t f_n(k_t, n_t) + (1-\eta) \cdot \chi + \eta(1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\gamma_N'(v_{Nt+1}) \cdot \theta_{Nt+1} \right) \right\} \tag{31}$$

in non-intermediated *random search* labor markets.

3.4 Aggregate Employment

The aggregate law of motion for employment

$$n_t = (1-\rho) n_{t-1} + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) + m(s_{Nt}, v_{Nt}) \tag{32}$$

takes into account both new job matches produced by the intermediated labor market — which, as described in the partial equilibrium model in Section 2, leads to aggregate increasing returns in matching — and the non-intermediated labor market.

3.5 Government

The (symmetric equilibrium) flow budget constraint of the government is

$$T_t = g_t + (1 - k^h(\theta_t)) \cdot s_t \cdot N_{Mt} \cdot \chi + (1 - k^h(\theta_{Nt})) \cdot s_{Nt} \cdot \chi, \tag{33}$$

in which lump-sum taxes T_t levied on households finance government-provided unemployment benefits and government spending g_t.

3.6 Aggregate Goods Resource Constraint

The decentralized economy's aggregate goods resource constraint

$$\begin{aligned} & c_t + k_{t+1} - (1-\delta) k_t + \gamma(v_t) \cdot N_{Mt} \\ + \quad & \gamma_N(v_{Nt}) + \Gamma_{Mt} N_{MEt} - \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t f(k_t, n_t), \end{aligned} \tag{34}$$

the derivation of which appears in Appendix E. Note that the aggregate increasing returns term in matching, $\rho(N_{Mt})N_{Mt}m(s_t,v_t)$, appears in the goods resource constraint.

3.7 Private-Sector Equilibrium

The period-t state of the economy is $S_t \equiv [n_{t-1}, N_{Mt-1}, k_t, z_t]$. A symmetric private-sector general equilibrium is made up of seventeen endogenous state-contingent processes $\{c_t, n_t, lfp_t, k_{t+1}, N_{Mt}, N_{MEt}, s_t, v_t, \theta_t, w_t, s_{Nt}, v_{Nt}, \theta_{Nt}, w_{Nt}, mc_t, p_{v_t}, p_{s_t}\}_{t=0}^{\infty}$ that satisfy the following seventeen conditions: the aggregate resource constraint

$$\begin{aligned} & c_t + k_{t+1} - (1-\delta)k_t + \gamma(v_t)\cdot N_{Mt} \\ + \quad & \gamma_N(v_{Nt}) + \Gamma_{Mt}N_{MEt} - \rho(N_{Mt})\cdot N_{Mt}\cdot m(s_t,v_t) = z_t f(k_t,n_t), \end{aligned} \tag{35}$$

the aggregate law of motion for labor

$$n_t = (1-\rho)n_{t-1} + m(s_{Nt},v_{Nt}) + \rho(N_{Mt})\cdot N_{Mt}\cdot m(s_t,v_t), \tag{36}$$

the definition of aggregate LFP

$$lfp_t = (1-\rho)n_{t-1} + s_{Nt} + s_t\cdot N_{Mt}, \tag{37}$$

the aggregate law of motion for recruiters

$$N_{Mt} = (1-\omega)N_{Mt-1} + N_{MEt}, \tag{38}$$

the capital Euler condition

$$1 = E_t\left\{\Xi_{t+1|t}\left(1 + z_{t+1}f_k(k_{t+1},n_{t+1}) - \delta\right)\right\}, \tag{39}$$

the free-entry condition for recruiters

$$\Gamma_{Mt} = (\rho(N_{Mt}) - mc(N_{Mt}))\cdot m(s_t,v_t) + (1-\omega)E_t\left\{\Xi_{t+1|t}\Gamma_{Mt+1}\right\}, \tag{40}$$

the vacancy creation condition directed towards monopolistically-competitive labor markets

$$\gamma'(v_t) = p_{v_t} + k_t^f\cdot\underbrace{\left(z_t f_n(k_t,n_t) - w_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma'(v_{t+1}) - p_{v_{t+1}}}{k_{t+1}^f}\right)\right\}\right)}_{\equiv \mathbf{J}(w_t,\theta_t)}, \tag{41}$$

the vacancy creation condition for non-intermediated random search labor markets

$$\gamma_N'(v_{Nt}) = k_{Nt}^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma_N'(v_{Nt+1})}{k_{Nt+1}^f}\right\}\right)}_{\equiv \mathbf{J}(w_{Nt},\theta_{Nt})}, \tag{42}$$

the active job search condition directed towards monopolistically-competitive labor markets

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= p_{s_t} + (1-k_t^h)\underbrace{\chi}_{\equiv \mathbf{U}} \\ &+ \quad k_t^h \underbrace{\left[w_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\left(\frac{1-k_{t+1}^h}{k_{t+1}^h}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right) - \frac{p_{s_{t+1}}}{k_{t+1}^h}\right]\right\}\right]}_{\equiv \mathbf{W}(w_t,\theta_t)} \ \forall ij,\end{aligned} \tag{43}$$

the active job search condition for non-intermediated random search labor markets

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= (1-k_{Nt}^h)\underbrace{\chi}_{\equiv \mathbf{U}} \\ &+ \quad k_{Nt}^h \underbrace{\left[w_{Nt} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k_{Nt+1}^h}{k_{Nt+1}^h}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right]}_{\equiv \mathbf{W}(w_{Nt},\theta_{Nt})},\end{aligned} \tag{44}$$

the surplus-sharing rule that determines wages w_t in monopolistic labor markets

$$\xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + \mathbf{W}(w_t) - \mathbf{U}_t = \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_t), \tag{45}$$

the surplus-sharing rule that determines Nash-bargained wages (with η denoting the employee's Nash bargaining power) in non-intermediated labor markets

$$\mathbf{W}(w_{Nt}) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w_{Nt}), \tag{46}$$

the monopolistic matching-market pricing condition

$$\rho(N_{Mt}) = \mu(N_{Mt}) \cdot mc(N_{Mt}), \tag{47}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t = \frac{v_t}{s_t}, \tag{48}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_{Nt} = \frac{v_{Nt}}{s_{Nt}}, \tag{49}$$

along with the equilibrium input prices

$$p_{v_t} = m_v(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \tag{50}$$

and

$$p_{s_t} = m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right), \tag{51}$$

taking as given the exogenous stochastic process for z_t and the initial conditions $n_{-1}, N_{M,-1}, k_0$.

4 General Equilibrium I — Analytical Results

Well known by those who work in the labor search literature is that (sans other distortions) the Mortensen-Hosios condition $\eta = \xi$ for Nash-bargained wages supports efficient allocations because it eliminates congestion externalities.[10] Well known by those who work with monopolistically-competitive models with endogenous entry is that (sans other distortions) Dixit-Stiglitz aggregation is efficient in that it eliminates the competing effects of incentives for entry and aggregate increasing returns, leading to an *efficient* number of monopolistically-competitive firms.[11]

In our model, *both* search-based congestion externalities *and* possible misalignment of aggregate increasing returns and incentives for entry are possible *in the same market.* As is formally stated below, a congestion externality (i.e., $\eta \neq \xi$) in the random-search and bargaining channel of new job creation *directly* leads to an *inefficient* number of monopolistically-competitive recruiting agencies *even if Dixit-Stiglitz aggregation holds.* We note that the results in Proposition 2 and Lemma 1 are for the static version of the model (in which the separation rates for employment and recruiters are, respectively, $\rho = 1$ and $\omega = 1$), not for the steady state of the fully dynamic model.

4.1 Spillover Effects on Monopolistic Recruiting Markets

As stated, respectively, in Proposition 2 and Lemma 1 for the static model, efficient Nash-bargained wages in new job matches via the random-search channel create *no* distortions in efficient monopolistic recruiting markets, whereas *in*efficient Nash-bargained wages create *inefficiencies* in monopolistic recruiting markets even though matching aggregation is Dixit-Stiglitz.

[10] As in Section 3, $\eta \in (0, 1)$ denotes the generalized Nash-bargaining power of new workers and $\xi \in (0, 1)$ denotes the elasticity of new matches with respect to the number of job seekers.

[11] A helpful recent review of this latter result is provided by Bilbiie, Ghironi, and Melitz (2008).

Proposition 2. ***Efficient Non-Intermediated Labor*** $\Rightarrow$ ***Efficient Intermediated Labor Markets.*** *Suppose the matching aggregator in monopolistic recruiting markets is Dixit-Stiglitz, and assume that* $\rho = 1$ *and* $\omega = 1$*. Both* N_M *and* θ *are efficient and are maximized if wages are determined efficiently* ($\eta = \xi$) *in new job matches created through random search,*

$$\frac{\partial N_M^*}{\partial \eta} = \frac{\partial \theta^*}{\partial \eta} = 0 \text{ if } \eta = \xi \text{ (Hosios)} \tag{52}$$

in which the asterisks denote maximized values.

Proof. See Appendix G. □

Lemma 1. ***Inefficient Non-Intermediated Labor*** $\Rightarrow$ ***Inefficient Intermediated Labor Markets.*** *Suppose the matching aggregator in monopolistic recruiting markets is Dixit-Stiglitz, and assume that* $\rho = 1$ *and* $\omega = 1$*. Both* N_M *and* θ *are below their efficient, maximized values if wages are inefficiently low in new job matches created through random search,*

$$\frac{\partial N_M^*}{\partial \eta} > 0 \text{ and } \frac{\partial \theta^*}{\partial \eta} > 0 \text{ iff } \eta < \xi \tag{53}$$

or are above their efficient, maximized levels if wages are inefficiently high in new job matches created through random search,

$$\frac{\partial N_M^*}{\partial \eta} < 0 \text{ and } \frac{\partial \theta^*}{\partial \eta} < 0 \text{ iff } \eta > \xi, \tag{54}$$

in which the asterisks denote maximized values.

Proof. See Appendix G. □

As mentioned above, the results in Proposition 2 and Lemma 1 are for the static version of the model, not for the steady state of the fully dynamic model. While it may not be impossible to prove the results in Proposition 2 and Lemma 1 for the steady state of the fully dynamic model, it is, from a technical standpoint, challenging because the steady-state counterpart requires use of implicit differentiation in the very first step of the proof, whereas the static model does not immediately require (i.e., in the first step of the proof) implicit differentiation.[12]

From an economic standpoint, though, Proposition 2 and Lemma 1 shed much insight because the prime focus of the search and matching framework regards *new* job creation. The intuition behind the non-monotonic patterns in the number of recruiting firms and recruiting-market tightness that Proposition 2 and Lemma 1 jointly imply is as follows. As the bargaining power in the non-intermediated market increases from an initially low level (i.e., η is lower than ξ), households

[12] Footnote 25 in Appendix G elaborates further on this point.

prefer to direct their search towards the market where their share of the surplus is expanding (i.e., the non-intermediated market). As a result, search in intermediated markets falls.

To offset this fall, recruiting firm entry rises since firms continue to post vacancies across markets, with the end result being a rise in intermediated market tightness. As the workers' bargaining power becomes increasingly higher, it becomes increasingly difficult to find employment in non-intermediated markets. There are two forces at play. First, as workers' bargaining power gets closer to "take-it-or-leave-it" offers (i.e., $\eta = 1$), this encourages potential new employees to continue to search in non-intermediated markets. Second, this same fact simultaneously leads to decreased job-finding probabilities as firms further reduce their non-intermediated market vacancies. This latter effect pushes households to start increasing their search in intermediated markets, ultimately leading to an increase in intermediated-market household searchers.

A similar rationale holds when we consider the behavior of vacancies in these markets. As the bargaining power of workers initially increases, firms decide to hire via the market where the bargaining power is not affecting how the employment surplus is split, but for high levels of the bargaining power, the job-filling probabilities are increasingly influenced by the high measure of searchers, implying that firms do not need to post as many vacancies to generate a given number of matches. As a result, intermediated-market vacancies start to decline as the bargaining power of workers approaches. Finally, as the bargaining power of workers increases and gets closer to 1, unemployment increases, and recruiting firms find it less profitable to participate in matching markets. This ultimately leads to a decline in the number of recruiting firms.

The economic insights of the "static" analytical results remain intact in the quantitative results, as shown quantitatively in Section 5.

4.2 Lack of Spillover Effects on Random Search

Before proceeding to the quantitative results, however, a next natural question is whether causality runs in the other direction. In our model, the causality of inefficiencies in Proposition 2 and Lemma 1 does *not* run in the opposite direction. More precisely, suppose that Nash bargaining power is $\eta = \xi$. If the matching aggregator in monopolistic recruiting markets were *not* Dixit-Stiglitz — suppose it were instead the Benassy aggregator or the translog aggregator, in which the incentives for entry and the welfare benefits of aggregate increasing returns are misaligned[13] — the inefficient aggregation does *not* lead to an inefficiency in non-intermediated new job creation.

Intuitively, this uni-directional causality is due to the presence of endogenous (monopolistically-competitive) recruiting firm entry in intermediated markets, and incidentally, the *asymmetry* in the degree of competition between labor markets. Indeed, while both markets have vacancies and

[13] We again refer to Bilbiie, Ghironi, and Melitz (2008) for a useful review.

the measure of searchers as key margins of adjustment amid changes in the degree of congestion externalities in non-intermediated labor markets (and/or the degree of inefficiency in intermediated markets), intermediated markets have a third *critical* margin, which is the *endogenous* measure of recruiting firms. This implies that, in relative terms, the non-intermediated market will be less responsive to changes in intermediated markets. Of note, as suggested by our quantitative analysis based on a fully dynamic model, this uni-directional causality holds beyond a static environment.

5 General Equilibrium II — Quantitative Results

5.1 Empirical Targets and Calibration

We assume log utility with respect to consumption, $u(c) = \log c$. In turn, the disutility from participation is given by $h(lfp) = \kappa \cdot lfp^{1+\frac{1}{\iota}} / \left(1 + \frac{1}{\iota}\right)$, where $\kappa, \iota > 0$. The production function is Cobb-Douglas, $f(k, n) = k^{\alpha} n^{1-\alpha}$, with $0 < \alpha < 1$. All matching functions are also Cobb Douglas, $m(s, v) = m \cdot s^{\xi} v^{1-\xi}$ and $m(s_N, v_N) = m_N \cdot (s_N)^{\xi} (v_N)^{1-\xi}$, in which ξ is the matching elasticity with respect to active jobs searchers and m and m_N denote, respectively, the exogenous matching efficiency parameters in the intermediated and non-intermediated labor market. This implies that the matching probabilities in the non-intermediated labor market are given by $k_N^h = m(s_N, v_N)/s_N$ and $k_N^f = m(s_N, v_N)/v_N$. The corresponding matching probabilities in the intermediated labor market take into account the increasing-returns-to-scale nature of the market, so that $k^h = N_M \cdot m(s, v)/s$ and $k^f = N_M \cdot m(s, v)/v$. Finally, we allow for the possibility of convex vacancy posting costs by using the functions $\gamma(v) = \gamma \cdot v^{\eta_v}$ and $\gamma_N(v_N) = \gamma_N \cdot v_N^{\eta_v}$, with $\gamma, \gamma_N > 0$ and $\eta_v \geq 1$.

A period in the model represents a quarter. Following the search and matching and business cycle literatures, we set the capital share to $\alpha = 0.40$, the subjective discount factor to $\beta = 0.99$, the capital depreciation rate $\delta = 0.02$, and the participation elasticity parameter $\iota = 0.18$ (Arseneau and Chugh (2012)). Turning to the labor market parameters, we set the quarterly exogenous separation probability to $\rho = 0.10$, the matching elasticity $\xi = 0.40$, and the Nash bargaining power for workers in non-intermediated labor markets to $\eta = 0.40$. We normalize steady-state aggregate productivity z to 1. Finally, we assume linear vacancy creation costs so that $\eta_v = 1$.

The novel block of the model is monopolistically-competitive intermediation in one of the two matching markets. We set the exit rate of recruiting firms to $\omega = 0.05$. For the matching aggregator function, we set the elasticity of substitution $\varepsilon = 6$ (which results in a 20 percent steady-state markup in the recruiting sector) when using Dixit-Stiglitz and, when using the translog case, we calibrate the translog parameter σ to target the same 20 percent steady-state net markup, which yields $\sigma = 7.1$, and the potential space of the universe of recruiting agencies $\tilde{N}_M$ to a sufficiently high value so that it affects neither the model's steady state nor its dynamics.[14]

We initially assume that $\gamma = \gamma_N$. Then, we calibrate the remaining parameters $\gamma(= \gamma_N)$, χ, κ, m, m_N, and Γ_{Mt} to match the following steady-state targets: a job-finding probability in the non-intermediated market of 0.6, a job-filling probability in the non-intermediated market of 0.7, a labor force of 0.74, a value for unemployment benefits representing 0.40 of average wages, a share of intermediated-market matches in total matches of 0.40, and an entry cost of 0.1. This calibration implies that the total resource cost from vacancy postings and recruiting-firm creation is close to 5 percent of total output. For ease of reference, Table 4 summarizes the baseline parameters. A point that will be discussed further in Section 6 is that the baseline parameters are *not* chosen in a way that purposefully allows for endogeneity of the intermediated labor market. Endogeneity of the monopolistic recruiting sector is an inherent property of the model (both in general equilibrium and in the partial equilibrium labor market).

5.2 Steady-State Analysis

To understand how potential asymmetries between intermediated and non-intermediated labor markets affect labor market and macro outcomes, we consider changes in two key sets of parameters: the Nash bargaining power of workers in the non-intermediated labor market, and the cost of posting vacancies in non-intermediated and intermediated markets, respectively.

Nash bargaining power of workers in the non-intermediated labor market.

As shown in Figures 3, 4, and 5, larger bargaining power for workers generates monotonic increases in unemployment and search in non-intermediated labor markets as well as monotonic reductions in vacancy postings and market tightness. These results are well known from standard search models and are intuitive: higher bargaining power implies that households extract a larger share of the surplus from employment relationships, which leads to not only increased household search behavior but also to a reduction in firms' incentive to create vacancies and ultimately sectoral market tightness.

In contrast, the intermediated labor market exhibits non-monotonic changes in its corresponding variables (this also applies to labor force participation). More importantly, the sign of the slope changes *when the Hosios condition in the non-intermediated market holds* ($\xi = \eta = 0.4$), as was formalized (albeit in a static framework) in Proposition 2 and Lemma 1. As discussed in Section 4, for low bargaining power, the number of recruiting firms, labor-market tightness intermediated markets, and vacancies are all increasing in the bargaining power of workers. Conversely, for high levels of bargaining power, all three of these variables are decreasing in bargaining power.

[14] The precise setting is $\tilde{N}_M = 10^8$, which is orders of magnitude larger than needed so that its precise setting does affect the model's steady state or dynamics.

Parameter	Value	Description
		Recruiting Sector
ε	6	Elasticity of substitution for Dixit-Stiglitz aggregator
σ	7.1	Calibrating parameter for translog aggregator
ω	0.05	Quarterly exogenous exit rate of recruiters
		Utility
β	0.99	Quarterly subjective discount factor
κ	4.58	Scaling parameter for $h(.)$
ι	0.18	Wage elasticity of lfp
		Goods Production
α	0.40	Elasticity of Cobb-Douglas goods production function $f(k, n)$ with respect to k
δ	0.02	Quarterly depreciation rate of physical capital
		Labor Market
ρ	0.10	Quarterly exogenous separation of jobs
ξ	0.40	Elasticity of Cobb-Douglas matching technology $m(s, v)$ with respect to s
η	0.40	Generalized Nash bargaining power for workers in non-intermediated labor markets

Table 4: **Baseline Parameters.**

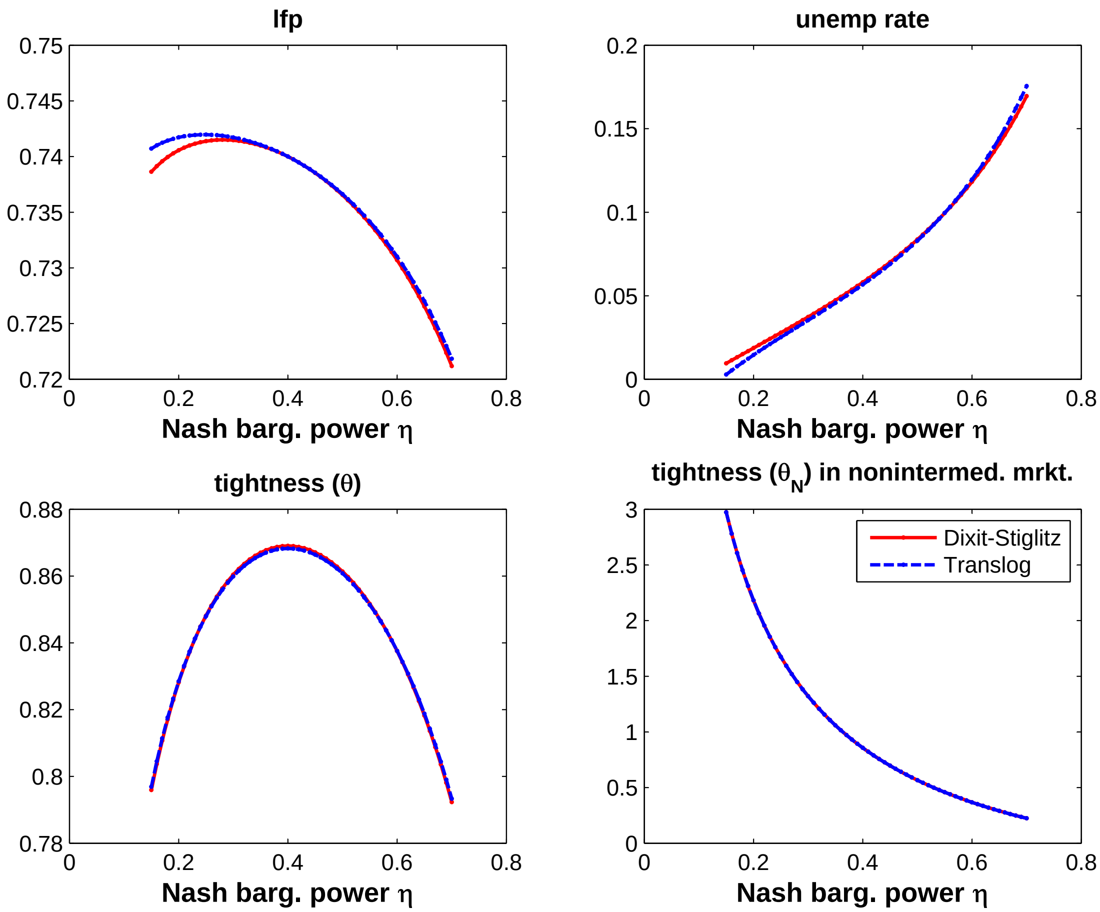

Figure 3: **Steady state as function of worker Nash wage bargaining power η in non-intermediated labor market I.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides η are held at their baseline values.

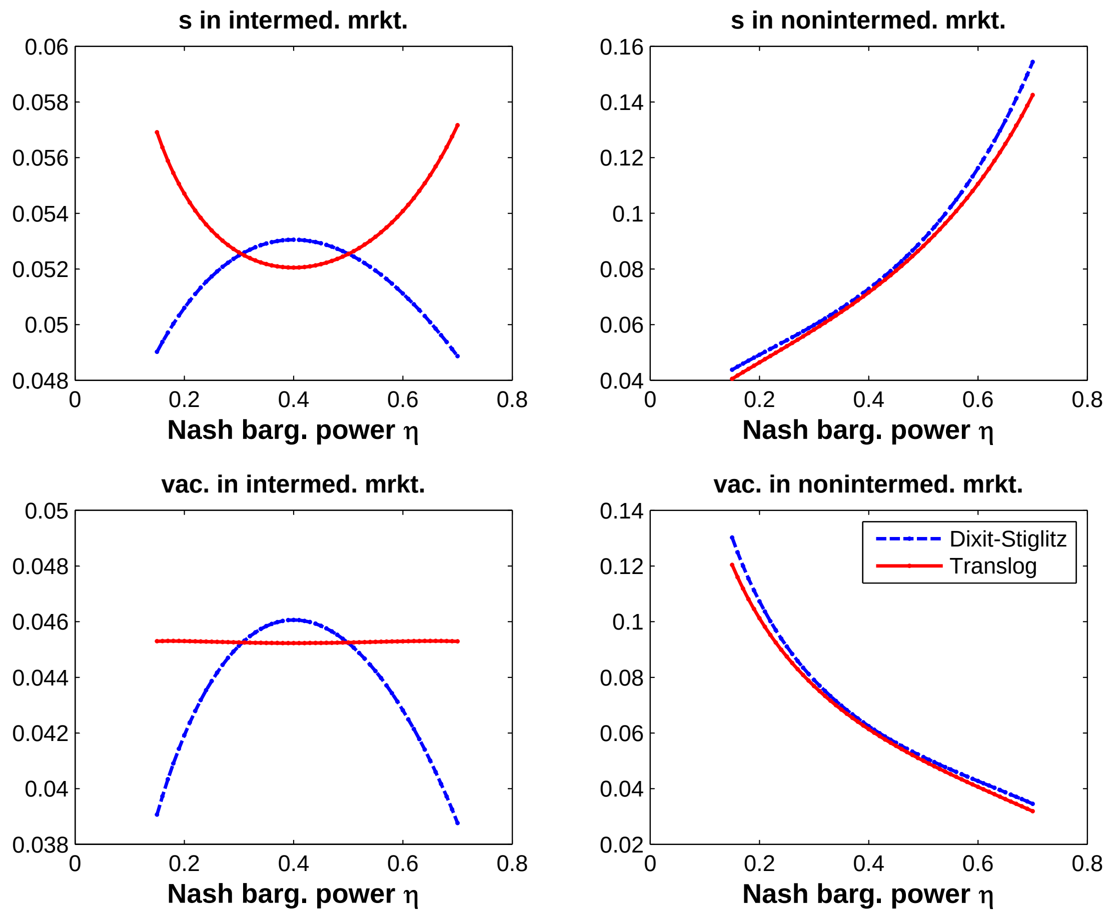

Figure 4: **Steady state as function of worker Nash wage bargaining power η in non-intermediated labor market II.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides η are held at their baseline values.

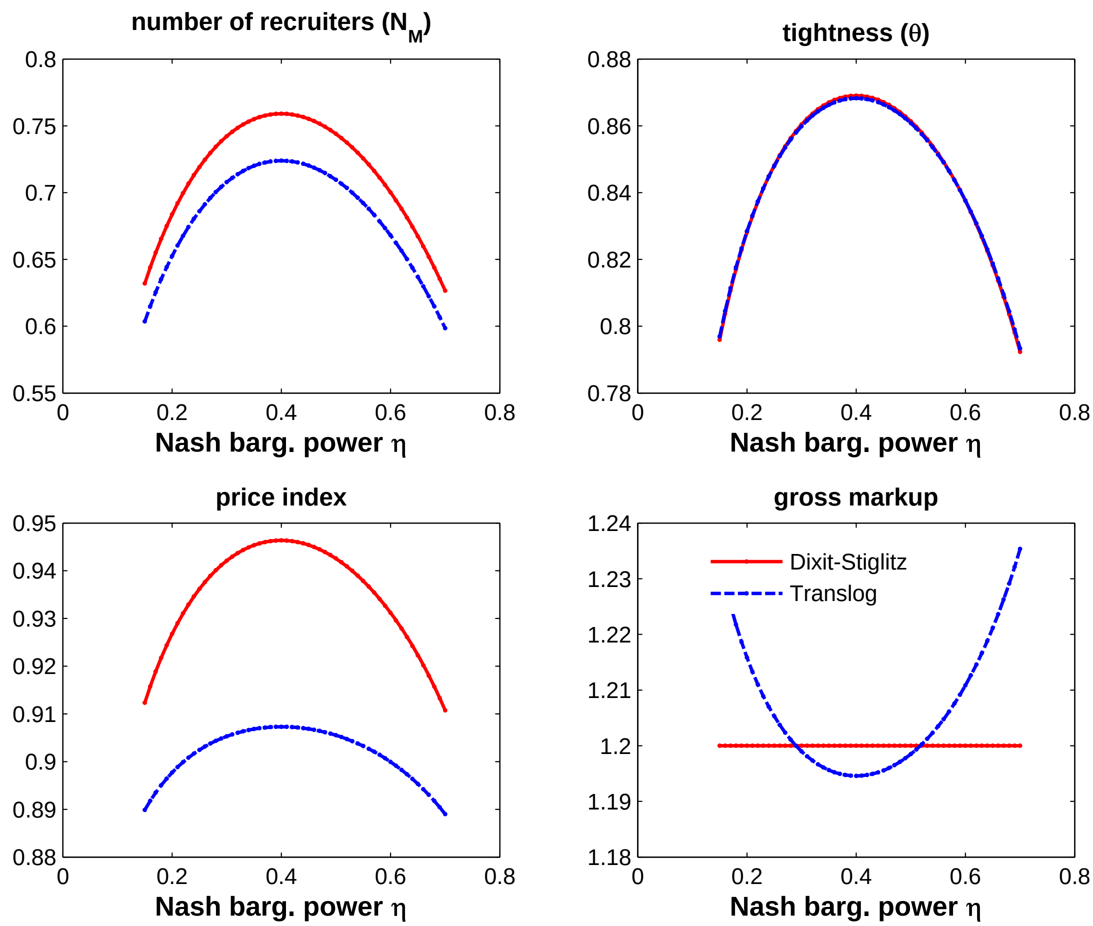

Figure 5: **Steady state as function of worker Nash wage bargaining power η in non-intermediated labor market III.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides η are held at their baseline values.

Vacancy Posting Costs in Non-Intermediated Markets.

For illustrative purposes, we consider a reduction in the flow vacancy posting cost γ_N of roughly one third from its baseline value (of 3.5). The steady-state implications (under both Dixit-Stiglitz and translog aggregation) are shown in Figures 6, 7, and 8. Specifically, a *reduction* in γ_N generates marginal changes in labor force participation and a reduction (increase) in unemployment (market tightness across matching markets).[15] Interestingly, lower vacancy posting costs in *non-intermediated* markets stimulates firm entry into the recruiting sector, leading to an expansion in the number of recruiter firms. Intuitively, this implies that for a given *individual* matching probability in intermediated markets, the likelihood of a match with a larger number of recruiting firms is higher, which rationalizes the behavior of vacancy postings. In particular, for the translog case, the rise in intermediated-market vacancies is such that, in equilibrium, firms prefer to reduce vacancies in non-intermediated markets, such that the rise in employment under this scenario is driven by the intermediated market.[16]

[15]While the increase in market tightness in both markets under lower vacancy posting costs in non-intermediated markets is driven by a reduction in searchers under Dixit-Stiglitz aggregation, the rise in intermediated market tightness under translog aggregation is explained by a sharper rise in intermediated-market vacancies. In contrast, the rise in non-intermediated market tightness under the same aggregation is driven by a sharper reduction in searchers.

[16]A similar claim can be made under Dixit-Stiglitz aggregation since the expansion in recruiting firms lead to an overall expansion in *total* intermediated markets even if household searchers search less under a lower γ_N.

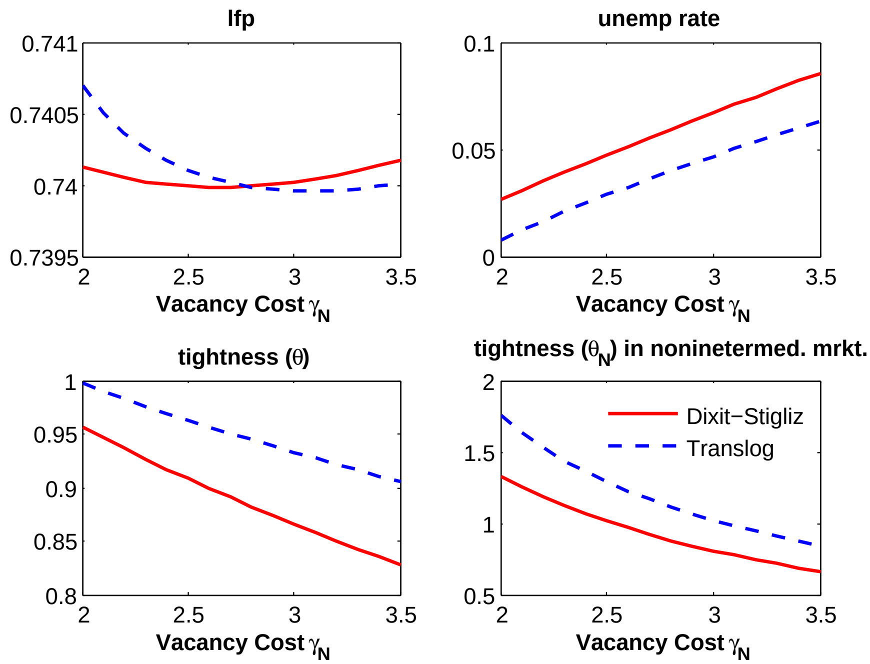

Figure 6: **Steady state as function of Vacancy Posting Cost γ_N in non-intermediated labor market I.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides γ_N are held at their baseline values.

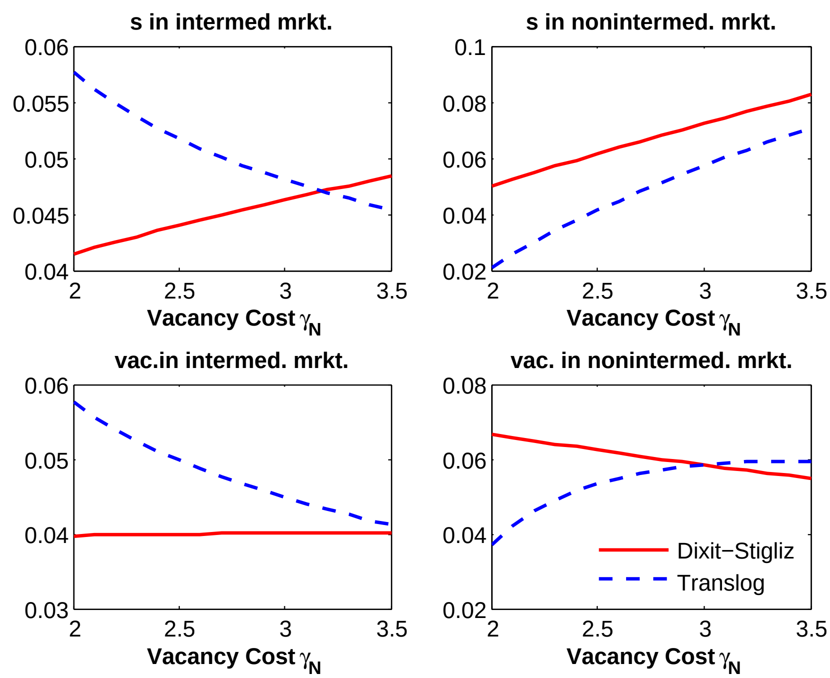

Figure 7: **Steady state as function of Vacancy Posting Cost γ_N in non-intermediated labor market II.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides γ_N are held at their baseline values.

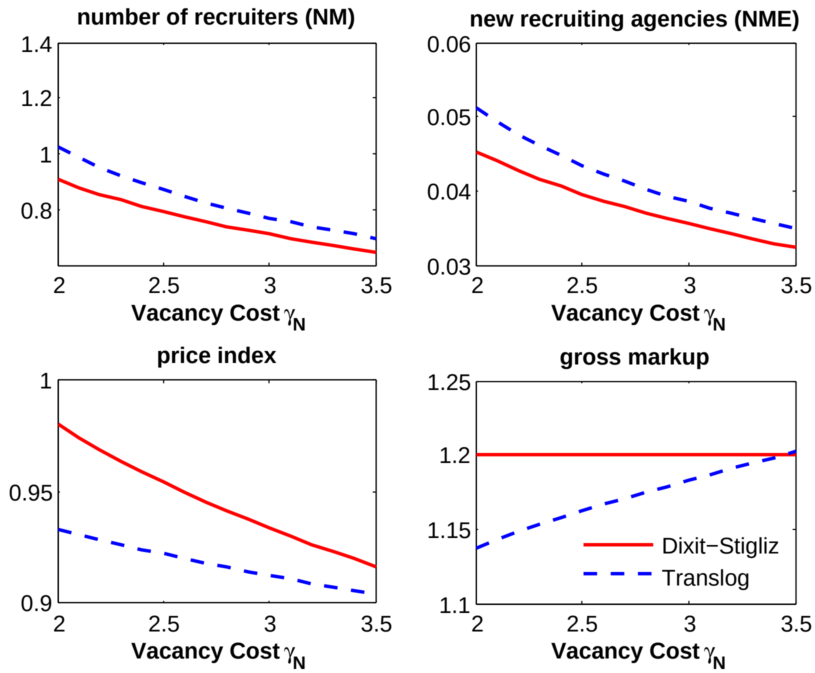

Figure 8: **Steady state as function of Vacancy Posting Cost γ_N in non-intermediated labor market III.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides γ_N are held at their baseline values.

Vacancy Posting Costs in Intermediated Market.

Similar to the change in γ_N, we now consider a reduction in the vacancy posting cost γ of roughly one third from its baseline value (of 3.5). The steady-state implications are shown in Figures 9, 10, and 11. Importantly, note that compared to a reduction in the cost of posting vacancies in non-intermediated markets, a reduction in γ leads to *qualitatively* different changes in unemployment, non-intermediated market tightness, searchers, and vacancies, and the number of recruiting firms. Indeed, a reduction in γ leads to virtually no changes in unemployment (or to a small increase under translog aggregation), an increase in both non-intermediated vacancies and searchers, and a reduction in recruiting firms. Intuitively, non-intermediated-market wages (not shown) are less responsive, in relative terms, compared to intermediated-market wages, where the latter increase. This implies that, despite the reduction in γ, firms increase non-intermediated vacancies by comparatively more, thereby pushing households to redirect their search into non-intermediated markets. As a result, the incentive to create recruiting firms falls. Importantly, the contraction in the recruiting sector is largely responsible for the negligible change in unemployment, where the latter stands in stark contrast to the results under a reduction in γ_N.

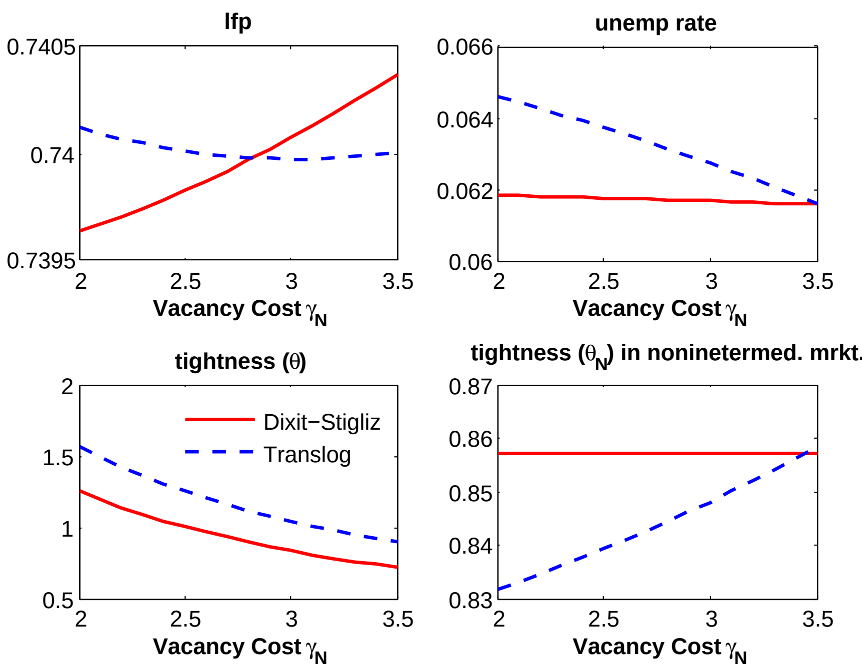

Figure 9: **Steady state as function of Vacancy Posting Cost γ in intermediated labor market I.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides γ_N are held at their baseline values.

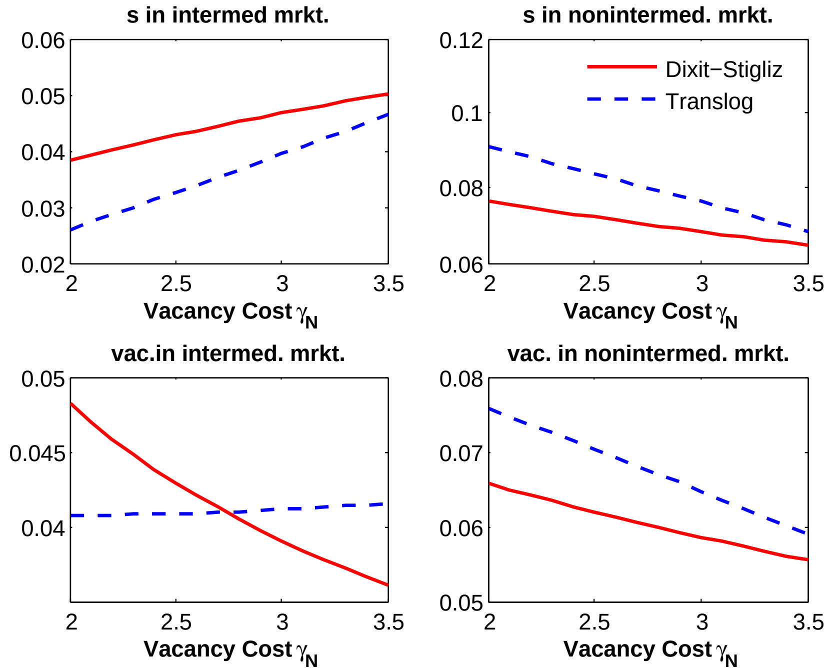

Figure 10: **Steady state as function of Vacancy Posting Cost γ in intermediated labor market II.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides γ_N are held at their baseline values.

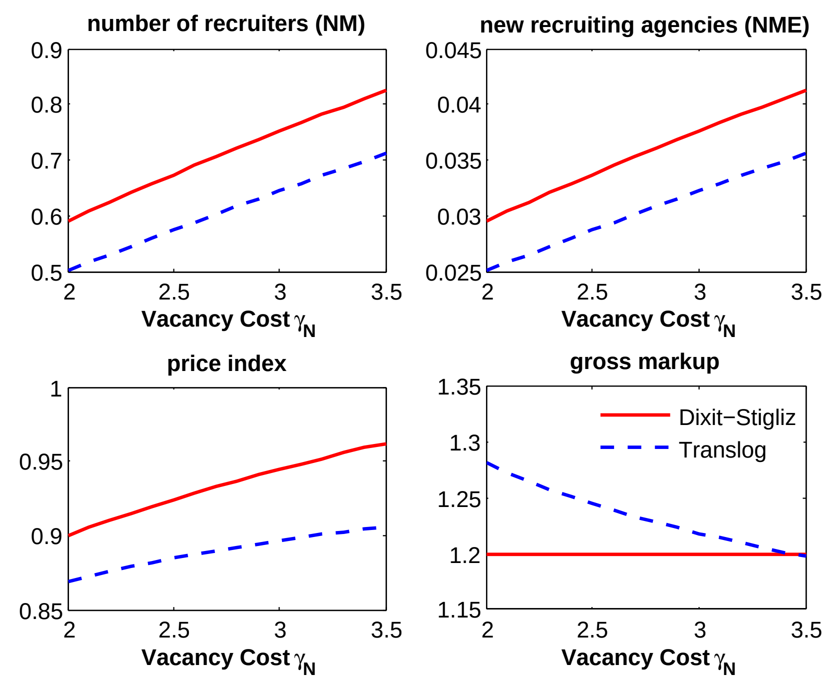

Figure 11: **Steady state as function of Vacancy Posting Cost γ in intermediated labor market III.** Solid red line is Dixit-Stiglitz aggregation, dashed blue line is translog aggregation. All other parameters besides γ_N are held at their baseline values.

5.3 Impulse Responses to a Positive TFP Shock

To analyze the response to a temporary shock to TFP, we follow the literature and consider an AR(1) process for TFP with persistence parameter 0.95 and a standard deviation of shocks equal to 0.007.[17]

Figure 12 and Figure 13 display impulse responses to a positive one-standard deviation TFP shock for, respectively, Dixit-Stiglitz aggregation in the recruiting sector and translog aggregation in the recruiting sector.

Baseline Economy. Consider a temporary increase in TFP under the baseline calibration of the model. An increase in TFP increases the marginal product of capital and labor and therefore pushes production firms to increase vacancy postings. Note that firms increase both intermediated and non-intermediated market vacancies, with the latter increasing by more than the former (i.e., vacancy postings in intermediated matching markets are more sensitive to TFP shocks than those taking place via intermediated matching markets). Intuitively, the non-intermediated matching market has lower steady-state matching efficiency. As a result, for a given positive shock to TFP and in relative terms, production firms gain comparably more by posting vacancies via this market compared to the intermediated market, which in turn explains why non-intermediated market vacancies increase by more. In response to the sharper increase in these vacancies, households redirect their searchers towards non-intermediated markets, resulting in an increase in searchers in the latter and a reduction in searchers in intermediated matching markets (not shown).

The above response in search behavior on the part of households explains the fact that the measure of new recruiters falls, leading to a reduction in active recruiters as well. Intuitively, while production firms continue to post vacancies in both matching markets, households redirect their search towards non-intermediated markets which, all else equal, lowers the effective probability of a match from the perspective of recruiting firms. In turn, this leads to a lower incentive in recruiting firm creation, despite the fact that production firms continue to post vacancies across matching markets amid temporarily higher TFP.

Finally, note that increased labor demand leads to higher labor force participation, which is driven by the increase in search for employment via non-intermediated matching markets. However, the boost in non-intermediated vacancy postings is larger than the increase in search activity, which ultimately leads to a reduction in unemployment. All told, positive TFP shocks lead to increases in GDP, investment, consumption, wages, and labor force participation, as well as lower unemployment and to a smaller number of recruiting firms (i.e., a more concentrated recruiting sector).

[17] We log-linearize the model and implement a first-order approximation of the equilibrium conditions.

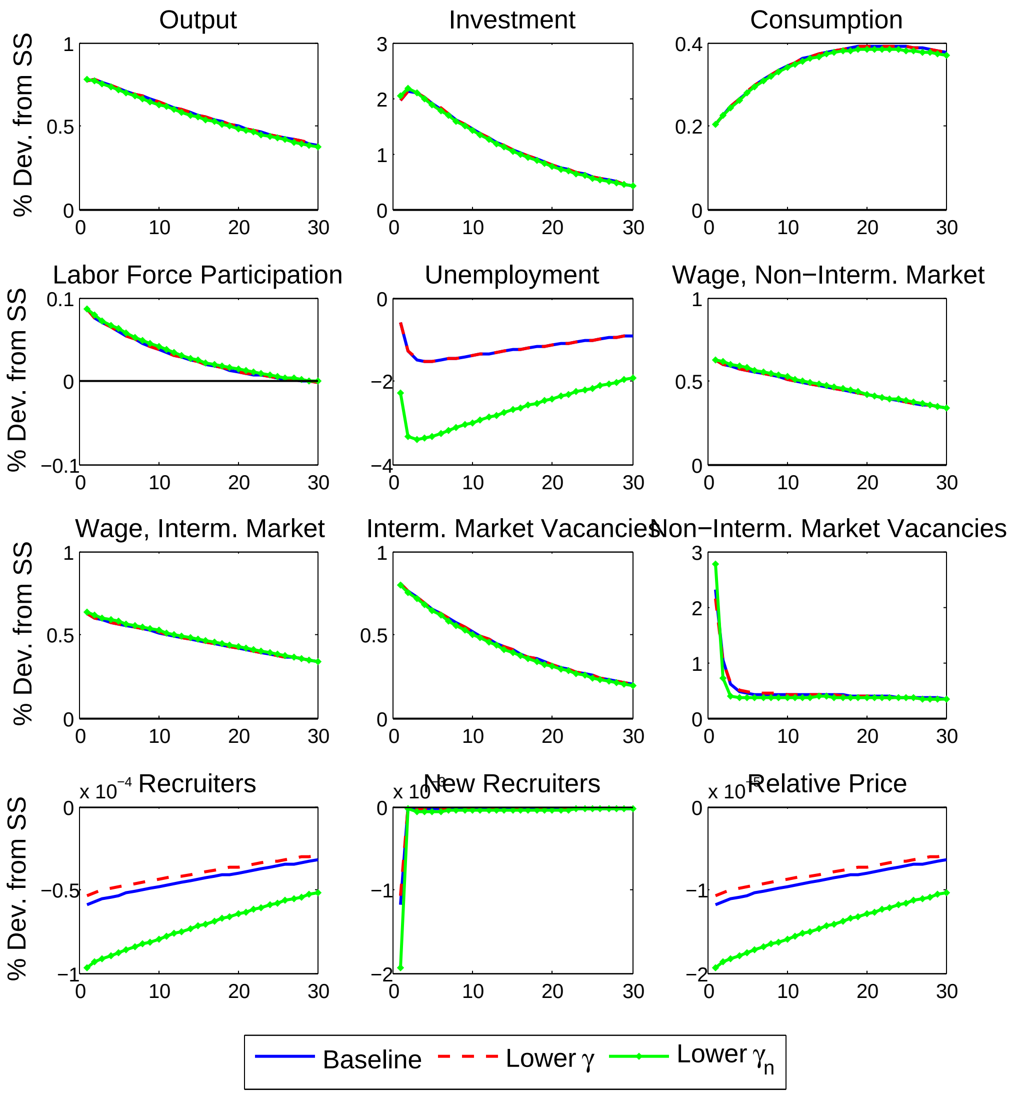

Figure 12: **Impulse Response Function to a Positive TFP Shock: Dixit-Stiglitz Aggregation.**

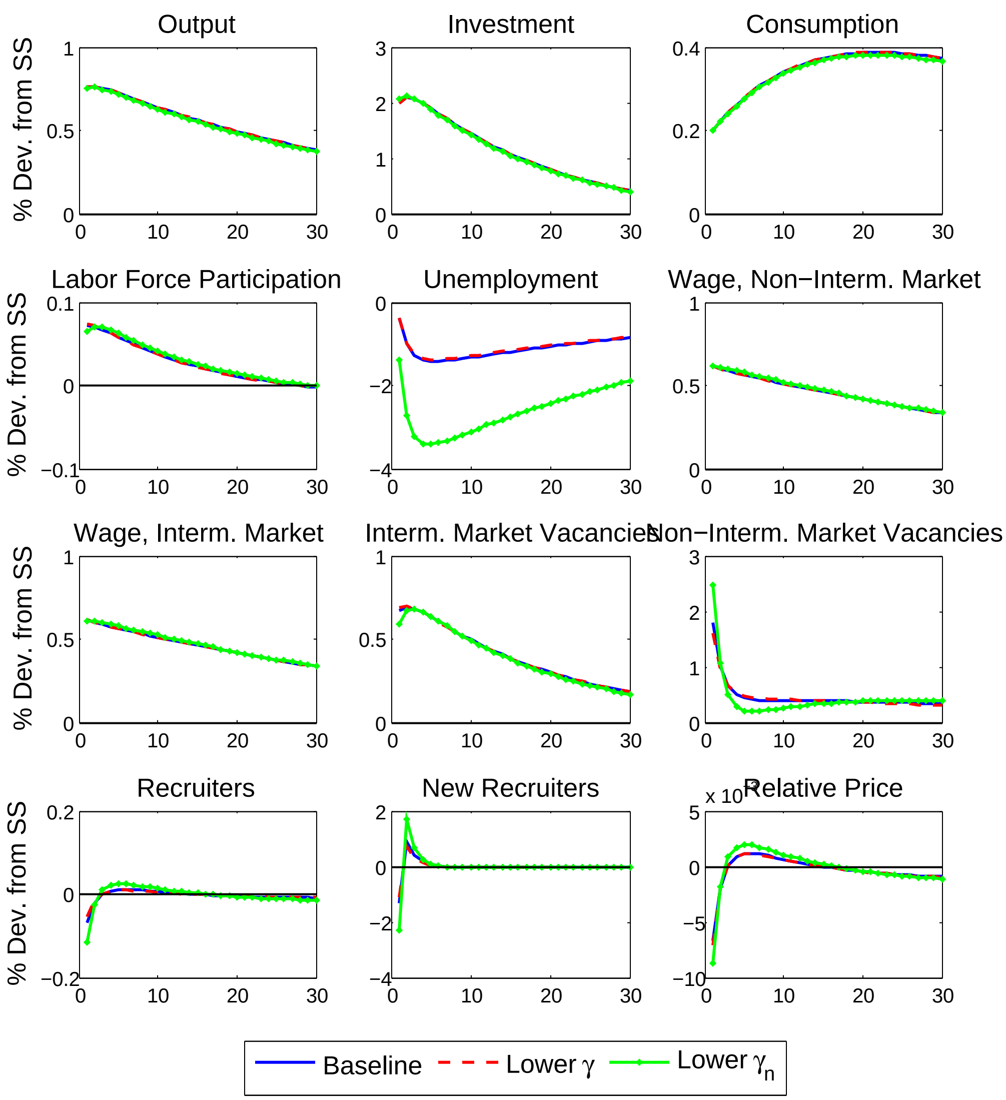

Figure 13: **Impulse Response Function to a Positive TFP Shock: translog Aggregation.**

Lower Vacancy Posting Costs in Intermediated Matching Markets. Relative to the baseline economy, an economy with a lower cost of posting vacancies via intermediated matching markets (roughly two-thirds of the cost in the baseline economy) implies a very similar response in terms of macro aggregates. In fact, the only variables that exhibit a somewhat different response are the number of recruiting firms and new recruiters, as well as these firms' relative price. Quantitatively, though, the differences across economies are negligible. Intuitively, with a lower cost of posting vacancies, production firms respond less forcefully by posting vacancies in the non-intermediated matching market relative to the baseline economy. In turn, this implies that the intermediated matching market becomes somewhat more stable, leading to a smaller reduction in the equilibrium measure of recruiting firms. Of note, differences in steady-state equilibria across the two economies are very small. The most notable result is that, under lower vacancy posting costs in the intermediated matching market, unemployment is not lower and is instead marginally higher (6.19 percent vs. the baseline 6.17 percent).

Lower Vacancy Posting Costs in Non-Intermediated Matching Markets. Relative to the baseline economy, an economy with a lower cost of posting vacancies via non-intermediated matching markets (roughly two-thirds of the cost in the baseline economy) generates sharper responses in non-intermediated vacancy postings, non-intermediated searchers, new recruiting firms, and ultimately unemployment. Importantly, in contrast to the case with lower vacancy posting costs in intermediated matching markets, steady-state unemployment is considerably lower (the steady state unemployment rate is slightly higher than 2 percent, in contrast to the baseline of 6.17 percent). In turn, this partly explains why unemployment falls by more in response to the same positive TFP shock. Intuitively, a lower vacancy posting cost in non-intermediated matching markets lowers the expected marginal cost of posting vacancies, leading to higher steady-state vacancy postings, thereby making these vacancies more sensitive to TFP shocks. At the same time, households face higher steady-state wages in non-intermediated markets, and therefore a higher incentive to redirect their search towards these markets. The reduction in search behavior in the intermediated matching market increases the probability of finding a job for those who continue to search in that market (vacancies in that market still respond positively as a result of higher TFP), leading to higher job finding rates across the board relative to the baseline economy. Ultimately, both search behavior and vacancy postings in non-intermediated markets become more responsive, leading to a sharper decline in unemployment.

All told, a main message from this experiment is that not all vacancies are created equal. While intermediated matching markets have higher matching efficiency, their presence does not bring about lower labor market volatility and unemployment is as responsive as the baseline economy. Conversely, a reduction in the cost of posting vacancies in the non-intermediated matching market makes unemployment considerably more responsive.

6 Discussion

Existence of Middlemen.

One potential criticism of our model is that it does not endogenize the emergence of potentially "costly" labor-market intermediation. This criticism is somewhat misleading. There is no reason that *both* an intermediated labor market *and* a non-intermediated labor market cannot co-exist *as long as matching probabilities appropriately adjust* between intermediated and non-intermediated labor markets. Matching probabilities across intermediated and non-intermediated labor markets *do* adjust in our model.[18] As but one example in which probabilities do not appropriately adjust, suppose that, for unmodeled reasons outside the scope of this framework, wages in the intermediated sector are "rigid" over time. The wage rigidity would cause a failure in matching probabilities in the intermediated sector to appropriately adjust. In this case, it is clear that (as long as outcomes such as, say, labor rationing do not occur) the existence of the "middlemen" sector is a waste of resources and would therefore shut down.

Relation to Literature.

Relative to the existing literature on intermediaries — several prominent examples of which are Rubinstein and Wolinksy (1987), Masters (2007), Wright and Wong (2014), Nosal, Wong, and Wright (2015), Gautier, Hu, and Watanabe (2016), and Farboodi, Jarosch, and Shimer (2017) — we stress our focus on market-structure imperfections in intermediated labor markets and endogenous entry among recruiting firms in the latter in a general equilibrium environment.[19] Importantly, our framework emphasizes endogenous entry among intermediaries, whereas the existing literature has generally modeled whether individuals become intermediaries as opposed to producers. Given our interest in labor markets, it is natural to consider the creation of labor market intermediaries through the lens of firm creation. As a result, our modeling approach centered on recruiting-firm entry complements existing theoretical work on middlemen and intermediaries.

More specifically, we put forth four main new results relative to existing work. First, the employment surplus between production firms and workers when matches take place via intermediated markets is influenced by the competitiveness of the recruiting sector, with important implications

[18] Note that "appropriate" adjustment is not synonymous with "efficient" adjustment.

[19] Farboodi, Jarosch, and Shimer (2017) highlight conditions under which intermediaries may arise in equilibrium. Other related studies on intermediation include Hall and Rust (2002) and Hendrickson (2016), who rationalize the existence of a minimum wage in a model where unions arise as middlemen, Gautier, Hu, and Watanabe (2016), who show that middlemen can arise in a directed search environment, and, among others, Chang and Zhang (2016) and Gregor and Menzio (2016).

for wages and therefore the incentive to search and post vacancies. Second, the presence of endogenous entry in the monopolistically-competitive recruiting sector gives rise to increasing returns to scale in intermediated-based matching. While this is, in a broad sense, related to the environment in Masters (2007) where, under increasing returns, the matching rate is increasing in the number of people who participate in the market, our framework instead posits that the matching probabilities for production firms and workers in intermediated labor markets depends on the measure of intermediaries (i.e., recruiting firms) in addition to the number of "individuals in the market (i.e., searchers and firm vacancies).[20] Then, the degree of increasing returns is intimately connected to (1) the cost of becoming a recruiting firm, and (2) the cost of posting vacancies in intermediated markets. This differs from the environment in Masters (2007).

Third, focusing on a quantitative application, we stress that the behavior of intermediated labor markets is affected by the degree of efficiency in non-intermediated labor markets, with important implications for sectoral and overall labor market conditions (i.e., labor market tightness), unemployment, and participation. Finally, we show that changes in firms' costs of attracting workers via vacancy posting can lead to widely different labor market outcomes, especially with respect to unemployment, depending on whether vacancy creation costs change in intermediated or non-intermediated labor markets

Perhaps closest to our work is Nosal, Wong, and Wright (2015), who establish the conditions under which middlemen arise as well as the characteristics under which efficiency is obtained in an environment with production. More specifically, their framework describes the distinct cases under which intermediaries arise in equilibrium (and, importantly, the cases under which intermediaries may be irrelevant). Moreover, the authors show that an appropriate calibration of bargaining powers (a la Hosios) amid endogenous production can lead to efficiency, and that deviations from Hosios generally lead to inefficient outcomes. Our work differs from Nosal, Wong, and Wright (2015) in two main ways. Relative to their environment, we allow for the coexistence of intermediated and non-intermediated markets which, importantly, implies that the Hosios condition alone may be insufficient amid monopolistic competition (and positive entry costs) in intermediated matching markets. Second, our work explicitly highlights a surplus sharing rule where the surplus from employment relationships is directly influenced to the degree of competitiveness of intermediated recruiting markets. In particular, this result suggests interesting and potentially important implications for optimal policy as standard policy instruments that tackle inefficiencies may not be easily implementable (an issue we leave for future work).

Furthermore, our results on the differential qualitative effects on the labor market that arise from changes in firms' search costs in intermediated vis-a-vis non-intermediated markets are, to

[20]The framework in Masters (2007) does not explicitly address labor markets, but his model can readily be applied to a labor market setting.

the best of our knowledge, also new relative to existing studies. Specifically, our findings suggest that changes in firms' search costs via non-intermediated markets can have large effects on unemployment, whereas the same cannot be said of similar changes in search costs in intermediated markets. This finding emerges in a general equilibrium environment and, importantly, as a result of the coexistence of intermediated and non-intermediated matching markets.[21] Finally, our results on the increasing-returns-to-scale nature of monopolistic intermediated markets and the effects of deviations from efficiency in non-intermediated markets on intermediated markets, as well as their implications for unemployment and participation, complement existing theory on intermediation in matching markets by quantitatively showing the relevance of the size of the recruiting sector.

7 Conclusion

Labor market intermediaries are playing an increasingly relevant role in job matching. We introduce a monopolistically-competitive recruiting (intermediated) sector with endogenous entry into a general equilibrium search and matching model with non-intermediated labor markets to explore the labor market and aggregate implications of intermediated labor markets. Our framework features endogenous labor force participation, endogenous recruiting firm entry, a standard non-intermediated labor market, and production firms that use both capital and labor to produce. Focusing on the intermediated labor market, we show that surplus-sharing from employment relationships is directly influenced by the degree of competition in intermediary labor markets. Our framework features increasing returns to scale in intermediary-based matching. These two results imply that, in general equilibrium, recruiting firm profits have aggregate implications by modifying the absorption of production. Finally, we numerically show that both deviations from efficiency in non-intermediated markets and differential changes in the cost of posting vacancies across labor markets have important implications for the behavior of (long-run and cyclical) unemployment, thereby highlighting the relevance of understanding the behavior of intermediated labor markets for aggregate labor market outcomes.

Our framework is tractable enough to be used to explore several additional experiments, including the implications of an expanding recruiting sector for unemployment fluctuations, the role of differential changes in hiring costs across intermediated and non-intermediated markets for unemployment dynamics, and both labor market policy and optimal fiscal policy. We plan to explore these and other issues in future work.

[21] While Nosal, Wong, and Wright (2015) consider production, our framework differs from theirs in several ways. First, our intermediation structure is tractably embedded in a standard business cycle model with labor search, which can easily be extended to consider important issues related to labor market dynamics and business cycles. Second, we consider endogenous labor force participation. Third, we allow for endogenous capital accumulation. Fourth, we quantitatively show the relevance of intermediated markets by considering changes in the costs of employment creation across labor markets.

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A Surplus Sharing

This Appendix provides the proof for Proposition 1.

A.1 Envelope Condition with respect to s_{ijt}

Recall that recruiting firm ij's value function is given by

$$\mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot) = \rho_{ijt} \cdot m(s_{ijt}, v_{ijt}) - mc_t \cdot m(s_{ijt}, v_{ijt}). \tag{55}$$

Recruiting firm ij's envelope condition with respect to s_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}_{Mij}(s_{ijt}, v_{ijt}; \cdot)}{\partial s_{ijt}} &= \rho_{ijt} \cdot m_s(s_{ijt}, v_{ijt}) - mc_t \cdot m_s(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_t) \cdot \xi \cdot k^h(\theta_{ijt}), \end{aligned} \tag{56}$$

in which the second line uses the Cobb-Douglas matching function.[22] As per Moen (1997), recruiting firm ij chooses w_{ijt} and θ_{ijt} to optimize

$$\begin{aligned} &(\rho_{ijt} - mc_t) \cdot \xi \cdot k^h(\theta_{ijt}) + \varphi^f_{ijt} \cdot \left[\gamma - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^F\right] \\ &+ \;\; 1 \cdot \left[k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t - \mathbf{X}^H\right], \end{aligned} \tag{57}$$

with φ^f_{ijt} and 1 being the respective Lagrange multipliers on attracting vacancies towards submarket ij and on attracting actively searching individuals towards submarket ij.[23]

The first-order conditions with respect to w_{ijt} and θ_{ijt} are

$$-\varphi^f_{ijt} \cdot k^f(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + k^h(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0, \tag{58}$$

and

$$(\rho_{ijt} - mc_t) \cdot \xi \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} - \varphi^f_{ijt} \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0. \tag{59}$$

Noting that $\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} = -1$ and $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 1$ in our model, the multiplier φ^f_{ijt} is

$$\begin{aligned} \varphi^f_{ijt} &= -\frac{k^h(\theta_{ijt})}{k^f(\theta_{ijt})} \\ &= -\theta_{ijt}, \end{aligned} \tag{60}$$

[22]For ease of reference, the Cobb-Douglas matching function relationships are $m_s(s_{ijt}, v_{ijt}) = \xi \theta_{ijt}^{1-\xi}$, $k^f(\theta_{ijt}) = \theta_{ijt}^{-\xi}$, and $k^h(\theta_{ijt}) = \theta_{ijt}^{1-\xi}$.

[23]It is without of generality to normalize one of the multipliers due to the constant-returns matching function.

in which the second line follows due to Cobb-Douglas matching. Substituting φ^f_{ijt} in (59) gives

$$(\rho_{ijt} - mc_t) \cdot \xi \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} + \theta_{ijt} \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

which, after substituting the Cobb-Douglas expressions $\frac{\partial k^h(\theta)}{\partial \theta}$ and $\frac{\partial k^f(\theta)}{\partial \theta}$ gives

$$(\rho_{ijt} - mc_t) \cdot \xi \cdot (1-\xi)\theta_{ijt}^{-\xi} - \xi\theta_{ijt} \cdot \theta_{ijt}^{-\xi-1} \cdot \mathbf{J}(w_{ijt}) + (1-\xi)\theta_{ijt}^{-\xi} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

Dividing this expression by $(1-\xi)\,\theta_{ijt}^{-\xi}$ and slightly rearranging gives the surplus sharing rule

$$\xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{61}$$

If the matching aggregator were of Dixit-Stiglitz form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\frac{1}{\varepsilon-1}}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{62}$$

If the matching aggregator were of Benassy form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\varphi}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{63}$$

If the matching aggregator were translog, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{(\sigma N_{Mt})^{-1}}{1 + (\sigma N_{Mt})^{-1}} \right) \cdot \exp\left(-\frac{1}{2} \cdot \frac{\tilde{N}_M - N_{Mt}}{\sigma \tilde{N}_M N_{Mt}} \right)}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{64}$$

A.2 Envelope Condition with Respect to v_{ijt}

Recruiting firm ij's envelope condition with respect to v_{ijt} is

$$\begin{aligned} \frac{\partial \mathbf{V}_{ij}^{M}(s_{ijt}, v_{ijt}; \cdot)}{\partial v_{ijt}} &= \rho_{ijt} \cdot m_v(s_{ijt}, v_{ijt}) - mc_{jt} \cdot m_v(s_{ijt}, v_{ijt}) \\ &= (\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot k^f(\theta_{ijt}). \end{aligned} \tag{65}$$

As per Moen (1997), recruiting firm ij chooses w_{ijt} and θ_{ijt} to optimize

$$\begin{aligned} &(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot k^f(\theta_{ijt}) + 1 \cdot \left[\gamma - k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^F\right] \\ + \quad & \varphi_{ijt}^h \cdot \left[k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}) + (1 - k^h(\theta_{ijt})) \cdot \mathbf{U}_t - \mathbf{X}^H\right], \end{aligned} \tag{66}$$

with 1 and φ_{ijt}^h the respective Lagrange multipliers on attracting vacancies towards submarket ij and on attracting actively searching individuals towards submarket ij.[24]

The first-order conditions with respect to w_{ijt} and θ_{ijt} are

$$-k^f(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + \varphi_{ijt}^h \cdot k^h(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0 \tag{67}$$

and

$$(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \varphi_{ijt}^h \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0. \tag{68}$$

Noting that $\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} = -1$ and $\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 1$ in our model, the multiplier φ_{ijt}^h is

$$\begin{aligned} \varphi_{ijt}^h &= -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} \\ &= -\theta_{ijt}^{-1}, \end{aligned} \tag{69}$$

in which the second line follows due to Cobb-Douglas matching.

Substituting φ_{ijt}^h in (68) gives

$$(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} - \frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) - \theta_{ijt}^{-1} \cdot \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0,$$

which, after substituting the Cobb-Douglas expressions $\frac{\partial k^h(\theta)}{\partial \theta}$ and $\frac{\partial k^f(\theta)}{\partial \theta}$ gives

$$-(\rho_{ijt} - mc_{jt}) \cdot (1-\xi) \cdot \xi \theta_{ijt}^{-\xi-1} - \xi \theta_{ijt}^{-\xi-1} \cdot \mathbf{J}(w_{ijt}) + (1-\xi) \cdot \theta_{ijt}^{-1} \cdot \theta_{ijt}^{-\xi} \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = 0.$$

[24] It is without of generality to normalize one of the multipliers due to the constant-returns matching function.

Dividing this expression by $(1-\xi)\,\theta_{ijt}^{-\xi-1}$ and slightly rearranging gives the surplus sharing rule

$$\xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{70}$$

If the matching aggregator were of Dixit-Stiglitz form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\frac{1}{\varepsilon-1}}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{71}$$

If the matching aggregator were of Benassy form, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \frac{1}{\varepsilon} \underbrace{N_{Mjt}^{\varphi}}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{72}$$

If the matching aggregator were translog, the surplus-sharing condition is

$$\xi \cdot (1-\xi) \cdot \underbrace{\left(\frac{(\sigma N_{Mt})^{-1}}{1 + (\sigma N_{Mt})^{-1}} \right) \cdot \exp\left(-\frac{1}{2} \cdot \frac{\tilde{N}_M - N_{Mt}}{\sigma \tilde{N}_M N_{Mt}} \right)}_{=\rho(N_{Mjt})} + (1-\xi) \cdot (\mathbf{W}(w_{ijt}) - \mathbf{U}_t) = \xi \cdot \mathbf{J}(w_{ijt}). \tag{73}$$

B Firms

There is a continuum $[0,1]$ of identical goods-producing firms. The representative goods-producing firm's lifetime profit function is

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ z_t f(k_t, n_t) - r_t k_t - \gamma_N(v_{Nt}) - \int_0^1 \int_0^{N_{Mjt}} \gamma(v_{ijt})\ di\ dj + \int_0^1 \int_0^{N_{Mjt}} p_{v_{jt}} v_{ijt}\ di\ dj \right\}$$
$$- \quad E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left\{ w_t \cdot (1-\rho) n_{t-1} + w_{Nt} \cdot k^f_{Nt} \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k^f_{ijt} \cdot v_{ijt}\ di\ dj \right\} \quad (74)$$

subject to the period-t perceived law of motion of employment

$$n_t = (1-\rho) n_{t-1} + k^f_{Nt} \cdot v_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k^f_{ijt} \cdot v_{ijt}\ di\ dj. \quad (75)$$

Defining the Lagrange multiplier on the perceived law of motion (75) as μ_t, the first-order conditions with respect to k_t, v_{ijt}, v_{Nt}, and n_t are

$$z_t f_k(k_t, n_t) - r_t = 0, \quad (76)$$

$$\mu_t \cdot k^f_{ijt} - \gamma'(v_{ijt}) + p_{v_{jt}} - w_{ijt} \cdot k^f_{ijt} = 0\ \forall ij, \quad (77)$$

$$\mu_t \cdot k^f_{Nt} - \gamma'_N(v_{Nt}) - w_{Nt} \cdot k^f_{Nt} = 0, \quad (78)$$

and

$$-\mu_t + z_t f_n(k_t, n_t) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\mu_{t+1} - w_{t+1} \right) \right\} = 0. \quad (79)$$

Isolating the multiplier μ_t from expression (78) gives

$$\mu_t = w_{Nt} + \frac{\gamma'_N(v_{Nt})}{k^f_{Nt}}, \quad (80)$$

and isolating the multiplier μ_t from expression (77) gives

$$\mu_t = w_{ijt} + \frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f_{ijt}}\ \forall ij. \quad (81)$$

Substituting the value for μ_t from (81) into (79) gives

$$\frac{\gamma'(v_{ijt})}{k^f_{ijt}} = z_t f_n(k_t, n_t) - w_{ijt} + \frac{p_{v_{jt}}}{k^f_{ijt}} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f_{jt+1}} + w_{jt+1} - w_{t+1} \right) \right\}\ \forall ij. \quad (82)$$

Next, substituting the value for μ_t from (80) into (79) gives

$$\frac{\gamma_N'(v_{Nt})}{k^f_{Nt}} = z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma_N'(v_{Nt+1})}{k^f_{Nt+1}} + w_{Nt+1} - w_{t+1} \right) \right\}. \tag{83}$$

B.1 Job-Creation Conditions

Without loss of generality, assuming that wages for incumbent employees in the periods after they were first hired (regardless of whether they were first hired through intermediated or non-intermediated labor markets) are identical simplifies the pair of expressions above to

$$\gamma'(v_{ijt}) = p_{v_{jt}} + k^f_{ijt} \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f_{jt+1}} \right) \right\} \right)}_{\equiv \mathbf{J}(w_{ijt}, \theta_{ijt})} \quad \forall ij \tag{84}$$

and

$$\gamma_N'(v_{Nt}) = k^f_{Nt} \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\gamma_N'(v_{Nt+1})}{k^f_{Nt+1}} \right\} \right)}_{\equiv \mathbf{J}(w_{Nt}, \theta_{Nt})}, \tag{85}$$

which characterize, respectively, costly job vacancies directed towards any intermediated labor submarket ij and costly job vacancies in the non-intermediated labor market. Around the optimum, the firm is indifferent between directing new job vacancies to intermediated submarket i or intermediated submarket k, $k^f(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}, \theta_{ijt}) = k^f(\theta_{kjt}) \cdot \mathbf{J}(w_{kjt}, \theta_{kjt})$, $\forall i \neq k$.

C Households

There is a continuum $[0,1]$ of identical households. In each household, there is a continuum $[0,1]$ of family members. In period t, each family member in the representative household has a labor-market status of employed, unemployed and actively seeking a job, or being outside the labor force. Regardless of which labor-market status a particular family member is in, each family member receives the same exact amount of consumption c_t due to full risk-sharing within each household (see Andolfatto (1996) for formal details).

The representative household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - h \left(n_t + \underbrace{(1-k_{Nt}^h) \cdot s_{Nt}}_{=ue_t^N} + \int_0^1 \left(\int_0^{N_{Mjt}} \underbrace{(1-k_{ijt}^h) \cdot s_{ijt}}_{=ue_{ijt}} \, di \right) dj \right) \right], \tag{86}$$

subject to the budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta) k_t + w_t (1-\rho) n_{t-1} + w_{Nt} \cdot k_{Nt}^h \cdot s_{Nt} \\ &+ \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k_{ijt}^h \cdot s_{ijt} \, di \, dj + \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt} \, di \, dj \\ &+ (1-k_{Nt}^h) \cdot s_{Nt} \chi + \int_0^1 \int_0^{N_{Mjt}} \left(1 - k_{ijt}^h\right) \cdot s_{ijt} \chi \, di \, dj + \int_0^1 \Pi_{jt}^M \, dj + \Pi_t^F, \end{aligned} \tag{87}$$

and the period-t perceived law of motion of employment

$$n_t = (1-\rho) n_{t-1} + k_{Nt}^h \cdot s_{Nt} + \int_0^1 \int_0^{N_{Mjt}} k_{ijt}^h \cdot s_{ijt} \, di \, dj. \tag{88}$$

Defining the Lagrange multiplier on the flow budget constraint as λ_t and on the perceived law of motion as μ_t, the first-order conditions with respect to c_t, k_{t+1}, n_t, s_{Nt}, and s_{ijt} are

$$u'(c_t) - \lambda_t = 0, \tag{89}$$

$$-\lambda_t + \beta E_t \left\{ \lambda_{t+1} \left(1 + r_{t+1} - \delta\right) \right\} = 0, \tag{90}$$

$$-\mu_t - h'(lfp_t) + \beta(1-\rho) E_t \left\{ \lambda_{t+1} w_{t+1} + \mu_{t+1} \right\} = 0, \tag{91}$$

$$-(1-k_{Nt}^h) \cdot h'(lfp_t) + \lambda_t \cdot \left(k_{Nt}^h \cdot w_{Nt} + (1-k_{Nt}^h) \cdot \chi \right) + \mu_t \cdot k_{Nt}^h = 0, \tag{92}$$

and

$$-(1-k_{ijt}^h) \cdot h'(lfp_t) + \lambda_t \cdot \left(k_{ijt}^h \cdot w_{ijt} + (1-k_{ijt}^h) \cdot \chi \right) + \lambda_t \cdot p_{s_{jt}} + \mu_t \cdot k_{ijt}^h = 0 \ \forall ij. \tag{93}$$

Isolating the multiplier μ_t from (92) gives

$$\frac{\mu_t}{u'(c_t)} = \left(\frac{1-k^h_{Nt}}{k^h_{Nt}}\right) \cdot \left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_{Nt}, \tag{94}$$

and isolating the multiplier μ_t from (93) gives

$$\frac{\mu_t}{u'(c_t)} = \left(\frac{1-k^h_{ijt}}{k^h_{ijt}}\right) \cdot \left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - \frac{p_{s_{jt}}}{k^h_{ijt}} - w_{ijt}\ \forall ij, \tag{95}$$

in which both of these expressions have substituted the marginal utility of income $\lambda_t = u'(c_t)$ from (89).

Substituting the multiplier as stated in expression (94) into (91) yields

$$\begin{aligned}
\left(\frac{1-k^h_{Nt}}{k^h_{Nt}}\right) \cdot \left(\frac{h'(lfp_t)}{u'(c_t)} - \chi\right) - w_{Nt} &= -\frac{h'(lfp_t)}{u'(c_t)} \\
+ \quad (1-\rho)E_t &\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} + \left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right) - w_{Nt+1}\right)\right\}.
\end{aligned}$$

Cancelling the $-h'(lfp_t)/u'(c_t)$ terms and multiplying by k^h_{Nt} gives

$$\begin{aligned}
\frac{h'(lfp_t)}{u'(c_t)} &= k^h_{Nt}w_{Nt} + (1-k^h_{Nt})\chi \\
+ \quad k^h_{Nt}(1-\rho)E_t &\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} - w_{Nt+1} + \left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right)\right\}.
\end{aligned}$$

Next, substituting the multiplier as stated in expression (95) into (91) and following the same steps of algebra as above yields

$$\begin{aligned}
\frac{h'(lfp_t)}{u'(c_t)} &= k^h_{ijt}w_{ijt} - \frac{p_{s_{jt}}}{k^h_{ijt}} + (1-k^h_{ijt})\chi \\
+ \quad k^h_{ijt}(1-\rho)E_t &\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(w_{t+1} - w_{jt+1} + \left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}}\right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right)\right\}\ \forall ij.
\end{aligned}$$

C.1 Labor Force Participation Conditions

Without loss of generality, assuming that wages for incumbent employees in the periods after they were first hired (regardless of whether they were first hired through intermediated or non-intermediated labor markets) are identical simplifies the pair of expressions above to

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h_{Nt} \underbrace{\left[w_{Nt} + (1-\rho)E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) \right\} \right]}_{\equiv \mathbf{W}(w_{Nt}, \theta_{Nt})} + (1-k^h_{Nt}) \underbrace{\chi}_{\equiv \mathbf{U}} \tag{96}$$

and

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= p_{s_{jt}} + (1-k^h_{ijt}) \underbrace{\chi}_{\equiv \mathbf{U}} \\ &+ \quad k^h_{ijt} \underbrace{\left[w_{ijt} + (1-\rho)E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left[\left(\frac{1-k^h_{jt+1}}{k^h_{jt+1}} \right) \cdot \left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi \right) - \frac{p_{s_{jt+1}}}{k^h_{jt+1}} \right] \right\} \right]}_{\equiv \mathbf{W}(w_{ijt}, \theta_{ijt})} \ \forall ij, \end{aligned} \tag{97}$$

which characterize, respectively, active job search in the non-intermediated labor market and active job search directed towards intermediated labor submarket ij. Given the household-level envelope conditions, around the optimum, active job search in all submarkets must yield the same value $k^h(\theta_{ijt}) \cdot \mathbf{W}(w_{ijt}, \theta_{ijt}) + \left(1 - k^h(\theta_{ijt})\right) \cdot \mathbf{U}(\cdot) = k^h(\theta_{kjt}) \cdot \mathbf{W}(w_{kjt}, \theta_{kjt}) + \left(1 - k^h(\theta_{kjt})\right) \cdot \mathbf{U}(\cdot), \forall i \neq k$.

For use in Appendix D, the participation conditions (96) and (97) can, respectively, be equivalently expressed as

$$\frac{h'(lfp_t) - u'(c_t)\chi}{k^h_{Nt} \cdot u'(c_t)} = w_{Nt} - \chi + (1-\rho)E_t \left\{ \Xi_{t+1|t} (1-k^h_{Nt+1}) \left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi}{k^h_{Nt+1} \cdot u'(c_{t+1})} \right) \right\} \tag{98}$$

and

$$\begin{aligned} \frac{h'(lfp_t) - u'(c_t)\chi - u'(c_t) \cdot p_{s_{jt}}}{k^h_{ijt} \cdot u'(c_t)} &= w_{ijt} - \chi \\ &+ \quad (1-\rho)E_t \left\{ \Xi_{t+1|t} (1-k^h_{jt+1}) \left(\frac{h'(lfp_{t+1}) - u'(c_{t+1})\chi - u'(c_{t+1}) \cdot p_{s_{jt+1}}}{k^h_{jt+1} \cdot u'(c_{t+1})} \right) \right\} \ \forall ij. \end{aligned} \tag{99}$$

D Derivation of Real Wage in Intermediated Market

Recall that the labor force participation condition can be written as

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} - p_{s_{jt}} &= k^h(\theta_{ijt})\left[w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\mu_{jt+1}}{u'(c_{t+1})}\right\}\right] + (1-k^h(\theta_{ijt}))\cdot\chi \\ &= k^h(\theta_{ijt})\cdot\mathbf{W}(w_{ijt},\theta_{ijt}) + (1-k^h(\theta_{ijt}))\cdot\mathbf{U}_t, \end{aligned} \tag{100}$$

and

$$\mathbf{W}(w_{ijt},\theta_{ijt}) - \mathbf{U}_t = \frac{h'(lfp_t) - u'(c_t)\cdot\chi - u'(c_t)\cdot p_{s_{jt}}}{k^h(\theta_{ijt})\cdot u'(c_t)}. \tag{101}$$

In turn, the job creation condition is given by

$$\frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f(\theta_{ijt})} = \underbrace{z_t f_n(k_t,n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})}\right\}}_{=\mathbf{J}(w_{ijt},\theta_{ijt})}. \tag{102}$$

In recursive form, the surplus earned by the household is

$$\mathbf{W}(w_{ijt},\theta_{ijt}) - \mathbf{U}_t = w_{ijt} - \chi + (1-\rho)E_t\left\{\Xi_{t+1|t}\cdot(1-k^h(\theta_{jt+1}))\cdot(\mathbf{W}(w_{jt+1},\theta_{jt+1}) - \mathbf{U}_{t+1})\right\}, \tag{103}$$

and the surplus earned by the goods-producing firm is

$$\mathbf{J}(w_{ijt},\theta_{ijt}) = z_t f_n(k_t,n_t) - w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{J}(w_{jt+1},\theta_{jt+1})\right\}. \tag{104}$$

Inserting expression (103) into the surplus-sharing condition

$$\xi\cdot(\rho_{ijt} - mc_t) + \mathbf{W}(w_{ijt},\theta_{ijt}) - \mathbf{U}_t = \left(\frac{\xi}{1-\xi}\right)\cdot\mathbf{J}(w_{ijt},\theta_{ijt}) \tag{105}$$

gives

$$\begin{aligned}&\xi\cdot(\rho_{ijt} - mc_t) + w_{ijt} - \chi \\ &+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\cdot(1-k^h(\theta_{jt+1}))\cdot(\mathbf{W}(w_{jt+1},\theta_{jt+1}) - \mathbf{U}_{t+1})\right\} = \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_{ijt},\theta_{ijt}).\end{aligned} \tag{106}$$

Next, using the period-$t+1$ sharing rule gives

$$\begin{aligned}&\xi\cdot(\rho_{ijt} - mc_t) + w_{ijt} - \chi \\ &+ \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\cdot(1-k^h(\theta_{jt+1}))\cdot\left[\left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_{jt+1},\theta_{jt+1}) - \xi\cdot(\rho_{Mjt+1} - mc_{t+1})\right]\right\} \\ &= \quad \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_{ijt},\theta_{ijt}).\end{aligned} \tag{107}$$

Substituting $\mathbf{J}(w_{ijt}, \theta_{ijt}) = \frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f(\theta_{ijt})}$ and $\mathbf{J}(w_{jt+1}, \theta_{jt+1}) = \frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})}$ yields

$$\begin{aligned} &\xi \cdot (\rho_{ijt} - mc_t) + w_{ijt} - \chi \\ + \quad & (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ = \quad & \left(\frac{\xi}{1-\xi} \right) \cdot \left(\frac{\gamma'(v_{ijt}) - p_{v_{jt}}}{k^f(\theta_{ijt})} \right). \end{aligned} \tag{108}$$

Next, use the job-creation condition to substitute on the right-hand side, which gives

$$\begin{aligned} &\xi \cdot (\rho_{ijt} - mc_t) + w_{ijt} - \chi \\ + \quad & (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ = \quad & \left(\frac{\xi}{1-\xi} \right) \cdot \left(z_t f_n(k_t, n_t) - w_{ijt} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} \right). \end{aligned} \tag{109}$$

Grouping terms in w_{ijt},

$$\begin{aligned} & w_{ijt} \cdot \left(1 + \frac{\xi}{1-\xi} \right) = \left(\frac{\xi}{1-\xi} \right) z_t f_n(k_t, n_t) + \chi - \xi \cdot (\rho_{ijt} - mc_t) \\ - \quad & (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ + \quad & \left(\frac{\xi}{1-\xi} \right) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \end{aligned} \tag{110}$$

Rearranging,

$$\begin{aligned} & w_{ijt} \cdot \left(\frac{1}{1-\xi} \right) = \left(\frac{\xi}{1-\xi} \right) z_t f_n(k_t, n_t) + \chi - \xi \cdot (\rho_{ijt} - mc_t) \\ - \quad & (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ + \quad & \left(\frac{\xi}{1-\xi} \right) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \end{aligned} \tag{111}$$

Next, multiply by $(1-\xi)$, which gives

$$\begin{aligned} & w_{ijt} = \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi - (1-\xi) \cdot \xi \cdot (\rho_{ijt} - mc_t) \\ - \quad & (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left[\left(\frac{\xi}{1-\xi} \right) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) - \xi \cdot (\rho_{Mjt+1} - mc_{t+1}) \right] \right\} \\ + \quad & \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \end{aligned} \tag{112}$$

Expanding the terms that appear in the second line yields

$$
\begin{aligned}
w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi - (1-\xi) \cdot \xi \cdot (\rho_{ijt} - mc_t) \\
&- \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} \\
&+ \quad \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \qquad (113)
\end{aligned}
$$

Next, collect the terms that contain the monopolistic term $(\rho_{ijt} - mc_t)$, which gives

$$
\begin{aligned}
w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi \\
&- \quad \xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\} \\
&- \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \qquad (114)
\end{aligned}
$$

Expanding the term in the third line yields

$$
\begin{aligned}
w_{ijt} &= \xi \cdot z_t f_n(k_t, n_t) + (1-\xi) \cdot \chi \\
&- \quad \xi \cdot (1-\xi) \cdot (\rho_{ijt} - mc_t) + \xi \cdot (1-\xi) \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\} \\
&- \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\} + \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot \frac{k^h(\theta_{jt+1})}{k^f(\theta_{jt+1})} \cdot \left(\gamma'(v_{jt+1}) - p_{v_{jt+1}} \right) \right\} \\
&+ \quad \xi \cdot (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma'(v_{jt+1}) - p_{v_{jt+1}}}{k^f(\theta_{jt+1})} \right) \right\}. \qquad (115)
\end{aligned}
$$

After cancelling terms in the third and fourth lines and using the Cobb-Douglas functional form $\frac{k^f(\theta)}{k^f(\theta)} = \theta$, the submarket ij wage is

$$
\begin{aligned}
w_{ijt} &= \xi z_t f_n(k_t, n_t) + (1-\xi)\chi + \xi(1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot \theta_{jt+1} \cdot \left(\gamma'(v_{jt+1}) - p_{v_{jt+1}} \right) \right\} \qquad (116) \\
&- \quad \xi(1-\xi) \left(\rho_{ijt} - mc_t \right) + \xi(1-\xi)(1-\rho) E_t \left\{ \Xi_{t+1|t} \cdot (1 - k^h(\theta_{jt+1})) \cdot (\rho_{Mjt+1} - mc_{t+1}) \right\}.
\end{aligned}
$$

E Aggregation

The (symmetric equilibrium) flow budget constraint of the government is

$$T_t = g_t + (1 - k^h(\theta_t)) \cdot s_t \cdot N_{Mt} \cdot \chi + (1 - k^h(\theta_{Nt})) \cdot s_{Nt} \cdot \chi, \tag{117}$$

in which lump-sum taxes T_t levied on households finance government-provided unemployment benefits and exogenous government spending g_t.

E.1 Aggregate Goods Resource Constraint

To construct the aggregate symmetric equilibrium household budget constraint, begin with expression (87), which, for convenience, is repeated here:

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ \int_0^1 \int_0^{N_{Mjt}} w_{ijt} \cdot k^h_{ijt} \cdot s_{ijt} \; di \; dj + \int_0^1 \int_0^{N_{Mjt}} p_{s_{jt}} \cdot s_{ijt} \; di \; dj \\ &+ (1 - k^h_{Nt}) \cdot s_{Nt}\chi + \int_0^1 \int_0^{N_{Mjt}} \left(1 - k^h_{ijt}\right) \cdot s_{ijt}\chi \; di \; dj + \int_0^1 \Pi^M_{jt} \; dj + \Pi^F_t, \end{aligned} \tag{118}$$

Integrating over the i intermediated submarkets in each labor market j gives

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ \int_0^1 N_{Mjt} \cdot w_{jt} \cdot k^h_{jt} \cdot s_{jt} \; dj + \int_0^1 N_{Mjt} \cdot p_{s_{jt}} \cdot s_{jt} \; dj \\ &+ (1 - k^h_{Nt}) \cdot s_{Nt}\chi + \int_0^1 N_{Mjt} \cdot \left(1 - k^h_{jt}\right) \cdot s_{jt}\chi \; dj + \int_0^1 \Pi^M_{jt} \; dj + \Pi^F_t. \end{aligned}$$

Next, integrating over the measure $j \in (0, 1)$ of recruiting markets gives the symmetric equilibrium household budget constraint

$$\begin{aligned} c_t + k_{t+1} + T_t &= (1 + r_t - \delta)k_t + w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ w_t \cdot k^h_t \cdot s_t \cdot N_{Mt} - p_{s_t} \cdot s_t \cdot N_{Mt} + (1 - k^h_{Nt}) \cdot s_{Nt}\chi + N_{Mt} \cdot \left(1 - k^h_t\right) \cdot s_t\chi + \Pi^M_t + \Pi^F_t. \end{aligned}$$

Combining this with the government budget (117) gives

$$\begin{aligned} c_t + k_{t+1} + (1 - \delta)k_t &= w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k^h_{Nt} \cdot s_{Nt} \\ &+ w_t \cdot k^h_t \cdot s_t \cdot N_{Mt} + p_{s_t} \cdot s_t \cdot N_{Mt} + r_t k_t + \Pi^M_t + \Pi^F_t. \end{aligned} \tag{119}$$

In symmetric equilibrium, the period-t aggregate flow profits for goods-producing firms Π_t^F are

$$\begin{aligned}\Pi_t^F &= z_t f(k_t, n_t) - w_t(1-\rho)n_{t-1} - w_{Nt} \cdot k_{Nt}^f \cdot v_{Nt} \\ &- \quad w_t \cdot k_t^f \cdot v_t \cdot N_{Mt} + p_{v_t} \cdot v_t \cdot N_{Mt} - r_t k_t - \gamma(v_t) \cdot N_{Mt} - \gamma_N(v_{Nt}) \end{aligned} \tag{120}$$

and aggregate recruiting-firm profits Π_t^M are

$$\Pi_t^M = [\rho(N_{Mt}) \cdot m(s_t, v_t) - mc(N_{Mt}) \cdot m(s_t, v_t)] \cdot N_{Mt} - \Gamma_{Mt} N_{MEt} \tag{121}$$

$$= [\rho(N_{Mt}) \cdot m(s_t, v_t) - p_{s_t} s_t - p_{v_t} v_t] \cdot N_{Mt} - \Gamma_{Mt} N_{MEt} \tag{122}$$

$$= \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) - (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} - \Gamma_{Mt} N_{MEt}.$$

Substituting Π_t^F into (119) gives

$$\begin{aligned} c_t + k_{t+1} + (1-\delta)k_t &= w_t(1-\rho)n_{t-1} + w_{Nt} \cdot k_{Nt}^h \cdot s_{Nt} \\ &+ \quad w_t \cdot k_t^h \cdot s_t \cdot N_{Mt} + p_{s_t} \cdot s_t \cdot N_{Mt} + r_t k_t + \Pi_t^M \\ &+ \quad z_t f(k_t, n_t) - w_t(1-\rho)n_{t-1} - w_{Nt} \cdot k_{Nt}^f \cdot v_{Nt} - w_t \cdot k_t^f \cdot v_t \cdot N_{Mt} \\ &+ \quad p_{v_t} \cdot v_t \cdot N_{Mt} - r_t k_t - \gamma(v_t) \cdot N_{Mt} - \gamma_N(v_{Nt}). \end{aligned}$$

Next, cancelling several terms and grouping the remaining terms informatively gives

$$\begin{aligned} c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} + \gamma_N(v_{Nt}) &= z_t f(k_t, n_t) \\ + \quad w_{Nt} \cdot \underbrace{\left(k_{Nt}^h s_{Nt} - k_{Nt}^f v_{Nt}\right)}_{=0} + w_t \cdot N_{Mt} \cdot \underbrace{\left(k_t^h s_t - k_t^f v_t\right)}_{=0} &+ (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} + \Pi_t^M. \end{aligned}$$

Due to matching-market clearing in both the non-intermediated labor market and the intermediated labor market ($k_{Nt}^h s_{Nt} = k_{Nt}^f v_{Nt}$ and $k_t^h s_t = k_t^f v_t$, respectively), the second and third terms on the right-hand side vanish. Next, substituting aggregate recruiting-sector profits Π_t^M from (121) gives

$$\begin{aligned} c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} + \gamma_N(v_{Nt}) &= z_t f(k_t, n_t) \\ + \quad (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} + \rho(N_{Mt}) \cdot m(s_t, v_t) \cdot N_{Mt} &- (p_{s_t} s_t + p_{v_t} v_t) \cdot N_{Mt} - \Gamma_{Mt} N_{MEt}. \end{aligned}$$

Cancelling terms gives the decentralized economy's aggregate goods resource constraint

$$c_t + k_{t+1} + (1-\delta)k_t + \gamma(v_t) \cdot N_{Mt} + \gamma_N(v_{Nt}) + \Gamma_{Mt} N_{MEt} = z_t f(k_t, n_t) + \rho(N_{Mt}) \cdot m(s_t, v_t) \cdot N_{Mt}. \tag{123}$$

E.2 Private-Sector Equilibrium

A symmetric private-sector general equilibrium is made up of seventeen endogenous state-contingent processes $\{c_t, n_t, lfp_t, k_{t+1}, N_{Mt}, N_{MEt}, s_t, v_t, \theta_t, w_t, s_{Nt}, v_{Nt}, \theta_{Nt}, w_{Nt}, mc_t, p_{v_t}, p_{s_t}\}_{t=0}^{\infty}$ that satisfy the following eighteen sequences of conditions: the aggregate resource constraint

$$c_t+k_{t+1}+(1-\delta)k_t+\gamma(v_t)\cdot N_{Mt}+\gamma_N(v_{Nt})+\Gamma_{Mt}N_{MEt} = z_t f(k_t,n_t)+\underbrace{\rho(N_{Mt})\cdot N_{Mt}}_{=N_{Mt}^{\frac{\varepsilon}{\varepsilon-1}}}\cdot m(s_t,v_t). \quad (124)$$

the aggregate law of motion for labor

$$n_t = (1-\rho)n_{t-1} + m(s_{Nt}, v_{Nt}) + N_{Mt}^{\frac{\varepsilon}{\varepsilon-1}} \cdot m(s_t, v_t), \quad (125)$$

the definition of aggregate LFP

$$lfp_t = (1-\rho)n_{t-1} + s_{Nt} + s_t \cdot N_{Mt}, \quad (126)$$

the aggregate law of motion for recruiters

$$N_{Mt} = (1-\omega)N_{Mt-1} + N_{MEt}, \quad (127)$$

the capital Euler condition

$$1 = E_t\left\{\Xi_{t+1|t}\left(1 + z_{t+1}f_k(k_{t+1}, n_{t+1}) - \delta\right)\right\}, \quad (128)$$

the free-entry condition for recruiters

$$\Gamma_{Mt} = \left(\rho(N_{Mt}) - mc(N_{Mt})\right) m(s_t, v_t) + (1-\omega)E_t\left\{\Xi_{t+1|t}\Gamma_{Mt+1}\right\}, \quad (129)$$

the vacancy creation condition for intermediated labor markets

$$\gamma'(v_t) = p_{v_t} + k_t^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma'(v_{t+1}) - p_{v_{t+1}}}{k_{t+1}^f}\right)\right\}\right)}_{\equiv \mathbf{J}(w_t, \theta_t)}, \quad (130)$$

the vacancy creation condition for non-intermediated labor markets

$$\gamma_N'(v_{Nt}) = k_{Nt}^f \cdot \underbrace{\left(z_t f_n(k_t, n_t) - w_{Nt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\gamma_N'(v_{Nt+1})}{k_{Nt+1}^f}\right\}\right)}_{\equiv \mathbf{J}(w_{Nt}, \theta_{Nt})}, \quad (131)$$

the active job search condition for non-intermediated labor markets

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h_{Nt} \underbrace{\left[w_{Nt} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(\frac{1-k^h_{Nt+1}}{k^h_{Nt+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right)\right\}\right]}_{\equiv \mathbf{W}(w_{Nt},\theta_{Nt})} + (1-k^h_{Nt}) \underbrace{\chi}_{\equiv \mathbf{U}}, \tag{132}$$

the active job search condition directed towards intermediated labor markets

$$\begin{aligned} \frac{h'(lfp_t)}{u'(c_t)} &= p_{s_t} + (1-k^h_t)\underbrace{\chi}_{\equiv \mathbf{U}} \\ &+ \quad k^h_t \underbrace{\left[w_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[\left(\frac{1-k^h_{t+1}}{k^h_{t+1}}\right)\cdot\left(\frac{h'(lfp_{t+1})}{u'(c_{t+1})} - \chi\right) - \frac{p_{s_{t+1}}}{k^h_{t+1}}\right]\right\}\right]}_{\equiv \mathbf{W}(w_t,\theta_t)}, \end{aligned} \tag{133}$$

the surplus-sharing rule that determines wages w_t in monopolistic labor markets

$$\xi \cdot (\rho(N_{Mt}) - mc(N_{Mt})) + \mathbf{W}(w_t) - \mathbf{U}_t = \left(\frac{\xi}{1-\xi}\right)\mathbf{J}(w_t), \tag{134}$$

the surplus-sharing rule that determines Nash-bargained wages (with η denoting the employee's Nash bargaining power) in non-intermediated labor markets

$$\mathbf{W}(w_{Nt}) - \mathbf{U}_t = \left(\frac{\eta}{1-\eta}\right)\mathbf{J}(w_{Nt}), \tag{135}$$

the monopolistic matching-market pricing expression

$$\rho(N_{Mt}) = \mu(N_{Mt}) \cdot mc(N_{Mt}), \tag{136}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_t = \frac{v_t}{s_t}, \tag{137}$$

the definition of labor-market tightness in monopolistic recruiting markets

$$\theta_{Nt} = \frac{v_{Nt}}{s_{Nt}}, \tag{138}$$

along with the equilibrium input prices

$$p_{v_t} = m_v(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \tag{139}$$

and

$$p_{s_t} = m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right). \tag{140}$$

F Recruiting Sector Empirics: Employment in Staffing Firms

Figure 14: **Employment in Staffing Firms.** Source: https://americanstaffing.net/staffing-research-data/asa-data-dashboard/asa-employment-sales/. Similar patterns emerge if we consider temporary help employment (a common measure of employment in the recruiting sector)s.

G Proofs of Proposition 2 and Lemma 1.

This Appendix provides the proofs of Proposition 2 and Lemma 1. For simplicity, suppose $\rho = 1$ and $\omega = 1$ and that total vacancy posting costs are linear in vacancies. The following are the equilibrium conditions: aggregate LFP (which could potentially be fixed at $l\bar{f}p$) is

$$lfp_t = l\bar{f}p = s_{Nt} + s_t \cdot N_{Mt}, \tag{141}$$

the (symmetric equilibrium) LFP condition directed towards recruiting markets is

$$\frac{h'(lfp_t)}{u'(c_t)} = p_{s_t} + k^h(\theta_t) \cdot w_t + (1 - k^h(\theta_t)) \cdot \chi, \tag{142}$$

the LFP condition for matching via through random-search-and-bargaining is

$$\frac{h'(lfp_t)}{u'(c_t)} = k^h(\theta_{Nt}) \cdot w_{Nt} + (1 - k^h(\theta_{Nt})) \cdot \chi, \tag{143}$$

the (symmetric equilibrium) job-creation condition directed towards recruiting markets is

$$\gamma = p_{v_t} + k^f(\theta_t)\left(z_t - w_t\right), \tag{144}$$

the job-creation condition for matching via random-search-and-bargaining is

$$\gamma = k^f(\theta_{Nt})\left(z_t - w_{Nt}\right), \tag{145}$$

the (symmetric equilibrium) wage in the monopolistically-competitive recruiting sector stated in explicit form is

$$\begin{aligned} w_t &= \xi z_t + (1-\xi)\chi - \xi(1-\xi)\left(\rho(N_{Mt}) - mc(N_{Mt})\right) \\ &= \xi z_t + (1-\xi)\chi - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right), \end{aligned} \tag{146}$$

the input factor prices are

$$\begin{aligned} p_{v_t} &= m_v(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \\ &= (1-\xi) \cdot \theta_t^{-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right), \end{aligned} \tag{147}$$

and

$$
\begin{aligned}
p_{s_t} &= m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \\
&= \xi \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right), \qquad (148)
\end{aligned}
$$

the Nash wage stated in explicit form is

$$
w_{Nt} = \eta \cdot z_t + (1-\eta) \cdot \chi, \qquad (149)
$$

the free-entry condition for the monopolistically-competitive recruiting market is

$$
\begin{aligned}
\Gamma_{Mt} &= (\rho(N_{Mt}) - mc(N_{Mt})) \cdot m(s_t, v_t) \qquad (150) \\
&= \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \cdot m(s_t, v_t), \qquad (151)
\end{aligned}
$$

the law of motion for recruiters is

$$
N_{Mt} = (1-\omega) N_{Mt-1} + N_{MEt}, \qquad (152)
$$

which, with $\omega = 1$, is

$$
N_{Mt} = N_{MEt}, \qquad (153)
$$

the law of motion for labor is

$$
n_t = m(s_{Nt}, v_{Nt}) + \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t), \qquad (154)
$$

and the aggregate goods resource constraint is

$$
c_t + \gamma \cdot v_t \cdot N_{Mt} + \gamma_N v_{Nt} + \Gamma_{Mt} N_{MEt} - \rho(N_{Mt}) \cdot N_{Mt} \cdot m(s_t, v_t) = z_t n_t. \qquad (155)
$$

Analysis.

Job-Creation Conditions. The first step is to substitute the Nash wage (149) into (145), which gives

$$
\begin{aligned}
\gamma &= k^f(\theta_{Nt})\,(z_t - w_{Nt}) \\
&= m_N^{EFF} \cdot \theta_{Nt}^{-\xi} \cdot (1-\eta) \cdot (z_t - \chi), \qquad (156)
\end{aligned}
$$

in which the expression on the third line allows us to compute θ_{Nt}

$$\theta_{Nt} = \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)}\right]^{-1/\xi} \tag{157}$$

in closed form as a function of only exogenous parameters, which makes clear that $\partial\theta_{Nt}/\partial\eta < 0$.[25] Then, the second step is substitution of both the price p_{v_t} from (147) and the recruiting-market wage (146) in the job-creation condition (144) which yields, after several steps of algebra,

$$\gamma = (1-\xi) \cdot m^{EFF} \cdot \theta_t^{-\xi} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right], \tag{158}$$

which is an equilibrium restriction between recruiting-market tightness θ_t and the measure N_{Mt} of monopolistic recruiters, which can equivalently be written in closed form as

$$\theta_t = \left(\frac{(1-\xi) \cdot m^{EFF} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right]}{\gamma}\right)^{1/\xi}. \tag{159}$$

Labor Force Participation Conditions. Substitution of both the price p_{s_t} from (148) and the recruiting-market wage (146) into the LFP condition (142) for recruiting markets gives

$$\begin{aligned}
\frac{h'(lfp_t)}{u'(c_t)} &= p_{s_t} + k^h(\theta_t)w_t + (1-k^h(\theta_t))\chi \\
&= m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left[\xi z_t + (1-\xi)\chi - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right] \\
&\quad + \left(1 - m^{EFF} \cdot \theta_t^{1-\xi}\right)\chi \\
&= m_s(s_t, v_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left[\xi z_t - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right] \\
&\quad + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \chi - \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \chi + \chi - m^{EFF} \cdot \theta_t^{1-\xi} \cdot \chi \\
&= \xi \cdot k^h(\theta_t) \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left[\xi(z_t - \chi) - \xi(1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right] \\
&= \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) + \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot (z_t - \chi) \\
&\quad - \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right) \\
&= \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right).
\end{aligned} \tag{160}$$

[25] In the analogous first step of the steady-state analysis of the dynamic model, it is also clear that $\partial\theta_{Nt}/\partial\eta < 0$, but the result arises from implicit differentiation because there is no closed-form solution; the lack of a closed-form solution in the very first step of the counterpart analysis greatly complicates matters.

Next, substituting the Nash wage (149) into the LFP condition (143) gives

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= k^h(\theta_{Nt}) \cdot w_{Nt} + (1 - k^h(\theta_{Nt})) \cdot \chi \\ &= m_N^{EFF} \cdot {\theta_{Nt}}^{1-\xi} \cdot \eta \cdot (z_t - \chi). \end{aligned} \tag{161}$$

Substituting (157) into the θ_{Nt} term on the right-hand side gives

$$\begin{aligned}\frac{h'(lfp_t)}{u'(c_t)} &= m_N^{EFF} \cdot \left(\left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-1/\xi} \right)^{1-\xi} \cdot \eta \cdot (z_t - \chi) \\ &= m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \cdot \eta \cdot (z_t - \chi). \end{aligned} \tag{162}$$

Then, dividing (160) by (162) gives

$$\begin{aligned} & m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \eta \cdot (z_t - \chi) \\ = \;& \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right). \end{aligned} \tag{163}$$

Dividing by the expression on the left-hand side and dividing by $\theta_t^{1-\xi}$ gives

$$\theta_t^{\xi-1} = \frac{\xi \cdot m^{EFF} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right)}{m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \eta \cdot (z_t - \chi)}, \tag{164}$$

which is an equilibrium restriction between recruiting-market tightness θ_t and the measure N_{Mt} of monopolistic recruiters, which can equivalently be written in closed form as

$$\theta_t = \left(\frac{\xi \cdot m^{EFF} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right)}{m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \eta \cdot (z_t - \chi)} \right)^{-1/(1-\xi)}. \tag{165}$$

Summary.

We have constructed two closed-form equilibrium restrictions between θ_t and N_{Mt}, one which arises from job-creation directed towards recruiting markets

$$\theta_t = \left(\left(\frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right] \right)^{1/\xi} \tag{166}$$

and the other arises from labor-force participation directed towards recruiting markets

$$\theta_t = \left(\frac{\xi \cdot m^{EFF} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right)}{m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \eta \cdot (z_t - \chi)} \right)^{-1/(1-\xi)}, \tag{167}$$

which thus implies the condition

$$\left(\left(\frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right] \right)^{1/\xi} \tag{168}$$

$$= \left(\frac{\xi \cdot m^{EFF} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right)}{m_N^{EFF} \cdot \underbrace{\left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}} \eta \cdot (z_t - \chi)} \right)^{-1/(1-\xi)}$$

in which the only endogenous variable is N_{Mt} and, note, in which the Nash-bargaining parameter η appears.

Once N_{Mt} is determined from expression (168), recruiting-market tightness θ_t is determined (from either (166) or (167)), which in turn jointly pin down the factor input prices p_{s_t} and p_{v_t} and the recruiting-market wage w_t.

The remaining variables to be determined are s_t, s_{Nt}, v_t, v_{Nt}, and n_t. Substituting (141) — which, as a reminder, is $lfp_t = l\bar{f}p = s_{Nt} + s_t \cdot N_{Mt}$ — in the marginal utility function $h'(lfp_t)$ in the LFP condition for random-search bargaining markets (143) gives

$$\frac{h'(l\bar{f}p)}{u'(c_t)} = m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \cdot \eta \cdot (z_t - \chi), \tag{169}$$

which in turn gives

$$\frac{h'(s_{Nt} + s_t \cdot N_{Mt})}{u'(c_t)} = m_N^{EFF} \cdot \left[\frac{\gamma}{m_N^{EFF} \cdot (1-\eta) \cdot (z_t - \chi)} \right]^{-\frac{1-\xi}{\xi}} \cdot \eta \cdot (z_t - \chi), \tag{170}$$

which (because N_{Mt} has already been determined) is an equilibrium restriction between s_t and s_{Nt}. Next, analogously, substitution of (141) in the LFP condition for recruiting markets (142) gives

$$\frac{h'(l\bar{f}p)}{u'(c_t)} = \xi \cdot m^{EFF} \cdot \theta_t^{1-\xi} \cdot \left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) - (1-\xi) \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right), \tag{171}$$

which in turn gives

$$\frac{h'(s_{Nt}+s_t\cdot N_{Mt})}{u'(c_t)}=\xi\cdot m^{EFF}\cdot\theta_t^{1-\xi}\cdot\left(\frac{\rho(N_{Mt})}{\mu(N_{Mt})}+(z_t-\chi)-(1-\xi)\cdot\left(\rho(N_{Mt})-\frac{\rho(N_{Mt})}{\mu(N_{Mt})}\right)\right), \tag{172}$$

which (because N_{Mt} has already been determined) is a second equilibrium restriction between s_t and s_{Nt}. The equilibrium restrictions (170) and (172) thus jointly determine s_t and s_{Nt}. Given that θ_t, θ_{Nt}, s_t, and s_{Nt} have now been determined, the determination of v_t, v_{Nt}, and n_t easily follow from definitions of market tightness and the aggregate law of motion for employment. These results presume that $lfp_t=l\bar{f}p$, which in turn implies that disutility of participation $h'(l\bar{f}p)$ is fixed. To allow for endogenous lfp_t (rather than fixed $l\bar{f}p$), relax the restriction of fixed disutility of participation $h'(\cdot)$. Finally, if we are considering general equilibrium, the determination of c_t easily follows from the goods resource constraint.

Dixit-Stiglitz Aggregation.

For Dixit-Stiglitz aggregation, $\rho(N_{Mt})=N_{Mt}^{\frac{1}{\varepsilon-1}}$, $\mu(N_{Mt})=\frac{\varepsilon}{\varepsilon-1}$, and $\frac{\rho(N_{Mt})}{\mu(N_{Mt})}=\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}$. Substituting these expressions in (168) gives

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)+\xi\cdot\left(N_{Mt}^{\frac{1}{\varepsilon-1}}-\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)\right)\right)^{1/\xi} \tag{173}$$

$$=\left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)-(1-\xi)\cdot\left(N_{Mt}^{\frac{1}{\varepsilon-1}}-\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)\right)}{\eta\cdot m_N^{EFF}\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}(z_t-\chi)}\right)^{-1/(1-\xi)}$$

Simplifying terms step-by-step for clarity, we first have

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)+\xi\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\cdot\left(1-\frac{\varepsilon-1}{\varepsilon}\right)\right)\right)^{1/\xi} \tag{174}$$

$$=\left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)-(1-\xi)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\cdot\left(1-\frac{\varepsilon-1}{\varepsilon}\right)\right)}{\eta\cdot m_N^{EFF}\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}(z_t-\chi)}\right)^{-1/(1-\xi)}$$

Second

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)+\xi\cdot\frac{1}{\varepsilon}\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)\right)^{1/\xi} \tag{175}$$
$$= \left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)-(1-\xi)\cdot\frac{1}{\varepsilon}\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}\right)}{\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}}\right)^{-1/(1-\xi)}$$

Then,

$$\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{1/\xi} \tag{176}$$
$$= \left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)}{\eta\cdot m_N^{EFF}\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}(z_t-\chi)}\right)^{-1/(1-\xi)}.$$

Define the implicit function

$$\Upsilon(N_{Mt},\eta;\cdot)\equiv\left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}} \tag{177}$$
$$-\left(\frac{\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)}{\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\cdot\underbrace{\left[\frac{\gamma}{m_N^{EFF}\cdot(1-\eta)\cdot(z_t-\chi)}\right]^{-\frac{1-\xi}{\xi}}}_{=\theta_{Nt}}}\right)^{\frac{1}{\xi-1}}$$
$$= 0. \tag{178}$$

The partial to be computed is

$$\frac{\partial N_{Mt}}{\partial\eta}=-\frac{\Upsilon_\eta(\cdot)}{\Upsilon_{N_{Mt}(\cdot)}}. \tag{179}$$

Before proceeding, we rewrite, for the sake of ease of calculation, the implicit function in a couple of steps. First,

$$
\begin{aligned}
\Upsilon(N_{Mt},\eta;\cdot) &\equiv \left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}} \qquad (180)\\
&- \left(\xi\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi-1}}\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\cdot\left[\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}\cdot(z_t-\chi)}\right]^{\frac{\xi-1}{\xi}}\right)^{\frac{1}{1-\xi}}\\
&= 0.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\Upsilon(N_{Mt},\eta;\cdot) &\equiv \left(\left(\frac{1-\xi}{\gamma}\right)\cdot m^{EFF}\cdot\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}} \qquad (181)\\
&- \underbrace{\left(\xi m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi-1}}}_{\equiv\mathbf{D}>0}\underbrace{\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{\frac{1}{1-\xi}}}_{\equiv f(\cdot)}\underbrace{\left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}}_{\equiv g(\cdot)}\\
&= 0. \qquad (182)
\end{aligned}
$$

Computation of $\Upsilon_{N_{Mt}}(\cdot)$.

$$
\begin{aligned}
\Upsilon_{N_{Mt}}(\cdot) &= \frac{1}{\xi}\left(\left(\frac{1-\xi}{\gamma}\right)m^{EFF}\left(\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi}-1}\left(\frac{1}{\varepsilon-1}\right)\left(\frac{1-\xi}{\gamma}\right)m^{EFF}\left(\frac{\varepsilon-1+\xi}{\varepsilon}\right)N_{Mt}^{\frac{1}{\varepsilon-1}-1}\\
&+ \frac{\left(\frac{1}{1-\xi}\right)\left(\xi m^{EFF}\cdot\left(\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)N_{Mt}^{\frac{1}{\varepsilon-1}}+(z_t-\chi)\right)\right)^{\frac{1}{\xi-1}-1}\left(\frac{1}{\varepsilon-1}\right)\xi m^{EFF}\left(\frac{\varepsilon-1-(1-\xi)}{\varepsilon}\right)\cdot N_{Mt}^{\frac{1}{\varepsilon-1}-1}}{\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{-\frac{1}{1-\xi}}\left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{\frac{1}{\xi}}}\\
&> 0 \qquad (183)
\end{aligned}
$$

Because the first term is strictly positive and the second term is also strictly positive (more precisely, both the numerator and the denominator of the second term are strictly positive), the partial $\Upsilon_{N_{Mt}}(\cdot)$ is unambiguously positive ($\Upsilon_{N_{Mt}}(\cdot)>0$).

Computation of $\Upsilon_{\eta}(\cdot)$.

For the computation of the partial $\Upsilon_{\eta}(\cdot)$, it is only the second line of (182) that is needed because η only appears in the second line. Moreover, for the sake of simplicity of notation, we use the term that is as defined $\mathbf{D}$ in the second line of (182) and define temporarily the functions

$$f(\cdot)=\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{\frac{1}{1-\xi}}>0 \qquad (184)$$

and

$$g(\cdot)=\left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}>0, \qquad (185)$$

which imply the partials

$$f_\eta(\cdot) = \frac{m_N^{EFF}\cdot(z_t-\chi)}{1-\xi}\cdot\left(\eta\cdot m_N^{EFF}\cdot(z_t-\chi)\right)^{\frac{1}{1-\xi}-1} > 0 \tag{186}$$

$$\begin{aligned}
g_\eta(\cdot) &= -\frac{1}{\xi}\cdot\left(\frac{\gamma\cdot(1-\eta)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}-1}\cdot\left(\frac{\gamma\cdot(1-\eta)^{-2}}{m_N^{EFF}(z_t-\chi)}\right)\\
&= -\frac{1}{\xi}\cdot\left(\frac{\gamma}{(1-\eta)\cdot m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot\left(\frac{\gamma}{(1-\eta)\cdot m_N^{EFF}(z_t-\chi)}\right)^{-1}\cdot\left(\frac{\gamma}{(1-\eta)^2\cdot m_N^{EFF}(z_t-\chi)}\right)\\
&= -\frac{1}{\xi}\cdot\left(\frac{\gamma}{(1-\eta)\cdot m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot\left(\frac{(1-\eta)\cdot m_N^{EFF}(z_t-\chi)}{\gamma}\right)\cdot\left(\frac{\gamma}{(1-\eta)^2\cdot m_N^{EFF}(z_t-\chi)}\right)\\
&= -\frac{1}{\xi}\cdot\left(\frac{\gamma}{(1-\eta)\cdot m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{-1}\\
&= -\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1}{\xi}}\cdot(1-\eta)^{-1}\\
&= -\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1}{\xi}-1}\\
&= -\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1-\xi}{\xi}}\\
&< 0
\end{aligned} \tag{187}$$

The Nash bargaining parameter η is intentionally emphasized in each of the functions $f(\cdot)$, $g(\cdot)$, $f_\eta(\cdot)$, and $g_\eta(\cdot)$, because it is the parameter around which the comparative static exercise is being conducted. With the natural restrictions on the model parameters ($\xi\in(0,1)$, $z_t>0$, $m_N^{EFF}>0$, $1<\varepsilon<\infty$, $\gamma>0$,, $z_t-\chi>0$, $\eta\in(0,1)$), the functions $f(\cdot)$ and $g(\cdot)$ are both unambiguously positive, the partial $f_\eta(\cdot)$ is unambiguously positive, and the partial $g_\eta(\cdot)$ is unambiguously negative. Stated in terms of $\mathbf{D}$, $f(\cdot)$, $g(\cdot)$, $f_\eta(\cdot)$, and $g_\eta(\cdot)$, the partial of the implicit function $\Upsilon(\cdot)$ with respect to Nash bargaining power η is

$$\begin{aligned}
\Upsilon_\eta(\cdot) &= -\underbrace{\mathbf{D}}_{>0}\cdot\left(\underbrace{f_\eta(\cdot)}_{>0}\cdot\underbrace{g(\cdot)}_{>0}+\underbrace{f(\cdot)}_{>0}\cdot\underbrace{g_\eta(\cdot)}_{<0}\right)\\
&\underbrace{=}_{?} 0
\end{aligned} \tag{188}$$

whose sign depends on whether $\eta<\xi$, $\eta=\xi$, or $\eta>\xi$.

Proof of Proposition 2.

Starting with the simple case, evaluating the function (188) at $\eta = \xi$,

$$
\begin{aligned}
f_\eta(\cdot)\cdot g(\cdot) + f(\cdot)\cdot g_\eta(\cdot) = {} & \underbrace{\frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}-1}}_{=f_\eta(\cdot)}\cdot\underbrace{\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}}_{=g(\cdot)} \\
& -\underbrace{\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}}_{=f(\cdot)}\cdot\underbrace{\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}}}_{=-g_\eta(\cdot)} \\
= {} & \frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}-1}\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \\
& -\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}} \\
= {} & \frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{-1}\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}} \\
& -\Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}} \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left[\left(\frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\right)\cdot\left(\frac{1}{\xi\cdot m_N^{EFF}(z_t-\chi)}\right)\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}-\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}}\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}-\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1-\xi}{\xi}}\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}-\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot(1-\xi)^{-1}\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot(1-\xi)^{\frac{1}{\xi}}-\frac{1}{\xi}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot(1-\xi)^{-1}\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot(1-\xi)^{\frac{1}{\xi}}-\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\cdot(1-\xi)^{\frac{1}{\xi}}\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot\left[\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)-\left(\frac{1}{1-\xi}\right)\cdot\left(\frac{1}{\xi}\right)\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot\left[\frac{1}{(1-\xi)\cdot\xi}-\frac{1}{(1-\xi)\cdot\xi}\right] \\
= {} & \Big(\xi\cdot m_N^{EFF}(z_t-\chi)\Big)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\xi)^{\frac{1}{\xi}}\cdot\underbrace{\left[\frac{1}{\xi-\xi\cdot\xi}-\frac{1}{\xi-\xi\cdot\xi}\right]}_{=0} \\
= {} & 0
\end{aligned}
$$

leads, as the last line clearly shows, to

$$\Upsilon_\eta^{\eta=\xi}(\cdot) = 0, \tag{190}$$

which in turns immediately implies

$$
\begin{aligned}
\frac{\partial N_{Mt}}{\partial \eta} &= -\frac{\Upsilon_\eta(\cdot)}{\Upsilon_{N_{Mt}(\cdot)}} \\
&= 0.
\end{aligned} \tag{191}
$$

The conclusion is that the equilibrium measure of monopolistically-competitive recruiters N_{Mt} is maximized if Nash-bargained wages in the random-search channel lead to **efficient** outcomes in the sense of Mortensen (1982) and Hosios (1990).[26]

Proof of Lemma 1.

Replacing the highlighted ξ terms that appear in expression (189) with η gives

$$\begin{aligned} f_\eta(\cdot)\cdot g(\cdot)+f(\cdot)\cdot g_\eta(\cdot) = {} & \underbrace{\frac{m_N^{EFF}(z_t-\chi)}{1-\xi}\cdot\left(\eta\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}-1}}_{=f_\eta(\cdot)}\cdot\underbrace{\left(\frac{\gamma(1-\xi)^{-1}}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}}_{=g(\cdot)} \\ & -\underbrace{\left(\eta\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}}_{=f(\cdot)}\cdot\underbrace{\frac{1}{\xi}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1-\xi}{\xi}}}_{=-g_\eta(\cdot)} \\ = {} & \left(\eta\cdot m_N^{EFF}(z_t-\chi)\right)^{\frac{1}{1-\xi}}\cdot\left(\frac{\gamma}{m_N^{EFF}(z_t-\chi)}\right)^{-\frac{1}{\xi}}\cdot(1-\eta)^{\frac{1}{\xi}}\cdot\left[\frac{1}{\eta-\eta\cdot\xi}-\frac{1}{\xi-\eta\cdot\xi}\right] \end{aligned} \tag{192}$$

from which it follows that the sign of $\Upsilon_\eta(\cdot)$, and hence the sign of $\frac{\partial N_{Mt}}{\partial\eta}$, depends only on whether the term in square brackets is positive or negative, which, in turn (and as is clear from observation of the term in square brackets) depends only on whether $\eta>\xi$ or $\eta<\xi$. If $\eta>\xi$, then, based on (188),

$$\begin{aligned} \frac{\partial N_{Mt}}{\partial\eta} &= -\frac{\Upsilon_\eta(\cdot)}{\Upsilon_{N_{Mt}}(\cdot)} \\ &< 0, \end{aligned} \tag{193}$$

whereas if $\eta<\xi$, then, based on (188),

$$\begin{aligned} \frac{\partial N_{Mt}}{\partial\eta} &= -\frac{\Upsilon_\eta(\cdot)}{\Upsilon_{N_{Mt}}(\cdot)} \\ &> 0. \end{aligned} \tag{194}$$

Limiting Argument.

This pair of results are limiting arguments that prove that N_{Mt} is indeed maximized (rather than minimized) at $\eta=\xi$; formally, the fact that the left-hand limit

$$\lim_{\eta\to\xi^-}\frac{\partial N_{Mt}}{\partial\eta}=0 \tag{195}$$

and the right-hand limit

$$\lim_{\eta\to\xi^+}\frac{\partial N_{Mt}}{\partial\eta}=0 \tag{196}$$

[26] Or, potentially, minimized; we rule out the minimization outcome below through a limiting argument.

are identical, N_{Mt} is (at least locally) maximized at $\eta = \xi$.

Recruiting Market Tightness and Remaining Variables.

Having solved the comparative static results for N_{Mt} with respect to η, $\frac{\partial N_{Mt}}{\partial \eta}$, the next step is to understand the comparative static results for (equilibrium) recruiting-market tightness θ_t. The comparative static result requires computation of

$$\frac{\partial \theta_t}{\partial \eta} = \frac{\partial \theta_t}{\partial N_{Mt}} \cdot \frac{\partial N_{Mt}}{\partial \eta}. \tag{197}$$

Based on the closed-form equilibrium restriction between θ_t and N_{Mt} that arises from job-creation directed towards recruiting markets that is stated in general form in expression (166), which, for the sake of convenience, is repeated here,

$$\theta_t = \left(\left(\frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left[\frac{\rho(N_{Mt})}{\mu(N_{Mt})} + (z_t - \chi) + \xi \cdot \left(\rho(N_{Mt}) - \frac{\rho(N_{Mt})}{\mu(N_{Mt})} \right) \right] \right)^{1/\xi}, \tag{198}$$

the Dixit-Stiglitz version[27] is

$$\theta_t = \left(\left(\frac{1-\xi}{\gamma} \right) \cdot m^{EFF} \cdot \left(\left(\frac{\varepsilon - 1 + \xi}{\varepsilon} \right) \cdot N_{Mt}^{\frac{1}{\varepsilon-1}} + (z_t - \chi) \right) \right)^{\frac{1}{\xi}}.$$

The partial with respect to N_{Mt} is

$$\begin{aligned} & \frac{\partial \theta_t}{\partial N_{Mt}} \\ = \; & \frac{1}{\xi} \left(\left(\frac{1-\xi}{\gamma} \right) m^{EFF} \left(\left(\frac{\varepsilon - 1 + \xi}{\varepsilon} \right) \cdot N_{Mt}^{\frac{1}{\varepsilon-1}} + (z_t - \chi) \right) \right)^{\frac{1}{\xi}-1} \left(\frac{1}{\varepsilon - 1} \right) \left(\frac{1-\xi}{\gamma} \right) m^{EFF} \left(\frac{\varepsilon - 1 + \xi}{\varepsilon} \right) N_{Mt}^{\frac{1}{\varepsilon-1}-1} \\ > \; & 0, \end{aligned}$$

which, as stated in the last line, is strictly positive because each term is strictly positive. Thus, based on (197), the sign of $\frac{\partial \theta_t}{\partial \eta}$ is the same as the sign of $\frac{\partial N_{Mt}}{\partial \eta}$.

The signs of the remaining variables (p_{s_t}, p_{v_t}, w_t, s_t, v_t, s_{Nt}, v_{Nt}, n_t, c_t, and lfp_t) with respect to η then easily follow from the conditions stated at the beginning of Appendix G.

[27] After substituting the Dixit-Stiglitz functions $\rho(N_{Mt}) = N_{Mt}^{\frac{1}{\varepsilon-1}}$, $\mu(N_{Mt}) = \frac{\varepsilon}{\varepsilon-1}$, and $\frac{\rho(N_{Mt})}{\mu(N_{Mt})} = \left(\frac{\varepsilon-1}{\varepsilon}\right) \cdot N_{Mt}^{\frac{1}{\varepsilon-1}}$.