Efficiency and Labor Market Dynamics in a Model of Labor Selection *

Sanjay K. Chugh †
Boston College
Kiel Institute for the World Economy

Christian Merkl‡
Friedrich-Alexander-University Erlangen-Nuremberg
Kiel Institute for the World Economy
IZA Institute

First Draft: September 2010
This Draft: August 9, 2015

Abstract

This paper characterizes efficient labor-market allocations in a labor selection model. The model's crucial aspect is cross-sectional heterogeneity for new job contacts, which leads to an *endogenous* selection threshold for new hires. With cross-sectional dispersion calibrated to *microeconomic* data, 40 percent of empirically-relevant fluctuations in the job-finding rate arise, which contrasts with results in an efficient search and matching economy. The efficient selection model's results hold in partial and general equilibrium, as well as with sequential search.

Keywords: labor market frictions, hiring costs, sequential search, efficiency, amplification
JEL Classification: E24, E32, J20

*We thank Sushant Acharya, Susanto Basu, Matteo Cacciatore, Ryan Chahrour, Timothy Fuerst, Simon Gilchrist Veronica Guerrieri, Joseph Kaboski, Wolfgang Lechthaler, Guido Lorenzoni, Steven Lugauer, Michael Pries, Federico Ravenna, Claire Reicher, Aysegul Sahin, Nicolas Vincent, and Henning Webber for useful feedback at various stages of the project. Sanjay K. Chugh thanks the "Schöller Forschungszentrum" for financial support. Christian Merkl thanks the Fritz Thyssen foundation for financial support during his visit at the National Bureau of Economic Research where the project was initiated and the German Science Foundation (SPP 1764). Ana Lariau, Verena Maier, and Killian Ruppert have provided brilliant research assistance. We received helpful comments at presentations at the Federal Reserve Board, the Federal Reserve Bank of New York, the 2011 SED Meetings, the German Economic Association, Sveriges Riksbank, the Federal Reserve Bank of Cleveland, the Ifo Macro Seminar, the Boston College Macro Lunch Workshop, the University of Notre Dame, the Kiel Institute, and HEC Montréal.

†Email: sanjay.chugh@bc.edu.
‡Corresponding author. Email: christian.merkl@fau.de.

Contents

1 Introduction

This paper uses *microeconomic* data on heterogeneity in training costs (or, more generally, idiosyncratic productivity for new matches) to show that a labor selection model displays large fluctuations in *aggregate* labor markets. Based on our micro-calibration, the main result is that, conditional on productivity shocks, roughly 40 percent of empirically-relevant fluctuations in the U.S. job-finding rate can be described by an *efficient* labor selection mechanism. These results are several times larger than in an efficient search and matching model, and the efficient selection model's results hold in both partial equilibrium fluctuations and in general equilibrium fluctuations.

Selection as an important margin of adjustment in firms' hiring decisions is a long-standing realistic idea, but has not been much emphasized in macro-labor analysis. An early important empirical firm-level contribution was Barron, Bishop, and Dunkelberg (1985, p. 50), who adopt and find strong evidence for the view that "...most employment is the outcome of an employer selecting from a pool of job applicants..." due to cross-sectional heterogeneity in the pool of applicants. Davis, Faberman, and Haltiwanger (2013) add further evidence to the view that, in their terminology, "hiring standards" play an important role among the many margins of labor adjustment. Selection issues seemingly would be an important component of hiring standards.

Our model's analysis revolves around one critical *microeconomic* parameter, which is the cross-sectional dispersion of new hires' idiosyncratic training costs (or, more generally, idiosyncratic productivity for new workers). Taking this cross-sectional dispersion as an *exogenous* primitive, the model economy delivers an *endogenous* selection threshold for new employees. The primary data source used to discipline this cross-sectional dispersion is the 1982 U.S. Employer Opportunity Pilot Project (EOPP), which many researchers continue to use to this day. Our focus, unlike many others who use the rich EOPP data, is on the cross-section of new hires' training costs; to that effect, we use the cross-sectional moments calculated by Barron, Black, and Loewenstein (1989, Table 1).

The social planner framework we construct intentionally says nothing about decentralization via wages. However, we show in Appendix D that efficiency can be decentralized. For the sake of robustness, we also use a second, independent, data source to discipline the cross-section, which are micro-level wage data for newly-hired employees constructed by Haefke, Sonntag, and van Rens (2013). We show in two different model based ways that the wage dispersion for newly hired employees is a very useful information to discipline our calibration exercise. Regardless of which micro data source is used, macro-level volatility in labor markets is an order of magnitude larger than in an efficient search and matching framework.

We show analytically that the outside option plays a different role in efficient selection models compared to efficient search and matching models. In the selection model, the outside option has *zero* first-order effect on the steady-state elasticity of the job-finding rate with respect to productivity. In contrast, the outside option appears directly in the search and matching model's steady-state elasticity of the job-finding rate with respect to productivity. In the selection framework, the driving force for amplification is the shape of the idiosyncratic distribution at the endogenous selection threshold.

The selection model's results depend on a distributional assumption about heterogeneous training characteristics (which could be interpreted as "match quality" characteristics), and, in turn, how large the mass of individuals is that moves across the endogenously time-varying selection threshold conditional on aggregate productivity shocks. We thus consider several different distributions. We also allow for sequential search (à la McCall (1970) and Mortensen (1970)) to permit an arbitrary number of "job arrivals" or "interviews" (and hence an arbitrary number of "match-quality" realizations) during a given time period. The quantitative results of course differ slightly as we vary distributions and vary the number of per-period contacts (aka, the job arrival rate) via sequential search within an empirically plausible range, but the main result remains the same: an efficient labor selection model provides powerful amplification effects on job-finding rates and unemployment rates in response to productivity shocks.

Focusing the model on selection effects and efficiency allows us to highlight that no other frictions are needed to deliver sharp fluctuations in the labor market, which makes the intuition easy to understand.[1,2] This focus on selection does not deny that other frictions, be they on quantities or on prices, do not or cannot play an important role in macro-labor dynamics. Rather, our focus on efficient selection is meant as a first step.

Extensively studied in the literature is the role of search and matching frictions, along with frameworks such as the involuntary unemployment framework of Christiano, Trabandt, and Walentin (2010), the rigid-wage class of models as in Gali (2011) and Gali, Smets, and Wouters (2012), as well as earlier versions of the labor selection model incorporating New Keynesian pricing frictions and wage setting frictions, such as Lechthaler, Merkl, and Snower (2010). These various models, based on different primitives and distortions, do not disentangle the amplification effects that are due to efficient surplus splitting versus inefficient surplus splitting. The point of our work is to analytically and quantitatively analyze efficient allocations in a tractable dynamic selection model, in a way that can be traced all the way back to the early indivisible (and efficient) labor models of Hansen (1985) and Rogerson (1988). Our model and its workings has interpretations in terms

[1] Carlsson, Eriksson, and Gottfries (2013) find that the market tightness of local labor markets in Sweden does not affect job creation. This suggests that other margins such as labor selection may be at work.

[2] While this focus on one margin of the labor market (namely, selection) may seem extreme at first sight, it is similar to many search and matching papers that assume that all workers that make a contact with a firm get matched. This is the case if the matching function is interpreted as a contact function (i.e., contact is endogenous, while selection is exogenous). For a further discussion of this issue see Brown, Merkl, and Snower (2015).

of the indivisible labor models of Hansen (1985) and Rogerson (1988), and it has connections with their generalization in terms of heterogeneity developed in Mulligan (2001).

The rest of the paper is organized as follows. Section 2 describes the details of the selection model, derives analytical results for the efficient partial equilibrium version and provides some intuition. Section 3 shows quantitative results in partial equilibrium and checks for the robustness of these results. Section 4 enlarges the model to general equilibrium by including endogenous labor-force participation and an aggregate goods resource constraint, in which the labor-market results carry through. Section 5 concludes.

2 Labor Markets I: Analytics

This section defines the basic labor market setup and notation that will be used throughout the paper. To focus on the main results of the model, both this Section 2 and Section 3 abstract from consumption markets and hence could be thought of "partial equilibrium."

2.1 Structure of Labor Market

There are two central features of the model. First, each newly selected worker imposes training costs on his employer (or, more generally, he draws a match-specific idiosyncratic shock).[3] The training costs consist of a fixed component, $\gamma^h > 0$, and an idiosyncratic component, ε_t^i, for newly-hired individual i. The idiosyncratic component is revealed only when a potential worker makes contact with a firm, as shown in Figure 1. The idiosyncratic component of training is the "match quality" cost in our analysis.[4] There is a continuum of production units, each of which is constant-returns-to-scale and homogenous in all other respects.

This is the second central feature of the model: depending on his training costs realized at a specific production unit, the potential new worker may or may not be hired. If not hired, the individual makes finitely many more "meetings with" or "contacts with" (we use the terms "meeting," "contact," "job arrival," and"job offer" synonymously) another firm, at which he may have had a lower contact-specific training cost, during the same period. This latter aspect of the model is the well-known *sequential search* model. Until we incorporate sequential search in Section 3 below, suppose that each individual available to be selected for work in a given time period makes contact with one arbitrary production unit with probability one. The one-contact-

[3]We will interchangeably refer to "firms" and "production units" in what follows because of the linearity of the production technology and the fact that we study only efficient allocations, so the manner in which employment surpluses are split via wages in a decentralized economy does not affect the results.

[4]There is no traditional matching function in the model as in the Pissarides (1985) framework. One could suppose that a simple matching, or "contact," function exists in which the measure of matches equals the number of searching individuals.

per-period framework conveys virtually all of the economic intuition of the paper.

From a planning perspective, it is thus efficient to hire only those individuals with sufficiently attractive characteristics. Sufficiency is characterized by an endogenous state-contingent selection threshold, $\tilde{\varepsilon}_t$. Because individual i's idiosyncratic characteristics are defined as a *cost*, he is hired only if $\varepsilon_t^i \leq \tilde{\varepsilon}_t$. Training costs are i.i.d. across worker-firm pairs within any given period and across time periods, with probability density function $f(\varepsilon_t)$. Selection occurs with endogenous probability $\eta(\tilde{\varepsilon}_t)$, which is the cumulative distribution function of ε; all searching individuals being considered for a job draw from the same distribution.

Both the fixed component of training costs γ^h and the idiosyncratic component of training costs ε_t^i are measured in units of output, and they are incurred in only the *first* period of a new employment relationship.[5] In contrast, each incumbent worker has zero training costs and produces stochastic output z_t in period t. Newly hired worker i produces output net of training costs $z_t - \varepsilon_t^i - \gamma^h$.

An individual who is not selected for work instead receives an outside option b. Absent general equilibrium, this outside option can interchangeably be thought of as the utility value of leisure, government-provided unemployment benefits, or home-produced goods. There is no intensive margin of labor. Each unit of labor is thus to be be thought of as indivisible in the sense of Hansen (1985) and Rogerson (1988), as well as in the baseline search and matching model.

Define $H(\tilde{\varepsilon}_t) \equiv \int_{\varepsilon \leq \tilde{\varepsilon}_t} \varepsilon f(\varepsilon) d\varepsilon$ (with $f(\varepsilon)$ as the probability density at ε), and define r as the net real interest rate across consecutive time periods. From an efficiency perspective, the dynamic surplus maximization problem for the representative production unit is thus

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[z_t n_t + s_t(1-\eta(\tilde{\varepsilon}_t))b - s_t \eta(\tilde{\varepsilon}_t) \left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right)\right] \tag{1}$$

subject to the endogenous law of motion for employment

$$n_t = (1-\rho) n_{t-1} + \eta(\tilde{\varepsilon}_t) s_t \tag{2}$$

and an expression defining the measure of searching individuals who are available for work during the period-t selection process,

$$s_t = lfp - (1-\rho) n_{t-1}, \tag{3}$$

in which labor force participation (lfp) is an exogenous constant in this section, but is endogenous in the full general equilibrium model of Section 4.

In the surplus function (1), $(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)})$ measures the average training cost across all new

[5] Individuals who have been employed for more than one period are identical in their characteristics.

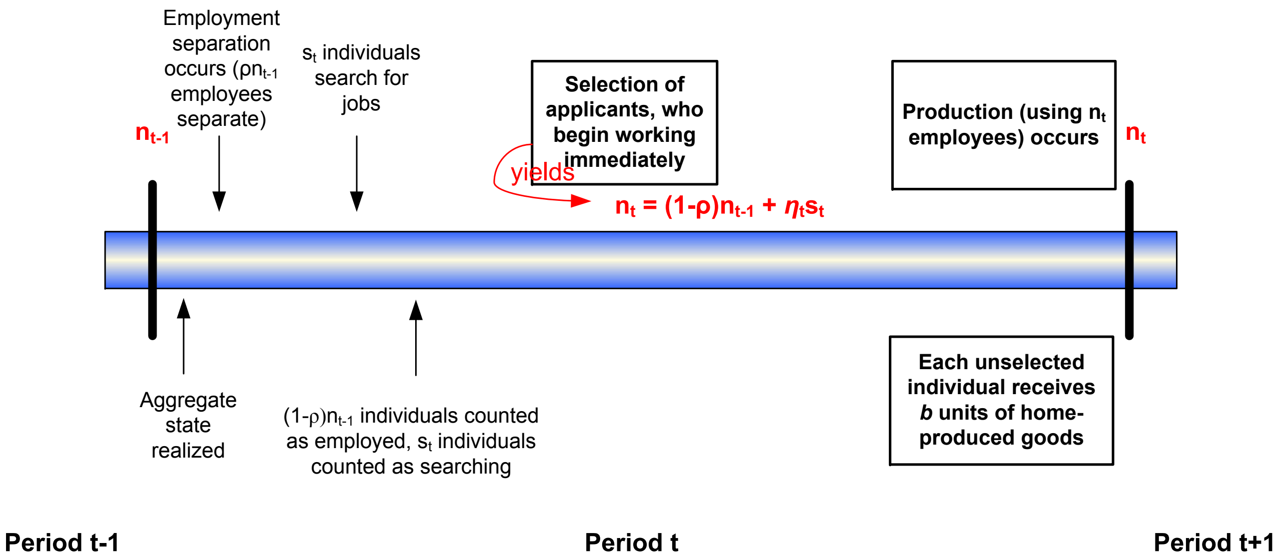

Figure 1: Timing of events in labor market.

employees, with $H(\tilde{\varepsilon}_t)/\eta(\tilde{\varepsilon}_t)$ measuring the average *idiosyncratic* component of training costs. As the law of motion (2) shows, there are "instantaneous transitions" into employment, in which new employees begin producing right away, rather than with a one-period delay. The number of individuals that receive the outside option is thus the unselected searchers, $s_t(1 - \eta(\tilde{\varepsilon}_t))$.[6] These unselected searchers are counted as unemployed.

The law of motion (2) also shows that each individual has a fixed probability ρ of separation. While heterogeneity in principle may also be empirically important for separations, the assumption of fixed ρ allows the model to clearly separate heterogeneity as a basis for hiring versus for firing.[7] Finally, (1) and (2) formally show that constant-returns-to-scale production of goods means it is without loss of generality to examine the problem of a single production unit that hires one worker or that hires many workers. We adopt the latter setup because it easily fits into the full model in Section 4. Figure 1 summarizes the model timing.

[6]This has also become a standard in models featuring labor matching frictions, so we adopt it for comparability. The main results do not depend on the particular timing of labor transitions, however.

[7]The seminal theoretical reference in the matching literature for endogenous separation is Mortensen and Pissarides (1994).

2.2 Efficient Allocations

The state-contingent first-order conditions of the surplus maximization problem with respect to n_t and $\tilde{\varepsilon}_t$ yield the efficient selection condition

$$\gamma^h + \tilde{\varepsilon}_t = z_t - b + \left(\frac{1-\rho}{1+r}\right) E_t \left\{ H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \tilde{\varepsilon}_{t+1} \right\}, \tag{4}$$

which is the central expression of the paper.[8] Efficient allocations are thus defined as state-contingent functions $\{\tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ that satisfy the sequence of selection conditions (4) and aggregate laws of motion of labor (2).

The intertemporal condition (4) characterizes the dynamics of $\tilde{\varepsilon}_t$. It corresponds to the matching model's vacancy-creation (or job-creation) condition; it also corresponds to the real business cycle (RBC) model's Euler equation for efficient capital accumulation. Even though our model does not have "physical capital" in the strict RBC sense, the creation of an employment match is an investment activity that yields a long-lasting asset. The selection condition is the intuitive heart of the model, so it is worth examining further.

To build intuition, first consider the stark case of $\rho = 1$, which makes employment a one-period, though not a frictionless, phenomenon. This yields a simple version of the selection condition, $\gamma^h + \tilde{\varepsilon}_t = z_t - b$, which implies that the elasticity of the selection threshold with respect to aggregate productivity along the business cycle is

$$\frac{\partial \ln \tilde{\varepsilon}_t}{\partial \ln z_t} = \frac{z_t}{\tilde{\varepsilon}_t}. \tag{5}$$

Given that γ^h is the average fixed training cost for every new hire, the distribution of ε_t, without loss of generality, can be centered around zero.

With the more realistic $\rho < 1$, an explicit elasticity can be derived only in the steady state. From the deterministic steady state version of (4), the elasticity of (steady-state) $\tilde{\varepsilon}$ with respect to (steady-state) z is

$$\frac{\partial \ln \tilde{\varepsilon}}{\partial \ln z} = \left(\frac{z}{\tilde{\varepsilon}}\right)\left(\frac{1+r}{r+\rho+(1-\rho)\eta(\tilde{\varepsilon})}\right). \tag{6}$$

Two observations based on (6) are important. First, the second term on the right-hand side is larger than unity given that $\rho \in [0,1]$ and $\eta(\tilde{\varepsilon}) \in [0,1]$. This elasticity is thus even larger than the one-period elasticity in (5). Second, the sensitivity of the threshold $\tilde{\varepsilon}$ to aggregate productivity does *not* directly depend on b and hence does not depend directly on the social surplus $z - b$.[9] Similarly,

[8] Its derivation appears in Appendix A.

[9] At higher-order approximations, the sensitivity of $\tilde{\varepsilon}$ to productivity shocks in general will depend on b because $\tilde{\varepsilon}$ is fundamentally a function of b (albeit a nonlinear one) through the selection condition. But these effects are indirect (i.e., of second- and higher order), unlike the effects of small surpluses in matching models, which have first-order effects. The latter effect can be seen in, for example, Hagedorn and Manovskii (2008, p. 1695), which highlights

the sensitivity does not depend on the average level of training costs γ^h.[10] Local fluctuations of the threshold training cost around the steady state are thus insensitive to the size of the social surplus and can be large, which is in sharp contrast to an efficient version of a labor search and matching model, as described in Section 2.4.

It is not the sensitivity of fluctuations of just the selection threshold, but, more importantly, of the sensitivity of the finding rate around the threshold $\eta(\tilde{\varepsilon}_t)$ that are relevant for the analysis. Using the implicit function theorem, the steady-state elasticity of the finding rate with respect to aggregate productivity is

$$\begin{aligned} \frac{\partial \ln \eta(\tilde{\varepsilon})}{\partial \ln z} &= \frac{\partial \ln \eta(\tilde{\varepsilon})}{\partial \ln \tilde{\varepsilon}} \frac{\partial \ln \tilde{\varepsilon}}{\partial \ln z} \\ &= \frac{\eta'(\tilde{\varepsilon})}{\eta(\tilde{\varepsilon})} \cdot z \cdot \left(\frac{1+r}{r+\rho+(1-\rho)\eta(\tilde{\varepsilon})} \right). \end{aligned} \tag{7}$$

While most of the parameters and variables in this elasticity expression have commonly-used values in the literature (or at least a range of plausible values) for the U.S., (e.g. r, $\eta(\tilde{\varepsilon})$, ρ), the first derivative of the selection rate ($\eta'(\tilde{\varepsilon})$) is less straightforward. Intuitively, the shape of the distribution of training costs at the cutoff point matters substantially. With a large mass of workers around the cutoff point $\tilde{\varepsilon}$, aggregate productivity shocks can be strongly amplified

2.3 Analytical Example

To illustrate the selection model's power, let's abstract for a moment from the dependence of $\eta'(\tilde{\varepsilon})$ on $\tilde{\varepsilon}$ by assuming a uniform distribution. A uniform distribution of course displays constant $\eta'(\tilde{\varepsilon})$ $(= f(\tilde{\varepsilon}))$ $\forall$ ε. That is, the probability density does not vary across the distribution. The $\mathcal{U}$ assumption in principle *weakens* our results precisely because $\eta'(\tilde{\varepsilon})$ does *not* depend on $\tilde{\varepsilon}$.[11]

To convey the intuition as clearly as possible, take the $\mathcal{U}[-x, x]$ distribution, as sketched in Figure 2. Setting $x = 1.2$ (reasons for this calibration are described in Section 3.1), the probability density is $f(\varepsilon) = \frac{1}{2.4}$ and the selection rate is $\eta(\tilde{\varepsilon}) = \frac{\tilde{\varepsilon}+1.2}{2.4}$. In the elasticity expression (7), insert U.S. economy quarterly parameter values of $\rho = 0.1$, $r = 0.01$, and a quarterly finding rate of $\eta(\tilde{\varepsilon}) = 0.58$ (and, without loss of generality, normalize $z = 1$). With these numerical values, the

that the social surplus $z - b$ *does* strongly affect the matching model's dynamics, even though it does not affect the selection model's dynamics.

[10] However, there is an indirect effect that larger γ^h and larger b reduce the steady state value of $\eta(\tilde{\varepsilon})$, which may lead to larger log-deviations for a given absolute deviation. Because the long-run selection rate in the model is calibrated to the empirically relevant long-run finding rate in U.S. data in all partial equilibrium and general equilibrium simulations below, this indirect effect is excluded.

[11] The argument in this statement supposes that, for a non-uniform distribution, the endogenous selection point $\tilde{\varepsilon}$ is in a region of the distribution in which the probability density is declining — that is, in a region around $\tilde{\varepsilon}$ in which $\eta''(\varepsilon) < 0$, which turns out to be true in our quantitative experiments below for non-uniform distributions.

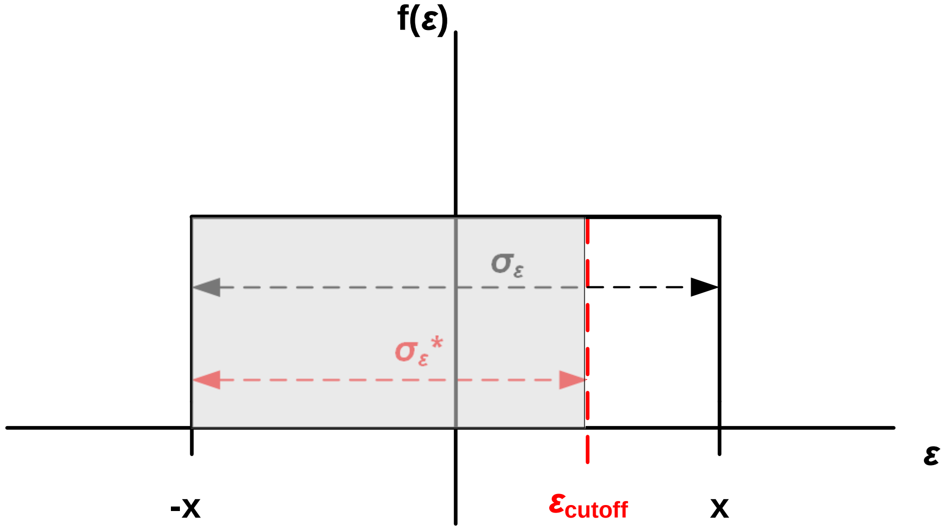

Figure 2: **Analytical example.** Illustration of the $\mathcal{U}[-x,x]$ density from which the match-specific training cost of potential new hires are drawn. Higher values of ε mean higher training costs. The endogenously-determined selection threshold is denoted by $\tilde{\varepsilon}$, the endogenous finding rate (the cumulative distribution at the cutoff $\tilde{\varepsilon}$) is $\eta(\tilde{\varepsilon})$ (the grey shaded area), the standard deviation for those hired is denoted by σ^*_ε, which is smaller than the population standard deviation σ_ε.

elasticity of the finding rate with respect to aggregate productivity around the steady state is

$$\frac{\partial \ln \eta(\tilde{\varepsilon})}{\partial \ln z} = 1.15, \tag{8}$$

which is an order of magnitude *larger* than the comparable elasticity in the efficient version of the labor matching model.

Vis-à-vis the data, this elasticity of 1.15 (in the simplest analytical case of the efficient labor selection framework) turns out to be roughly 40 percent of that in U.S. data. Using data constructed for the period 1951:Q1 - 2007:Q1, and measuring the cyclical component after HP detrending with a smoothing parameter of 1,600, the empirical value of the elasticity of the finding rate with respect to aggregate productivity is 2.9.[12,13] To the best of our knowledge, this empirical elasticity has not

[12]This elasticity corresponds to the coefficient obtained in an OLS regression of log job finding probability on log aggregate productivity. The data on unemployment, vacancies, output and productivity were downloaded from the webpage of Pascal Michaillat (https://sites.google.com/site/pmichaillat/) and correspond to the quarterly average of the series compiled by Michaillat from underlying BLS and JOLTS/Conference Board data. The quarterly job-finding probability constructed from underlying BLS data by Shimer was downloaded from his research webpage (https://sites.google.com/site/robertshimer/research/flows). Both Michaillat (2012) and Shimer (2005) use HP smoothing parameter 10^5 — neither reports this particular elasticity of the finding rate with respect to aggregate productivity,

[13]We are grateful to Ana Lariau for collecting these data in a way consistent with both the Shimer and Michailliat analyses and for computing the cyclical elasticities.

directly appeared in the existing literature.[14]

2.4 Comparison with Search and Matching

We compare the amplification effects in an efficient selection model to an efficient search and matching model. The matching function is assumed to be Cobb-Douglas, with $\alpha \in (0,1)$ denoting the matching elasticity with respect to unemployed individuals. The job-creation condition is

$$\frac{\kappa}{(1-\alpha)\,\theta_t^{-\alpha}} = z_t - b - \frac{\kappa\alpha\theta_t^{1-\alpha}}{(1-\alpha)\,\theta_t^{-\alpha}} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{\kappa}{(1-\alpha)\,\theta_{t+1}^{-\alpha}}\right\}, \tag{9}$$

where κ denotes the cost of posting a vacancy and θ stands for market tightness.

Around the deterministic steady state, the efficient elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln \eta}{\partial \ln z} = \left(\frac{z}{z-b}\right)\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1-\beta\,(1-\rho)+\alpha}{1-\beta\,(1-\rho)+1}\right) \tag{10}$$

(see Appendix F for details). In contrast to the efficient selection model, the outside value of b plays an important role. Targeting a long-run job-finding rate of $\eta(\tilde{\varepsilon}) = 0.58$, and using $\alpha = 0.72$ (as in Shimer (2005)) and $b = 0$, the efficient matching model delivers an elasticity of 0.29, which is roughly four times smaller than in the baseline version of our efficient baseline selection model with the uniform distribution. Section 3 shows quantitatively that amplification in the selection model is somewhat larger with other distributional forms.

With moderate levels of the outside option (e.g. $b = 0.4$, as in Shimer (2005)) and continuing to set a long-run job-finding rate of $\eta(\tilde{\varepsilon}) = 0.58$, the steady-state elasticity in the efficient search and matching model is still only less than a half compared to the efficient selection model. Thus, our brief comparison to the efficient search and matching model shows that efficient selection generates larger amplification effects.

3 Labor Markets II: Quantitative

We now describe the calibration of the model.

[14] Mortensen and Nagypal (2007) measure the elasticity of labor market tightness with respect to productivity, which is not exactly the elasticity we are estimating. Furthermore, as in Michaillat (2012) and Shimer (2005), they also use an HP smoothing parameter 10^5.

3.1 Micro-Empirical Evidence

We now impose *microeconomic* discipline to calibrate the key parameters of the model, which are the pair of distributional parameters $(\gamma^h, \sigma_\varepsilon^*)$ and the non-market "outside option" b.[15] The focus is on the steady state calibration, hence we omit time indices and normalize $z = 1$ without loss of generality.

To highlight the two important distributional parameters, for this section (and Section 3.5 below) we include both $\tilde{\varepsilon}$ and σ_ε^* as arguments to the endogenous outcomes $H(\tilde{\varepsilon}, \sigma_\varepsilon^*)$, $\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)$, and so on. Next, to further ease economic interpretation, we rearrange the terms inside the square brackets in (1) in two steps. First, rewrite it slightly as

$$n + s \cdot (1 - \eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)) \cdot b - s \cdot \eta(\tilde{\varepsilon}, \sigma_\varepsilon^*) \left(\gamma^h + \frac{H(\tilde{\varepsilon}, \sigma_\varepsilon^*)}{\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)} \right). \tag{11}$$

Second, after defining the average steady-state training cost of a new hire as

$$TC(\tilde{\varepsilon}, \sigma_\varepsilon^*) \equiv \gamma^h + \frac{H(\tilde{\varepsilon}, \sigma_\varepsilon^*)}{\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)}, \tag{12}$$

we can further rewrite (11) as

$$n + s \cdot (1 - \eta(\tilde{\varepsilon}, \sigma_\varepsilon^*) \cdot b - s \cdot \eta(\tilde{\varepsilon}, \sigma_\varepsilon^*) \cdot TC(\tilde{\varepsilon}, \sigma_\varepsilon^*). \tag{13}$$

The second term in (12) is the average *idiosyncratic* component of training costs across *all* new workers, which arises (refer to Section 2) from integration across all newly-hired workers' flow surpluses $1 - (\gamma^h + \varepsilon^i)\ \forall \varepsilon^i \leq \tilde{\varepsilon}$.

Our baseline calibration uses *microeconomic* evidence on training costs from Barron, Black, and Loewenstein (1989) to pin down the two critical parameters γ^h and σ_ε^*. There is a treasure trove of papers in the micro-labor literature that document *first moments* for various components of the hiring process, including short-term training costs for new workers. To the best of our knowledge, Barron, Black, and Loewenstein (1989) is the only one that also measures second moments.

Barron, Black, and Loewenstein's (1989) study reports that the standard deviation of training costs during a new hire's first three months of employment is 207 hours, and the mean training cost for those individuals is 151 hours.[16] Translating these into days, the standard deviation is 26 days, and the mean is roughly 19 days during the first quarter of work.[17]

[15] For the sake of illustration, one can refer to Figure 2, but keep in mind that the distinction between σ_ε^* and the population standard deviation σ_ε is true for any distributional form considered (we consider below several distributional shapes).

[16] See Barron, Black, and Loewenstein (1989, Table 1, p. 5).

[17] These translations assume an eight-hour workday for new hires (as well as incumbent workers, but that is irrelevant for the calibration at hand). An eight-hour workday is tantamount to the "intensive margin" of labor being

Parameter	Interpretation	Value	Notes
	Distributional Parameters		
$TC(\cdot)$	Average of new hires' training costs	0.30	use to calibrate γ^h (from Barron et al (1989))
σ_ε^*	Cross-sectional SD of distribution for for new hires' idiosyncratic characteristics	0.40	from Barron et al (1989)
	Typical Parameters		
r	Real interest rate	0.01	
ρ	Exogenous separation rate	0.10	
z	Steady-state labor productivity	1	
ρ_z	AR(1) persistence of exogenous log z_t productivity	0.95	
σ_{ϵ_z}	Standard deviation of exogenous innovation to log productivity	0.01	$\epsilon_z \sim N(0, \sigma^2_{\epsilon_z})$
lfp	Labor force participation	0.74	from Veracierto (2008)

Table 1: **Parameter values for partial equilibrium model.** Parameter values for the partial equilibrium model in Section 2 and Section 3 for several distributional forms. The distributional parameter σ_ε is a population moment, whereas σ_ε^* is the sample moment. For any given distribution, the population moment σ_ε is chosen so that the endogenous σ_ε^* that emerges is consistent with the micro evidence.

The model described in Section 2 requires training costs denominated in units of goods. One quarter has roughly $\frac{5}{7} \cdot 90 \approx 65$ working days; with an eight-hour work day, total hours in market work ≈ 500 per quarter. Due to the linearity of market production in labor in the model built thus far and given that every worker's steady-state marginal product is $z = 1$ goods, every unit of time spent in market work produces one unit of market output. Barron et al's evidence can thus be easily stated through the lens of our framework as $TC(\cdot) = \frac{151}{500} \approx 0.30$ and $\sigma_\varepsilon^* = \frac{207}{500} \approx 0.40$. The first two lines of Table 1 summarize this.

Aside from the uncontroversial "typical parameters" at the bottom of Table 1 (including parameters for the exogenous forcing process z_t), the other main model parameter to be calibrated is the outside option b. For any given distributional form (which the Barron et al evidence does not pin down) considered in our experiments, b is chosen to target a steady-state quarterly job-finding probability of $\eta(\tilde{\varepsilon}) = 0.58$.

fully utilized. But, as noted in Section 2.1, our framework abstracts from the intensive margin.

3.2 Quantitative Verification of Analytical Example

Returning to the analytical example in Section 2, Figure 3 displays impulse responses for the finding rate and the unemployment rate for the $\mathcal{U}[-1.2, 1.2]$ distribution. On impact of the one-percent persistent productivity shock, the job-finding rate rises by about 1.2 percent (in blue in the upper left panel of Figure 3), which is in line with the analytical elasticity computed in expression (8). In order to achieve the steady-state quarterly finding rate $\eta(\tilde{\varepsilon}) = 0.58$, the outside option needed for the $\mathcal{U}[-1.2, 1.2]$ case is $b = 0.53$.

A natural question is whether or not this outside option is "large." The natural benchmark is the efficient search and matching model. The well-known Shimer (2005) analysis supposes an outside option of roughly 40 percent; our 50 percent outside option is in line with it. In turn, the choice of b in the selection framework is *conditional on the assumed distributional form*, which in Figure 3 is the $\mathcal{U}[-1.2, 1.2]$ case.

3.3 Quantitative Results for Several Distributional Forms

We now consider other distributional forms, in particular the normal distribution $N(0, \sigma_\varepsilon^2)$ and the log-normal $\ln N(\mu, \sigma_\varepsilon^2)$ that allow for curvature in $\eta'(\tilde{\varepsilon})$. Compared to the $\mathcal{U}[-1.2, 1.2]$ distribution, Figure 3 shows that the impact amplification based on $N(0, \sigma_\varepsilon^2)$ is somewhat larger, and that for the $\ln N(\mu, \sigma_\varepsilon^2)$ is quite similar.[18] The log-normal distribution allows us to capture the fact that the cross-sectional training cost distribution in Barron, Black, and Loewenstein (1989) is skewed (no firm reports negative training costs).

Table 2 reports results for the same experiment, but now allows for smaller and larger different sample standard deviations (σ_ε^*) (across rows) and different distributional forms (across columns). Not surprisingly, for each distribution, the volatility of the finding rate decreases as σ_ε^* increases. As discussed in Section 3.7 below, $\sigma_\varepsilon^* = 0.6$ should be considered as an extreme upper bound for the model calibration. Despite the bit of amplification differences amongst the various distributional assumptions, the main message that Figure 3 and Table 2 convey is that the *micro-disciplined efficient selection model* achieves meaningful *macro-level* labor-market volatility.

3.4 The Role of the Outside Option

The outside option b plays a different role in the selection framework, as now demonstrated. Recall that b does not appear in the analytical steady-state elasticity computed in (7). The implication is that, up to first order, the sensitivity of the finding rate $\eta(\tilde{\varepsilon})$ to aggregate productivity does *not*

[18] In order to maintain a steady-state job-finding rate $\eta(\tilde{\varepsilon}) = 0.58$, the outside options required for the $N(0, \sigma_\varepsilon^2)$ case and the $\ln N(\mu, \sigma_\varepsilon^2)$ case are, respectively, $b = 0.62$ and 0.57.

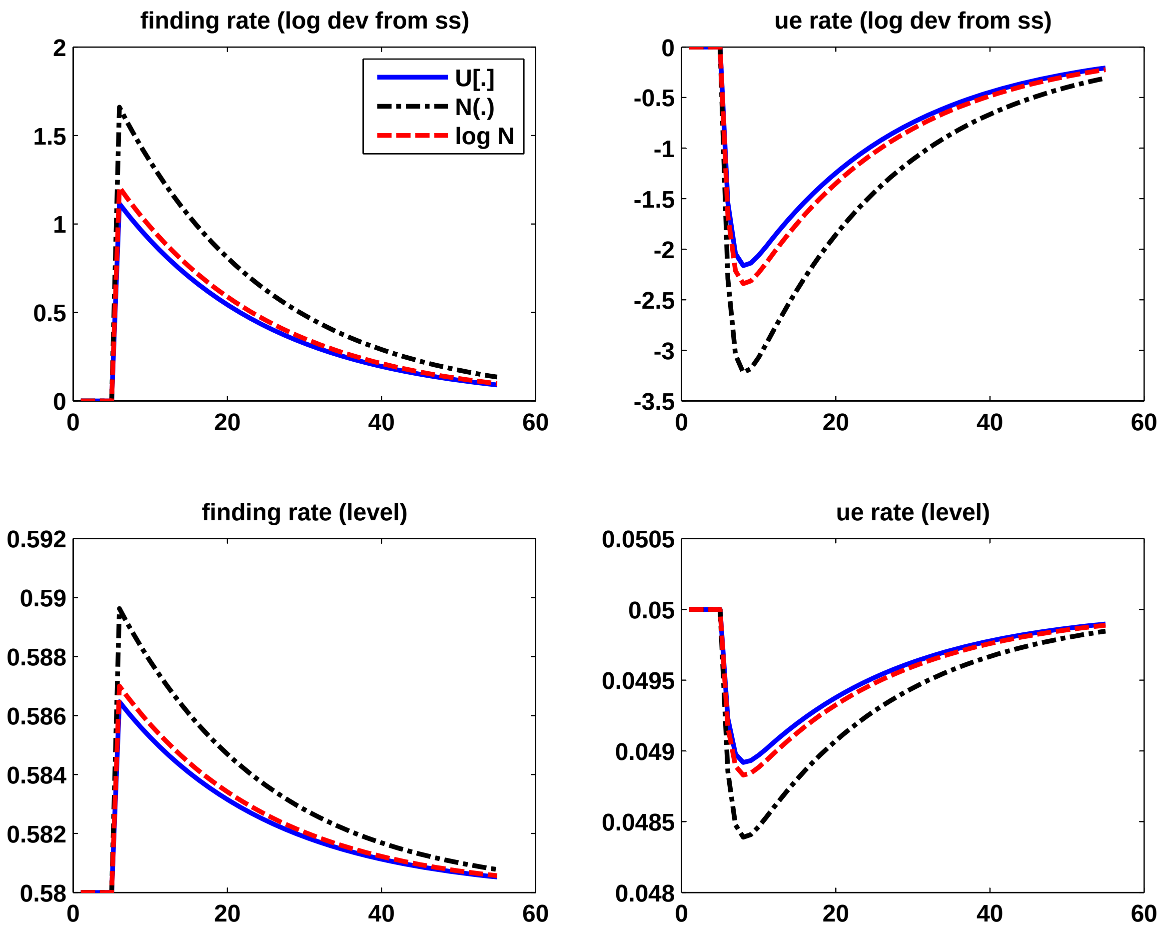

Figure 3: **Impulse response profiles, with various distributional forms for idiosyncratic characteristics.** The blue lines display impulse responses for the $\mathcal{U}[-1.2, 1.2]$ distribution, the black dashed-dotted lines for the $N(\cdot)$ distribution, and the red dashed lines for the $\ln N(\cdot)$ distribution. Upper row displays log deviations from steady state. Lower row displays levels. The endogenous sample standard deviation in the selection model is $\sigma_\varepsilon^* = 0.40$.

σ_ε^*	$\mathcal{U}[-1.2, 1.2]$	$N(0, \sigma_\varepsilon^2)$	$\ln N(\mu, \sigma_\varepsilon^2)$
0.2	2.23	3.49	2.31
0.4	**1.12**	**1.66**	**1.21**
0.6	0.74	1.10	0.81

Table 2: **Volatility of Hiring Rate $\eta(\hat{\varepsilon})$ for Different Distributions and Sample Standard Deviations σ_ε^*.** The bolded middle row highlights the micro-evidence provided in Barron et al (1989).

depend on b and hence does not depend directly on the social surplus $z-b$. This is in sharp contrast to an efficient version of a labor search and matching model, as demonstrated in Section 2.4.[19]

From here on, we use the $\ln N(\mu, \sigma_\varepsilon^2)$ distribution for idiosyncratic characteristics due to its skewness. Using log-normal traits, Figure 4 shows that, so long as the calibration of the selection model is such that the long-run finding rate is $\eta(\tilde{\varepsilon}) = 0.58$, variations in b do not affect dynamics. Figure 4 illustrates impulse response function with a targeted finding rate $\eta(\tilde{\varepsilon}) = 0.58$ and $b = 0$. The impulse response function is literally the same as with $b = 0.53$ (see our baseline scenario). An alternative experiment is to hold γ^h constant as b varies. Figure 4 displays results for $b = 0, 0.2, 0.4$. When b increases, but all other parameters of the model remain constant, the social planner sends fewer people to work due to the smaller gap between market production and the value of the outside option. Thus, the long-run job-finding rate declines and unemployment rises. The upper left panel of Figure 4 shows that a larger b increases the absolute deviations from steady state. When the job-finding rate drops, the cutoff point moves to a point in the log-normal distribution where the density function has more mass. The upper left panel shows that the log-deviations for $b = 0.2$ and $b = 0.4$ increase by a lot more than the absolute deviations. The reason is that larger values of b reduce the job-finding rate from 58% to 40% to 14%, respectively.

While this second exercise (varying b and keeping all other parameters fixed) is conceptually interesting, the first exercise (fixing the job-finding rate and varying b and γ^h) corresponds more closely to experiments performed in the search and matching literature. Hagedorn and Manovksii (2008) target, for example, a similar steady state as Shimer (2005), but vary two parameters (bargaining power and value of leisure). Note that we focus exclusively on the efficient benchmark of the selection model.

3.5 Average Output Lost in Training

In the analysis so far, training costs on average cause (referring to expression (12)) a reduction of

$$TC(\tilde{\varepsilon}_t, \sigma_\varepsilon^*) = \gamma^h + \frac{H(\tilde{\varepsilon}_t, \sigma_\varepsilon^*)}{\eta(\tilde{\varepsilon}_t, \sigma_\varepsilon^*)} \tag{14}$$

units of output.[20] We relax this assumption by introducing an "average output loss" parameter $\phi \in (0, 1]$. Including the output loss parameter, the social planner maximizes

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[z_t n_t + s_t(1 - \eta(\tilde{\varepsilon}_t, \sigma_\varepsilon^*))b - \phi \cdot s_t \cdot \eta(\tilde{\varepsilon}_t, \sigma_\varepsilon^*) \left(\gamma^h + \frac{H(\tilde{\varepsilon}_t, \sigma_\varepsilon^*)}{\eta(\tilde{\varepsilon}_t, \sigma_\varepsilon^*)}\right)\right] \tag{15}$$

[19] For the role of training costs in search and matching models, see Silva and Toledo (2009).

[20] Expression (14) with time subscripts is equivalent to expression (12) without time indices because all training costs for new employees are incurred within a period.

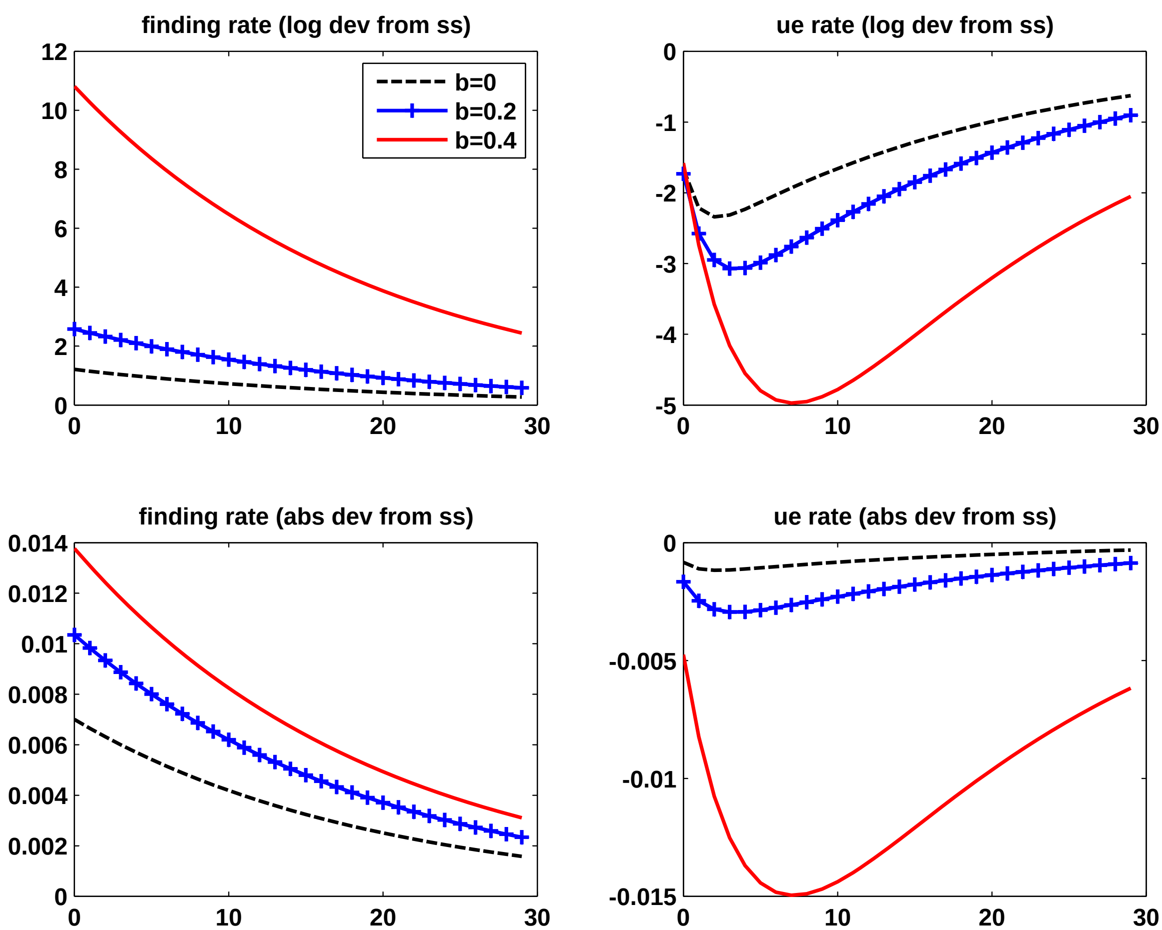

Figure 4: **Impulse response profiles for several values of b.** Upper row displays log deviations from respective steady states, bottom row displays absolute deviations from respective steady states. Black dashed line: $b = 0$. Blue crossed line: $b = 0.20$. Solid red line: $b = 0.40$. Idiosyncratic characteristics $\sim \ln N(1, 0.30^2)$.

subject to

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t, \sigma_\varepsilon^*)\left(lfp - (1-\rho)n_{t-1}\right). \tag{16}$$

For the sake of economic intuition provided at the end of this sub-section, we intentionally include σ_ε^* as an argument to the endogenous outcomes $H(\tilde{\varepsilon}, \sigma_\varepsilon^*)$ and $\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)$.

Maximization leads to a slightly generalized efficient selection condition,

$$\phi \cdot \left(\gamma^h + \tilde{\varepsilon}_t\right) = z_t - b + \phi \cdot \left(\frac{1-\rho}{1+r}\right) E_t\left\{H(\tilde{\varepsilon}_{t+1}, \sigma_\varepsilon^*) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}, \sigma_\varepsilon^*) + \gamma^h + \tilde{\varepsilon}_{t+1}\right\}, \tag{17}$$

compared to the selection condition (4). In turn, the steady-state elasticity of the finding rate with respect to aggregate productivity also slightly generalizes to

$$\frac{\partial \ln \eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)}{\partial \ln z} = \frac{1}{\phi} \cdot \frac{\eta'(\tilde{\varepsilon}, \sigma_\varepsilon^*)}{\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)} \cdot z \cdot \left(\frac{1+r}{r+\rho+(1-\rho)\,\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)}\right). \tag{18}$$

For any given ϕ, we target the empirically-relevant long-run finding rate $\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)$. Thus, $\eta'(\tilde{\varepsilon}, \sigma_\varepsilon^*)$ is unchanged by ϕ because the selection threshold $\tilde{\varepsilon}$ is at the same location in the density function. Expression (18) then informs us that as output loss ϕ declines, the elasticity of hiring increases. As noted in Section 3.3, a decrease in ϕ qualitatively affects $\eta(\tilde{\varepsilon}, \sigma_\varepsilon^*)$ in a similar manner as would a decrease in the population dispersion parameter σ_ε^*. However, as described in Section 3.1, the latter would contradict the micro-discipline we use regarding σ_ε^*. Intuitively, then, we consider $\phi = 1$ as establishing a *lower bound* for the sensitivity of the job-finding rate with respect to productivity shocks. Thus, for the remainder of our analysis, we will continue with $\phi = 1$.

3.6 Sequential Search

So far, we have assumed that each unemployed individual makes one contact per period. It is natural and perhaps realistic to consider the case of multiple contacts per period. Supposing the number of contacts per period is exogenous, as is the case in the well-known sequential search framework à la McCall (1970), Mortensen (1970), and others, this is easily captured in our framework.

Let i denote the exogenous number of contacts per period that occur in Poisson fashion. Our analysis above (implicitly) considered the case of $i = 1$. Now let $i \in \{2, 3, 4, ...\}$. For ease of discussion, let's consider the simple case of $i = 3$ because it corresponds to a monthly frequency.[21] As mentioned in the Introduction and described in Section 3.1, the microeconomic data used to discipline the cross-section of the distribution of idiosyncratic match quality is at the *quarterly* frequency. To the best of our knowledge, corresponding data for the distribution of match quality at finer levels (monthly, say) do not exist. A formal analysis of the $i \in \{2, 3, 4, ...\}$ cases, though,

[21] Alternatively, we could consider $i = 12$ if we were interested in the weekly frequency.

would require it.

A natural inclination might be to assume that the distributional form of monthly-level match-quality dispersion has the same distributional form as is assumed for the quarterly level. However, the micro-disciplined distributional assertion at the quarterly level implies *nothing* about the *convolution* between the monthly-level distribution and the quarterly-level distribution. What follows in the remainder of this paragraph is essentially a proof by contradiction, which could be formalized as a short lemma. Taking the case of the uniform for the sake of simplicity, the reason the assumption is incorrect is that the convolution of (monthly) uniform distributions leads to a *triangular distribution*, rather than to a uniform distribution. But this then contradicts the initial assertion of the uniform distribution at the quarterly level.

This "proof by contradiction" is of course not true for every distribution. For example, convolution of normal distributions leads to a normal distribution. But for, say, the more realistic log-normal distribution which displays the skewness shown by Barron et al, convolution theory does not hold.

This argument makes it seem that the framework cannot generate any results at all upon inclusion of sequential search. This is not true, because we can use another data source to discipline cross-sectional evidence at the monthly level.

3.7 Micro-Empirical Evidence on Wages and Sequential Search

In addition to being uninformative about the monthly frequency, the EOPP training cost survey may provide an over- or underestimation for our purposes. On the one hand, there are other idiosyncratic heterogeneities for newly-hired workers besides training costs (e.g., differences in idiosyncratic productivity). This leads to a potential underestimation for the overall idiosyncratic heterogeneity. On the other hand, the training cost survey does not differentiate between idiosyncratic and systematic components (e.g., related to education).[22] However, in the context of our model, only the idiosyncratic component is relevant; we abstract from the systematic component.[23] Since the survey does not differentiate these two components, this leads to a potential overestimation of the dispersion of the idiosyncratic component.[24] We thus turn to a second reference point: analyzing the hourly wages for entrants (i.e. workers who move from unemployment to employment), based on Haefke, Sonntag, and van Rens (2013) who use the Current Population Survey (CPS).[25]

[22]Barron, Black, and Loewenstein (1989) show, for example, that high-skilled workers require more time to be trained.

[23]We would, for example, assume that low-skilled and high-skilled workers act in different labor markets and both draw idiosyncratic training costs.

[24]Ideally, we would have a dataset which allows us to quantify the dispersion of match-specific idiosyncratic productivity shocks for new jobs in the United States (controlling for worker and establishment/firm fixed effects). However, to the best of our knowledge, such a dataset is not available.

[25]The data is available online http://www.thijsvanrens.com/wage/

In order to be able to use wage data, we show for the decentralized model with exogenous labor force participation and standard Nash bargaining that the entire surplus has to go to the new worker in order to decentralize the social planner solution (see Appendix D for the proof).[26] Atomistic firms do not take into account that their hiring behavior affects the future pool of searching workers (as the social planner does). If part of the bargaining surplus goes to firms, this leads to a suboptimal level of searching workers. If the entire surplus goes to workers, they internalize this externality.

Thus, in an efficient economy the wages of entrants correspond to their respective productivity (aggregate productivity and the idiosyncratic component). This allows us to look at wage data through the lens of our model. We analyze the hourly wages for entrants, based on Haefke, Sonntag, and van Rens (2013). See Appendix H for details. The coefficient of variation for new entrants is roughly 0.6, both on the monthly and quarterly level. Note that we consider the coefficient of variation of entrant wages to be an extreme upper bound for match-specific idiosyncratic productivity because it does not control for all sorts of observable and unobservable characteristics (in particular, not for establishment and worker fixed effects), which are abstracted from in our model. Card, Heining and Kline (2013, see their Figure IV) show with a rich employer-employee linked dataset for Germany that adding industry, occupation and establishment fixed effects reduces the dispersion of wages substantially, namely by roughly one third.[27] Given that it is impossible to control for fixed effects in our dataset (which represent systematic components that we abstract from), our benchmark of $\sigma_{\varepsilon}^{*} = 0.4$ from the EOPP continues to appear reasonable.

In reality, the bargaining power of households may be smaller than one (i.e., the social planner solution may not be decentralized). Would the coefficient of variation of wage nevertheless provide some guidance for our purposes? In order for our model to be stationary in the long run (i.e., for unemployment not to converge to 0 or 1), we require the following conditions. The dispersion of idiosyncratic training costs (productivity) has to grow proportionally with aggregate productivity. In addition, in the long-run wages would have to co-move proportionally with productivity to establish stationarity.[28] We show in Appendix E that under these conditions, the long-run coefficients of variation for productivity (including idiosyncratic productivity) and for wages are the same. Thus, the coefficient of variation for wages continues to provide an extreme upper bound for the dispersion of idiosyncratic productivity.

From the sequential search literature, we know the relationship between the per-period job-

[26] In the model with endogenous labor force participation, the entire surplus also has to go the worker in order to decentralize efficiency. However, the intuition is different. This condition is necessary for achieving efficient labor-force participation.

[27] Recent evidence for the United States by Barth et al. (2014) also emphasizes the importance of establishment fixed effects for wage dispersion (although not for new jobs, as would be necessary for our work).

[28] More generally, conventional business cycle and growth models require that productivity and wages comove one to one in the long run in order to make labor input (employment) stationary.

finding rate $\eta(\tilde{\varepsilon}_t)$ and the per-contact finding rate, which we define as $\omega(\tilde{\varepsilon}_t)$,

$$\eta(\tilde{\varepsilon}_t) = \sum_{j=1}^{i} (1 - \omega(\tilde{\varepsilon}_t))^{j-1} \omega(\tilde{\varepsilon}_t), \tag{19}$$

in which i is an integer number that denotes the number of interviews in a given quarter.

For the identification of the standard deviation of idiosyncratic productivity of those hired (σ_{ε}^*), it is important to know i. Unfortunately, there is no dataset that provides the number of job offers over time. However, there are various datasets that provide the job arrival/offer rate for a given point in time and for a given group (depending on the survey). This allows us to determine a plausible range for i.

Blau and Robins (1990) report a weekly job offer rate for unemployed workers of 18 percent. Holzer (1987, Table 2) reports a quarterly job offer for unemployed workers of 34 percent. According to Faberman et al. (2015, Table 8) the number of offers per week of search (conditional on those actively searching for a job) is 17 percent.

The differences are due to different samples and different points in time when the surveys were performed. Blau and Robins (1990) use the 1980 household survey of the EOPP. Note that the contact definition[29] in the survey does not correspond to ours, i.e. it is too broad in order to serve as revelation of information. Thus, we use the job offer rates. Holzer (1987) uses the 1981 New Youth Cohort of the National Longitudinal Survey. Holzer (1987) argues that (despite the focus on young workers) the survey is very suitable for the analysis of search behavior. Faberman et al. (2015) use a recent supplement to the Survey of Consumer Expectations (SCE), administered monthly by the Federal Reserve Bank of New York.

For our purposes, we require the average number of job offers per quarter (i.e., i in the above equation). Thus, we assume that job offers arrive with Poisson probability and we calculate how long it takes on average to obtain one job offer, which we denote by D (for duration). With probability (1-job arrival rate), there will not be a job offer for one period, with probability (1-job arrival rate)^2, there will be no offer for two periods, etc. Thus, in steady state, the average waiting time for a job offer is

$$D = \sum_{k=1}^{\infty} (1 - jar)^k \tag{20}$$

$$= \frac{1}{jar}, \tag{21}$$

[29] "How many places of employment did you contact by phone, mail or in person as a result of a certain search method?" This is a stage of the application process where no actual job application is filled out and thus there is insufficient information about the actual realization of idiosyncratic shocks. The survey does not contain any question on the number of actual interviews.

where jar is the job arrival rate.

According to Blau and Robins' (1990) numbers it would on average take 1/0.18=5.6 weeks to obtain a job offer. According to Faberman et al. (2015), it would on average take 1/0.17=5.7 weeks to obtain a job offer. According to Holzer (1987) it would on average take 1/0.34=2.9 months to obtain a job offer. Thus, these different surveys provide a plausible range for the average time to get one job offer for in between 5.6 and almost 12 weeks. The latter corresponds to our baseline calibration with one contact per quarter. The former is within our range of robustness checks below where we also assume that a job offer arrives on average every month. Given that our robustness checks do not deliver considerably different results, we argue that we have chosen plausible assumptions in this dimension.

To illustrate the effects of multiple meetings in the selection framework, let's start with an approximation of the non-linear expression (19), which is

$$\eta(\tilde{\varepsilon}_t) = m \cdot \omega(\tilde{\varepsilon}_t). \tag{22}$$

This approximation is useful for conveying the main messages of the framework upon inclusion of sequential search.

Using (22), the social planning problem is

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[z_t n_t + s_t(1 - m \cdot \omega(\tilde{\varepsilon}_t)) \cdot b - s_t \cdot m \cdot \omega(\tilde{\varepsilon}_t) \cdot \left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)}\right)\right] \tag{23}$$

subject to

$$n_t = (1-\rho)n_{t-1} + m \cdot \omega(\tilde{\varepsilon}_t) \cdot (lfp - (1-\rho)n_{t-1}), \tag{24}$$

which yields the efficient selection condition

$$\gamma^h + \tilde{\varepsilon}_t = z_t - b + \left(\frac{1-\rho}{1+r}\right) E_t \left\{ \left(\frac{H(\tilde{\varepsilon}_{t+1})}{\omega(\tilde{\varepsilon}_{t+1})} - \tilde{\varepsilon}_{t+1}\right) m \cdot \omega(\tilde{\varepsilon}_{t+1}) + \gamma^h + \tilde{\varepsilon}_{t+1} \right\}. \tag{25}$$

Given efficient selection as characterized in (25), the steady-state elasticity of the finding rate with respect to productivity is now

$$\frac{\partial \ln \eta(\tilde{\varepsilon})}{\partial \ln z} = \frac{\omega'(\tilde{\varepsilon})}{\omega(\tilde{\varepsilon})} \cdot z \cdot \left(\frac{1+r}{r + \rho + (1-\rho)\,\eta(\tilde{\varepsilon})}\right). \tag{26}$$

The term in parentheses on the right-hand side is exactly the same as in expression (7), which (implicitly) was for the $m = 1$ case. However, $\frac{\eta'(\tilde{\varepsilon})}{\eta(\tilde{\varepsilon})}$ is replaced by $\frac{\omega'(\tilde{\varepsilon})}{\omega(\tilde{\varepsilon})}$. Thus, the relative size of these two expressions ($\frac{\eta'(\tilde{\varepsilon})}{\eta(\tilde{\varepsilon})}$ versus $\frac{\omega'(\tilde{\varepsilon})}{\omega(\tilde{\varepsilon})}$) will determine whether $m > 1$ increases or decreases $\frac{\partial \ln \eta(\tilde{\varepsilon})}{\partial \ln z}$. There turn out to be two countervailing effects. Both the numerator and the denominator become smaller when m rises.[30] Analytically, it is unclear which effect dominates.

Thus, we proceed with a numerical illustration. We choose $m = 1.65$ and $m = 2.30$.[31] With the log-normal distribution, the job-finding rate is now 1.7 and 1.8 times more volatile, respectively, than productivity. This is somewhat larger than our baseline results (which was 1.2) with the log-normal distribution.

The fixed m in the approximation (25) allows us to easily convey the main economic intuition. If we instead use the *exact* nonlinear expression (19) in the social planning problem stated in (23) and (24), Appendix B displays the *exact* efficient selection condition, which delivers quantitatively similar results (based on a numerical solution). Based on the exact efficient selection condition, computing analytically the steady-state elasticity of the finding rate with respect to productivity is very difficult and does not provide an interpretable expression.

Given the similar numerical results we obtain from the approximated model for $i = 2$ and $i = 3$ job arrivals, and the difficulty of computing the steady-state elasticity from the exact selection condition, in what follows we will proceed with $i = 1$ (one contact per quarter). In addition, we have shown above that $i = 1$ is within the range of plausible empirical values.

4 Labor Markets in General Equilibrium

All of the analysis in Section 2 and 3 is partial equilibrium in nature. We now show that the intuition and main results carry over once endogenous labor supply and an aggregate goods resource constraint are introduced. In general equilibrium, the amplification effects of the partial equilibrium analysis carry over with little change. Figure 5 displays the timing of events in general equilibrium.

4.1 Preferences

As in the model in Sections 2 and 3, there is a measure one of individuals in the economy. However, in this extended model, individuals may be in one of three labor-market states: employed, not working but searching for employment, or outside the labor force. Three labor-market states allow for broader generality of the efficiency results. Regardless of labor-market status, each individual has full consumption insurance, which is modeled by assuming a representative household that pools income and shares consumption equally amongst all individuals. This "large household" assumption is a tractable and often-used way of modeling perfect consumption-risk insurance and is standard in the DSGE literature.

[30] Remember that we target a certain job-finding rate on the quarterly level $\eta(\tilde{\varepsilon})$. With multiple interviews per quarter, the selection rate per interview drops. Thus, $\frac{H(\tilde{\varepsilon}_{t+1})}{\omega(\tilde{\varepsilon}_{t+1})}$ falls. And for a targeted σ_{ε}^{*}, the dispersion of the idiosyncratic shock has to be larger. This explains why $\omega'(\tilde{\varepsilon})$ should drop. Given that $\eta(\tilde{\varepsilon}) = m \cdot \omega(\tilde{\varepsilon}_t)$, the denominator also declines.

[31] This corresponds to $i = 2$ and $i = 3$ in the exact non-linear expression (19).

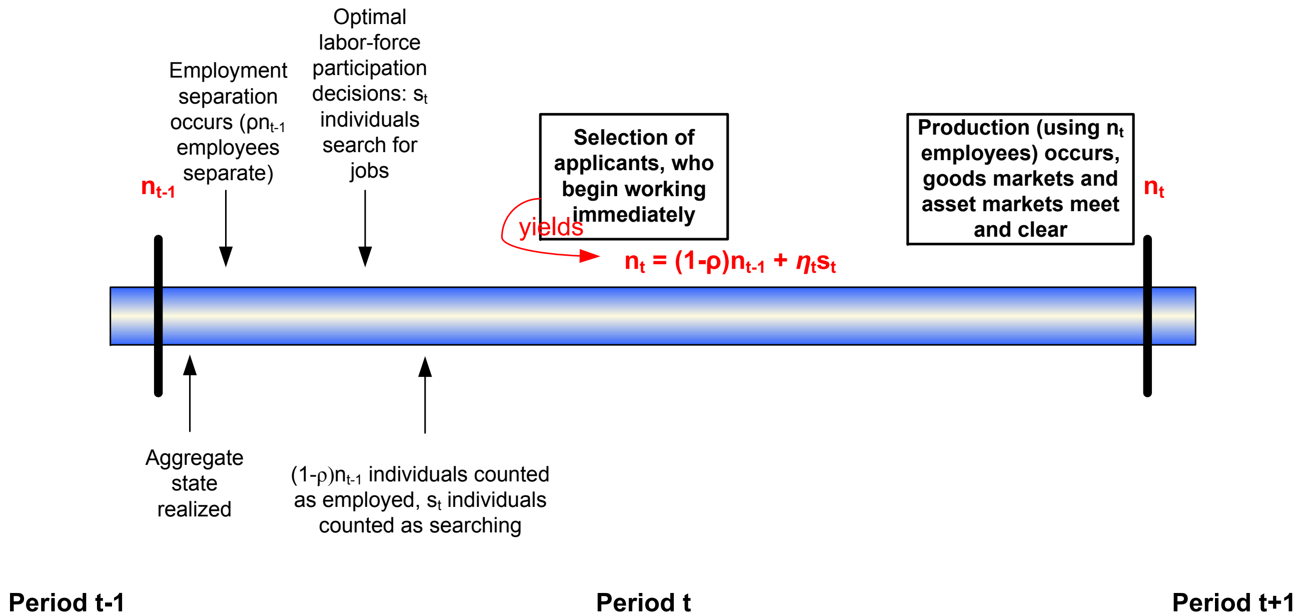

Figure 5: Timing of events.

The representative household has lifetime expected utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(n_t + (1-\eta_t)s_t\right) \right], \tag{27}$$

in which $lfp_t \equiv n_t + (1-\eta_t)s_t$. The subjective discount factor is $\beta \equiv (1+r)^{-1}$, the function $u(.)$ is a standard strictly-increasing and strictly-concave subutility function over consumption, and the function $v(.)$ is strictly increasing and strictly convex in the size of the labor force. The convexity of $v(.)$ at the level of the representative household can be shown in exactly the same way as in Mulligan (2001), with individual-level heterogeneity of the disutility of indivisible units of labor. Thus, $v(.)$ can be flexibly parameterized, rather than displaying quasi-linearity.[32] The measure of individuals in the labor force is endogenous, which is not the case in Sections 2 and 3, but has become fairly common in the recent matching literature.[33] For intuition and because it facilitates analogy with both the RBC model and the basic matching model, it will be helpful to interpret the measure $1 - lfp_t$ of individuals outside the labor force as enjoying leisure. We thus use the terms leisure and non-participation interchangeably, or, respectively, labor supply and labor force participation interchangeably.

[32]Given the definitions presented above, we sometimes will write $v(lfp_t)$.

[33]In a variety of applications, Veracierto (2008), den Haan and Kaltenbrunner (2009), Krusell, Mukoyama, Rogerson, and Sahin (2009), Ebell (2011), Haefke and Reiter (2011), and Arseneau and Chugh (2012), among others, have introduced participation margins into matching models.

4.2 Resource Use

The aggregate goods resource constraint

$$c_t = z_t n_t - s_t \eta(\tilde{\varepsilon}_t)\left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right). \tag{28}$$

Note that the b term from Sections 2 and 3 does not appear in the resource constraint. Recall that b can be interpreted in a few different ways: 1) the utility value of leisure; 2) government-provided unemployment benefits; and 3) home production. The preference specification (27) already embeds a value of leisure, which is the utility of being outside the labor force (which, without loss of generality, is normalized to zero). Second, even if we included a government in the general equilibrium model that provides unemployment benefits, these benefits would not appear in (28).[34] Finally, meaningfully including "home production" in general equilibrium would require an additional home production resource constraint along with utility from "home-consumption" (in the sense of, say, Greenwood, Rogerson, and Wright (1995)), which is outside the scope of our analysis and better left for future research.

The evolution of aggregate employment continues to be described by

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)s_t, \tag{29}$$

with n_t and s_t now interpreted as economy-wide measures.

4.3 Efficient Allocations

Efficient allocations $\{c_t, s_t, \tilde{\varepsilon}_t, n_t\}_{t=0}^{\infty}$ are characterized by four (sequences of) conditions: a labor-force participation (LFP) condition

$$\frac{v'(lfp_t)}{u'(c_t)} = \tilde{\varepsilon}_t \eta(\tilde{\varepsilon}_t) - H(\tilde{\varepsilon}_t), \tag{30}$$

a general equilibrium version of the selection condition

$$\gamma^h + \tilde{\varepsilon}_t = z_t + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \tilde{\varepsilon}_{t+1}\right]\right\}, \tag{31}$$

and conditions (28) and (29). The efficiency conditions (30) and (31) are obtained by maximizing household welfare (27) subject to the technological frontier defined by the sequence of goods resource constraints (28) and laws of motion for employment (29). The formal analysis of this problem is

[34] Due to the fact that we are considering only efficient allocations, which in turn would require lump-sum taxation to provide those benefits.

virtually identical to that in the partial equilibrium model.[35]

Condition (30) is a within-period dimension of efficiency and is analogous to consumption-leisure efficiency in the RBC model. Condition (31) is an intertemporal dimension of efficiency, and it extends the partial equilibrium selection condition (4) to general equilibrium by bringing the stochastic kernel $\beta u'(c_{t+1})/u'(c_t)$ in to replace $(1+r)^{-1}$.

4.4 Quantitative Results

4.4.1 Calibration

As in the partial equilibrium model, the model frequency is quarterly, so the subjective discount factor is set to $\beta = 0.99$ and the separation rate to $\rho = 0.1$. For utility, standard functional forms are used, $u(c_t) = \ln c_t$ and $v(lfp_t) = \frac{\kappa}{1+1/\phi} lfp_t^{1+1/\phi}$. The parameter ϕ is the elasticity of labor-force participation with respect to the real wage, which is set to $\phi = 0.18$ following Arseneau and Chugh's (2012) calibration of a matching model with endogenous participation fit to U.S. data. The scale parameter is set to $\kappa = 6.35$ to deliver a steady-state participation rate of 74 percent, the long-run U.S. empirical measure reported by Veracierto (2008).

The distribution of random training costs is assumed to be log-normal, $\ln N\left(\mu, \sigma_\varepsilon^2\right)$. We target $\sigma_\varepsilon^* = 0.4$ in our baseline version and $\sigma_\varepsilon^* = 0.2$ and $\sigma_\varepsilon^* = 0.6$ as robustness checks.[36]

The steady state selection rate is calibrated to $\eta(\tilde{\varepsilon}) = 0.58$ to generate an unemployment rate of 5 percent. We choose the fixed training cost parameter γ^h is chosen to obtain the targeted steady state selection rate.[37] Note that the unemployment rate is normalized by the entire population (i.e. also those who are out of the labor force). By dividing this number by the labor force participation rate 0.74, we obtain an unemployed / active population ratio of 0.068, which is in line with the conventionally reported unemployment rate.

Finally, as in the partial equilibrium analysis in Section 2, the only source of aggregate risk remains aggregate productivity shocks, which follows the same AR(1) process $\ln z_t = \rho_z \ln z_{t-1} + \epsilon_t^z$, with identical parameters $\rho_z = 0.95$ and $\sigma_{\epsilon^z} = 0.01$.

4.4.2 Results

Table 3 provides business-cycle statistics for the U.S. economy, to which we compare our model's results. The top panel of Table 4 displays results for the baseline calibration of the general equilibrium model, while Figure 6 presents results in impulse-response form.

[35]The formal analysis appears in Appendix G.

[36]The baseline quantitative values we thus set are $\mu = 1$ and a population standard deviation $\sigma_\varepsilon = 0.3$. In our robustness checks, we have $\mu = 0.35$ and population standard deviation $\sigma_\varepsilon = 0.3$ (which delivers sample standard deviation $\sigma_\varepsilon^* = 0.2$) and $\mu = 1.4$ and population standard deviation $\sigma_\varepsilon = 0.3$ (which delivers $\sigma_\varepsilon^* = 0.6$).

[37]As discussed in Section 2, γ^h is required to fix the steady-state level of the job-finding rate. But for a given job-finding rate γ^h is not important for the elasticity with respect to aggregate productivity.

	gdp	n	ue	lfp	$\eta(\tilde{\varepsilon})$
Relative standard deviation ($/gdp$)	1	0.60	5.15	0.20	3.72
Autocorrelation	0.87	0.94	0.91	0.68	0.82
Correlation with gdp	1	0.78	-0.86	0.39	0.79

Table 3: **Cyclical Dynamics of U.S. Labor Markets.** Quarterly business-cycle statistics for the job-finding rate $\eta(\tilde{\varepsilon})$ computed by authors' research assistants. Remainder of quarterly business-cycle statistics from Arseneau and Chugh (2012, Table 1).

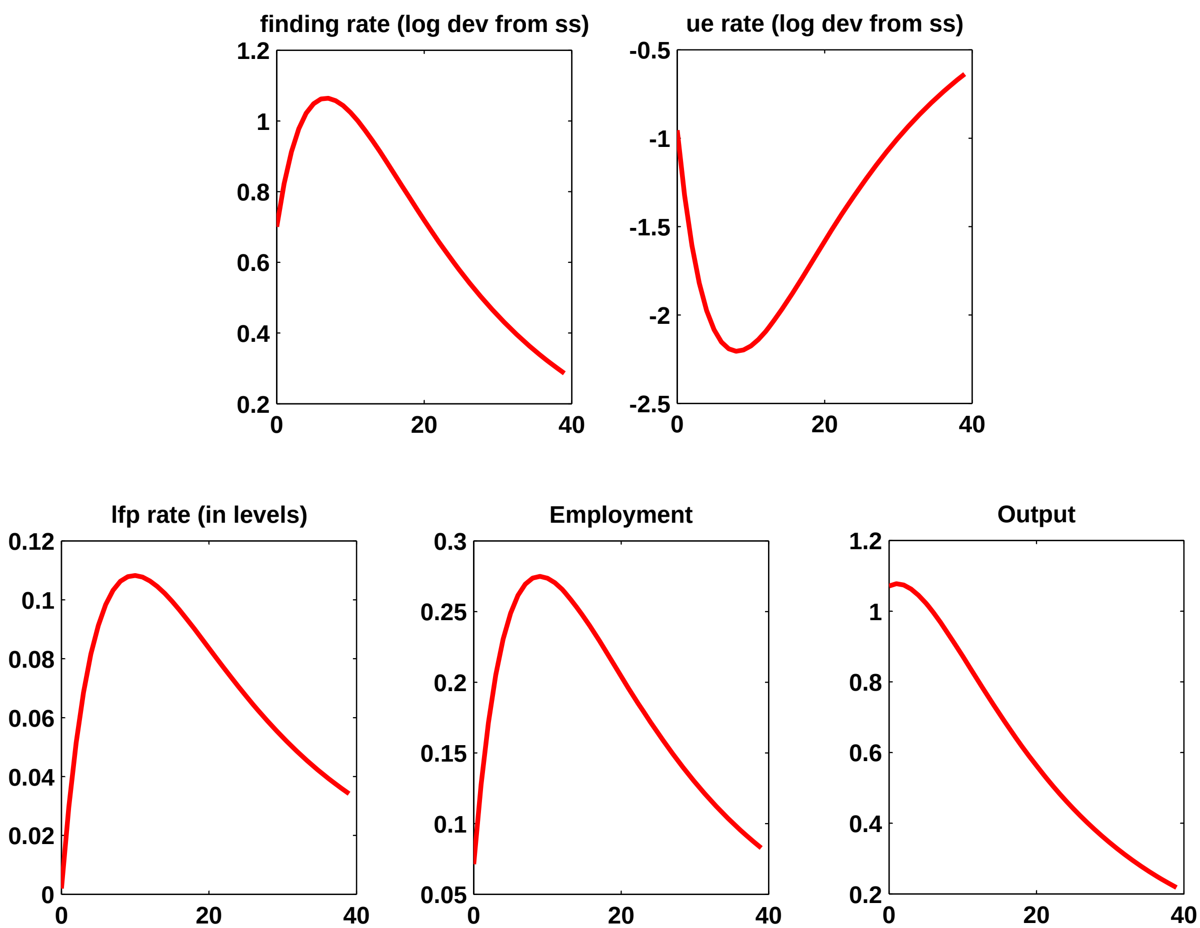

Figure 6: **Impulse response profiles in general equilibrium.** Idiosyncratic characteristics $\sim \ln N(1, 0.30^2)$.

As in the partial equilibrium version in Section 3, empirically realistic fluctuations of the finding rate and unemployment arise. The participation rate, which is also now endogenous, fluctuates slightly less relative to GDP fluctuations than reported by Veracierto (2008) for U.S. labor markets, and slightly less than in an efficient version of the labor search and matching framework with endogenous participation as shown in Arseneau and Chugh (2012). Regarding cyclicality, both Table 4 and the impulse responses in Figure 6 show that the model generates participation fluctuations that are procyclical with GDP and are in line with the U.S. data presented in Table 3.

Figure 6 shows that the amplification of the job-finding rate has the same order of magnitude in general equilibrium as in partial equilibrium (compare to Figure 4). In contrast to the partial equilibrium model, the job-finding rate is somewhat more persistent due to movements of the stochastic discount factor. The autocorrelation of the job-finding rate is very close to the one observed in the data (compare Tables 3 and 4). The lower two panels of Table 4 provide results for smaller and larger values of σ_ε^* than our benchmark $\sigma_\varepsilon^* = 0.6$, in which the main messages continue to hold.

5 Conclusion

We have proposed a simple labor-selection model, based on the idea that firms select workers from a pool of applicants. We have solved the social planner problem of this economy, both in partial and in general equilibrium. In both cases, our model generates realistic business cycle statistics, most importantly strong amplification effects of the finding rate and unemployment. This is in sharp contrast to the efficient version of the search and matching framework.

This paper is the starting point for a variety of new questions. By adding match-specific shocks for incumbent workers and thereby generating endogenous separations, our framework is a natural framework for the analysis of frictional wage dispersion. Search models (without on-the job-search) have problems to replicate a sufficient degree of wage dispersion (Hornstein, Krusell and Violante (2011)). Our model may shed new light on this issue.

In addition, the primitives in our proposed framework are observable. While it is impossible to observe the matching function directly, it is possible to provide survey evidence on training costs (although the existing evidence is somewhat scarce) or idiosyncratic productivity. Thus, this allows a tighter connection of micro-evidence and macro-modeling. The dispersion of training costs may for example have shifted over time due to technological advances. Further microeconomic evidence would generate testable assumptions for macroeconomic dynamics.

	gdp	n	ue	lfp	$\eta(\tilde{\varepsilon})$	z
Efficient allocation, baseline cross-sectional SD ($\sigma_{\varepsilon}^{*} = 0.40$)						
Mean	0.69	0.69	0.05	0.74	0.58	1
Volatility (SD%)	1.48	0.27	2.29	0.11	1.23	1.30
Relative standard deviation ($/gdp$)	1	0.20	1.54	0.07	0.83	0.88
Autocorrelation	0.75	0.95	0.91	0.95	0.85	0.71
Correlation with gdp	1	0.80	-0.91	0.58	0.98	0.99
Efficient allocation, smaller cross-sectional SD ($\sigma_{\varepsilon}^{*} = 0.20$)						
Mean	0.69	0.69	0.05	0.74	0.58	1
Volatility (SD%)	1.48	0.36	2.64	0.18	1.47	1.30
Relative standard deviation ($/gdp$)	1	0.25	1.78	0.12	0.99	0.88
Autocorrelation	0.75	0.96	0.91	0.96	0.86	0.71
Correlation with gdp	1	0.58	-0.80	0.30	0.91	1.00
Efficient allocation, larger cross-sectional SD ($\sigma_{\varepsilon}^{*} = 0.60$)						
Mean	0.69	0.69	0.05	0.74	0.58	1
Volatility (SD%)	1.47	0.20	1.94	0.06	1.02	1.30
Relative standard deviation ($/gdp$)	1	0.14	1.32	0.04	0.69	0.89
Autocorrelation	0.75	0.92	0.91	0.94	0.83	0.71
Correlation with gdp	1	0.84	-0.88	0.74	0.98	1.00

Table 4: **Business Cycle Statistics.** Upper panel presents second moments for the baseline calibration of cross-sectional dispersion of newly-hired workers, $\sigma_{\varepsilon}^{*} = 0.40$. Second panel presents second moments for a tighter cross-sectional dispersion, $\sigma_{\varepsilon}^{*} = 0.20$. Third panel presents second moments for a wider cross-sectional dispersion, $\sigma_{\varepsilon}^{*} = 0.60$. Business-cycle statistics obtained using HP parameter 1,600.

References

Arseneau, David M. and Sanjay K. Chugh. 2012. "Tax Smoothing in Frictional Labor Markets." *Journal of Political Economy,* Vol. 120, pp. 926-985.

Barron, John M., John Bishop, and William C. Dunkelberg. 1985. "Employer Search: The Interviewing and Hiring of New Employees." *Review of Economics and Statistics,* Vol. 67, pp. 43-52.

Barron, John M., Dan A. Black, and Mark A. Loewenstein. 1989. "Job Matching and On-the-Job Training." *Journal of Labor Economics,* Vol. 7, pp. 1-19.

Barth, Erling, Alex Bryson, James C. Davis and Richard Freeman. 2014. "It's Where You Work: Increases in Earnings Dispersion across Establishments and Individuals in the U.S.." NBER Working Paper 20447.

Blau, David M, and Philip K. Robins. 1990. "Job Search Outcomes for the Employed and Unemployed." *Journal of Political Economy,* Vol. 98, pp. 637-655.

Brown, Alessio, Christian Merkl, and Dennis Snower. 2015. "An Incentive Theory of Matching." *Macroeconomic Dynamics,* Vol. 19, pp. 643-668.

Card, David, Jörg Heining and Patrick Kline. 2013. "Workplace Heterogeneity and the Rise of West German Inequality." *Quarterly Journal of Economics,* Vol. 128, pp. 967-1015.

Carlsson, Mikael, Stefan Eriksson and Nils Gottfries. 2013. "Product Market Imperfections and Employment Dynamics." *Oxford Economic Papers,* Vol. 65(2), pp. 447-470.

Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. 2010. "Involuntary Unemployment and the Business Cycle." European Central Bank Working Paper 1202.

Davis, Steven J., R. Jason Faberman, and John C. Haltiwanger. 2013. "The Establishment-Level Behavior of Vacancies and Hiring." *Quarterly Journal of Economics,* Vol. 128, pp. 581-622.

den Haan, Wouter and Georg Kaltenbrunner. 2009. "Anticipated Growth and Business Cycles in Matching Models." *Journal of Monetary Economics,* Vol. 56, pp. 309-327.

Dolfin, Sarah. 2006. "An Examination of Firms' Employment Costs." *Applied Economics,* Vol. 38, pp. 861-878.

Ebell, Monique. 2011. "On the Cyclicality of Unemployment: Resurrecting the Participation Margin." *Labour Economics,* Vol. 18, pp. 822-836.

Faberman, Jason R., Andreas Mueller, Aysegül Sahin and Giorgio Topa. 2015. "Job Search Behavior among the Employed and Non-Employed." Federal Reserve Bank of New York, mimeo.

Faia, Ester, Wolfgang Lechthaler and Christian Merkl. 2014. "Labor Selection, Turnover Costs, and Optimal Monetary Policy." *Journal of Money, Credit, and Banking,* Vol.

46, pp. 115-144.

Gali, Jordi. 2011. "The Return of the Wage Phillips Curve." *Journal of the European Economic Association,* Vol. 9, pp. 431-461.

Gali, Jordi, Frank Smets, and Rafael Wouters. 2012. "Unemployment in an Estimated New Keynesian Model." NBER Macroeconomics Annual 2011.

Gertler, Mark and Antonella Trigari. 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Journal of Political Economy,* Vol. 117, pp. 38-86.

Haefke, Christian and Michael Reiter. 2011. "What Do Participation Fluctuations Tell Us About Labor Supply Elasticities?." IZA Discussion Papers, No. 6039

Haefke, Christian, Marcus Sonntag, and Thijs van Rens. 2013. "Wage Rigidity and Job Creation." *Journal of Monetary Economics,* Vol. 60, pp. 887-899.

Hagedorn, Marcus, and Iourii Manovskii. 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." *American Economic Review,* Vol. 98, pp. 1692-1706.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics,* Vol. 16, pp. 309-327.

Holzer, Harry J.. 1987. "Job Search by Employed and Unemployed Youth." *Industrial and Labor Relations Review,* Vol. 40, pp. 601-611.

Hornstein, Andreas, Per Krusell, and Giovanni Violante. 2011. "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." *American Economic Review,* Vol. 101, pp. 2873-98.

Lechthaler, Wolfgang, Christian Merkl, and Dennis J. Snower. 2010. "Monetary Persistence and the Labor Market: A New Perspective." *Journal of Economic Dynamics and Control,* Vol. 34, pp. 968-983.

Krusell, Per, Toshihiko Mukoyama, Richard Rogerson, and Aysegul Sahin. 2009. "Labor Supply in a Frictional Labor Market." Princeton University.

McCall, John J.. 1970. "Economics of Information and Job Search." *Quarterly Journal of Economics,* Vol. 84 (1), pp. 113–126.

Mortensen, Dale T. 1970. "A Theory of Wage and Employment Dynamics. In *Microeconomic Foundations of Employment and Inflation Theory,* edited by E. Phelps. W.W. Norton and Co.

Mortensen, Dale T., and Eva Nagypal. 2007. "More on Unemployment and Vacancy Fluctuations." *Review of Economic Dynamics,* Vol. 10, pp. 327-347.

Mortensen, Dale T., and Christopher A. Pissarides. 1994. "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies,* Vol. 61, pp. 397-415.

Mulligan, Casey B. 2001. "Aggregate Implications of Indivisible Labor." *The B.E. Journals* *in Macroeconomics: Advances in Macroeconomics,* Vol. 1, pp. 1-33.

Pissarides, Christopher A. 1985. "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages." *American Economic Review,* Vol. 75, pp. 676-690.

Rogerson, Richard. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics,* Vol. 21, pp. 3-16.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review,* Vol. 95, pp. 25-49.

Silva, José and Manuel Toledo. 2009. "Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment." *Macroeconomic Dynamics,* Vol. 13 , pp. 76-96.

Veracierto, Marcelo. 2008. "On the Cyclical Behavior of Employment, Unemployment and Labor Force Participation." *Journal of Monetary Economics,* Vol. 55, pp. 1143-1157.

A Partial Equilibrium Model

The social planner optimization problem is

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t \left[z_t n_t + s_t(1-\eta(\tilde{\varepsilon}_t))b - s_t \eta(\tilde{\varepsilon}_t) \left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \right) \right] \tag{32}$$

subject to the endogenous law of motion for employment

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t) \left(lfp - (1-\rho)n_{t-1} \right), \tag{33}$$

in which lfp is an exogenous labor force participation rate, and the initial condition n_{-1} and the exogenous law of motion for aggregate productivity are taken as given.

The social planner takes into account the dependence of the finding rate and the average operating cost of a newly-selected worker on the threshold $\tilde{\varepsilon}_t$, which is made explicit in the notation here. Given that $\eta(\tilde{\varepsilon}_t) \equiv \int_{\varepsilon \leq \tilde{\varepsilon}_t} \eta'(\varepsilon) d\varepsilon$ and $H(\tilde{\varepsilon}_t) \equiv \int_{\varepsilon \leq \tilde{\varepsilon}_t} \varepsilon \eta'(\varepsilon) d\varepsilon$, we have $H'(\tilde{\varepsilon}_t) = \tilde{\varepsilon}_t \eta'(\tilde{\varepsilon}_t)$, by the Fundamental Theorem of Calculus.

Let μ_t denote the Lagrange multiplier. The first-order conditions of the social planner problem (in which we have substituted $s_t = lfp - (1-\rho)n_{t-1}$) with respect to n_t and $\tilde{\varepsilon}_t$ are, respectively,

$$z_t - \mu_t + \left(\frac{1-\rho}{1+r} \right) E_t \left\{ \mu_{t+1} \left[1 - \eta(\tilde{\varepsilon}_{t+1}) \right] + \left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + H(\tilde{\varepsilon}_{t+1}) - (1-\eta(\tilde{\varepsilon}_t))\, b \right] \right\} = 0 \tag{34}$$

and

$$-s_t \left[\gamma^h \eta'(\tilde{\varepsilon}_t) + H'(\tilde{\varepsilon}_t) + \eta'(\tilde{\varepsilon}_t) b \right] + \mu_t s_t \eta'(\tilde{\varepsilon}_t) = 0. \tag{35}$$

Isolating the multiplier μ_t from (35),

$$\mu_t = \gamma^h + \tilde{\varepsilon}_t + b. \tag{36}$$

Next, substituting expression (36) for μ_t (and its time $t+1$ counterpart) in (34), we have

$$\begin{aligned} \gamma^h + \tilde{\varepsilon}_t + b \;\; &= \;\; z_t \\ &+ \;\; \left(\frac{1-\rho}{1+r} \right) E_t \left\{ u'(c_{t+1}) \left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + H(\tilde{\varepsilon}_{t+1}) - (1-\eta(\tilde{\varepsilon}_t))\, b \right] \right\} \\ &+ \;\; \left(\frac{1-\rho}{1+r} \right) E_t \left\{ u'(c_{t+1}) \left[\gamma^h + \tilde{\varepsilon}_{t+1} + b \right] \left[1 - \eta(\tilde{\varepsilon}_{t+1}) \right] \right\}. \end{aligned} \tag{37}$$

Thus, we have the selection condition

$$\gamma^h + \tilde{\varepsilon}_t = z_t - b + \left(\frac{1-\rho}{1+r} \right) E_t \left\{ H(\tilde{\varepsilon}_{t+1}) - \tilde{\varepsilon}_{t+1} \eta(\tilde{\varepsilon}_{t+1}) + \gamma^h + \tilde{\varepsilon}_{t+1} \right\}. \tag{38}$$

B Partial Equilibrium Model with Sequential Search

The social planner optimization problem is

$$\max E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[z_t n_t + s_t(1-\eta(\tilde{\varepsilon}_t))b - s_t \eta(\tilde{\varepsilon}_t)\left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)}\right)\right] \tag{39}$$

subject to the law of motion for employment

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)\left(lfp - (1-\rho)n_{t-1}\right), \tag{40}$$

and the sequential-search-based definition of the quarterly job-finding rate

$$\eta(\tilde{\varepsilon}_t) = \sum_{j=1}^{i} \left(1-\omega(\tilde{\varepsilon}_t)\right)^{j-1} \omega(\tilde{\varepsilon}_t), \tag{41}$$

in which i is an integer that denotes the number of interviews in a given quarter.

The first derivative of the quarterly job-finding rate with respect to the cutoff point is

$$\eta(\tilde{\varepsilon}_t)' = \omega'(\tilde{\varepsilon}_t) \cdot \sum_{j=1}^{i} \left[\left(1-\omega(\tilde{\varepsilon}_t)\right)^{j-1} - (j-1)\left(1-\omega(\tilde{\varepsilon}_t)\right)^{j-2} \omega(\tilde{\varepsilon}_t)\right]. \tag{42}$$

For analytical convenience, we define

$$\eta'(\tilde{\varepsilon}_t) = \omega'(\tilde{\varepsilon}_t) \cdot F(\tilde{\varepsilon}_t), \tag{43}$$

with $F(\tilde{\varepsilon}_t) \equiv \sum_{j=1}^{i} \left[\left(1-\omega(\tilde{\varepsilon}_t)\right)^{j-1} - (j-1)\left(1-\omega(\tilde{\varepsilon}_t)\right)^{j-2} \omega(\tilde{\varepsilon}_t)\right]$.

Let $\beta^t \mu_t$ be the Lagrange multiplier on the period-t law of motion for employment, and, for ease of notation, $\Omega(\tilde{\varepsilon}_t) \equiv \frac{\eta(\tilde{\varepsilon}_t)H(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)}$. The first-order conditions of the social planner problem with respect to n_t and $\tilde{\varepsilon}_t$ are, respectively,

$$z_t - \mu_t + \left(\frac{1-\rho}{1+r}\right) E_t \left\{\mu_{t+1}\left[1-\eta(\tilde{\varepsilon}_{t+1})\right] + \left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + \Omega(\tilde{\varepsilon}_{t+1}) - (1-\eta(\tilde{\varepsilon}_{t+1}))\, b\right]\right\} = 0 \tag{44}$$

and

$$-s_t \left[\gamma^h \eta'(\tilde{\varepsilon}_t) + \Omega'(\tilde{\varepsilon}_t) + \eta'(\tilde{\varepsilon}_t) b\right] + \mu_t s_t \eta'(\tilde{\varepsilon}_t) = 0. \tag{45}$$

Isolating the multiplier μ_t from (45),

$$\mu_t \quad = \quad \frac{\gamma^h \eta'(\tilde{\varepsilon}_t) + \Omega'(\tilde{\varepsilon}_t) + \eta'(\tilde{\varepsilon}_t) b}{\eta'(\tilde{\varepsilon}_t)}$$

$$
\begin{aligned}
&= \gamma^h + b + \frac{\Omega'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\
&= \gamma^h + b + \frac{H(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)}\left(1 - \frac{\eta(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)F(\tilde{\varepsilon}_t)}\right) + \frac{\eta(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)F(\tilde{\varepsilon}_t)}\tilde{\varepsilon}_t. \qquad (46)
\end{aligned}
$$

Using this expression immediately above, the selection condition is

$$
\begin{aligned}
&\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)}\left(1 - \frac{\eta(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)F(\tilde{\varepsilon}_t)}\right) + \frac{\eta(\tilde{\varepsilon}_t)}{\omega(\tilde{\varepsilon}_t)F(\tilde{\varepsilon}_t)}\tilde{\varepsilon}_t = z_t - b \qquad (47) \\
&+ \left(\frac{1-\rho}{1+r}\right) E_t\left\{\gamma^h + \frac{\eta(\tilde{\varepsilon}_{t+1})H(\tilde{\varepsilon}_{t+1})}{\omega(\tilde{\varepsilon}_{t+1})}\right\} \\
&+ \left(\frac{1-\rho}{1+r}\right) E_t\left\{\left(\frac{H(\tilde{\varepsilon}_{t+1})}{\omega(\tilde{\varepsilon}_{t+1})}\left(1 - \frac{\eta(\tilde{\varepsilon}_{t+1})}{\omega(\tilde{\varepsilon}_{t+1})F(\tilde{\varepsilon}_{t+1})}\right) + \frac{\eta(\tilde{\varepsilon}_{t+1})}{\omega(\tilde{\varepsilon}_{t+1})F(\tilde{\varepsilon}_{t+1})}\tilde{\varepsilon}_{t+1}\right)(1 - \eta(\tilde{\varepsilon}_{t+1}))\right\}.
\end{aligned}
$$

C Decentralized Economy

Consider a decentralized economy in which a representative "large firm" hires and makes wage payments to many workers. To establish some notation, define w_t^I as the period-t wage earned by any incumbent worker. An incumbent worker is one that has completed at least one full period of employment and, because they are identical, all incumbent workers earn the same wage. Also define

$$\omega_e(\tilde{\varepsilon}_t) \equiv \int_{\infty}^{\tilde{\varepsilon}_t} w(\varepsilon) f(\varepsilon) d\varepsilon. \tag{48}$$

C.1 Firms

In period zero, the representative firm chooses state-contingent decision rules for its desired employment stock and the threshold operating cost $\tilde{\varepsilon}_t$ below which it is willing to hire in order to maximize discounted profits

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[z_t n_t - \gamma^h \eta(\tilde{\varepsilon}_t) s_t - \frac{\omega_e(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t - (1-\rho) n_{t-1} w_t^I - \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)} \eta(\tilde{\varepsilon}_t) s_t \right]. \tag{49}$$

In (49), $\Xi_{t|0}$ is the period-0 value to the representative household of period-t goods, which the firm uses to discount profit flows because households are the ultimate owners of firms. Without any confusion between firm-level variables and aggregate variables, the hiring rate η_t is understood in this section to be a consequence of the firm's decisions, while the firm takes as given the number of job-seekers s_t as well as the wage-setting process. Because output is sold in a perfectly-competitive market, the firm's problem is to choose $\tilde{\varepsilon}_t$ and n_t, $\forall t$, to maximize (49) subject to a sequence of perceived laws of motion for its employment level,

$$n_t = (1-\rho) n_{t-1} + s_t \eta(\tilde{\varepsilon}_t). \tag{50}$$

Letting $\beta^t \mu_{ft}$ denote the Lagrange multiplier on the period-t law of motion (50), the first-order conditions with respect to $\tilde{\varepsilon}_t$ and n_t are

$$\mu_{ft} \eta'(\tilde{\varepsilon}_t) s_t - \omega_e'(\tilde{\varepsilon}_t) s_t - \gamma^h \eta'(\tilde{\varepsilon}_t) s_t - H'(\tilde{\varepsilon}_t) s_t = 0 \tag{51}$$

and

$$z_t - \mu_{ft} + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left(\mu_{ft+1} - w_{t+1}^I \right) \right\} = 0, \tag{52}$$

in which $\Xi_{t+1|t} \equiv \Xi_{t+1|0} / \Xi_{t|0}$ is the one-period stochastic discount factor. From (51), the value to the firm of an employee can be measured as

$$\begin{aligned}\mu_{ft} &= \frac{\omega_e'(\tilde{\varepsilon}_t) + \gamma^h \eta'(\tilde{\varepsilon}_t) + H'(\tilde{\varepsilon}_t)}{\eta'(\tilde{\varepsilon}_t)} \\ &= \frac{w(\tilde{\varepsilon}_t) f(\tilde{\varepsilon}_t) + \gamma^h f(\tilde{\varepsilon}_t) + \tilde{\varepsilon}_t f(\tilde{\varepsilon}_t)}{f(\tilde{\varepsilon}_t)} \\ &= w(\tilde{\varepsilon}_t) + \gamma^h + \tilde{\varepsilon}_t, \end{aligned} \tag{53}$$

where the second line follows from the Fundamental Theorem of Calculus. Substituting (53) into (52),

$$\gamma^h + \tilde{\varepsilon}_t = z_t - w(\tilde{\varepsilon}_t) + (1-\rho) E_t \left\{ \Xi_{t+1|t} \left[\gamma^h + \tilde{\varepsilon}_{t+1} + w(\tilde{\varepsilon}_{t+1}) - w^I_{t+1} \right] \right\}, \tag{54}$$

which is the firm's hiring (selection) condition.

C.2 Nash Bargained Wages

C.2.1 Value Equation for the Firm

$$\mathbf{J}_{It} = z_t - w^I_t + (1-\rho) E_t \Xi_{t+1|t} \mathbf{J}_{It+1} \tag{55}$$

is the equilibrium value to the firm of an incumbent worker and

$$\mathbf{J}_E(\varepsilon^i_t) = z_t - w(\varepsilon^i_t) - \gamma^h - \varepsilon^i_t + (1-\rho) E_t \Xi_{t+1|t} \mathbf{J}_{It+1} \tag{56}$$

is the equilibrium value to the firm of a newly-hired worker with idiosyncratic characteristics ε^i_t.

C.2.2 Value Equations for the Household

An incumbent worker in period t has value (measured in period-t goods) to the household

$$\mathbf{W}_{It} = w^I_t + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho) \mathbf{W}_{It+1} + \rho \left(\eta(\tilde{\varepsilon}_{t+1}) \bar{\mathbf{W}}_{Et+1} + (1 - \eta(\tilde{\varepsilon}_{t+1})) \mathbf{U}_{t+1} \right) \right] \right\}.$$

The value of an unemployed worker is

$$\mathbf{U}_t = b + E_t \left\{ \Xi_{t+1|t} \left[\eta(\tilde{\varepsilon}_{t+1}) \left(\frac{\int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}}{\eta(\tilde{\varepsilon}_{t+1})} \right) + (1 - \eta(\tilde{\varepsilon}_{t+1})) \mathbf{U}_{t+1} \right] \right\}. \tag{57}$$

To conserve on notation, denote hereafter $\bar{\mathbf{W}}_{Et+1} \equiv \int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}$. The surplus between these two values is

$$\mathbf{W}_{It} - \mathbf{U}_t$$

$$= \quad w_t^I - b + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} + (\rho-1)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}. \quad (58)$$

Based on the same logic as above, a newly-hired worker with idiosyncratic characteristics ε_t^i in period t has value (measured in period-t goods) to the household

$$\mathbf{W}_E(\varepsilon_t^i) = w(\varepsilon_t^i) + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} + (\rho-1)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}, \quad (59)$$

and thus the surplus between it and $\mathbf{U}_t$ is

$$\begin{aligned} &\mathbf{W}_E(\varepsilon_t^i) - \mathbf{U}_t \\ &= \quad w(\varepsilon_t^i) - b + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} + (\rho-1)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}. \end{aligned}$$

C.2.3 Bargaining Solution

From generalized Nash bargaining,

$$\mathbf{W}_{It} - \mathbf{U}_t = \left(\frac{\alpha}{1-\alpha}\right)\mathbf{J}_{It} \quad (60)$$

is the Nash sharing rule for incumbent workers where α is the bargaining power of workers, and

$$\mathbf{W}_E(\varepsilon_t^i) - \mathbf{U}_t = \left(\frac{\alpha}{1-\alpha}\right)\mathbf{J}_E(\varepsilon_t^i) \quad (61)$$

is the Nash sharing rule for a newly-hired individual with idiosyncratic characteristics ε_t^i.

To obtain an expression for the period-t bargained wage of an incumbent, first substitute (58) in the Nash sharing rule (60), which gives

$$\begin{aligned} &w_t^I - b + (1-\rho)E_t\left\{\Xi_{t+1|t}\left[\mathbf{W}_{It+1} - \mathbf{U}_{t+1}\right]\right\} \quad (62) \\ &- \quad (1-\rho)E_t\left\{\Xi_{t+1|t}\eta(\tilde{\varepsilon}_{t+1})\left[\bar{\mathbf{W}}_{Et+1} - \mathbf{U}_{t+1}\right]\right\} = \left(\frac{\alpha}{1-\alpha}\right)\left(z_t - w_t^I + (1-\rho)E_t\Xi_{t+1|t}\mathbf{J}_{It+1}\right). \end{aligned}$$

Isolating w_t^I, we have

$$\begin{aligned} w_t^I \quad &= \quad \alpha\left(z_t + (1-\rho)E_t\Xi_{t+1|t}\mathbf{J}_{It+1}\right) \quad (63) \\ &+ (1-\alpha)\left[b + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} - (1-\rho)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}\right]. \end{aligned}$$

Similarly,

$$w(\varepsilon_t^i) \quad = \quad \alpha\left(z_t - \gamma^h - \varepsilon_t^i + (1-\rho)E_t\Xi_{t+1|t}\mathbf{J}_{It+1}\right) \quad (64)$$

$$+ (1-\alpha)\left[b + E_t\left\{\Xi_{t+1|t}\left[(1-\rho)\mathbf{W}_{It+1} - (1-\rho)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}\right].$$

C.3 Equilibrium

A symmetric private-sector equilibrium is made up of endogenous state-contingent processes $\{c_t, n_t, \tilde{\varepsilon}_t, w_t^I, w(\tilde{\varepsilon}_t)\}_{t=0}^{\infty}$ that satisfy the sequences of conditions: the representative firm's selection condition (54), the goods resource constraint (65),

$$c_t = z_t n_t - s_t \eta(\tilde{\varepsilon}_t)\left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right), \tag{65}$$

the aggregate law of motion for employment (66),

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)s_t, \tag{66}$$

and the two conditions that determine the wage processes $\{w_t^I, w(\tilde{\varepsilon}_t)\}$.

D Proof for Decentralization of Efficiency

The wage at the cutoff point $w(\tilde{\varepsilon}_t)$ is equal to the reservation wage (i.e. the point where $\mathbf{W}_E(\tilde{\varepsilon}_t) - U_t = 0$):

$$\mathbf{W}_E(\tilde{\varepsilon}_t) - U_t = w(\tilde{\varepsilon}_t) - b + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho)\mathbf{W}_{It+1} - (1-\rho)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\} = 0 \tag{67}$$

Thus:

$$w(\tilde{\varepsilon}_t) - b = E_t \left\{ \Xi_{t+1|t} \left[(1-\rho)\mathbf{W}_{It+1} - (1-\rho)\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\} \tag{68}$$

Now let's compare the decentralized job creation condition and the efficient one. For this purpose, we denote the social planner solution with SP and the decentralized solution with D.

$$\gamma^h + \tilde{\varepsilon}_t^D = z_t - w(\tilde{\varepsilon}_t^D) + (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[\gamma^h + \tilde{\varepsilon}_{t+1}^D + w(\tilde{\varepsilon}_{t+1}^D) - w_{t+1}^I \right] \right\}, \tag{69}$$

and

$$\gamma^h + \tilde{\varepsilon}_t^{SP} = z_t - b + (1-\rho)\, E_t \Xi_{t+1|t} \left\{ H(\tilde{\varepsilon}_{t+1}^{SP}) - \tilde{\varepsilon}_{t+1}^{SP}\eta(\tilde{\varepsilon}_{t+1}^{SP}) + \gamma^h + \tilde{\varepsilon}_{t+1}^{SP} \right\}, \tag{70}$$

For efficiency to be decentralized, we require $\tilde{\varepsilon}_t^D = \tilde{\varepsilon}_t^{SP}$ for all t. In our proof, we will equalize these two thresholds and check under which condition they are equal. Thus, we denote $\tilde{\varepsilon}_t = \tilde{\varepsilon}_t^D = \tilde{\varepsilon}_t^{SP}$ and subtract (70) from (69), which gives

$$w(\tilde{\varepsilon}_t) - b = (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\}. \tag{71}$$

Subtracting this equation from equation (68) gives

$$\begin{aligned} & (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w_{t+1}^I \right] \right\} & (72) \\ = \; & -(1-\rho)E_t \left\{ \Xi_{t+1|t} \left[\mathbf{W}_{It+1} - \left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}. & (73) \end{aligned}$$

For convenience, we repeat the present value expressions, which are

$$\mathbf{W}_t^I = w_t^I + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho)\mathbf{W}_{It+1} + \rho\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}, \tag{74}$$

$$\mathbf{W}_E(\varepsilon_t^i) = w(\varepsilon_t^i) + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho)\mathbf{W}_{It+1} + \rho\left(\eta(\tilde{\varepsilon}_{t+1})\bar{\mathbf{W}}_{Et+1} + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1}\right)\right]\right\}, \tag{75}$$

and

$$\mathbf{U}_t = b + E_t \left\{ \Xi_{t+1|t} \left[\eta(\tilde{\varepsilon}_{t+1}) \left(\frac{\int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}}{\eta(\tilde{\varepsilon}_{t+1})} \right) + (1-\eta(\tilde{\varepsilon}_{t+1}))\mathbf{U}_{t+1} \right] \right\}. \tag{76}$$

Substituting these value expressions into equation (72), we have

$$\begin{aligned} & -(1-\rho)E_t \left\{ \Xi_{t+1|t} \left[\begin{array}{c} w^I_{t+1} + E_t \left\{ \Xi_{t+2|t+1} \left[\begin{array}{c} (1-\rho)\mathbf{W}_{It+2} + \\ \rho\left(\eta(\tilde{\varepsilon}_{t+2})\bar{\mathbf{W}}_{Et+2} + (1-\eta(\tilde{\varepsilon}_{t+2}))\mathbf{U}_{t+2}\right) \end{array} \right] \right\} \\ -\eta(\tilde{\varepsilon}_{t+1}) \left(\begin{array}{c} \left(\frac{\int_{-\infty}^{\tilde{\varepsilon}_{t+1}} \mathbf{W}_E(\varepsilon^i_{t+1}) f(\varepsilon^i_{t+1}) d\varepsilon^i_{t+1}}{\eta(\tilde{\varepsilon}_{t+1})} \right) + \\ \left\{ \Xi_{t+2|t+1} \left[(1-\rho)\mathbf{W}_{It+2} + \rho\eta(\tilde{\varepsilon}_{t+2})\bar{\mathbf{W}}_{Et+2} + \rho(1-\eta(\tilde{\varepsilon}_{t+2}))\mathbf{U}_{t+2} \right] \right\} \end{array} \right) \\ -(1-\eta(\tilde{\varepsilon}_{t+1})) \left[b + \left\{ \Xi_{t+2|t+1} \left[\begin{array}{c} \eta(\tilde{\varepsilon}_{t+2})\bar{\mathbf{W}}_{Et+2} \\ +(1-\eta(\tilde{\varepsilon}_{t+2}))\mathbf{U}_{t+2} \end{array} \right] \right\} \right] \end{array} \right] \right\} \\ = \; & (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - w^I_{t+1} \right]. \right\} \end{aligned} \tag{77}$$

Collecting terms, we obtain

$$\begin{aligned} & -(1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\eta(\tilde{\varepsilon}_{t+1})) \left\{ \Xi_{t+2|t+1}(1-\rho) \left[\mathbf{W}_{It+2} - \eta(\tilde{\varepsilon}_{t+2})\bar{\mathbf{W}}_{Et+2} - (1-\eta(\tilde{\varepsilon}_{t+2}))\,\mathbf{U}_{t+2} \right] \right\} \right] \right\} \\ = \; & (1-\rho)E_t\Xi_{t+1|t} \left\{ \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - \bar{\mathbf{W}}_{Et+1} - (1-\eta(\tilde{\varepsilon}_{t+1}))b \right\}. \end{aligned} \tag{78}$$

Finally, replace the term in square brackets in the first line by equation (68), iterated one period forward:

$$\begin{aligned} & (1-\rho)E_t \left\{ \Xi_{t+1|t} \left[(1-\eta(\tilde{\varepsilon}_{t+1})) \left\{ [w(\tilde{\varepsilon}_{t+1}) - b] \right\} \right] \right\} \\ = \; & (1-\rho)E_t\Xi_{t+1|t} \left\{ \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) + w(\tilde{\varepsilon}_{t+1}) - \bar{\mathbf{W}}_{Et+1} - (1-\eta(\tilde{\varepsilon}_{t+1}))b \right\}. \end{aligned} \tag{79}$$

Further simplification yields

$$(1-\rho)E_t\Xi_{t+1|t} \left\{ \tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) + \eta(\tilde{\varepsilon}_{t+1})w(\tilde{\varepsilon}_{t+1}) - \bar{\mathbf{W}}_{Et+1} \right\} = 0. \tag{80}$$

Thus, efficiency in the decentralized economy requires

$$\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1}) - H(\tilde{\varepsilon}_{t+1}) = \bar{\mathbf{W}}_{Et+1} - \eta(\tilde{\varepsilon}_{t+1})w(\tilde{\varepsilon}_{t+1}). \tag{81}$$

Thus, whenever the difference of epsilon between marginal and average worker is the same as the wage differential, we obtain a decentralization of the social planner solution.

Let's take our Nash solution (64) and see what the wage difference between the marginal and average entrant is. The difference between $w(\tilde{\varepsilon}_{t+1}) - \bar{\mathbf{W}}_E$ is

$$w(\tilde{\varepsilon}_{t+1}) - \frac{\bar{\mathbf{W}}_{Et+1}}{\eta(\tilde{\varepsilon}_{t+1})} = \alpha \left(-\tilde{\varepsilon}_{t+1} + \frac{H(\tilde{\varepsilon}_{t+1})}{\eta(\tilde{\varepsilon}_{t+1})} \right). \tag{82}$$

For the efficiency condition (81) to hold, we require $\alpha = 1$. When the entire surplus goes to the worker the existing externality will be internalized.[38]

[38] See Faia, Lechthaler, and Merkl (2014) for a similar proof in the context of a different modeling assumption.

E Long-Run Wage Dispersion

So far, we have written our model in a stationary environment. Productivity shocks are mean-reverting and can thus be interpreted as deviations from some long-run trend. Under the presence of long-run productivity growth, our model has to modified in order to guarantee stationarity. To illustrate this, assume that productivity growth at a constant rate ψ (where ψ is one plus the period's net growth rate). In the decentralized economy, profits of an entrant would then be

$$\pi_t^E(\varepsilon_t) = z_t - w(\varepsilon_t) - \varepsilon_t - \gamma^h + (1-\rho)E_t\left\{\Xi_{t+1|t}\pi_{t+1}^I\right\}, \tag{83}$$

In steady state and by assuming that the wage of entrants is a function of the long-run productivity, we obtain:

$$\pi^E(\varepsilon) = z - w(\varepsilon) - \varepsilon - \gamma^h + \beta(1-\rho)\left(\psi z - w^I(\psi z)\right) + \beta^2(1-\rho)^2\left(\psi^2 z - w^I\left(\psi^2 z\right)\right) + ..., \tag{84}$$

When we use the standard assumption that wages comove 1:1 with long-run productivity (as common in RBC models), we can simplify the equation above to

$$\pi^E(\varepsilon) = z - w(\varepsilon) - \varepsilon - \gamma^h + \beta(1-\rho)\psi\frac{z - w^I(z)}{1-\beta\psi(1-\rho)} \tag{85}$$

It is straightforward to see that introducing long-run productivity growth ($\psi > 1$) profits would increase over time and thus the cutoff point would shift every period (which can be obtained by setting $\pi^E(\tilde{\varepsilon}) = 0$). Thus, the model would not be stationary.

Stationarity can be established by a very simple assumption. Besides wages, the idiosyncratic productivity ε and the fixed component γ^h also have to trend with aggregate productivity. It is then straighforward to see that the cutoff remains unaffected. To see this, compare the cutoff point in period t and in some future period $t+x$.

In period t:

$$\tilde{\varepsilon} = z - w(\tilde{\varepsilon}) - \gamma^h + \beta(1-\rho)\psi\frac{z - w^I(z)}{1-\beta\psi(1-\rho)} \tag{86}$$

In period $t+x$:

$$\psi^x\tilde{\varepsilon} = \psi^x z - \psi^x w(\tilde{\varepsilon}) - \psi^x\gamma^h + \beta(1-\rho)\psi^x\frac{z - w^I(z)}{1-\beta\psi(1-\rho)} \tag{87}$$

By dividing equation (87) by ψ^x, it is straightforwad to see that the cutoff point remains the same and thus the model becomes stationary.

How does this help us? In order to guarantuee stationarity, we required two assumptions: i) idiosyncratic productivity trends with aggregate productivity, ii) wages are a proportional function of productivity. Let's assume a general wage mechanism: $w(a, \varepsilon_i) = g(z - \varepsilon_i) + h(w(a, \varepsilon_i) b)$, where g and h are linear functions. As shown above, z and ε_i have to trend with long productivity to obtain stationarity. In addition, we define the fall-back option as a replacement rate (which generates stationarity). Thus, the wage in period $t + x$ is $w(a, \varepsilon_i) = \frac{\psi^x g(z-\varepsilon_i)}{1-h(b)}$.

Let's calculate the coefficient of variation *cov* for wages:

$$cov(w(a, \varepsilon_i)) = \frac{SD(w(a, \varepsilon_i))}{mean(w(a, \varepsilon_i))} = \frac{\sqrt{\sum_i (w(a, \varepsilon_i) - mean(w(a, \varepsilon_i)))^2}}{mean(w(a, \varepsilon_i))}$$

and use the two conditions from above:

$$\begin{aligned} cov(w(\varepsilon_i)) &= \frac{\sqrt{\sum_i \left(\frac{\psi^x g(z-\varepsilon_i)}{1-h(b)} - mean\left(\frac{\psi^x g(z-\varepsilon_i)}{1-h(b)}\right)\right)^2}}{mean\left(\frac{\psi^x g(z-\varepsilon_i)}{1-h(b)}\right)} \\ &= \frac{\sqrt{\sum_i (g(z-\varepsilon_i) - mean(g(z-\varepsilon_i)))^2}}{mean(g(z-\varepsilon_i))} \end{aligned}$$

Given that g is a linear function, the second line is exactly equal to the coefficient of variation of aggregate and idiosyncratic productivity:

$$cov(z - \varepsilon_i) = \frac{\sqrt{\sum_i (z - \varepsilon_i - mean(z - \varepsilon_i))^2}}{mean(z - \varepsilon_i)}$$

Thus, we look at the data through the lens of our model (remaining agnostic about the actual underlying wage formation mechanism) where the cross-sectional coefficient of variation (i.e. standard deviation divided by mean) for wages would have to be same as the coefficient of variation for idiosyncratic productivity. In the dataset of Haefke, Sonntag, and van Rens (2013), the coefficient of variation for new entrants is on average 0.63 on the quarterly level (from 1989 to 2004) and remarkably stable over time (as required by our model).

F Search and Matching

The dynamic job-creation condition in an efficient search and matching model is

$$\frac{\kappa}{(1-\alpha)\,\theta_t^{-\alpha}} = z_t - b - \frac{\kappa\alpha\theta_t^{1-\alpha}}{(1-\alpha)\,\theta_t^{-\alpha}} + (1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\frac{\kappa}{(1-\alpha)\,\theta_{t+1}^{-\alpha}}\right\}. \tag{88}$$

In steady state:

$$\kappa\,(1-\beta\,(1-\rho))\,\theta^{\alpha} = (1-\alpha)\,(z-b) - \alpha\kappa\theta \tag{89}$$

Elasticity of the job-finding rate with respect to productivity:

$$\frac{\partial \ln \eta}{\partial \ln z} = \frac{\partial \ln \eta}{\partial \theta}\frac{\partial \theta}{\partial z}\frac{\partial z}{\partial \ln z}$$

Intermediate steps:

$$\alpha\theta^{\alpha-1}\frac{\partial\theta}{\partial z}\kappa\,(1-\beta\,(1-\rho)) = (1-\alpha) - \alpha\kappa\frac{\partial\theta}{\partial z} \tag{90}$$

$$\frac{\partial\theta}{\partial z} = \frac{1-\alpha}{\alpha}\frac{1}{(\theta^{\alpha-1}\kappa\,(1-\beta\,(1-\rho)) + \kappa)} \tag{91}$$

Thus:

$$\frac{\partial \ln \eta}{\partial \ln z} = (1-\alpha)\,\frac{1}{\theta^{1-\alpha}}\theta^{-\alpha}\frac{1-\alpha}{\alpha}\frac{z}{(\theta^{\alpha-1}\,(1-\beta\,(1-\rho))\,\kappa + \kappa)} \tag{92}$$

$$= \frac{(1-\alpha)^2}{\alpha}\frac{\theta^{-1}z}{(\theta^{\alpha-1}\,(1-\beta\,(1-\rho)) + 1)}\frac{1}{\kappa} \tag{93}$$

Given our calibration strategy to target a certain η, we can back out the steady state market tightness $\theta = \eta^{\frac{1}{1-\alpha}}$ and thereby calculate the required vacancy posting costs to obtain this steady state, namely:

$$\kappa = \frac{(1-\alpha)\,(z-b)}{(1-\beta\,(1-\rho))\,\theta^{\alpha} + \alpha\theta} \tag{94}$$

Substituting this into equation (92), we obtain

$$\begin{aligned}\frac{\partial \ln \eta}{\partial \ln z} &= \frac{(1-\alpha)^2}{\alpha}\frac{\theta^{-1}z}{(\theta^{\alpha-1}\,(1-\beta\,(1-\rho)) + 1)}\frac{(1-\beta\,(1-\rho))\,\theta^{\alpha} + \alpha\theta}{(1-\alpha)\,(z-b)}\\ &= \frac{1-\alpha}{\alpha}\frac{z}{z-b}\frac{(1-\beta\,(1-\rho)) + \alpha}{(1-\beta\,(1-\rho)) + 1}\end{aligned}$$

G General Equilibrium Model

A social planner in this economy optimally allocates the measure one of individuals in the representative household to leisure, unemployment, and employment. There are several representations of the planning problem available: suppose that c_t, lfp_t, n_t, and $\tilde{\varepsilon}_t$ are the formal objects of choice. Given the accounting identities of the model, the measure of individuals available for work can thus be expressed $s_t = lfp_t - (1-\rho)n_{t-1}$.

The social planner problem is to maximize lifetime expected utility of the representative household

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v\left(lfp_t\right)\right] \tag{95}$$

subject to the sequence of goods resource constraints

$$c_t = z_t n_t - [lfp_t - (1-\rho)n_{t-1}] \cdot \eta(\tilde{\varepsilon}_t) \left(\gamma^h + \frac{H(\tilde{\varepsilon}_t)}{\eta(\tilde{\varepsilon}_t)}\right), \tag{96}$$

and laws of motion for the employment stock

$$n_t = (1-\rho)n_{t-1} + \eta(\tilde{\varepsilon}_t)[lfp_t - (1-\rho)n_{t-1}]. \tag{97}$$

The social planner takes into account the dependence of the finding rate and the average operating cost of a newly-selected worker on the threshold $\tilde{\varepsilon}_t$, which is made explicit in the notation here. Recalling that $\eta(\tilde{\varepsilon}_t) \equiv \int_{\varepsilon \le \tilde{\varepsilon}_t} \eta'(\varepsilon) d\varepsilon$ and $H(\tilde{\varepsilon}_t) \equiv \int_{\varepsilon \le \tilde{\varepsilon}_t} \varepsilon \eta'(\varepsilon) d\varepsilon$, we have $H'(\tilde{\varepsilon}_t) = \tilde{\varepsilon}_t \eta'(\tilde{\varepsilon}_t)$, by the Fundamental Theorem of Calculus.

Let $\beta^t \lambda_t$ be the Lagrange multiplier on the period-t goods resource constraint, and $\beta^t \mu_t$ be the Lagrange multiplier on the period-t law of motion for employment. The first-order conditions of the social planner problem with respect to c_t, lfp_t, n_t, and $\tilde{\varepsilon}_t$ are, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{98}$$

$$-v'(lfp_t) - \lambda_t \left[\gamma^h \eta(\tilde{\varepsilon}_t) + H(\tilde{\varepsilon}_t)\right] + \mu_t \eta(\tilde{\varepsilon}_t) = 0, \tag{99}$$

$$\lambda_t z_t - \mu_t + (1-\rho)\beta E_t \left\{\mu_{t+1} \left[1 - \eta(\tilde{\varepsilon}_{t+1})\right] + \lambda_{t+1} \left[\gamma^h \eta(\tilde{\varepsilon}_{t+1}) + H(\tilde{\varepsilon}_{t+1})\right]\right\} = 0, \tag{100}$$

and

$$-\lambda_t s_t \left[\gamma^h \eta'(\tilde{\varepsilon}_t) + H'(\tilde{\varepsilon}_t)\right] + \mu_t s_t \eta'(\tilde{\varepsilon}_t) = 0. \tag{101}$$

Isolating the multiplier μ_t from (101),

$$\begin{aligned}\mu_t &= \frac{u'(c_t)\left[\gamma^h\eta'(\tilde{\varepsilon}_t)+H'(\tilde{\varepsilon}_t)\right]}{\eta'(\tilde{\varepsilon}_t)}\\ &= u'(c_t)\left[\gamma^h+\tilde{\varepsilon}_t\right],\end{aligned} \tag{102}$$

in which we have substituted (98). Substituting this expression for μ_t in (99) gives

$$\begin{aligned}\frac{v'(lfp_t)}{u'(c_t)} &= \tilde{\varepsilon}_t\eta(\tilde{\varepsilon}_t)-H(\tilde{\varepsilon}_t)\\ &= \int_{\varepsilon\leq\tilde{\varepsilon}_t}\left[\tilde{\varepsilon}_t-\varepsilon\right]f(\varepsilon)d\varepsilon,\end{aligned} \tag{103}$$

in which the second line substitutes the definitions of $H(\tilde{\varepsilon}_t)$ and $\eta(\tilde{\varepsilon}_t)$. The term in square brackets in the integral is unambiguously positive. Expression (103) is the static efficiency condition that appears as condition (30) in the main text.

Next, substituting expression (102) for μ_t (and its time $t+1$ counterpart) in (100), we have

$$\begin{aligned}u'(c_t)\left(\gamma^h+\tilde{\varepsilon}_t\right)z_t &= u'(c_t)z_t+(1-\rho)\beta E_t\left\{u'(c_{t+1})\left[\gamma^h\eta(\tilde{\varepsilon}_{t+1})+H(\tilde{\varepsilon}_{t+1})+(\gamma^h+\tilde{\varepsilon}_{t+1})(1-\eta(\tilde{\varepsilon}_{t+1}))\right]\right\}\\ &= u'(c_t)z_t+(1-\rho)\beta E_t\left\{u'(c_{t+1})\left[H(\tilde{\varepsilon}_{t+1})-\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})+\gamma^h+\tilde{\varepsilon}_{t+1}\right]\right\}.\end{aligned} \tag{104}$$

Dividing by $u'(c_t)$ gives the selection condition

$$\gamma^h+\tilde{\varepsilon}_t = z_t+(1-\rho)E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[H(\tilde{\varepsilon}_{t+1})-\tilde{\varepsilon}_{t+1}\eta(\tilde{\varepsilon}_{t+1})+\gamma^h+\tilde{\varepsilon}_{t+1}\right]\right\}. \tag{105}$$

H Extended Description of Micro-Empirical Evidence on Wages

This Appendix expands the discussion on the empirical micro-evidence discussed in Section 3.7, which is based on Haefke, Sonntag, and van Rens (2013) who use the CPS. Ideally, we would have a dataset which allows us to quantify the dispersion of match-specific idiosyncratic productivity shocks for new jobs in the United States (controlling for worker and establishment/firm fixed effects). However, to the best of our knowledge, such a dataset is not easily available.

We restrict ourselves to full time workers (i.e. those who worked more than 35 hours per week). How can we use wages to approximate idiosyncratic productivity? In order for our model to be stationary in the long run, the dispersion of idiosyncratic training costs (productivity) has to be proportional to aggregate productivity. In addition, in a decentralized version of the model, in the long-run wages would have to co-move proportionally with productivity to establish stationarity.[39] Thus, we look at the data through the lens of our model (remaining agnostic about the actual underlying wage formation mechanism) where the cross-sectional coefficient of variation (i.e. standard deviation divided by mean) for wages would have to be same as the coefficient of variation for idiosyncratic productivity. In the dataset of Haefke, Sonntag, and van Rens (2013), the coefficient of variation for new entrants is on average 0.63 on the quarterly level (from 1989 to 2004) and fairly stable over time (as required by our model).

Note that we consider the coefficient of variation of entrant wages to be an extreme upper bound for match-specific idiosyncratic productivity because it does not control for all sorts of observable and unobservable characteristics (in particular, not for establishment and worker fixed effects), which are abstracted from in our model. Card, Heining and Kline (2013, see Figure IV) show with a rich employer-employee linked dataset for Germany that adding industry, occupation and establishment fixed effects reduces the dispersion of wages substantially, namely by roughly one third. This brings us in a similar order of magnitude as suggested by the EOPP.

Interestingly, the coefficient of variation in the wage data is also roughly 0.6 on the monthly level. Given that establishment fixed effects are not included, we also proceed with a coefficient of variation of 0.4 when we calibrate to the monthly level.

[39]More generally, conventional business cycle and growth models require that productivity and wages comove one to one in the long run in order to make labor input (employment) stationary.