Firm Risk and Leverage-Based Business Cycles *

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Abstract

I characterize cyclical fluctuations in the cross-sectional dispersion of firm-level productivity. Using the micro-estimated dispersion, or “risk,” stochastic process as an input to a baseline DSGE financial accelerator model, I assess how well the model reproduces cyclical movements in both real and financial conditions of the economy. In the model, risk shocks calibrated to micro data lead to empirically-relevant steady-state leverage, a financial measure typically thought to be closely associated with real activity. In terms of aggregate quantities, however, pure risk shocks account for only a small share of GDP fluctuations in the model, less than one percent. Instead, standard aggregate productivity shocks explain virtually all of the model’s real fluctuations. These results do not necessarily reveal a dichotomy at the core of a popular class of DSGE financial frictions models. Rather, it is the magnitude of micro-estimated vs. macro-estimated risk shocks that are critical for aggregate quantity fluctuations.

Keywords: second-moment shocks, time-varying volatility, credit frictions, financial accelerator

JEL Classification: E10, E20, E32, E44

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1 Introduction

There are two distinct components of this paper that contribute to the literature on macro-financial accelerator models. The first is an estimation of time-series volatility of “risk shocks” using microeconomic data, and the second is an application of the empirical results to a well-known macroeconomic framework. The notion of “risk” studied is the standard deviation in any given time period of firm-level idiosyncratic productivity. “Risk shocks” are thus exogenous time variations in this cross-sectional dispersion. On the estimation front, the time-series volatility of idiosyncratic productivity risk is consistent with or a bit larger than other existing microeconomic estimates. However, it is an order of magnitude smaller than recent macroeconomic estimates.

The estimation result leads to the second component of the study, which is a quantitative application to a small-scale general equilibrium financial accelerator framework. Two main results emerge here. First, the endogenous steady-state leverage ratio in the model nearly matches that in the data (0.67 vs. 0.56, respectively), even though the calibration was not designed to do so. Second, using the estimated time-series volatility of cross-sectional risk as a driving force of the model, risk shocks alone generate little movement in standard macro aggregates, such as GDP. Variance decomposition shows that at most 0.1% of GDP volatility is accounted for by risk shocks, and the rest is driven by total factor productivity (these are the only two shocks in the model). Risk shocks lead to miniscule real-side fluctuations because of, from a macroeconomic perspective, the “small” estimate of volatility of risk. Nonetheless, risk shocks generate fluctuations in financial aggregates such as leverage, and their inclusion along with average (total factor) productivity shocks improves the fit of the model along the dimension of business-cycle co-movements with leverage.

The empirical side of this paper itself consists of two different parts. First, I characterize fluctuations in firm-level dispersion using U.S. micro data for the period 1973-1988. Specifically, based on data constructed by Cooper and Haltiwanger (2006), I estimate the mean, the persistence, and the time variation in the cross-sectional dispersion of firm-level productivity. The estimates for the mean and persistence parameters are broadly similar to those in existing literature.

The estimated time variation, which is identified in this paper as risk fluctuations, is comparable to existing microeconomic evidence, but is an order of magnitude smaller than recent macroeconomic estimates. The measure of firm risk I base on Cooper and Haltiwanger (2006) is strongly countercyclical with respect to GDP, consistent with the micro-level evidence of Bloom, Floetotto, Jaimovich, Saportka-Ekstein, and Terry (2012) — henceforth, BFJST (2012) — and Bachmann and Bayer (2010). In a microeconomic sense, firm risk is quite volatile over the business cycle: measured by the ratio of the standard deviation of innovations in risk to average risk, the volatility of annual firm risk is 17 percent. By this metric, volatility of firm risk is similar to that measured by BFJST (2012), but is substantially larger than that measured by Bachmann and Bayer (2010).
Comparisons must be made with caution, because the U.S. micro data I examine are different from the U.S. micro data examined by BFJST (2012), which in turn are different from the German micro data examined by Bachmann and Bayer (2010). Nonetheless, the evidence I present complements these and other emerging empirical measures of firm-level risk. The estimated risk shock process is used as an input to the theoretical model.

Before proceeding to theory, though, the second empirical aspect is an extension of the leverage measure provided in Masulis (1988) to cover the time period 1973-1988, which has two advantages. One advantage is to allow for clean comparability with the period over which the risk shock process is estimated. The other is that the Masulis-based leverage series also permits some comparability with a component of the Christiano, Motto, and Rostagno (2014) macroeconomic estimation, about which more is described soon.

In terms of theory, I deploy the estimated risk shock process in the Carlstrom and Fuerst (1998) agency-cost “output model.” A closely-related existing study is Dorofeenko, Lee, and Salyer (2008) — henceforth, DLS — which analyzes the importance of risk shocks in the Carlstrom and Fuerst (1997) “investment model.” The marginal contribution relative to DLS is the estimation of risk shock parameters rather than assumptions about them. Remarkably, my estimated risk shock parameters are very similar to their assumed parameters. The quantitative importance of risk shocks alone in generating fluctuations is minuscule in both my results and those of DLS.

Based on these results, in order for risk shocks to be important, either exogenous shocks to risk must be larger, or risk shocks must interact with other frictions and rigidities, or both. The Carlstrom and Fuerst (1997, 1998) investment and output models are small-scale frameworks compared to recently-emerging studies. A prominent recent study is the medium-scale model of Christiano, Motto, and Rotagno (2014) — henceforth, CMR — which estimates, among a variety of other parameters, a risk shock process based on macroeconomic data. CMR’s estimated volatility of risk is an order of magnitude larger than what I find in micro data. Risk shocks turn out to be the most

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1I thank Ana Lariau for her tenacity in locating the data in the IRS Corporate Income Tax Returns database in order to be able to extend the Masulis (1988) series.

2Leverage, defined as the debt-to-asset ratio, is often thought to play a central role in connecting financial and real activity. In principle, model-based leverage fluctuations have the potential to drive, or at least be associated with, real fluctuations. Such “leverage-based business cycles” could arise through fluctuations in firms’ balance sheet conditions that are induced by risk shocks. The transmission channel that the model emphasizes and tests is thus explicitly financial: if there were no agency costs in financial markets, there is no channel by which risk shocks could affect real fluctuations at all. This aspect of the model is similar to the qualitative business cycle model of Williamson (1987) and the quantitative model of DLS.

3I have used my estimated parameters in the investment model, and the results remain the same as in DLS. Results available upon request.

5The CMR framework, whose starting point is Bernanke, Gertler, and Gilchrist (1999), is of the “investment model” variety.
important driving force of business cycles in the CMR framework. In the small-scale framework considered below, counterfactually substituting this order-of-magnitude larger volatility of risk also generates meaningful business-cycle volatility.

Taken together, my results (which impose microeconomic discipline), the results of DLS, and the results of CMR (which impose macroeconomic discipline) may raise an issue about the “proper” way to parameterize risk shocks in agency-cost accelerator models — use of micro data vs. macro data. That is, the contrasting results seem to raise a tension between a micro-calibration approach and a macro-calibration approach. This tension does not have to be portrayed in a negative light. Rather, it indicates that further research is required.

Finally, two points regarding modeling approach are in order. First, as should be clear from the discussion so far, the idea of “risk shocks” in this paper is variations over time in the cross-sectional standard deviation of firm-level productivity, holding constant average productivity. This is the same notion of idiosyncratic “second-moment shocks” that BFJST (2012), Bachmann and Bayer (2010), CMR, and DLS study. However, it is distinct from an aggregate notion of “second-moment shocks” emphasized by Justiniano and Primiceri (2008), Fernández-Villaverde and Rubio-Ramirez (2007), Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2010), Gourio (2011), Basu and Bundick (2011), and others, in which the standard deviation of the innovations affecting aggregate driving processes such as productivity, real interest rates, and monetary disturbances vary over time. Crucial in this latter group of studies is that they are all representative-agent economies, so there is no meaningful concept of cross-sectional dispersion. Focusing on the cross section is the main idea in BFJST (2012), Bachmann and Bayer (2010), CMR, DLS, and this paper. Second, echoing the well-articulated argument in Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramirez, and Uribe (2010), I treat cross-sectional risk fluctuations as exogenous. How to endogenize such fluctuations is an interesting question, but because the empirical evidence in this area is so new and fast moving, I adopt the view that this is not the first natural question to consider. Instead, I focus on the consequences of such fluctuations as mediated through agency-cost frictions, which sharply focuses attention on the transmission mechanism between the financial and real sectors in the model. Regarding terminology, I use the terms “risk shocks,” “firm-level risk,” “second-moment shocks,” and “dispersion shocks” interchangeably.

The rest of the paper is organized as follows. Section 2 presents new empirical evidence on firm-level risk and its business cycle properties. Section 3 briefly presents the aggregate leverage data used as a financial point of comparison for the model’s results. Section 4 presents the baseline model, in which shocks to average productivity and risk shocks are independent exogenous processes. Section 5 presents quantitative results, which are supplemented by further analysis in the Appendix. Section 6 concludes.
2 Risk Fluctuations

The main goal of this section is to document the properties of business cycle fluctuations in firm-level dispersion. The analysis is based on a balanced panel, constructed by Cooper and Haltiwanger (2006), from the Longitudinal Research Database (LRD). The data are annual observations of plant-level measures such as revenue, materials and labor costs, and investment at approximately 7,000 large U.S. manufacturing plants over the period 1974-1988. The starting point for the analysis is Cooper and Haltiwanger’s (2006) measures of plant-level profitability residuals from this panel.\(^6\)

Briefly, Cooper and Haltiwanger (2006) compute for each plant \(i\) in year \(t\) a residual \(A_{it}\) that reconciles exactly the observations of plant \(i\)’s profits and capital stock in year \(t\) when described by a profit function that depends only on the capital stock.\(^7\) The year-specific aggregate residual \(\omega_{mt}\) is computed as the mean of \(A_{it}\) across firms in year \(t\). Plant \(i\)’s profit function in year \(t\) is viewed as being shifted by both the aggregate shock \(\omega_{mt}\) and an idiosyncratic shock \(\omega_{it} \equiv A_{it}/\omega_{mt}\). In each year, there is thus a cross-sectional distribution of \(\omega_{it}\). Denote by \(\sigma_{\omega t}^2\) the cross-sectional standard deviation in year \(t\) of the idiosyncratic component of profitability \(\omega_{it}\). I make three identifying assumptions regarding \(\omega_{it}\) and thus the interpretation of its cross-sectional dispersion \(\sigma_{\omega t}^2\). These assumptions align the analysis of the data with the model into which they will be an input.

First, although \(\sigma_{\omega t}^2\) measures cross-firm dispersion, I treat it as measuring true cross-firm risk.\(^8\) The two concepts are identical only if each firm’s idiosyncratic component \(\omega_{it}\) has zero persistence. Cooper and Haltiwanger (2006, p. 622-623) estimate an AR(1) coefficient of the idiosyncratic component of 0.885, hence \(\omega_{it}\) is actually quite persistent (recall the data are annual). However, it is computationally very difficult to handle persistent idiosyncratic shocks in the theoretical model developed below, so the model assumes \(iid\) idiosyncratic shocks.\(^9\) To align the empirical analysis of \(\sigma_{\omega t}^2\) with its role in the model, I thus proceed by assuming zero idiosyncratic persistence.

There are both advantages and potential drawbacks of this approach. An advantage is that the dispersion of firm-level outcomes in the model are thus calibrated to the data. A potential drawback is that \(\sigma_{\omega t}^2\) is thus an overestimate of firm-level risk, which, when input as an exogenous process to the model, in principle gives risk shocks the largest possible role in driving the model’s fluctuations. As the quantitative results in Section 5 show, risk shocks alone turn out to not drive much of aggregate fluctuations.

The second identifying assumption is that firm-level profitability shocks are true productivity shocks. Because plant-level price deflators are unavailable in the dataset, it is impossible to dis-

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\(^6\)I thank John Haltiwanger for providing their aggregative data on profitability residuals.

\(^7\)The Appendix in Cooper and Haltiwanger (2006) describes in detail the construction of the data and the residuals.

\(^8\)Which is the basis for my interchangeable references to firm-level “dispersion” and firm-level “risk.”

\(^9\)To my knowledge, no DSGE models based on the agency-cost framework have been solved assuming persistent idiosyncratic shocks.
tistinguish cost shocks from revenue shocks, so the $\omega$ residuals mix both supply and demand shifts (hence the term “profitability” shocks).\textsuperscript{10} As an identifying assumption for the theoretical model, I simply interpret these profitability shocks as true productivity shocks. A model-based justification for this is that the relative price of all goods in the model is always unity due to perfect competition in goods markets. Thus, one can think of this aspect of the data analysis as also being conducted strictly through the lens of the model.

Third, when deploying the evidence documented here in the model, I identify “plants” as “firms,” abstracting from the fact that a non-negligible share of plant-level output in the LRD represents output of multi-plant firms. With these three identifying assumptions, I characterize the business cycle behavior of both $\omega_{mt}$ and of $\sigma_t^\omega$, aspects of the data not studied by Cooper and Haltiwanger (2006).

2.1 Productivity Risk

I first compute the cross-sectional coefficient of variation of productivity (profitability) for each of the 15 years of the sample. Cross-sectional coefficients of variation are used because the residually-computed aggregate mean level of productivity ($\omega_{mt}$) is not unity in the data, but it is normalized to unity in the model below. The time-averaged mean of the cross-sectional coefficient of variation is 0.156, hence I normalize long-run dispersion in the model to $\bar{\sigma}^\omega = 0.156$. Given the discussion above, true long-run “risk” is smaller than $\bar{\sigma}^\omega = 0.156$. Specifically, taking a stationary AR(1) view of idiosyncratic productivity and using the Cooper and Haltiwanger (2006, p. 622-623) estimate of idiosyncratic persistence of 0.885, true long-run firm-level risk is $\sqrt{1 - 0.885^2 \bar{\sigma}^\omega} = 0.0726$. Aligning the empirical analysis with the model thus overstates firm-level risk by roughly a factor of two.

Figure 1 plots the time series $\sigma_t^\omega$, which suggests a modest upward trend in dispersion. Figure 2 displays the HP-filtered components of $\sigma_t^\omega$ and GDP over the period 1974-1988. A clear negative cyclical correlation between the two series is apparent — the contemporaneous correlation between the two series is -0.83, hence expansions are associated with a decrease in dispersion of firms’ idiosyncratic productivity, and recessions are associated with an increase in dispersion of firms’ idiosyncratic productivity. Strongly countercyclical firm-level risk is also a robust finding in the micro evidence of Bachmann and Bayer (2010) — hereafter, BB (2010) — and BFJST (2012). In terms of volatility, the standard deviation of the cyclical component of $\sigma_t^\omega$ is 3.15 percent over the sample period. With an innocuous abuse of notation, I hereafter use $\sigma_t^\omega$ to denote the cyclical component of cross-sectional dispersion.

In the model presented below, I suppose that $\sigma_t^\omega$ follows the exogenous AR(1)

\[
\ln \sigma_{t+1}^\omega = (1 - \rho_\sigma) \ln \bar{\sigma}^\omega + \rho_\sigma \ln \sigma_t^\omega + \epsilon_{t+1}^\omega,
\]

\textsuperscript{10}More precisely, they are available only at five-year intervals, too low a frequency for business cycle analysis.
with $\epsilon \sim N(0, \sigma^2)$. Given $\sigma^2 = 0.156$, the point estimate (using OLS) of the AR(1) parameter is $\rho_{\sigma^2} = 0.48$, with a t-statistic of 1.93. With this estimate of $\rho_{\sigma^2}$ and the standard deviation of $\sigma_t^2$ of 3.15 percent, the standard deviation of the (annual) innovations to the cross-firm dispersion process can be computed to be 0.0276. This implies a coefficient of variation (with respect to the mean dispersion $\bar{\sigma} = 0.156$) of 17.7 percent, which can be directly compared to the empirical evidence reported by BB (2010) and BFJST (2012). Computed in a variety of ways, BB (2010) find a coefficient of variation of innovations to firm-level productivity for their entire sample of German firms between two and three percent. However, because the Cooper and Haltiwanger (2006) analysis is of large manufacturing plants, the most comparable result in BB (2010) is their finding for the largest (ranked by employment) five percent of firms in their sample. For this sample, BB (2010) find a coefficient of variation of firm-level innovations of 5.5 percent (see their Table 8). The 17.7 percent coefficient of variation of plant-level innovations in the Cooper and Haltiwanger (2006) sample is thus substantially larger than the largest firms in BB (2010)'s sample. However, this degree of volatility of firm risk lines up much better with the evidence of BFJST (2012), who document using a variety of cross-sectional measures that dispersion of firm outcomes rises very sharply during recessions.

2.2 Average Productivity

For further consistency in the way the firm-level data are used as an input to the model, I also characterize the time-series behavior of $\omega_{mt}$, the average productivity (profitability) residual. In the model, this measure will correspond conceptually to the standard notion of aggregate productivity (i.e., the first moment of the productivity distribution). Figures 3 and 4 display the actual series, its HP trend, and the cyclical component of average productivity.\(^{11}\) The cyclical component of $\omega_{mt}$ is highly correlated with the cyclical component of GDP, as Figure 4 shows — the contemporaneous correlation between the two is 0.87. The volatility of the cyclical component of $\omega_{mt}$ is 1.26 percent (at an annual horizon). Again with an innocuous abuse of notation, I hereafter use $\omega_{mt}$ to denote the cyclical component of average productivity.

In the model presented below, I suppose that $\omega_{mt}$ follows the exogenous AR(1)

$$\ln \omega_{mt+1} = \rho_{\omega} \ln \omega_{mt} + \epsilon_{\omega_{mt+1}},$$

with $\epsilon_{\omega_{mt}} \sim N(0, \sigma_{\omega_{mt}})$. Estimation gives a point estimate $\rho_{\omega} = 0.48$, with a t-statistic of 1.84.\(^{12}\) With this estimate of $\rho_{\omega}$ and the standard deviation of $\omega_{mt}$ of 1.26 percent, the standard deviation

\(^{11}\)As noted above, long-run average productivity is normalized to unity in the model, so the vertical scale in Figure 3 is arbitrary. In the empirical analysis of Cooper and Haltiwanger (2006), mean productivity was not normalized.

\(^{12}\)This differs from Cooper and Haltiwanger’s (2006, p. 623) estimate of the persistence of mean productivity because they do not detrend; the AR(1) coefficient of the unfiltered $\omega_{mt}$ series is 0.76.
of the (annual) innovations to the average productivity process can be computed to be 0.0111. Finally, the cyclical correlation between average productivity and the dispersion of productivity (i.e., the concept of firm risk) is -0.97; this extremely strong negative correlation is part of the motivation of the “bundled-shock” model extension considered in Appendix B.

In the model developed in Section 4, I pursue annualizations of a quarterly calibration because the leverage evidence documented in Section 3 is annual. Because the evidence presented in this section is from annual data, I use persistence parameters of $\rho_{\sigma^\omega} = 0.48^{0.25} = 0.83$ and $\rho_{\omega^m} = 0.48^{0.25} = 0.83$, which assumes smoothness in the processes during the year. How this inference of quarterly persistence from annual estimates affects the model calibration of the innovation parameters $\sigma_{\sigma^\omega}$ and $\sigma_{\omega^m}$ is discussed in Section 5.2.

One final note is helpful: a concern may be the slight downward trend in “productivity” in the manufacturing sector during 1974-1988. Keep in mind, however, that (apart from the sharp recessions during this period) what is being measured is actually profitability residuals. If the relative prices of the inputs, capital and labor, trended during this period, this would show up as a trend in profitability. As the results above show, the final AR(1) stochastic process that describes average profitability/productivity is very similar to a simple RBC model’s average productivity process. So, if one prefers, one can think of the AR(1) process as an illustrative, off-the-shelf RBC-style process, the precise parameter settings for which are not crucial to the main conclusions of the paper.

3 Leverage Data

Table 1 displays aggregate leverage data for the manufacturing sector. The data through 1986 were obtained from Masulis (1988) and extended to include the years 1987 and 1988 so that it covers the time period used in estimating the risk shock process. As noted in the Introduction, CMR’s estimation is partly based on the Masulis (1988) data.

As also noted in the Introduction, the risk-shock calibration of Section 2 turns out to endogenously match quite well the time-series average of empirical leverage as provided in Masulis (1988) and extended to include the years 1987 and 1988 so that it covers the time period used in estimating the risk shock process. The average leverage ratio during 1973-1988 was 0.56, which is quite close to the 0.67 in the calibrated model of Section 5, even though leverage was not a calibration target.

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13I thank Larry Ball and Chris Carroll for raising these points.
4 Model

As described in the introduction, the model is based on the well-known agency-cost frameworks used by Carlstrom and Fuerst (1997, 1998).\textsuperscript{14} The model is most directly based on the “output model” of Carlstrom and Fuerst (1998), in which all prices are flexible, a homogenous final good is used for both consumption and investment purposes, firms require short-term working capital (formally, intraperiod) to finance their production costs, and there are no other rigidities or frictions whatsoever. This provides the cleanest model to evaluate the role of empirically-relevant shocks to firm risk, so I refer to the Carlstrom and Fuerst (1998) — henceforth, CF — output model as “the” underlying model, recognizing that it is meant to capture an entire literature of work. In a study with a very similar motivation, DLS study the role of risk shocks in the Carlstrom and Fuerst (1997) “investment model,” in which it is only capital-goods producers that are subject to financing constraints. Besides this difference in the applicability of agency frictions, DLS parameterize the risk process in an illustrative way, rather than calibrating it to micro data as I do.\textsuperscript{15}

As an aid to the ensuing description of the model, Figure 5 illustrates the timing of events in the model. Because the model is virtually identical to the CF output model, with only a couple of modifications made to align the model with the data analysis in Sections 2 and 3, readers familiar with the CF model may prefer to skip to the analysis beginning in Section 5.

4.1 Households

A representative household maximizes expected lifetime discounted utility over streams of consumption $c_t$ and labor $n_t$,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + v(n_t) \right], \quad (3)$$

subject to the sequence of flow budget constraints

$$c_t + k_{ht+1} = w_t n_t + k_{ht} [1 + r_t - \delta] + \Pi_t. \quad (4)$$

The functions $u(.)$ and $v(.)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and labor, respectively. The rest of the notation is as follows. The household’s subjective discount factor is $\beta \in (0, 1)$, $k_{ht}$ denotes the household’s capital holdings at the start of period $t$, $w_t$ is the real wage that is taken as given, $r_t$ is the market rental rate on capital that

\textsuperscript{14}Bernanke, Gertler, and Gilchrist (1999) recasts the Carlstrom and Fuerst (1997) framework to allow for nominal rigidities and monetary policy.

\textsuperscript{15}Before the reader reaches the quantitative analysis in Section 5, I note that I have also used the risk shock estimates described in Section 2 in the DLS investment model — that is, in the Carlstrom and Fuerst (1997) model — and obtain virtually the same results they find. For the sake of brevity, those results are not presented, but are available upon request.
is also taken as given, and $\delta$ is the per-period depreciation rate of capital. The capital good and consumption good are identical and thus have a unit relative price. The household also receives aggregate dividend payments $\Pi_t$ from firms as lump-sum income, the determination of which is described below.\footnote{I could also introduce shares in order to directly price streams of dividends paid by firms to households; but this extra detail is unnecessary for the main points, so it is omitted.}

Emerging from household optimization is a completely standard labor supply condition

$$\frac{v'(n_t)}{u'(c_t)} = w_t, \quad (5)$$

and a standard capital Euler condition

$$u'(c_t) = \beta E_t \{u'(c_{t+1}) [1 + r_{t+1} - \delta]\}, \quad (6)$$

which follow as usual from the household’s period-$t$ first-order conditions with respect to $c_t$, $n_t$, and $k_{ht+1}$. The one-period-ahead stochastic discount factor is defined as $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which firms, in equilibrium, discount profit flows.

### 4.2 Firms

There is a continuum of unit mass of firms, each of which produces output by operating a constant-returns technology. Firms are heterogeneous in their productivity. Firm $i$ produces output using the technology $\omega_i F(k_{it}, n_{it})$: $k_{it}$ is the firm’s purchase of physical capital on spot markets, $n_{it}$ is the firm’s hiring of labor on spot markets, and $\omega_i$ is a firm-specific productivity realization.

Each period, firm $i$’s idiosyncratic productivity is a draw from a distribution with cumulative distribution function $\Phi(\omega)$, which has a time-varying mean $\omega_{mt}$, a time-varying standard deviation $\sigma_{\omega t}$, and associated density function $\phi(\omega)$, all of which are identical across firms. Time-variation in $\omega_{mt}$ corresponds to the usual notion of aggregate productivity shocks, in the sense of exogenous variation in the mean of firms’ technology. The time-varying volatility $\sigma_{\omega t}^2$ is the key innovation in the model compared to CF. Given the first and second moments $\omega_{mt}$ and $\sigma_{\omega t}^2$ common across firms, idiosyncratic productivity for a given firm is i.i.d. over time, an assumption made for tractability.\footnote{The assumption of zero persistence of the idiosyncratic component of a firm’s productivity was noted in Section 2, and it greatly simplifies the computation of the model because the firm sector essentially can be analyzed as a representative agent. This point is discussed further below when I come to the aggregation of the model. This simplification still allows me to illustrate the main point of the model, which is that variations in cross-sectional productivity dispersion can lead to large fluctuations in aggregate leverage and possibly, in turn, to fluctuations in economic activity. In addition to greatly reducing the computational burden, the assumption of zero persistence in idiosyncratic shocks also retains the simplicity of the CF and Bernanke and Gertler (1989) contracting specifications. If persistent shocks were allowed, it is not clear that the simple debt contracts of these models could not be improved upon by the contracting parties by, say, multi-period contracts. Sidestepping this issue is yet another reason to...}
Firms are owned by households, and the objective of firms is to maximize the expected present discounted value of dividends remitted to households. Denote by \( \Pi_{it} \) the dividend payment made by firm \( i \) to households. For descriptive convenience, I decompose \( \Pi_{it} \) into a “non-retained earnings” component \( \Pi_{it}^e \) and an “expected operating profit” component \( E_\omega \Pi_{it}^f \); the notation \( E_\omega \) indicates an expectation conditional on the period-\( t \) aggregate state but before idiosyncratic realizations are revealed to any firm.\(^{18}\) Thus, \( \Pi_{it} \equiv \Pi_{it}^e + E_\omega \Pi_{it}^f \). As described below, the component \( E_\omega \Pi_{it}^f \) essentially corresponds to static profits as in a simple RBC model.

Because they are owned by households, firms apply the representative household’s stochastic discount factor (the one-period-ahead discount factor is \( \Xi_{t+1|t} \), as defined above) to their intertemporal optimization problem. However, firms are also assumed to be “more impatient” than households by the factor \( \gamma < 1 \), which can be thought of as a principal-agent problem that prevents perfect alignment of the firms’ managerial objectives with households’ intertemporal preferences. At a technical level, \( \gamma < 1 \) ensures that firms cannot accumulate enough assets to become self-financing, which would render irrelevant the financial frictions described below. This device for avoiding self-financing outcomes is common in models of financial frictions.

The intertemporal objective function of firm \( i \) is thus
\[
E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left[ \Pi_{it}^e + E_\omega \Pi_{it}^f \right].
\]
(7)

The firm problem is now further developed and analyzed.

### 4.2.1 Firm Financing and Contractual Arrangement

This subsection describes the financial arrangements of the model. To facilitate comparison of general equilibrium with firm-level optimization, the following intuitive description of state variables is helpful. From a general equilibrium perspective, financial outcomes are contingent on the exogenous aggregate state \( (\omega_{mt}, \sigma_\omega^m) \) of the economy. From firm \( i \)’s partial equilibrium perspective, financial outcomes also take as given net worth \( nw_{it} \) and the markup \( p_t \), each of which is determined in other markets (which in turn are contingent on the aggregate state \( (\omega_{mt}, \sigma_\omega^m) \)).

In period \( t \), total operating costs of firm \( i \), which are the sum of capital rental costs and wage payments, are
\[
M_{it} = w_t n_{it} + r_t k_{it}.
\]
(8)

\(^{18}\)As Figure 5 indicates, firm decisions are made in the first “subperiod” of period \( t \), before idiosyncratic shocks have been realized but after aggregate shocks have been realized, hence the need for \( E_\omega \).
As in CF and as shown in Figure 5, the firm is assumed to commit to all of its input costs after observing the aggregate exogenous state \((\omega_{mt}, \sigma_\omega_t)\), but before observing its idiosyncratic realization \(\omega_{it}\) and thus before any output or revenue are created.

Part of the financing of the firm’s costs comes from its own accumulated net worth, which is held primarily in the form of capital. The capital that each firm accumulates is rented on spot markets to (other) firms, just like households rent their capital on spot markets. Firm \(i\)’s capital holdings at the start of period \(t\) are \(k_{it}^e\). Thus, note that \(k_{it}^e\), which reflects the firm’s savings decisions, is distinct from \(k_{it}\), which reflects the firm’s capital demand decisions for production purposes.

However, the firm’s internal funds (which I refer to interchangeably as its net worth or its equity) are insufficient to cover all input costs. To finance the remainder, a firm borrows short-term — formally, intraperiod — working capital. A firm requires external financing because of the assumption that it is more impatient than households, as described above.\(^{19}\) By acquiring external funds, the firm is able to leverage its net worth in period \(t\),

\[
nw_{it} = k_{it}^e [1 + r_t - \delta] + e_t,
\]

into coverage of its operating costs \(M_{it}\). Total borrowing by the firm is thus \(M_{it} - nw_{it}\). The component \(e_t\) of net worth is a small amount of “endowment income” that each firm receives to ensure its continued operations in the event that it was unable to repay its debt and thus had to undergo costly reorganization in the previous period. In closing the model, this endowment is absorbed into the payout \(\Pi_{it}\) the firm pays to its owners, which is the representative household. The payout \(\Pi_{it}\) is thus interpreted as net of the endowment \(e_t\).\(^{20}\)

I describe only briefly the outcome of the contracting arrangement between borrowers (firms) and lenders (households) because it is standard in this class of models.\(^{21}\) The financial contract is a debt contract, which is fully characterized by a reorganization threshold \(\bar{\omega}_t\) and a loan size \(M_{it} - nw_{it}\). A firm must be “reorganized” if its realized productivity \(\omega_{it}\) is below the contractually specified threshold \(\bar{\omega}_t\). Below this endogenous threshold, firm \(i\) does not have enough resources to fully repay its loan. In that case, the firm is declared insolvent and receives nothing, while

\(^{19}\)As noted above, this is a standard assumption in this class of models and avoids the self-financing outcome. See, for example, Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999).

\(^{20}\)Thus, \(e_t\) can loosely be interpreted as a lump-sum transfer of “startup funds” provided by households to firms, as in Gertler and Karadi (2011). By allowing a “firm’s” operations to continue in the event of bankruptcy, the assumption of a startup fund brings great analytical tractability to the model. Thus, the “costs of bankruptcy” in the model are more properly interpreted as “costs of reorganization” without any disruption of its output-producing activities (i.e., bringing in new management to oversee ongoing operations).

\(^{21}\)In the context of general equilibrium settings, familiar expositions appear in Carlstrom and Fuerst (1997, 1998), Bernanke, Gertler, and Gilchrist (1999), and, in an application to labor search and matching, Chugh (2013). In partial-equilibrium settings, analysis of this type of contractual arrangement traces back to Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).
the lender must pay reorganization costs that are proportional to the total output of the firm and receives, net of these reorganization costs, all of the output of the firm. Note that all firms, regardless of whether or not they end up requiring reorganization, do produce output up to their full (idiosyncratic) capacity.

Define by \( f(\bar{\omega}_t) \) the expected share of idiosyncratic output \( \omega_{it} F(k_{it}, n_{it}) \) the borrower (the firm) keeps after repaying the loan, and by \( g(\bar{\omega}_t) \) the expected share received by the lender.\(^{22}\) These expectations are conditional on the realization of the time-\( t \) exogenous aggregate state \( (\omega_{mt}, \sigma^\omega_t) \), but before revelation of a firm’s idiosyncratic productivity \( \omega_{it} \). The contractually-specified loan size is characterized by a zero-profit condition on the part of lenders,

\[
M_{it} = \frac{nw_{it}}{1 - p_t g(\bar{\omega}_t)},
\]

and the contractually-specified liquidation threshold is characterized by

\[
p_t f(\bar{\omega}_t) \frac{1}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)},
\]

in which \( p_t > 1 \) is a “markup” on input costs that arises solely from the external financing needs of the firm.\(^{23}\) Thus, for each unit of capital the firm rents, the cost, inclusive of financing costs, is \( p_t r_t \), not just \( r_t \). The same holds for payment of labor.

The loan size \( M_{it} - nw_{it} \) is firm-specific. However, the reorganization threshold \( \bar{\omega}_t \) is not because idiosyncratic productivity has zero persistence. Condition (11) thus implies \( p_t \) is also identical across firms, which is the key result that makes aggregation in the model simple, which in turn justifies omission of firm-\( i \) indexes for the variables \( p \) and \( \bar{\omega} \). The interpretation of \( p_t \) is an endogenous “markup” that drives a wedge between factor prices and marginal products. Finally, the contract multiplier \( \lambda_t \) that is associated with conditions (10) and (11) is

\[
\lambda_t = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)}.
\]

An interesting point regarding \( \lambda_t \) is quantified in Section 5.

\(^{22}\)Formally, \( f(\bar{\omega}_t) \equiv \int_0^\infty (\omega_i - \bar{\omega}_t) \phi(\omega_i) d\omega_i = \int_0^\infty \omega_i \phi(\omega_i) d\omega_i - [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t \) is the share received by the firm, and \( g(\bar{\omega}_t) \equiv \int_0^{\omega_i} (\omega_i - \mu) \phi(\omega_i) d\omega_i + \int_{\bar{\omega}_t}^\infty \bar{\omega}_t \phi(\omega_i) d\omega_i = \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i) d\omega_i + [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t - \mu [1 - \Phi(\bar{\omega}_t)] \) is the share received by the lender.

\(^{23}\)The background assumptions of the zero profit condition are that lending is a perfectly competitive activity and entry into lending is costless. Formally, the two conditions characterizing the optimal contract result from maximizing (the firm’s share of) the return on the financial contract (because the firm, if it remains solvent, is the residual claimant on output), \( p_t f(\bar{\omega}_t) M_{it} \), subject to the zero profit condition of the lender, \( p_t g(\bar{\omega}_t) M_{it} = M_{it} - nw_{it} \). Define \( \lambda_t \) as the shadow value on the zero-profit constraint.
4.2.2 Operating Profits and Asset Evolution

Firms take as given contractual outcomes when maximizing profits. The expected operating profit of firm $i$ in period $t$ is

$$E_\omega \Pi_{it}^f = \omega_{mt} F(k_{it}, n_{it}) - p_t [w_t n_{it} + r_t k_{it}].$$

(13)

As discussed above, this is an expected profit because it is measured before the realization of firm-specific idiosyncratic productivity but after the realization of the aggregate period-$t$ state of the economy, $(\omega_{mt}, \sigma^\omega_t)$. Because the mean of $\omega_{it}$ is $\omega_{mt}$, ex-ante revenue of the firm is $\omega_{mt} F(k_{it}, n_{it})$. The idiosyncratic risk $\omega_{it}$ and associated financing costs implied by it are captured by the inclusion of the external finance premium $p_t$ in the above expression.24 Firms take as given the competitively-determined factor prices $w_t$ and $r_t$.

Regarding the dynamic aspect of firms, firm $i$ begins period $t$ with assets $k_e^{it}$, whose beginning-of-period-$t$ market value determines the firm’s net worth $nw_{it}$, as shown in (9). The firm borrows $M_{it} - nw_{it}$ against the value of these assets, and it expects to keep $p_t f(\bar{\omega}_t) M_{it}$ after repaying its loan.25 Of these “excess” resources, the firm can either accumulate assets or make payments to households. That is,

$$\Pi_{it}^e + k_{it+1}^e = p_t f(\bar{\omega}_t) M_{it},$$

(14)

which highlights that $k_{it+1}^e$ can be thought of as retained earnings. Substituting the contractually-specified quantity of borrowing, $M = \frac{nw}{1 - p_t g(\bar{\omega})}$, this can be re-written as

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} nw_{it}. $$

(15)

Further substituting the definition of net worth from (9), the firm’s asset evolution is described by

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_{it}^e [1 + r_t - \delta] + e_t).$$

(16)

Finally substituting (13) and (16) into (7), the dynamic profit function of the firm is

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left\{ \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} (k_{it}^e [1 + r_t - \delta] + e_t) - k_{it+1}^e + \omega_{mt} F(k_{it}, n_{it}) - p_t [w_t n_{it} + r_t k_{it}] \right\}. $$

(17)

---

24 As is common in macro models, writing, for example, $p_t$, is shorthand for the state-contingent equilibrium function $p(\omega_{mt}, \sigma^\omega_t)$. If the distribution of $\omega$ were degenerate — that is, if there were no idiosyncratic component of technology — then we would have $p_t = 1 \forall t$, which simply has the interpretation that financing issues are irrelevant as in, say, a baseline RBC model in which $E_\omega$ is vacuous.

25 This is because, as noted in footnote 28, the firm keeps the entire (expected) surplus from the contractual arrangement. Hence, in expectation, the firm is left with $p_t f(\omega_t) M_{it}$ after the sequence of borrowing, renting factors of production, producing output, and repaying its loan.
4.2.3 Profit Maximization

Maximization of (17) with respect to capital rental $k_{it}$ and labor hiring $n_{it}$ gives rise to the capital demand condition

$$r_t = \frac{\omega_m F_k(k_{it}, n_{it})}{p_t}$$

and the labor demand condition

$$w_t = \frac{\omega_m F_n(k_{it}, n_{it})}{p_t}.$$  

In (18) and (19), the effective payments per unit of each factor are $p_t r_t$ for capital rental and $p_t w_t$ for labor, reflecting firms’ need for external financing. Financing costs drive an endogenous time-varying wedge between prices and marginal returns in factor markets, which, as noted above, leads to the interpretation of $p_t$ as an “endogenous markup.” Note that, although firms will differ in their levels of factor usage, each firm chooses an identical capital-labor ratio because the market prices $r_t$ and $w_t$ and the external premium $p_t$ are identical for all firms and the production technology $F(.)$ is constant-returns.

Maximization of (17) with respect to asset accumulation $k_{it+1}$ yields the capital Euler equation for firms,

$$1 = \gamma E_t \left\{ \Xi_{t+1} p_{t+1} f(\bar{\omega}_{t+1}) \left[ 1 + r_{t+1} - \delta \right] \right\},$$

which, note, is independent of firm-$i$ conditions.

4.2.4 Aggregation

Firms are heterogenous with respect to their net worth and differ (only) in size — a firm with a larger net worth receives a proportionately larger loan and so produces more output. However, the size distribution of firms is irrelevant for computing prices and hence aggregates in the economy, which makes the agency-cost framework tractable in a DSGE setting. The production side of the economy can thus be analyzed as if there were a representative firm that held the average quantity of net worth and hired the average quantity of labor and capital for production. The specific assumptions and results behind this aggregation result are: the constant-returns nature of the production function $F(.)$; the linearity of the monitoring technology (in the quantity monitored); and, crucially, the result that the prices $w_t$, $r_t$, and $p_t$ are identical for all firms.\(^{26}\)

The stand-in representative firm has a profit function identical to (17) (with firm indices dropped), which clearly gives rise to the same optimality conditions (18), (19), and (20).

\(^{26}\)The result that $p$ is identical for all firms is an implication of zero persistence of firms’ idiosyncratic productivity, which, as described above, makes it impossible to condition the contractually-specified liquidation threshold $\bar{\omega}$ on firm-specific variables. See also CF (1997, 1998) for further discussion. The result that $w$ and $r$ are identical for all firms follows simply from the assumption of perfectly-competitive rental markets for labor input and capital input.
(aggregate) profits that get transferred to households are thus

\[
\Pi_t = \Pi_t^e + \Pi_t^f = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left( k_t^e [1 + r_t - \delta] + e_t \right) - \Pi_{t+1}^e + k_{t+1}^e + \omega_{mt} F(k_t, n_t) - p_t \left[ w_t n_t + r_t k_t \right] \\
= \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left( k_t^e [1 + r_t - \delta] + e_t \right) - k_{t+1}^e + \omega_{mt} F(k_t, n_t) - \omega_{mt} F_n(k_t, n_t) n_t - \omega_{mt} F_k(k_t, n_t) k_t \\
= \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left( k_t^e [1 + r_t - \delta] + e_t \right) - k_{t+1}^e.
\] (21)

The second line makes use of the factor price conditions (18) and (19), and the third line follows because \(F(.)\) is constant-returns. Thus, note that in this representative-firm foundation of aggregates, firms earn zero aggregate operating profits, so \(\Pi_t = \Pi_t^e\). The capital Euler equation that arises from maximizing this representative-firm profit function with respect to aggregate entrepreneurial capital holdings \(k_{t+1}^e\) is clearly identical to (20).

Finally, the aggregate resource constraint of the economy is

\[
c_t + k_{t+1} - (1 - \delta)k_t = \omega_{mt} F(k_t, n_t) \left[ 1 - \mu \Phi(\omega_t) \right],
\] (22)
in which \(k_t = k_{ht} + k_t^e\) is the equilibrium quantity of physical capital at the beginning of period \(t\). Note that aggregate monitoring costs are a final use of output.

### 4.3 Private Sector Equilibrium

A symmetric private-sector equilibrium is made up of state-contingent endogenous processes \(\{c_t, n_t, k_{ht+1}, k_{t+1}^e, k_{t+1}, \Pi_t^e, w_t, r_t, p_t, \bar{\omega}_t\}\) that satisfy the following conditions: the labor-supply condition

\[
- \frac{v'(n_t)}{u'(c_t)} = w_t;
\] (23)

the labor-demand condition

\[
w_t = \frac{\omega_{mt} F_n(k_t, n_t)}{p_t};
\] (24)

the capital-demand condition

\[
r_t = \frac{\omega_{mt} F_k(k_t, n_t)}{p_t};
\] (25)

the representative household’s Euler equation for capital holdings

\[
1 = E_t \left\{ \Xi_{t+1|t} [1 + r_{t+1} - \delta] \right\};
\] (26)

the (representative) firm’s Euler equation for capital holdings

\[
1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} [1 + r_{t+1} - \delta] \right\};
\] (27)

aggregate capital market clearing

\[
k_t = k_{ht} + k_t^e;
\] (28)

15
the aggregate resource constraint

$$c_t + k_{t+1} - (1 - \delta) k_t = \omega_{mt} F(k_t, n_t) [1 - \mu \Phi(\tilde{\omega}_t)];$$

(29)

the contractually-specified loan size

$$M_t = \frac{nw_t}{1 - p_t g(\tilde{\omega}_t)},$$

(30)

in which expression (9) for $nw_t$ is substituted in; the contractually-specified liquidation threshold

$$\frac{p_t f(\tilde{\omega}_t)}{1 - p_t g(\tilde{\omega}_t)} = -\frac{f'(\tilde{\omega}_t)}{g'(\tilde{\omega}_t)};$$

(31)

and the evolution of the aggregate assets of firms (equivalently, the assets of the representative firm)

$$\Pi_t^e + k_{t+1}^e = \frac{p_t f(\tilde{\omega}_t)}{1 - p_t g(\tilde{\omega}_t)} (k_t^e [1 + r_t - \delta] + e_t).$$

(32)

The private sector takes as given the stochastic process for $\{\omega_{mt}, \sigma^2_t\}_{t=0}^\infty$. To emphasize, and as noted above, conditions (30) and (31) characterize the partial-equilibrium financial outcomes, and hence can be viewed (in partial equilibrium) as taking $p_t$ and net worth as given.

5 Quantitative Analysis

5.1 Computational Strategy

Changes in cross-sectional risk are aggregate, not idiosyncratic, shocks in the model economy. Because I track only aggregate outcomes and do not track any firm-specific outcomes, there is no reason to think that local approximation methods will misrepresent the model’s aggregate dynamics. To study the dynamics of the model, I thus compute a first-order approximation of the equilibrium. Because the main interest is in business cycle fluctuations, such methods are likely to accurately portray the model’s dynamic behavior, as the studies by Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) and Caldera, Fernandez-Villaverde, Rubio-Ramirez, and Yao (2009) suggest. This also reinforces the point made by DLS (2008, p. 386) that linearization does not impose certainty equivalence on this type of second-moment (a cross-sectional variance) shock. The quantitative results reported below are thus fundamentally driven by the model’s mechanism — changes in cross-sectional risk, which then potentially are transmitted to the real economy — rather than choice of approximation method.

Before presenting the dynamic results, I complete the description of the calibration of the model (which was begun in Section 2) and briefly describe some of its long-run predictions.

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27 Recall the discussion above that, given the maintained assumptions of the model, aggregates in the model do not depend on distributions of outcomes at the firm level.

28 The numerical algorithm is my own implementation of the perturbation method described by Schmitt-Grohe and Uribe (2004).
5.2 Calibration

The novel aspect of the model calibration is the risk shock process using micro data, as described in Section 2. As described there, long-run dispersion of firm productivity is \( \bar{\sigma}_\omega = 0.156 \). This is about half the value used by CF (1998, p. 590) and Bernanke, Gertler, and Gilchrist (1999, p. 1368), which are calibrated to aggregate financial data, not firm-level data: the former set \( \bar{\sigma}_\omega = 0.37 \), and the latter set \( \bar{\sigma}_\omega = 0.28 \). CMR’s estimate is \( \bar{\sigma}_\omega = 0.26 \). Thus, direct micro evidence indicates less cross-sectional dispersion than standard macro calibrations of agency-cost models.

As also discussed in Section 2, I assume sufficient smoothness in the average productivity and risk processes so that I can set quarterly persistence parameters \( \rho_{\omega_m} = 0.83 \) and \( \rho_{\sigma_\omega} = 0.83 \), even though the data on which the estimation is based are annual. This mismatch between (desired) model frequency and empirical frequency raises the question of the appropriate calibration of the standard errors of the quarterly innovations in the productivity and risk processes.\(^{29}\)

Given the quarterly frequency of the model and the annual frequency of the productivity data, I simply time aggregate the simulated data from the model, and set parameters \( \sigma_{\omega_m} \) and \( \sigma_{\sigma_\omega} \) so that the annualized volatilities of average productivity and dispersion of productivity in the model match their annual empirical counterparts. As documented in Section 2, the empirical volatilities are, respectively, 1.26 percent and 3.15 percent. This calibration procedure leads to \( \sigma_{\omega_m} = 0.008 \) and \( \sigma_{\sigma_\omega} = 0.0033 \).\(^{30}\) For the sake of comparison with DLS and CMR, their respective pairs of parameters are: \( \sigma_{\omega_m} = 0.007 \) and \( \sigma_{\sigma_\omega} = 0.007 \) (see DLS Table 4) and \( \sigma_{\omega_m} = 0.0046 \) and \( \sigma_{\sigma_\omega} = 0.07 \) (see CMR Table 2).

Besides the calibration of the exogenous processes, Table 3 lists all functional forms used in the quantitative experiments, and Table 4 lists all baseline parameter settings. The preference and production parameters are standard in business cycle models. The agency cost parameter is set to \( \mu = 0.15 \), which is the same as the calibrated value in Covas and den Haan (2011) and in line with the estimate \( \mu = 0.12 \) by Levin, Natalucci, and Zakrajsek (2004). The value for firms’ “additional” discount factor is set to \( \gamma = 0.98 \), which allows the model to match a long-run annualized external finance premium of two percent. This value of \( \gamma \) is quite close to CMR’s calibration \( \gamma = 0.985 \), and is a bit larger than the calibrations of CF and BGG.

\(^{29}\)Recall from Section 2 that the point estimates for annual persistence are \( \rho_{\omega_m} = 0.47 \) and \( \rho_{\sigma_\omega} = 0.48 \), and the standard deviation of the annual innovations in the average productivity and risk processes are, respectively, 0.0111 and 0.0276.

\(^{30}\)It is interesting to note that \( \sigma_{\omega_m} = 0.008 \) is quite similar to the calibration of the size of quarterly innovations in the aggregate productivity process in a baseline RBC model, in which a benchmark value is 0.007. Here, of course, \( \sigma_{\omega_m} = 0.008 \) is computed directly from micro data.
5.3 Long-Run Dispersion and Long-Run Equilibrium

The long-run deterministic (steady-state) equilibrium is computed numerically using a standard nonlinear equation solver. The main comparative static exercise is presented in Figure 7, which plots long-run equilibria as a function of long-run cross-sectional dispersion $\bar{\sigma}$. All other parameters are held fixed at those presented in Table 4.

Figure 7 shows that the long-run response of the economy to changes in $\bar{\sigma}$ is non-monotonic. For low dispersion of idiosyncratic productivity, GDP falls as dispersion rises, but for high dispersion, the comparative static result reverses. Other standard aggregate quantities such as gross investment and consumption also display this nonmonotonicity (for brevity, these are not shown in Figure 7). This effect is not due to any nonmonotonicity of the contract terms, as leverage (upper right panel) is strictly decreasing in $\bar{\sigma}$, the bankruptcy/reorganization rate (middle left panel) is strictly increasing in $\bar{\sigma}$, and the external finance premium is strictly increasing in $\bar{\sigma}$.

However, despite the nonmonotonicity of economic outcomes, they are relatively insensitive over a large region of long-run dispersion. The insensitivity seems to be due to the zero steady-state elasticity of the endogenous contract multiplier $\lambda$ with respect to $\bar{\sigma}$, which is quantitatively shown in the bottom panel of Figure 7. To the best of my knowledge, the steady-state invariance of the contract multiplier with respect to the dispersion of productivity has not previously been known in the CF (1997, 1998) models.

For the baseline calibration, the model’s long-run leverage ratio is 0.67, which is remarkably close to the average leverage ratio of 0.56 during the period 1973-1988, as Section 3 showed. The calibration of the model was not designed to match average leverage. Furthermore, given the Masulis (1988) definition of leverage in the first line, the subsequent lines characterize leverage in model notation

\[
\ell(\bar{\sigma}; \sigma) \equiv \frac{\text{debt}}{\text{assets}} = \frac{M - nw}{M - nw + nw} = 1 - \frac{nw}{M}.
\]

As long-run dispersion $\bar{\sigma}$ shrinks to zero, lenders face no risk whatsoever on their loans, which in turn implies entrepreneurs have to accumulate zero net worth. Leverage thus approaches its upper limit of unity as $\bar{\sigma} \rightarrow 0$.\footnote{The extreme case of $\bar{\sigma} = 0$ is simply the textbook RBC model.}

It is useful to also highlight the long-run values implied by the model of two other financial variables of interest: the (annualized) finance premium and the bankruptcy/reorganization rate.

\footnote{In results available upon request, the nonmonotonicity, yet relative insensitivity, of steady-state economic outcomes are also present in the CF 1998 investment model.}
These are collected in Table 5. The long-run bankruptcy rate is substantially lower than in the Dun & Bradstreet evidence cited by CF (1998, p. 590), while the finance premium is in line with most of the measures of premia presented in DeGraeve (2008).33

5.4 Business Cycle Dynamics

Now I turn to the model’s cyclical fluctuations.

5.4.1 Risk Shocks

The first set of experiments conducted is dynamics driven by risk shocks alone. Figure 8 presents impulse responses to a one-time, one-standard deviation positive shock to the cross-sectional dispersion of firm productivity, holding constant average productivity. Firm-level productivity thus has larger idiosyncratic risk. In a departure from the typical presentation of impulse responses, Figure 8 presents levels of variables. Complementing this impulse-response analysis are the simulated business cycle statistics reported in Table 6. There are two main results from these experiments from a purely risk-shock driven economy.

As Figure 8 highlights, a pure risk shock induces virtually zero GDP response: the peak response of GDP is about 1/125th of one percent! The same is true for aggregate consumption, investment, and labor, as shown in the second, third, and fourth panels, respectively. Thus, empirically-relevant micro risk shocks play virtually no role as an independent driver of aggregate macro fluctuations when mediated through a typical agency-cost friction.34 The small pass-through of risk shocks to macro fluctuations arises despite the fact that the risk innovations documented in Section 2 are larger than found in other micro-level evidence. This negative result is also in line with the findings of DLS.

However, Figure 8 also shows that financial variables do react more noticeably to the micro-disciplined risk shock: leverage declines modestly, the annualized credit premium rises from 2% to

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33 As discussed extensively by DeGraeve (2008), it is not clear what is the most relevant empirical counterpart to the model’s external finance premium. Many natural alternatives suggest themselves, such as the difference between the prime borrowing rate and the short-term T-bill rate, the interest spread between AAA-rated commercial paper and T-bills, the spread between BB (2010)B-commercial paper and T-bills, and so on. DeGraeve (2008) documents that these various empirical measures of “the external finance premium” behave differently enough over the business cycle that it remains an open question what the natural empirical counterpart of the model’s external finance premium is.

34 This is also one of the main messages of the theoretical model of BB (2010), even though their model does not situate financial frictions as part of the transmission channel for risk shocks. Examining just the role of financial frictions in the transmission channel leads to a broadly similar conclusion as BB (2010). The result here is even starker than in BB (2010), though, because I found innovations in firm risk to be five to ten times larger than found by BB (2010), as discussed in Section 2. However, as described next, aggregate leverage fluctuations induced by risk shocks in the model are small, but empirically meaningful, hence the overall conclusions are not as pessimistic.
2.2%, and the quarterly bankruptcy rate rises moderately from 1.6% to 1.67%, although quickly recede back to steady state. Viewed through the agency-cost lens, micro-estimated risk fluctuations matter more for the magnitude of financial fluctuations than aggregate macro fluctuations. Table 6 conveys the same idea by measuring standard business-cycle comovements. For example, the business-cycle volatility of GDP conditional on risk shocks is 0.0121%, which mirrors the top left panel of Figure 8.

5.4.2 Both Risk Shocks and Average Productivity Shocks

Next, consider the model economy’s dynamics when it is hit by independent shocks to both average productivity and cross-firm dispersion. Important to keep in mind here is that both of these shock processes, as described in Section 2, were measured using micro data. The remainder of the calibration was based on standard parameters in the macro literature. Whether or not these processes together with the parsimonious set of “standard macro parameters” portray well empirically-relevant aggregate business cycles is an open question.

Table 7 shows that it portrays fluctuations well, especially given the small-scale nature of the model. Referring back to the empirics in Table 2, the absolute standard deviations (the first row of Table 7) of GDP, investment, hours, and leverage broadly resemble the data. The volatility of consumption in Table 7 is smaller than in the data, but keep the recessionary nature of part of the 1973-1988 period. Finally, comparison of Table 7 with the results in Table 8, in which fluctuations are driven by only average productivity shock, shows that leverage fluctuations broadly resemble their empirical counterpart.

5.4.3 Micro vs. Macro Calibration of Risk Shocks

Taken together, the results seem to portray a negative view of the importance of risk shocks for macro fluctuations. However, the magnitude of macro fluctuations seems highly dependent on the volatility of risk shocks. As a point of comparison, consider the CMR medium-scale macro-estimated accelerator model. A main message of CMR is that risk shocks are important drivers of GDP fluctuations. Fully comparing and contrasting CMR’s results to the ones of this paper is challenging due to the richness of their model vs. the small-scale model considered here. However, most of the difference arises from the order-of-magnitude larger risk volatility CMR estimated, $\sigma_{\omega} = 0.07$, compared to what I estimate, $\sigma_{\omega} = 0.0033$. Table 9 shows that inserting $\sigma_{\omega} = 0.07$ into the small-scale model considered in this paper leads to much larger real-side fluctuations. The contrasting results raise the issue of a micro-calibration approach or a macro-calibration approach for use in this class of models. Thus, there is no disagreement or tension between the results, but rather an indication that continued research is required.
6 Conclusion

This paper measured the business-cycle properties of firm risk based on micro-level data. Micro-disciplined firm risk is fairly volatile over the cycle and highly countercyclical. Using a baseline quantitative financial accelerator model which is only two parameters removed from the frictionless RBC model, the main theoretical question was to assess the extent to which the former can explain the latter. Empirically-relevant risk shocks turn out to explain well steady-state empirical leverage, a financial measure commonly thought to be important for macro fluctuations, such as GDP movements. However, in the model, the leverage fluctuations that risk shocks induce lead to only tiny fluctuations of real activity — GDP volatility conditional on risk shocks alone is less than one percent of GDP volatility conditional on shocks to average productivity alone.

A broad idea that emerges is that understanding changes directly in the distribution of micro-level risk may be important for guiding the further development of business cycle models in which financial frictions are prominent. This paper and other recent works have exploited second-moment disturbances. Perhaps fluctuations in third- or higher-order moments may also need to be considered for understanding some aspects of the transmission channels of risk. This requires moving away from the symmetry of normally-distributed (log) productivity standard in macro models. Given the robust evidence that firm-level outcomes are distributed non-Gaussian, there seems reason to think that skewness and higher moments of the firm productivity distribution may be time-varying. Such “higher-moment shocks” would also be expected to affect leverage and so possibly real activity; the quantitative degree to which they do may be an interesting question. A tractable starting point might be Pareto-distributed productivity.
References


A Leverage: Empirics and Theory

As noted in the introduction, the risk-shock calibration of Section 2 turns out to endogenously match quite well the time-series average of empirical leverage as provided in Masulis (1988) and extended to include the years 1987 and 1988 so that it covers the time period used in estimating the risk shock process. Table 1 displays the leverage data, which is for the manufacturing sector. The average leverage ratio during 1973-1988 was 0.56, which is quite close to the 0.67 in the calibrated model of Section 5, even though leverage was not a calibration target.

Formally, given the nature of the CF model, in which both the ex-ante signing of contracts and ex-post resolutions of contracts occurs intraperiod, we only need to zoom in on the equilibrium contractual conditions (30) and (31). To emphasize that the contractual terms depend on $\sigma^\omega$, for this section only I write $f(\bar{\omega}, \sigma^\omega)$ and $g(\bar{\omega}, \sigma^\omega)$, omitting time subscripts because the contract is intraperiod.

Let $\ell(\bar{\omega}; \sigma^\omega)$ denote leverage. Defining leverage as in Masulis (1988) in the first line, the subsequent lines characterize leverage in model notation

$$
\ell(\bar{\omega}; \sigma^\omega) \equiv \frac{\text{debt}}{\text{assets}} = \frac{M - nw}{M - nw + nw} = 1 - \frac{nw}{M} = 1 - (1 - pg(\bar{\omega}; \sigma^\omega)) = pg(\bar{\omega}; \sigma^\omega),
$$

(33)

in which the next-to-last line uses the contractual version of the (binding) zero-profit condition (30) of the lender.

Inserting this expression for $\ell(\bar{\omega}; \sigma^\omega)$ into condition (31), which characterizes the terms of the financial contract, allows for yet another way of expressing leverage,

$$
\ell(\bar{\omega}; \sigma^\omega) = 1 + \frac{pf(\bar{\omega}; \sigma^\omega)g(\bar{\omega}; \sigma^\omega)}{f(\bar{\omega}; \sigma^\omega)g(\bar{\omega}; \sigma^\omega)}. 
$$

(34)

The expected share functions $f(.)$ and $g(.)$ and their derivatives depend on the cross-sectional dispersion $\sigma^\omega$ of firm productivity, hence the leverage ratio also depends on $\sigma^\omega$.

The contract maximization problem was briefly mentioned in Section 4, but not fully fleshed out. Formally, the lender and borrower $i$ maximize

$$
pf(\bar{\omega}, \sigma^\omega) M_i
$$

subject to

$$
pg(\bar{\omega}, \sigma^\omega) M_i \geq M_i - nw_i
$$

(35)
with respect to $M_i$ and $\bar{\omega}$. Letting $\lambda$ denote the multiplier on the lending constraint, the first-order conditions with respect to $M_i$ and $\bar{\omega}$ are

$$ pf (\bar{\omega}, \sigma^{\omega}) + \lambda [pg (\bar{\omega}, \sigma^{\omega}) - 1] = 0 \quad (37) $$

and

$$ pf (\bar{\omega}, \sigma^{\omega}) M_i + \lambda pg (\bar{\omega}, \sigma^{\omega}) M_i = 0. \quad (38) $$

The contract multiplier can thus be stated as either

$$ \lambda = \frac{f_{\omega} (\bar{\omega}, \sigma^{\omega})}{g_{\omega} (\bar{\omega}, \sigma^{\omega})} \quad (39) $$

or

$$ \lambda = \frac{pf (\bar{\omega}, \sigma^{\omega})}{1 - pg (\bar{\omega}, \sigma^{\omega})}. \quad (40) $$

Equalizing these two expressions for $\lambda$ gives exactly the equilibrium condition (31). (Important to emphasize is that conditions (30) and (31) are purely static (within-period) conditions. Thus, the financial contract is of one-period duration.)

Figure 6 sketches why changes in the cross-sectional dispersion of firms’ TFP would be expected to cause changes in leverage. Suppose the solid black curve in Figure 6 is the pdf $\phi(\omega)$ before a risk shock occurs. The liquidation threshold $\bar{\omega}$ shown is for this initial distribution. Suppose there is an exogenous reduction in dispersion. If the liquidation threshold $\bar{\omega}$ were to remain unchanged, fewer firms would draw an idiosyncratic $\omega < \bar{\omega}$, which lenders understand because the density $\phi(\omega)$ is common knowledge. This in turn means that fewer firms are expected to be unable to repay their loans, which reduces lenders’ risk. Conditional on a value for $\bar{\omega}$, lenders would be willing to extend more credit, which implies higher leverage ratios for firms (borrowers). In equilibrium, $\bar{\omega}$ will of course also change, which can only be determined quantitatively given the log-linear distribution of idiosyncratic productivity used in Section 5.
B Bundled Shocks: Productivity-Induced Risk Fluctuations

Countercyclicality of firm risk can be modeled by linking time-variation in average TFP directly to fluctuations in firm-level risk. Specifically, the cross-sectional dispersion of productivity across firms is now assumed to decline when average TFP improves. First-moment shocks are thus assumed to be bundled with second-moment shocks, and I refer to the entire bundle as an “aggregate shock.” The two processes are assumed to be linked according to

\[ \sigma_t^\omega = \bar{\sigma}^\omega + \varphi \ln \omega_{mt}. \]  

(41)

This condition replaces the exogenous law of motion (1) for \( \sigma_t^\omega \), and the evolution of \( \omega_{mt} \) is still described by (2). The rest of the model is exactly the same as above. The parameter \( \varphi \) is clearly the key parameter of this version of the model, with \( \varphi < 0 \) implying countercyclicality of firm-level risk.\(^{35}\) In terms of correlation between average TFP and dispersion of TFP, \( \varphi < 0 \) obviously implies a perfect negative correlation between the two, but this portrayal is not counterfactually stark compared to the data; recall from Section 2 that the contemporaneous cyclical correlation between average TFP and dispersion of TFP is -0.98.

Figure 10 illustrates why \( \varphi < 0 \) leads to countercyclical firm risk. A positive shift in average TFP will, all else equal, increase GDP. If at the same time cross-sectional dispersion declines due to \( \varphi < 0 \), and supposing initially that the bankruptcy threshold \( \bar{\omega} \) were fixed, fewer firms would be expected to go bankrupt. This in turn would induce lenders to extend more credit, hence leverage rises for given net worth. Indeed, the second part of the intuitive argument is exactly the same as that underlying Figure 6. What is different from the baseline model is the event that now induces the change in dispersion. In the baseline model, the change in dispersion itself was the exogenous event, whereas here it is a positive shock to average TFP.

This bundled aggregate shock is of course a reduced-form construct. However, I bring the same empirical evidence presented in Section 5.2 to bear on the calibration of the crucial elasticity parameter \( \varphi \). The calibration approach is to choose \( \varphi \) so that the model matches the observed time-series variation in cross-sectional dispersion. Section 5.2 documented that the time-series volatility in annual cross-sectional dispersion is 3.15 percent. Given this target and holding fixed all parameters in Table 4, this calibration procedure (with average TFP fluctuations now as the sole truly exogenous driving process) leads to \( \varphi = -1.43 \).

Figure 11 presents impulse responses to a positive bundled aggregate shock. The most salient comparison for these impulse responses are those presented in Figure 9, in which the same size first-moment shock is also the exogenous impulse except with no change in cross-firm dispersion.

\(^{35}\)Clearly, \( \varphi > 0 \) would deliver procyclical firm-level risk, and \( \varphi = 0 \) would recover the baseline CF model in which there are never any changes in firm risk.
Comparing Figure 11 with Figure 9 shows that the bundled aggregate shock induces very similar dynamics in most variables as does the unbundled first-moment shock alone. The only difference compared to Figure 9 is that equity rises by much less in response to the bundled shock.

Finally, Table 10 presents simulation-based business cycle statistics. The first row shows that the volatility of leverage (and debt) carries over from the baseline model’s results presented in Table 7.

To summarize, the bundled-shock model by construction is consistent with the empirically-observed countercyclicality of cross-sectional firm risk (see the last two rows of the lower panel of Table 10), and it retains the volatility predictions of the baseline model driven by independent first-moment and second-moment shocks. However, it fails to reproduce the countercyclicality that leverage exhibits in the data. On the other hand, the baseline model driven by a complete set of independent, “unbundled,” shocks performed well on the volatility dimension, but failed to capture the countercyclicality of firm-level risk. Although I do not take up this extension here, a conjecture is that a combination of bundled shocks along with independent, exogenous, shocks to firm risk may help in capturing all these dimensions of the data.\footnote{Of course, there are a host of other model features and/or shocks one could consider introducing to the model. Such analysis is left to future work.}
Figure 1: Cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Data are annual. Trend component constructed with HP filter (smoothing parameter 100). Based on profitability series from Cooper and Haltiwanger (2006).

Figure 2: Cyclical component of cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Vertical axis is percentage deviation from HP trend. Computed from profitability residuals constructed by Cooper and Haltiwanger (2006).
Figure 3: Mean level of firm-level profitability residuals over the period 1974-1988. Data are annual. Trend component constructed with HP filter (smoothing parameter 100). Based on profitability series from Cooper and Haltiwanger (2006).

Figure 4: Cyclical component of mean of firm-level profitability residuals over the period 1974-1988. Vertical axis is percentage deviation from HP trend. Computed from profitability residuals constructed by Cooper and Haltiwanger (2006).
Each firm commits to its capital rental and wage payments, and borrows funds against its net worth.

Factors of production (k and n) are rented in spot markets.

Household makes aggregate consumption and investment choices.

Net profit payouts to households.

Lenders incur costs to reorganize insolvent firms and seize all their output.

Solvent firms repay their entire debt.

Production occurs by all firms.

Firm-specific productivity realized.

Period t

Mean TFP and cross-sectional dispersion realized.

Period t-1

Period t+1

Figure 5: Timing of events in model.
Figure 6: An exogenous decrease in the dispersion of productivity across firms. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.
Figure 7: **Long-run equilibrium.** Steady-state equilibrium as long-run standard deviation of idiosyncratic productivity distribution, $\sigma^\omega$, varies; $\sigma^\omega$ plotted on horizontal axis. All other model parameters are held fixed as described in Section 5.
Figure 8: Impulse response to risk shock. Impact of a one-standard-deviation exogenous increase in the dispersion $\sigma^2$ of firm productivity, holding constant average productivity. Vertical scale measures actual values of variables. Shock occurs in period 5.
Figure 9: Impulse response to average productivity shock. Impact of a one-standard-deviation exogenous increase in average productivity, holding constant dispersion $\sigma^\omega$. Vertical scale measures actual values of variables. Shock occurs in period 5.
Figure 10: A positive shock to the mean of aggregate TFP causes a decrease in the dispersion of productivity across firms. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.
Figure 11: **Impulse response to a positive “bundled aggregate shock.”** Impact in which a one-standard-deviation exogenous increase in average TFP induces a decrease in cross-sectional dispersion. Except where noted, scale is percentage point deviation from steady state.
<table>
<thead>
<tr>
<th>Std. dev. (%)</th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.31</td>
<td>2.10</td>
<td>8.38</td>
<td>2.08</td>
<td>1.36</td>
<td></td>
</tr>
<tr>
<td>Relative std. dev (/GDP)</td>
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<td>0.90</td>
<td>3.62</td>
<td>0.89</td>
<td>0.58</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.51</td>
<td>0.72</td>
<td>0.18</td>
<td>0.55</td>
<td>0.01</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>0.88</td>
<td>0.86</td>
<td>0.83</td>
<td>0.02</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Functional Form</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ln \sigma_{t+1}^\omega = (1 - \rho_{\sigma^\omega}) \ln \sigma^\omega + \rho_{\sigma^\omega} \ln \sigma_t^\omega + \epsilon_{t+1}^{\sigma^\omega})</td>
<td>Exogenous process for firm productivity dispersion</td>
</tr>
<tr>
<td>(\ln \omega_{mt+1} = \rho_{\omega_m} \ln \omega_{mt} + \epsilon_{t+1}^{\omega_m})</td>
<td>Exogenous process for mean of TFP</td>
</tr>
<tr>
<td>(u(c) = \ln c)</td>
<td>Consumption subutility</td>
</tr>
<tr>
<td>(v(n) = -\psi n)</td>
<td>Labor subutility</td>
</tr>
<tr>
<td>(F(k, n) = k^\alpha n^{1-\alpha})</td>
<td>Production technology</td>
</tr>
</tbody>
</table>

Table 3: Functional forms for quantitative analysis.
### Parameter Values for Baseline Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description/Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Households’ quarterly subjective discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.98</td>
<td>Firms’ (additional) subjective discount factor</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2.90</td>
<td>Labor calibrating parameter</td>
</tr>
<tr>
<td><strong>Production Technology</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
<td>Capital’s share in production function</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.02</td>
<td>Depreciation rate of capital</td>
</tr>
<tr>
<td><strong>Financial Markets and Agency Costs</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.15</td>
<td>Per-unit monitoring cost</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>1</td>
<td>Long-run mean of idiosyncratic productivity</td>
</tr>
<tr>
<td>$\sigma^{\omega}$</td>
<td>0.156</td>
<td>Long-run standard deviation of distribution of $\ln \omega$</td>
</tr>
<tr>
<td>$\rho_{\sigma^\omega}$</td>
<td>0.83</td>
<td>Quarterly persistence of log firm risk process</td>
</tr>
<tr>
<td>$\sigma_{\sigma^\omega}$</td>
<td>0.0033</td>
<td>Standard deviation of innovations to log firm risk</td>
</tr>
<tr>
<td><strong>Exogenous Process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{\omega_m}$</td>
<td>0.83</td>
<td>Quarterly persistence of log mean-TFP process</td>
</tr>
<tr>
<td>$\sigma_{\omega_m}$</td>
<td>0.0081</td>
<td>Standard deviation of innovations to log mean-TFP</td>
</tr>
</tbody>
</table>

Table 4: Parameter values for baseline model.

<table>
<thead>
<tr>
<th>Financial Measure</th>
<th>Long-Run Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leverage ratio, $\ell(\bar{\omega})$</td>
<td>0.668</td>
</tr>
<tr>
<td>External premium</td>
<td>2.00 percent</td>
</tr>
<tr>
<td>Bankruptcy rate, $100\Phi(\bar{\omega})$</td>
<td>1.60 percent</td>
</tr>
<tr>
<td>Reorganization costs / GDP</td>
<td>0.24 percent</td>
</tr>
</tbody>
</table>

Table 5: Long-run financial variables for the baseline calibration of the model. External premium, $\bar{\omega} \cdot gdp/(M - nw)$, reported in annualized terms. Fourth line reports the percentage of GDP absorbed by reorganization costs, $\mu \Phi(\bar{\omega})$. 

39
### Table 6: Risk shocks only. Annualized simulation-based business cycle statistics.

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. dev. (%)</td>
<td>0.0121</td>
<td>0.0071</td>
<td>0.0494</td>
<td>0.0176</td>
<td>0.7597</td>
</tr>
<tr>
<td>Relative std. dev (/GDP)</td>
<td>1</td>
<td>0.59</td>
<td>4.07</td>
<td>1.44</td>
<td>62.55</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.59</td>
<td>0.52</td>
<td>0.51</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>-0.67</td>
<td>0.94</td>
<td>0.96</td>
<td>-0.95</td>
</tr>
</tbody>
</table>

### Table 7: Independent average productivity shocks and risk shocks. Annualized simulation-based business cycle statistics.

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. dev. (%)</td>
<td>3.00</td>
<td>1.35</td>
<td>10.34</td>
<td>2.32</td>
<td>0.82</td>
</tr>
<tr>
<td>Relative std. dev (/GDP)</td>
<td>1</td>
<td>0.45</td>
<td>3.44</td>
<td>0.77</td>
<td>0.27</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.59</td>
<td>0.91</td>
<td>0.50</td>
<td>0.48</td>
<td>0.51</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>0.66</td>
<td>0.94</td>
<td>0.90</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Annual std. dev (%) average productivity: 0 (Data: 1.26)
Annual std. dev (%) $\sigma^\omega$: 3.32 (Data: 3.16)
Annual correlation (GDP, $\sigma^\omega$): 0.99 (Data: -0.83)
Annual correlation (average productivity, $\sigma^\omega$): — (Data: -0.98)
<table>
<thead>
<tr>
<th>Std. dev. (%)</th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.00</td>
<td>1.35</td>
<td>10.43</td>
<td>2.32</td>
<td>0.30</td>
</tr>
<tr>
<td>Relative std. dev (/GDP)</td>
<td>1</td>
<td>0.45</td>
<td>3.44</td>
<td>0.77</td>
<td>0.10</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.60</td>
<td>0.91</td>
<td>0.50</td>
<td>0.48</td>
<td>0.06</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>0.67</td>
<td>0.95</td>
<td>0.90</td>
<td>0.71</td>
</tr>
</tbody>
</table>

| Annual std. dev (%) average productivity | 1.27 (Data: 1.26) |
| Annual std. dev (%) σω | — (Data: 3.16) |
| Annual correlation (GDP, σω) | — (Data: -0.83) |
| Annual correlation (average productivity, σω) | — (Data: -0.98) |

Table 8: **Average productivity shocks only.** Annualized simulation-based business cycle statistics.

<table>
<thead>
<tr>
<th>Std. dev. (%)</th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>σω = 0.0033</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All other calibrated parameters as in this paper</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. dev. (%)</td>
<td>0.0121</td>
<td>0.0071</td>
<td>0.0494</td>
<td>0.0176</td>
<td>0.7597</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.59</td>
<td>0.52</td>
<td>0.51</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>-0.67</td>
<td>0.94</td>
<td>0.96</td>
<td>-0.95</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Std. dev. (%)</th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>σω = 0.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All other calibrated parameters as in this paper</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. dev. (%)</td>
<td>0.2601</td>
<td>0.1513</td>
<td>1.06</td>
<td>0.3763</td>
<td>16.11</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.58</td>
<td>0.52</td>
<td>0.51</td>
<td>0.50</td>
<td>0.58</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>-0.67</td>
<td>0.93</td>
<td>0.94</td>
<td>-0.95</td>
</tr>
</tbody>
</table>

Table 9: **Risk shocks only.** Annualized simulation-based business cycle statistics. Top panel: results from Table 6. Bottom panel: risk shock volatility as measured in CMR (σω = 0.07), with all other parameters calibrated as in Table 6
<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>C</th>
<th>I</th>
<th>hours</th>
<th>leverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std. dev. (%)</td>
<td>2.31</td>
<td>0.32</td>
<td>9.34</td>
<td>2.12</td>
<td>5.01</td>
</tr>
<tr>
<td>Relative std. dev (/GDP)</td>
<td>1</td>
<td>0.14</td>
<td>4.05</td>
<td>0.92</td>
<td>2.17</td>
</tr>
<tr>
<td>Auto. corr.</td>
<td>0.62</td>
<td>0.91</td>
<td>0.61</td>
<td>0.61</td>
<td>0.30</td>
</tr>
<tr>
<td>Corr. w/ GDP</td>
<td>1</td>
<td>0.45</td>
<td>0.99</td>
<td>0.99</td>
<td>0.93</td>
</tr>
</tbody>
</table>

Annual std. dev (%) TFP    | 1.17 | (Data: 1.26) |
Annual std. dev (%) $\sigma^\omega$ | 3.16 | (Data: 3.16) |
Annual correlation (GDP, $\sigma^\omega$) | -0.99 | (Data: -0.83) |
Annual correlation (TFP, $\sigma^\omega$) | -1 | (Data: -0.98) |

Table 10: **Bundled aggregate shocks.** Annualized simulation-based business cycle statistics, in which average productivity (first moment) shocks induce changes in cross-sectional dispersion. Parameter $\varphi = -1.43$. 

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