# Costly External Finance and Labor Market Dynamics *

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**Abstract**

We study the role of agency frictions and costly external finance in cyclical labor market dynamics, with a focus on how credit-market frictions may amplify aggregate TFP shocks. The main result is that aggregate TFP shocks lead to large fluctuations of labor market quantities if the model is calibrated to the empirically-observed countercyclicality of the finance premium. A financial accelerator mechanism thus amplifies labor market fluctuations by rendering rigidity in real wage dynamics. In contrast, if the finance premium is procyclical, which the model can be parameterized to accomodate, amplification is absent, and labor-market fluctuations display the Shimer (2005) puzzle.

**Keywords:** credit frictions, financial accelerator, risk shocks, labor search and matching, volatility puzzle, business cycle modeling
**JEL Classification:** E32, E44, J63, J64

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# Contents

# 1 Introduction

This paper studies the role of costly external finance in the dynamics of labor markets. The starting point of the model is that firms require working capital to finance their operating costs, and the focus of the analysis is on how credit-market frictions may amplify neutral technology shocks. The environment in which this question is studied brings together a benchmark business-cycle model of financial frictions and a benchmark business-cycle model of labor search-and-matching frictions. The main result is that aggregate technology shocks can lead to large cyclical fluctuations of labor market quantities — in particular, unemployment, vacancies, and labor-market tightness, the quantities identified by Shimer (2005) as failing to be explained by standard search models. The framework quantitatively accounts for the empirically-observed large fluctuations of labor markets very well, even though it is calibrated to the cyclical nature of financial conditions rather than to the cyclicality of labor markets. The model thus provides a joint explanation of some salient financial-market and labor-market dynamics.

The property of the model economy that is crucial for amplification is a countercyclical external finance premium. In a version of the model featuring instead a procyclical external finance premium — which the model can be parameterized to accommodate — no amplification occurs, and the model displays the Shimer (2005) volatility puzzle. A broad message of the paper is thus that costly external finance can play an important role in amplifying shocks into the labor market, but it is not financing frictions per se that are important. Rather, the cyclical behavior of financing costs is crucial for the amplification mechanism; in particular, the mechanism imparts rigidity to the real wage. Real wage rigidity has been the main theme in the recent DSGE literature, as summarized by Rogerson and Shimer (2011).

The cyclicality of the finance premium is governed by a single parameter in the model economy, the elasticity of firms' idiosyncratic productivity with respect to aggregate total factor productivity (TFP). Once this parameter is selected via simulated method of moments to match U.S. empirical evidence on the dynamics of the finance premium — in particular, a contemporaneous cyclical correlation with GDP of about -0.50 — all other parameters regarding credit markets and labor markets hardly matter quantitatively for the response of the labor market to shocks to aggregate TFP. Furthermore, the model's predictions of the cyclical fluctuations of key labor-market quantities matches well cyclical fluctuations observed in the U.S., even though the model is calibrated to match the cyclical properties of the finance premium, not to match the cyclical properties of labor markets. The amplification the model displays is thus not merely qualitative in nature, but also a good quantitative fit.

The mechanism of the model turns on how fluctuations in aggregate TFP shift the distribution of firms' idiosyncratic productivity, an effect referred to as a "technology spillover" or a "productivity correlation." If there is no technology spillover, then the finance premium is (mildly) procyclical and labor-market dynamics in the face of TFP shocks are similar to those predicted by baseline DSGE search models such as Andolfatto (1996) and Merz (1995) despite the presence of credit market frictions. On the other hand, if technology spillovers are sufficiently positive — specifically, if an improvement in aggregate TFP raises sufficiently the mean of the distribution from which firms draw idiosyncratic productivity — the finance premium is countercyclical. Because firms borrow to finance their inputs, a countercyclical finance premium leads to sharper expansions of firm activity, including hiring activity, during aggregate upturns and sharper pullbacks of firm activity during aggregate downturns. A financial accelerator mechanism thus amplifies labor market fluctuations.

The financial accelerator effect accounts for 60 percent of the model's ability to improve on standard search models in explaining labor market fluctuations. This channel operates by sharply reducing a firm's idiosyncratic risk of bankruptcy for a given size positive aggregate TFP shock, which lowers the bankruptcy premium charged by the firm's lenders. A lower finance premium in turn allows net-worth-constrained firms to expand activity, including new job-vacancy creation, more than otherwise.

The other 40 percent of the model's mechanism operates through a direct productivity channel. At the firm level, productivity is the sum of an aggregate component and an idiosyncratic component. If aggregate TFP shocks shift positively the distribution of a firm's idiosyncratic productivity, a firm's effective productivity moves more than without the positive spillover. This direct productivity correlation induces sharper adjustments, including hiring adjustments, in response to shocks than if there were no productivity correlation between the macro level and the micro level, even if the cyclical behavior of the finance premium remained unchanged.

Nascent evidence from firm-level studies is suggestive of the type of positive technology correlation present in our model. By constructing new measures of firm-level productivity, Petrin, White, and Reiter (2011) document, among many other micro-macro supply-side empirical relationships, this type of productivity correlation. Of particular relevance for the calibration of our model, Petrin, White, and Reiter (2011) compute an annual correlation between aggregate productivity growth, as measured by an aggregate Solow residual, and growth in firms' technical efficiency, which is a measure of firm-specific technology, in the range of 0.79 to 0.89. Our model, which is driven by only an aggregate TFP shock, portrays this high correlation in the extreme, assuming a correlation of unity. Nonetheless, we are still left with the task of selecting the appropriate elasticity of idiosyncratic productivity with respect to aggregate TFP, which we do via simulated method of moments to match the cyclical properties of the finance premium.

The positive technology spillover in the model in this paper is also virtually identical to a key mechanism underlying Faia and Monacelli's (2007) study of optimal monetary policy in a New

Keynesian model featuring financial frictions and perfect labor markets. At a theoretical level, we think it is important to know that a modeling strategy that has proven useful in a very different branch of the business-cycle literature turns out to also be important for the question under study in this paper. At an empirical level, the work cited above by Petrin, White, and Reiter (2011), in addition to recent work by Foster, Haltiwanger, and Syverson (2008), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012) and Oberfield (2012) (the latter two document and model this positive relationship), adds some realistic foundation to the Faia and Monacelli (2007) assumption.[1] Given this positive relationship, the main question of this paper is how much amplification is induced in labor markets due to productivity shocks.

Regarding the question addressed in this paper, the study most closely related is Petrosky-Nadeau (2009). Our work shares with his the basic ideas that financing frictions may induce an amplified response of the labor market to aggregate TFP shocks and that the cyclicality of the finance premium is important for the transmission mechanism. In these respects, the two studies are highly complementary. Several modeling choices, however, most importantly the ones that govern the precise amplification mechanism, distinguish our work from Petrosky-Nadeau (2009).

First, as already noted, the way in which we construct our model allows for both a countercyclical as well as a procyclical external finance premium. The calibration of this central part of the transmission mechanism is guided by suggestive evidence on firm-level productivity and aggregate evidence on the dynamics of the finance premium; these features are intrinsically linked in our model because the cyclicality is governed by the degree of technology spillover. Because it can admit a procyclical finance premium, our setup thus especially highlights the centrality of a countercyclical finance premium, rather than simply the existence of financing frictions per se, in the amplification of TFP shocks. A second distinction between our work and Petrosky-Nadeau (2009) is that his model does not feature physical capital accumulation, whereas our model does. Third, in Petrosky-Nadeau (2009), financing frictions are assumed to affect only recruitment costs, which, at roughly 2 percent, are a small share of firms' total input costs.

A more reasonable empirical view is that a (much) larger share of firms' input costs are subject to working capital requirements, and that the costs subject to working capital requirements are not merely recruitment costs. I take the broader view that all of firms' ongoing operating costs — wage payments and capital rental payments — require short-term working capital, whereas recruitment costs may or may not require financing. Empirically, this view is more in line with Buera and Shin's (2008) finding that a majority of firms' costs require working capital. Theoretically, this broader

[1]However, opposite to the direction of causality in this paper, it is changes in idiosyncratic conditions affect aggregate conditions in the Acemoglu et al (2012) and Oberfield (2012) models. There is also evidence in this very active literature against the assumption used in this paper, such as Lee and Mukoyama (2012) and Clementi and Palazzo (2013), who highlight the firm entry and exit mechanism.

view motivates our adoption of Carlstrom and Fuerst's (1998) "output model" of all-encompassing financial constraints, rather than the more commonly-employed "investment model" specification of Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999), and much of the DSGE literature on financial frictions, in which it is only investment goods that are subject to financing frictions.

Another theoretical reason that leads us to adopt the output specification is that it has the most potential to interact with labor market frictions. Search and matching frictions directly impinge on the neoclassical "labor wedge" studied by Chari, Kehoe, and McGrattan (2007), Shimer (2009), and others. As Carlstrom and Fuerst (1998) show, if financial constraints affect firms' entire input bill, financing frictions appear in equilibrium as wedges in all factor markets, including labor markets. In contrast, with the investment specification, financing frictions dichotomize away from the labor wedge and instead appear only as an investment wedge. Of course, general equilibrium effects would likely cause financing frictions affecting investment wedges to interact with labor wedges arising from labor market frictions. But, in principle, a more promising and direct route to studying the interactions is if they both affect the same wedge. Moreover, this output-model formulation also allows us to connect to the results of Arseneau and Chugh (2012), who have developed a new notion of labor market wedges for general equilibrium labor search environments.

The rest of the paper is structured as follows. Section 2 develops the baseline model. Section 3 presents the concept of the external finance premium, which is standard in business-cycle agency cost models, and presents the intuition for why positive technology spillovers have the potential to lead to a countercyclical finance premium and in turn to amplify labor-market fluctuations. In Section 4, we draw on the concept of efficiency for labor search environments developed by Arseneau and Chugh (2012) to show that, in contrast to Carlstrom and Fuerst (1998), financing frictions do *not* create an independent wedge in the economy's consumption-leisure margin; rather, the surplus-sharing rule by which wages are determined completely absorbs the effects of financing costs. Section 5 presents our main quantitative results, including a detailed parsing of the results in Section 5.4. As an application, Section 6 briefly assesses how well our model performs in explaining the sharp decline in employment and sharp rise in unemployment experienced during the current credit-market-induced recession. Section 7 provides a number of quantitative robustness exercises with respect to the labor-market and credit-market parameters of the model. The results are hardly changed compared to the baseline model, which strengthens the case that the productivity correlation is the most important feature of the model. Section 8 concludes.

# 2 Model

## 2.1 Households

There is a representative household in the economy. Each household consists of a continuum of measure one of family members, and each individual family member is classified as either inside the labor force or outside the labor force. An individual family member that is outside the labor force enjoys leisure. An individual family member that is part of the labor force is engaged in one of two activities: working, or not working but actively searching for a job. The convenience of an "infinitely-large" household is that we can naturally suppose that each individual family member experiences the same level of consumption regardless of his personal labor-market status. This tractable way of modeling perfect consumption insurance in general-equilibrium search-theoretic models of labor markets has been common since Andolfatto (1996) and Merz (1995). We use the terms "individual" and "family member" interchangeably from here on. Given the basics of the environment, we also use the terms "leisure" and "outside the labor force" interchangeably from here on. In any period $t$, there is a measure $n_t^h$ of employed individuals, a measure $s_t^h$ of searching individuals, and thus a measure $1 - s_t^h - n_t^h$ of individuals outside the labor force.

The household maximizes its expected lifetime discounted utility

$$\sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + h(1 - s_t^h - n_t^h) \right] \tag{1}$$

subject to a sequence of flow budget constraints

$$c_t + k_{ht+1} + T_t = (1 - \tau_t^n) w_t n_t^h + k_{ht} \left[ 1 + (1 - \tau_t^k)(r_t - \delta) \right] + b s_t^h + \Pi_t \tag{2}$$

and a sequence of perceived laws of motion for the measure of family members that are employed,

$$n_t^h = (1 - \rho) n_{t-1}^h + s_t^h p^h(\theta_t). \tag{3}$$

The functions $u(.)$ and $h(.)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and leisure, respectively. The rest of the notation is as follows. The household's subjective discount factor is $\beta \in (0, 1)$, $c_t$ denotes the household's (and thus each family member's) consumption, $k_{ht}$ denotes the household's capital holdings at the start of period $t$, $w_t$ is the real wage, $\tau_t^n$ is a proportional labor income tax, $r_t$ is the market rental rate on capital, $\tau_t^k$ is a proportional capital income tax, $\delta$ is the depreciation rate of capital, $b$ is a fixed benefit that each actively-searching individual receives from the government, $\rho$ is the fraction of workers that are immediately separated from their jobs and $p^h(\theta)$ is the probability that one of the searching individuals will find employment. The household also pays a lump-sum tax $T_t$ to the government and receives aggregate dividends $\Pi_t$ from all firms as lump-sum income.

In the baseline model, the measure of participants in the labor force is endogenous and is chosen optimally by the household. This assumption is made for two reasons. First, while cyclical fluctuations in the participation rate are empirically small, they are not zero, as is assumed by most formulations of labor search models. As we show when we present results, the model's volatility along the participation rate is in line with the data. Second, allowing for endogenous participation means we can apply the concept of search-based labor-market wedges developed by Arseneau and Chugh (2012) to our environment. Doing so enables us to assess whether financing frictions appear as a "wedge" in the labor-market equilibrium of a search model as they do in the model of Carlstrom and Fuerst (1998), who assume a perfectly-competitive labor market. Section 7 shows that our main results remain intact if we instead assume a fixed participation rate over the business cycle.

Flows between employment and job search are governed by the search, matching, and job separation processes in the labor market, all of which are taken as given at the level of household optimization. The household takes as given the probability $p^h(\theta)$ that one of its searching individuals will find employment. As with matching probabilities for firms, $p^h$ depends only on aggregate labor-market tightness, $\theta$, given the assumption of a constant-returns-to-scale aggregate matching technology.

The formal analysis of this problem is presented in Appendix A; here we simply describe intuitively the optimality conditions that emerge. One condition is standard and thus requires no further discussion: the capital supply condition,

$$u'(c_t) = \beta u'(c_{t+1}) \left[ 1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta) \right], \tag{4}$$

which follows as usual from the household's period-$t$ first-order conditions with respect to $c_t$ and $k_{ht+1}$. This consumption-savings optimality condition defines the stochastic discount factor, $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which firms, in equilibrium, discount profit flows.

The other optimality condition is the household's labor-force participation condition

$$\frac{h'(1 - s^h_t - n^h_t) - u'(c_t)b}{p^h(\theta_t)} = u'(c_t)(1-\tau^n_t)w_t - h'(1-s^h_t-n^h_t) + \beta(1-\rho)E_t \left\{ \frac{h'(1 - s^h_{t+1} - n^h_{t+1}) - u'(c_{t+1})b}{p^h(\theta_{t+1})} \right\}, \tag{5}$$

which is derived in detail in Appendix A. The expected cost of participating in the labor market, given by the left hand side of (5), is measured by the marginal utility of leisure (each unit of search involves forgoing one unit of leisure) net of any unemployment benefits received during (unsuccessful) search. The expected benefit of participation is the marginal utility value of after-tax wage income net of the marginal disutility of work (the first two terms on the right-hand-side of (5)), along with the asset value to the household of having an additional family member engaged in an ongoing employment relationship (the last term on the right hand side of (5)). This asset value reflects the value to the household of sending one fewer family member out to search for a job in the future.

Despite the search frictions and long-lived nature of employment relationships, the labor-force participation condition (5) has the same interpretation as the labor-supply function in a simple neoclassical labor market, as exists in the standard RBC model. Indeed, we can recover a neoclassical labor market by setting $\rho = 1$ (all employment "relationships" are one-period, spot, transactions), setting the fixed benefit $b = 0$, and fixing the probability of "finding a job" to $p^h(\theta) = 1$ (because in a neoclassical market there of course is no friction in "finding a job"). Imposing these assumptions and logic on (5), we obtain $\frac{h'(.)}{u'(.)} = (1 - \tau_t^n)w_t$, which defines the labor-supply function in a neoclassical market. With matching frictions that create a meaningful separation of the labor force into those individuals that are employed and those individuals that are unemployed and searching, condition (5) defines transitions of individuals from outside the labor force (leisure, in our model) into the pool of searching unemployed, from where the aggregate matching process will pull some individuals into employment.

## 2.2 Firms

There is a continuum of unit mass of firms, each of which produces output by operating a constant-returns technology. Firms are heterogenous in their productivity. Firm $i$ produces output using the technology $\omega_{it} z_t F(k_{it}, n_{it})$: $k_{it}$ is the firm's purchase of physical capital on spot markets, $n_{it}$ is the firm's stock of employment used for production in period $t$, $z_t$ is aggregate TFP and $\omega_{it}$ is a firm-specific productivity realization.

Each period, firm $i$'s idiosyncratic productivity is a draw from a distribution with cumulative distribution function $\Phi(\omega)$, which has a time-varying mean $\omega_{mt}$, a time-varying standard deviation $\sigma_t^\omega$, and associated density function $\phi(\omega)$, all of which are identical across firms. Time-variation in $\omega_{mt}$ corresponds to the usual notion of TFP shocks, in the sense of exogenous variation in the mean of firms' technology. As in Chugh (2013), the time-varying volatility $\sigma_t^\omega$ is the key new shock in the model. Given the first and second moments $\omega_{mt}$ and $\sigma_t^\omega$ common across firms, idiosyncratic productivity for a given firm is i.i.d. over time, an assumption made for tractability.[2]

[2]The assumption of zero persistence of the idiosyncratic component of a firm's productivity is at odds with the evidence of Cooper and Haltiwanger (2006) and others, but it greatly simplifies the computation of the model because the firm sector essentially can be analyzed as a representative agent. This point is discussed further below when I come to the aggregation of the model. This device still allows me to illustrate the main point of the model, which is that variations in cross-sectional productivity dispersion can lead to large fluctuations in aggregate leverage and possibly, in turn, to fluctuations in economic activity. In addition to greatly reducing the computational burden, the assumption of zero persistence in idiosyncratic shocks also retains the simplicity of the Carlstrom and Fuerst (1997, 1998) and Bernanke and Gertler (1989) contracting specifications. If persistent shocks were allowed, it is not clear that the intraperiod loan contracts of these models could not be improved upon by the contracting parties by, say,

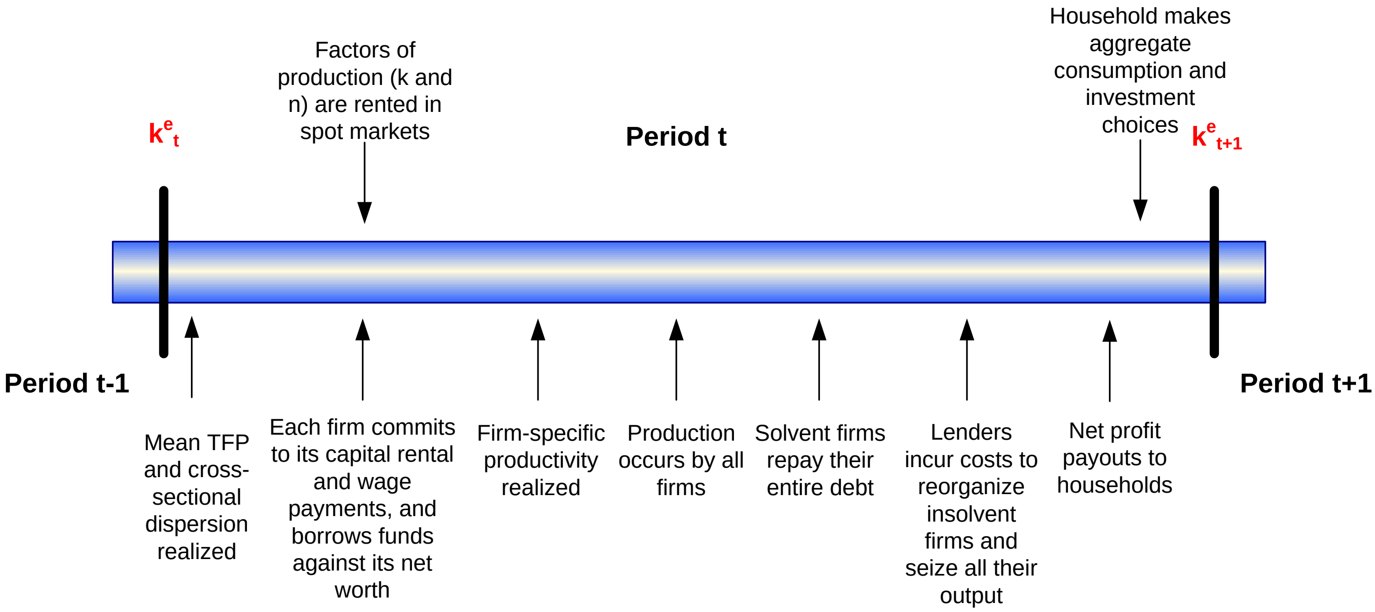


Figure 1: Timing of events in model.

Firms are owned by households, and the objective of firms is to maximize the expected present discounted value of dividends paid out to households. Denote by $\Pi_{it}$ the dividend payment made by firm $i$ to households. For descriptive convenience, I decompose $\Pi_{it}$ into a "non-retained earnings" component $\Pi^e_{it}$ and an "expected operating profit" component $E_\omega \Pi^f_{it}$; the notation $E_\omega$ indicates an expectation conditional on the period-$t$ aggregate state but before idiosyncratic realizations are revealed to any firm.[3] Thus, $\Pi_{it} \equiv \Pi^e_{it} + E_\omega \Pi^f_{it}$. As described below, the component $E_\omega \Pi^f_{it}$ essentially corresponds to static profits as in a simple RBC model.

Because firms are owned by households, they apply the representative household's stochastic discount factor (the one-period-ahead discount factor is $\Xi_{t+1|t}$, as defined above) to their intertemporal optimization problem. However, firms are also assumed to be "more impatient" than households by the factor $\gamma < 1$, which can be thought of as a reduced-form way of capturing some sort of principal-agent problem that prevents perfect alignment of the firms' objectives with households' preferences. At a technical level, $\gamma < 1$ ensures that firms cannot accumulate enough assets to become self-financing, which would render irrelevant the financial frictions described below. This

multi-period contracts. Sidestepping this issue is yet another reason to assume no persistence in realized idiosyncratic productivity. Note, however, that assuming persistence in shocks to $\sigma^\omega_t$, as I do, does not pose any of these problems; indeed, shocks to $\sigma^\omega_t$ really are aggregate shocks.

[3] As Figure 1 indicates, firm decisions are made in the first "subperiod" of period $t$, before idiosyncratic shocks have been realized but after aggregate shocks have been realized, hence the need for $E_\omega$.

device for avoiding self-financing outcomes is common in models of financial frictions.

The intertemporal objective function of firm $i$ is thus

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left[ \Pi_{it}^e + E_\omega \Pi_{it}^f \right], \tag{6}$$

which is now further developed and analyzed.

### 2.2.1 Hiring Employees

Hiring employees is a costly and time-consuming process. The firm begins period $t$ with employment stock $n_{it-1}$, a fraction $\rho$ of whom immediately separate.[4] Its period-$t$ productive employment stock, $n_{it}$, depends on its period-$t$ vacancy-posting choices as well as the matching process in labor markets. To hire a worker, the firm must first create a vacancy. The total costs of creating $v_{it}$ vacancies is $\varrho(v_{it})$ in terms of output goods. Vacancy creation costs may be concave, linear, or convex, reflecting possibly increasing, constant, or diminishing marginal returns, respectively, in the firms' recruitment technology; hence, $\varrho'(v_{it}) > 0$ and $\varrho''(v_{it}) \leq\geq 0$. Each vacancy is filled with probability $p^f(\theta_t) < 1$, a market-determined variable which the firm takes as given parametrically. Successful new recruits in period $t$ begin working at the firm in period $t$. Firm $i$'s perceived laws of motion for its employment stock is thus

$$n_{it} = (1-\rho) n_{it-1} + p^f(\theta_t) v_{it}. \tag{7}$$

### 2.2.2 Firm Financing and Contractual Arrangement

In period $t$, total operating costs of firm $i$, which include capital rental costs, wage payments, and hiring costs, are

$$M_{it} = w_t n_{it} + r_t k_{it} + \varrho(v_{it}), \tag{8}$$

in which the equilibrium result that all workers at firm $i$ earn the same wage is anticipated.[5] As shown in Figure 1, the firm is assumed to commit to all of its input costs after completing its period-$t$ recruiting activities, which itself is conditional on the aggregate exogenous state $(\omega_{mt}, \sigma_t^\omega)$, but before observing its idiosyncratic realization $\omega_{it}$ and thus before any output or revenue are created. In the baseline specification, recruitment costs, wage payments, and capital rental payments are thus all subject to the working capital requirement.

Part of the financing of the firm's costs comes from its own accumulated net worth, which is held primarily in the form of capital. The capital that each firm accumulates is rented on spot markets

[4] For ease of notation, I will denote firm $i$'s labor as $n_i$ rather than $n_i^f$.

[5] This follows because workers are assumed to be homogenous. Thus, there is no component of productivity that is idiosyncratic to any particular worker-job match. Firm $i$'s idiosyncratic component of productivity is common across all workers at that firm.

to (other) firms, just like households rent their capital on spot markets. Firm $i$'s capital holdings at the start of period $t$ are $k^e_{it}$. Thus, note that $k^e_{it}$, which reflects the firm's *savings* decisions, is distinct from $k_{it}$, which reflects the firm's *capital demand* decisions for production purposes.

However, the firm's internal funds (which I refer to interchangeably as its net worth or its equity) are insufficient to cover all input costs. To finance the remainder, a firm borrows short-term — formally, intraperiod — working capital.[6] By acquiring external funds, the firm is able to leverage its net worth in period $t$,

$$nw_{it} = k^e_{it}\left[1 + (1-\tau^k_t)(r_t - \delta)\right] + e_t, \tag{9}$$

into coverage of its operating costs $M_{it}$. Total borrowing by the firm is thus $M_{it} - nw_{it}$. The component $e_t$ of net worth is a small amount of "endowment income" that each firm receives to ensure its continued operations in the event that it becomes insolvent. In closing the model, this endowment is absorbed into the payout $\Pi_{it}$ the firm pays to its owners, which is the representative household. The payout $\Pi_{it}$ is thus interpreted as net of the endowment $e_t$.[7]

I describe only briefly the outcome of the contracting arrangement between borrowers (firms) and lenders (households) because it is well-known in this class of models. In the context of general-equilibrium settings, familiar expositions appear in Carlstrom and Fuerst (1997, 1998) — henceforth, CF — and Bernanke, Gertler, and Gilchrist (1999).[8] The financial contract is a debt contract, which is fully characterized by a liquidation threshold $\bar{\omega}_t$ and a loan size $M_{it} - nw_{it}$. A firm must be liquidated or "reorganized," at cost $\mu > 0$, if its realized productivity $\omega_{it}$ falls below the contractually-specified threshold $\bar{\omega}_t$. Below this threshold, the firm does not have enough resources to fully repay its loan. In that case, the firm is declared insolvent and receives nothing, while the lender must pay reorganization costs that are proportional to the total output of the firm and receives, net of these reorganization costs, all of the output of the firm. Note that all firms, regardless of whether or not they end up requiring reorganization, do produce output up to their full (idiosyncratic) capacity.

Define by $f(\bar{\omega}_t)$ the expected share of idiosyncratic output $\omega_{it} z_t F(k_{it}, n_{it})$ the borrower (the firm) keeps after repaying the loan, and by $g(\bar{\omega}_t)$ the expected share received by the lender.[9] These

[6]As described above, a firm requires external financing because of the assumption that it is more impatient than households, a standard assumption in this class of models that avoids the self-financing outcome.

[7]Thus, equivalently, $e_t$ can be interpreted as a lump-sum transfer of "startup funds" provided by households to firms, as in Gertler and Karadi (2011). By allowing a "firm's" operations to continue in the event of bankruptcy, the assumption of a startup fund brings great analytical tractability to the model. Thus, the "costs of bankruptcy" in the model are more properly interpreted as "costs of reorganization" without any disruption of its output-producing activities (i.e., bringing in new management to oversee ongoing operations).

[8]In partial-equilibrium settings, analysis of this type of contractual arrangement traces back to Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).

[9]Formally, $f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} (\omega_i - \bar{\omega}_t)\,\phi(\omega_i) d\omega_i = \int_{\bar{\omega}_t}^{\infty} \omega_i \phi(\omega_i) d\omega_i - [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t$ is the share received by the firm, and

expectations are conditional on the realization of the time-$t$ aggregate state, but before revelation of a firm's idiosyncratic productivity $\omega_{it}$. The contractually-specified loan size is characterized by a zero-profit condition on the part of lenders,

$$M_{it} = \frac{nw_{it}}{1 - p_t g(\bar{\omega}_t)}, \tag{10}$$

and the contractually-specified liquidation threshold is characterized by

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}, \tag{11}$$

in which $p_t > 1$ is a "markup" on input costs that arises solely from the external financing needs of the firm.[10] Thus, for each unit of capital the firm rents, the cost, inclusive of financing costs, is $p_t r_t$, rather than just $r_t$. The same is true for wage payments and recruiting costs. Note that neither $p_t$ nor $\bar{\omega}_t$ are firm-specific; this is simply asserted for now, but I return to this below when considering the aggregation of the model. Finally, all contractual outcomes are contingent on the aggregate state $(\omega_{mt}, \sigma_t^{\omega})$ of the economy.

CF interpret $p_t$ as a "markup" that drives a wedge between factor prices and marginal products. The analysis below shows that this interpretation also carries over here. However, another informative interpretation of $p_t$ is as an external finance premium. For every unit of cost firms incur for their inputs, they must pay $p > 1$ units inclusive of borrowing costs, which gives $p$ a natural interpretation as an external finance premium.

### 2.2.3 Asset Evolution and Dynamic Profit Function

Firms take as given contractual outcomes when maximizing profits. The *expected* operating profit of firm $i$ in period $t$ is

$$E_{\omega}\Pi_{it}^{f} = \omega_{mt} z_t F(k_{it}, n_{it}) - p_t \left[ w_t n_{it} + r_t k_{it} + \varrho(v_{it}) \right]. \tag{12}$$

As discussed above, this is an *expected* profit because it is measured before the realization of firm-specific idiosyncratic productivity but after the realization of the aggregate period-$t$ state of the economy, $(\omega_{mt}, \sigma_t^{\omega})$. Because $E(\omega_{it}) = \omega_{mt}$, ex-ante revenue of the firm is $\omega_{mt} z_t F(k_{it}, n_{it})$. The idiosyncratic risk $\omega_{it}$ and associated financing costs implied by it are captured by the inclusion of

---

$g(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} (\omega_i - \mu)\, \phi(\omega_i) d\omega_i + \int_{\bar{\omega}_t}^{\infty} \bar{\omega}_t \phi(\omega_i) d\omega_i = \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i) d\omega_i + [1 - \Phi(\bar{\omega}_t)]\bar{\omega}_t - \mu\Phi(\omega_i)$.

[10] The background assumptions of the zero profit condition are that lending is a perfectly competitive activity and entry into the lending market is costless. Formally, the two conditions characterizing the optimal contract result from maximizing (the firm's share of) the return on the financial contract (because, if it remains solvent, the firm is the residual claimant on output), $p_t f(\bar{\omega}_t) M_{it}$, subject to the zero profit condition of the lender, $p_t g(\bar{\omega}_t) M_{it} = M_{it} - nw_{it}$.

the external finance premium $p_t$ in the above expression.[11] Firms take as given the competitively-determined factor price $r_t$, and the outcome of the Nash wage-bargaining process.

Regarding the savings decisions of firms, firm $i$ begins period $t$ with assets $k^e_{it}$, whose period-$t$ market value determines the firm's net worth $nw_{it}$, as shown in (9). The firm borrows $M_{it} - nw_{it}$ against the value of these assets, and it expects to keep $p_t f(\bar{\omega}_t)M_{it}$ after repaying its loan.[12] Of these "excess" resources, the firm can either accumulate assets or make payments to households. That is,

$$\Pi^e_{it} + k^e_{it+1} = p_t f(\bar{\omega}_t) M_{it}, \tag{13}$$

which highlights that $k^e_{it+1}$ can be thought of as retained earnings. Substituting the contractually-specified quantity of borrowing, $M = \frac{nw}{1-pg(\bar{\omega})}$, this can be re-written as

$$\Pi^e_{it} + k^e_{it+1} = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} nw_{it}. \tag{14}$$

Further substituting the definition of net worth from (9), the firm's asset evolution is described by

$$\Pi^e_{it} + k^e_{it+1} = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left[ k^e_{it} \left[ 1 + (1 - \tau^k_t)(r_t - \delta) \right] + e_t \right]. \tag{15}$$

Finally substituting (12) and (15) into (6), the dynamic profit function of the firm is

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left\{ \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} \left[ k^e_{it} \left[ 1 + (1 - \tau^k_t)(r_t - \delta) \right] + e_t \right] - k^e_{it+1} + \omega_{mt} z_t F(k_{it}, n_{it}) - p_t \left[ w_t n_{it} + r_t k_{it} + \varrho(v_{it}) \right] \right\}. \tag{16}$$

### 2.2.4 Firm Optimality Conditions

Maximization of (16) subject to (7) with respect to vacancies $v_{it}$ and employment $n_{it}$ yields the vacancy creation condition,

$$\frac{\varrho'(v_t) p_t}{p^f(\theta_t)} = \omega_{mt} z_t F_n(k_t, n_t) - p_t w_t + (1 - \rho) E_t \left\{ \Xi_{t+1|t} \frac{\varrho'(v_{t+1}) p_{t+1}}{p^f(\theta_{t+1})} \right\}, \tag{17}$$

in which firm $i$ indexes are dropped because I analyze an equilibrium symmetric across all firms, which is justified by the aggregation of the model described below. As is standard in search models, the vacancy creation condition states that at the optimum, the marginal creation cost incurred by the firm is equated to the discounted expected value of profits from the marginal job match. Profits from a new match take into account contemporaneous (because hiring is within-period) marginal

[11] As is common in macro models, writing, for example, $p_t$, is shorthand for the state-contingent equilibrium function $p(\omega_{mt}, \sigma^\omega_t)$. If the distribution of $\omega$ were degenerate — that is, if there were no idiosyncratic component of technology — then we would have $p_t = 1 \; \forall t$, which simply has the interpretation that financing issues are irrelevant.

[12] This is because, as noted in footnote 26, the firm keeps the entire (expected) surplus from the contractual arrangement. Hence, in expectation, the firm is left with $p_t f(\bar{\omega}_t) M_{it}$ after the sequence of borrowing, renting factors of production, producing output, and repaying its loan.

profits from the match and the asset value of having a pre-existing relationship with an employee in period $t+1$. Financing frictions cause both $p_t$ and $p_{t+1}$, which reflect current and expected future financing needs, to affect the vacancy creation decision: $p_t$ because it affects period-$t$ creation costs and financing of wages, and $p_{t+1}$ because it measures the financing component of the period-$t+1$ replacement value of the employee.

Maximization of (16) with respect to capital rental $k_{it}$ gives rise to the capital demand condition

$$r_t = \frac{\omega_{mt} z_t F_k(k_{it}, n_{it})}{p_t}, \tag{18}$$

and maximization of (16) with respect to asset accumulation $k^e_{it+1}$ yields the capital Euler equation for firms,

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} \left[ 1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta) \right] \right\}. \tag{19}$$

Note that, although firms may differ in their levels of factor usage, each firm chooses an identical capital-labor ratio because the production technology $F(.)$ is constant-returns and because in equilibrium the prices $r_t$ and $w_t$ and the external premium $p_t$ are identical for all firms.

## 2.3 Wage Bargaining

The wage-determination mechanism is Nash bargaining. The wages of all workers, whether newly-hired or not, are set in period-by-period Nash negotiations. This assumption is common in DGE search models. A detailed derivation of the wage-bargaining problem is presented in Appendix B. However, none of our results is sensitive to wages being determined in an explicitly bilateral fashion, as is the case with Nash bargaining. An alternative equilibrium concept for search models, due to Moen (1997), is a *competitive search equilibrium.* In competitive search equilibrium, wages are determined by decentralized forces and taken as given by all market participants. Appendix E shows that competitive search delivers exactly the same wage outcome as period-by-period Nash negotiations; hence its cyclical implications are identical.[13]

In what follows, we simply present the bargaining outcome. Assuming that $\eta \in (0,1)$ is a worker's Nash bargaining power and $1-\eta$ a firm's Nash bargaining power, the Nash wage outcome is given by

$$\begin{aligned} w_t = {} & \eta \frac{\omega_{mt} z_t F_n(k_t, n_t)}{p_t} + (1-\eta) \frac{h'(1 - s^h_t - n^h_t)}{u'(c_t)(1 - \tau^n_t)} \\ & - \eta \frac{1-\rho}{1-\tau^n_t} \left( \Xi_{t+1|t} \frac{1 - \tau^n_{t+1}}{p_{t+1}} \frac{\varrho'(v_{t+1}) p_{t+1}}{p^f(\theta_{t+1})} \right) + \frac{\eta(1-\rho)}{p_t} \left( \Xi_{t+1|t} \frac{\varrho'(v_{t+1}) p_{t+1}}{p^f(\theta_{t+1})} \right), \end{aligned} \tag{20}$$

[13]This equivalence is a DSGE extension of the equivalence shown by Moen (1997) between competitive search outcomes and Nash bargaining under the well-known Hosios (1990) condition. As discussed below, all of the analysis of the bargaining economy satisfies the Hosios (1990) condition. See also Rogerson, Shimer, and Wright (2005) for more on competitive search.

The first two terms in (20) show that part of the period-$t$ wage payment is a convex combination of the contemporaneous values to the firm and the household, given, respectively, by the marginal revenue $\omega_{mt} z_t F_n(k_t, n_t)$ generated for the firm by a new employment match, which is deflated by the financing cost, and the after-tax MRS between consumption and leisure. The second line of (20) captures the forward-looking, relationship, aspect of employment, whose value is also capitalized in the period-$t$ wage payment. Along the dynamic stochastic equilibrium of the economy, expected changes in labor tax rates and expected changes in financing costs play a role in the determination of period-$t$ wages. If neither tax rates nor financing costs varied over the business cycle, $\tau^n_{t+1} = \tau^n_t = \tau^n$ and $p_{t+1} = p_t = p \ \forall t$, then the last two terms cancel with each other and the wage collapses to a simple static split,

$$w_t = \eta \frac{\omega_{mt} z_t F_n(k_t, n_t)}{p} + (1-\eta) \frac{h'(1 - s^h_t - n^h_t)}{u'(c_t)(1-\tau^n)}, \tag{21}$$

between the firm's marginal revenue product and the household's after-tax MRS.

## 2.4 Government

Unemployment benefits are provided by the government, and the government also has other exogenous spending $g_t$ that is not used for any productive purpose. Any excess (shortfall) of income tax revenue over government outlays is rebated to (raised from) households in a lump-sum manner. The flow government budget constraint is thus

$$bs^h_t + g_t = T_t + \tau^n_t w_t n_t + \tau^k_t (r_t - \delta) k_t. \tag{22}$$

## 2.5 Matching Technology

Matches between unemployed individuals searching for jobs and firms searching for workers for their vacancies are formed according to a constant-returns matching function, $m(s_t, v_t)$. A fraction $\rho$ of both matches that produced in period $t-1$ and that were newly formed in period $t-1$ are exogenously destroyed before period $t$. It is thus possible that a newly formed match never produces. The evolution of aggregate employment is thus given by

$$n_t = (1-\rho) n_{t-1} + m(s_t, v_t). \tag{23}$$

As is well-known in the search and matching literature, if $m(s_t, v_t)$ is constant-returns-to-scale, then the matching probabilities are $p^h(\theta_t) \equiv \frac{m(s_t, v_t)}{s_t}$ and $p^f(\theta_t) \equiv \frac{m(s_t, v_t)}{v_t}$.

## 2.6 Private Sector Equilibrium

A symmetric private-sector equilibrium is made up of state-contingent endogenous processes $\{c_t, n_t, s_t, k_{ht+1}, k^e_{t+1}, k_{t+1}, \Pi^e_t, w_t, \theta_t, p_t, \bar{\omega}_t\}$ that satisfy the following conditions: the labor-force participation condition

$$\frac{h'(1-s_t^h-n_t^h)-u'(c_t)b}{p^h(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - h'(1-s_t^h-n_t^h) + \beta(1-\rho)E_t\left\{\frac{h'(1-s_{t+1}^h-n_{t+1}^h)-u'(c_{t+1})b}{p^h(\theta_{t+1})}\right\}; \tag{24}$$

the vacancy creation condition

$$\frac{\varrho'(v_t)p_t}{p^f(\theta_t)} = \omega_{mt}z_tF_n(k_t,n_t) - p_tw_t + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right\}; \tag{25}$$

the Nash wage rule

$$\begin{aligned} w_t = {} & \eta\frac{\omega_{mt}z_tF_n(k_t,n_t)}{p_t} + (1-\eta)\frac{h'(1-s_t^h-n_t^h)}{u'(c_t)(1-\tau_t^n)} \\ - {} & \eta\frac{1-\rho}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right) + \frac{\eta(1-\rho)}{p_t}\left(\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right); \end{aligned} \tag{26}$$

the representative household's Euler equation for capital holdings

$$1 = E_t\left\{\Xi_{t+1|t}\left[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right]\right\}; \tag{27}$$

the firm/entrepreneur's Euler equation for capital holdings

$$1 = \gamma E_t\left\{\Xi_{t+1|t}\frac{p_{t+1}f\left(\bar{\omega}_{t+1}\right)}{1-p_{t+1}g\left(\bar{\omega}_{t+1}\right)}\left[1+(1-\tau_{t+1}^k)(r_{t+1}-\delta)\right]\right\}; \tag{28}$$

the aggregate law of motion for employment

$$n_t = (1-\rho)n_{t-1} + m(s_t,v_t); \tag{29}$$

aggregate capital market clearing

$$k_t = k_{ht} + k_t^e; \tag{30}$$

the aggregate resource constraint

$$c_t + k_{t+1} - (1-\delta)k_t + \varrho(v_t) = \omega_{mt}z_tF(k_t,n_t)\left[1-\mu\Phi(\bar{\omega}_t)\right]; \tag{31}$$

the contractually-specified loan size

$$M_t = \frac{nw_t}{1-p_tg\left(\bar{\omega}_t\right)}; \tag{32}$$

(in which expression (9) for $nw_t$ is substituted in); firms' asset evolution

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_tf(\bar{\omega}_t)}{1-p_tg(\bar{\omega}_t)}\left[k_{it}^e\left[1+(1-\tau_t^k)(r_t-\delta)\right]+e_t\right]; \tag{33}$$

and the contractually-specified bankruptcy threshold

$$\frac{p_tf(\bar{\omega}_t)}{1-p_tg(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}. \tag{34}$$

The private sector takes as given the stochastic process $\{z_t,\tau_t^n,\tau_t^k\}_{t=0}^{\infty}$.

# 3 Basic Analytics: Productivity Correlation, External Finance Premium, and Sensitivity of the Labor Market

In this section, we provide an analytical look at the model's amplification mechanism. These analytics do not formally prove our main results, which are quantitative in nature. But they shed light on the transmission mechanism, which is subsequently quantified in the following section.

As already emphasized, the dynamics of the external finance premium are crucial for the model's propagation of shocks to the labor market. Following Bernanke, Gertler, and Gilchrist (1999), the external finance premium is defined in the following way. Let

$$\mu G(\bar{\omega}_t) \equiv \mu \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i; .) d\omega_i \tag{35}$$

be the expected monitoring cost incurred by the lender per unit of output.[14] Given linearity of the monitoring technology, total — that is, summed over all units of output — expected bankruptcy costs are thus $\mu G(\bar{\omega}_t) p_t y_t$, and this total cost is reflected in the interest rate charged on risky loans. Because the total loan extended is $M_t - nw_t$, the gross external finance premium is thus defined as

$$R_t^y \equiv 1 + \frac{\mu G(\bar{\omega}_t) p_t y_t}{M_t - nw_t}. \tag{36}$$

Thus, our measure of the external finance premium is the ratio of total expected bankruptcy cost to quantity borrowed.

Following Faia and Monacelli (2007), the idiosyncratic productivity distribution is assumed to depend on aggregate TFP. More precisely, the mean $\omega_{mt}$ of a firm's idiosyncratic productivity draw is time-varying and a function of $z_t$, but the standard deviation of idiosyncratic productivity, $\sigma_\omega$ is constant. As in Faia and Monacelli (2007), the productivity correlation is characterized by

$$\ln \omega_{mt} = \nu \ln z_t, \tag{37}$$

in which $\nu \geq 0$ is the elasticity of the log-mean of idiosyncratic productivity with respect to log-TFP.

Figure 2 illustrates why $\nu > 0$ has the potential to lead to a countercyclical finance premium. Suppose the solid curve in black in Figure 2 is the pdf $\phi(\omega, .)$ before a shock occurs — in the long-run steady-state of the economy, say. The bankruptcy threshold $\bar{\omega}$ shown is for this initial distribution. If $\nu > 0$, then a positive shock to aggregate TFP causes a rightwards translation of the distribution. If the bankruptcy threshold $\bar{\omega}$ were to remain unchanged, fewer firms would draw an idiosyncratic $\omega_t < \bar{\omega}$, which lenders understand because $\phi(\omega, .)$ is common knowledge. Fewer bankruptcies mean smaller bankruptcy (monitoring) costs, which in turn means that the

[14] Note from the definition of $g(\bar{\omega}_t)$, the expected share of entrepreneurial output received by lenders, we can express $\mu G(\bar{\omega}_t) = \mu \left[ g(\bar{\omega}_t) + \mu \Phi(\bar{\omega}_t) - \bar{\omega}_t \left[1 - \Phi(\bar{\omega}_t)\right]\right]$, which makes a bit easier numerically computing expected bankruptcy costs.

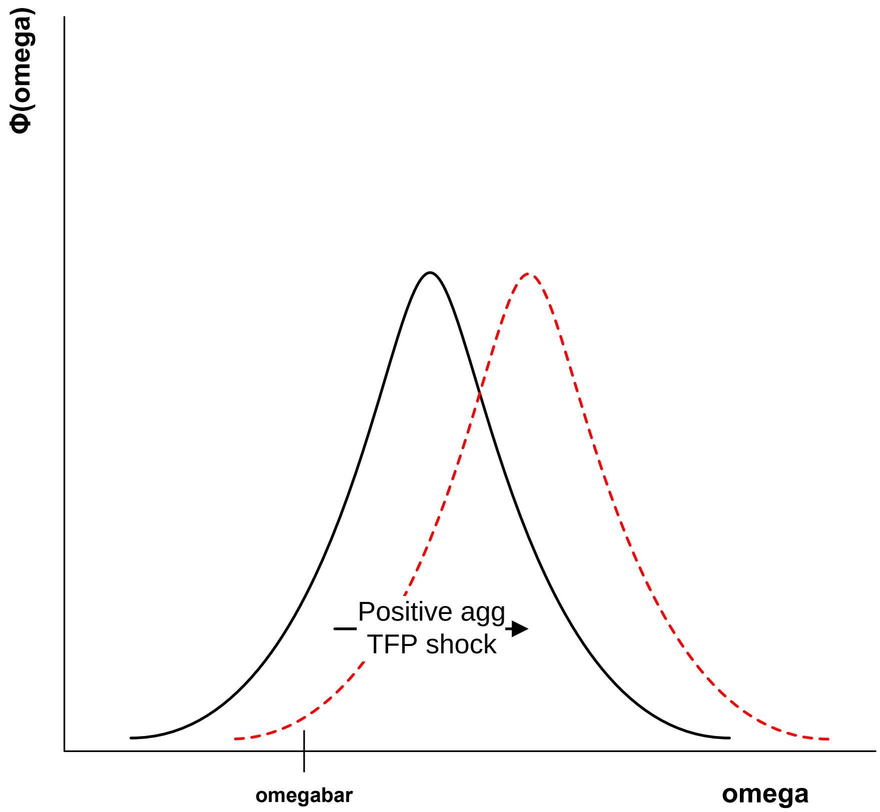


Figure 2: Intuition for the financing channel of the model. If there is a positive spillover from aggregate TFP shock to idiosyncratic productivity, a positive shock to aggregate TFP causes a rises in the mean of the distribution of firm-level productivity, without changing its variance. Thus, the distribution translates to the right. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would go bankrupt and hence the external finance premium would be lower.

external finance premium charged by competitive lenders declines. The decline in the external finance premium thus in equilibrium is associated with a rise in GDP (because productivity has improved), hence a countercyclical finance premium. In general equilibrium, $\bar{\omega}$ will of course also change. It is thus a quantitative question whether, for a given value of $\nu$, the finance premium is countercyclical.

As noted in the introduction, firm-level evidence suggests that such a positive spillover from aggregate TFP to idiosyncratic productivity may exist. Petrin, White, and Reiter (2011) use manufacturing plant-level data to measure idiosyncratic productivity and correlate it with constructed data on aggregate productivity. They find an annual correlation in the range of 0.79-0.89 between growth in idiosyncratic and aggregate productivity. A known issue in the literature is that firm-level productivity measures may confound productivity and profitability. Foster, Haltiwanger, and Syverson (2008) provide evidence that at least for some industries, firm-level "productivity" shocks are highly correlated with "profitability" shocks.

For our purposes, we interpret this evidence broadly because our model, which is effectively a representative-agent model, does not portray the rich firm-level heterogeneity present in analyses such as Petrin, White, and Reiter (2011) and Foster, Haltiwanger, and Syverson (2008). We thus interpret the micro evidence as supportive of a positive value of $\nu$, rather than $\nu = 0$ of standard formulations of agency cost models. In some sense, this is the extent to which we rely on the firm-level productivity evidence — that is, we do not draw a particular parameter value directly from firm-level studies, which would likely be even more problematic given the array of issues plaguing firm-level studies.

Given $\nu > 0$, our model's correlation between aggregate TFP and idiosyncratic productivity is unity, rather than the 0.79-0.89 computed by Petrin, White, and Reiter (2011). This is a byproduct of the presence of only one aggregate shock, the TFP shock, in our model. If we introduced other aggregate shocks, in particular demand shocks, and posited that idiosyncratic "productivity" also depended on them, this correlation would be lowered.[15] As we describe in our parameterization below, accepting that $\nu > 0$, we then select $\nu$ to match the degree of countercyclicality of the finance premium observed in U.S. data.

With the intuition behind why technology spillovers can lead to a countercyclical finance premium in hand, we now link this to the intuition for why labor-market volatility should be expected to increase compared to a benchmark search model. In Appendix C, we derive the following steady-state elasticity of labor-market tightness $\theta$ with respect to the mean of idiosyncratic productivity,

[15] Such exploration is left to future research, which one may be able to base on the type of evidence presented in papers such as Foster, Haltiwanger, and Syverson (2008), Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Oberfield (2012), and others.

$\omega_m$,

$$\epsilon_{\theta,\omega_m} = \frac{(1-\eta)z_t F_n(k,n)}{\xi[1-\beta(1-\rho)]p\varrho'(v)\theta^{\xi-1}}\frac{\omega_m}{\theta}, \tag{38}$$

in which $\xi \in (0,1)$ is the elasticity of aggregate matches with respect to the number of unemployed individuals searching for jobs. We focus on market tightness because it is a useful summary indicator of labor-market conditions in search-based models. The recent literature has shown that understanding the determination of $\theta$ can be as important for considering cyclical issues as it is for considering long-run issues.

This elasticity is informative for understanding the dynamics of the labor market for two reasons. First, even though (38) is a deterministic condition, to the extent that business-cycle magnitude fluctuations are fairly linear phenomena around steady states, it offers some guidance as to how the stochastic dynamics of the model may behave. Indeed, Hagedorn and Manovskii's (2008) analysis of labor-market dynamics is also intuitively premised on analysis of a key steady-state elasticity.[16] Second, considering (38) sheds light on both channels of amplification present in our model: the direct productivity channel and the financial conditions channel.

To see the direct productivity channel, note that the elasticity $\epsilon_{\theta,\omega_m}$ is increasing in $\omega_m$. Thus, the more $\omega_m$ increases in the face of an aggregate TFP shock, the sharper will be the response of the labor market. Outside of steady state, the parameter $\nu$ governs the contemporaneous responsiveness of $\omega_m$ to $z$.

To see the financial conditions channel, note that the elasticity $\epsilon_{\theta,p}$ is decreasing in $p$, which is the component of input costs that arises due to firms' financing needs. Intuitively, $p$ and the external finance premium $R^L$ can be thought of similarly because each is a measure of firms' financing costs. It thus makes sense that $p$ and $R^L$ should behave similarly. Indeed, Carlstrom and Fuerst (1998) explain the dynamics of their output model through the lens of cyclical fluctuations in $p_t$, rather than of $R^L_t$. While $p$ and $R^L$ are not identical measures of financing costs, simulations for all versions of the model studied below exhibit a high cyclical correlation between $p$ and $R^L$.[17] We frame all of our analysis below through the lens of movements in $R^L_t$, but expression (38) shows the intuition behind the financing channel of our model. Following a positive aggregate TFP shock, if $p$ were to *decline*, the response of the labor market would be more pronounced than otherwise because market tightness is more elastic to productivity. As discussed above, the productivity correlation has the ability to generate just such an effect.

[16] In their case, the steady-state elasticity of market tightness to workers' outside option in wage bargaining.

[17] The lowest average correlation was 0.70, in the version of the model featuring no technology correlation.

# 4 General Equilibrium Consumption-Labor Margin

In CF, the model of labor markets is neoclassical, hence the representative household's subutility function over leisure is simply $h(1-n_t)$. CF shows that financing frictions that affect firms' total input costs are reflected in equilibrium as part of the wedge between MRS and MRT in the labor market,

$$\frac{h'(1-n_t)}{u'(c_t)} = \frac{1-\tau_t^n}{p_t}\omega_{mt} z_t F_n(k_t, n_t). \tag{39}$$

In a neoclassical model, $\omega_{mt} z_t F_n(k_t, n_t)$ of course has the interpretation of the economy's MRT between leisure and consumption.

Arseneau and Chugh (2012) develop a broader concept of this MRT for general-equilibrium search models, given by

$$\varrho'(v_t)\theta_t \frac{\xi}{1-\xi}, \tag{40}$$

in which $\xi$ is the elasticity of aggregate labor-market matches with respect to the number of job searchers. As Arseneau and Chugh (2012) show in detail, this concept is a joint description of the technology embodied by *both* the matching process *and* the production process. The neoclassical notion of MRT captures only the latter. If search and matching frictions do not exist, condition (40) nests the neoclassical MRT, also shown by Arseneau and Chugh (2012) and Ravenna and Walsh (2012). As shown in Appendix D, the decentralized economy's consumption-leisure margin in this model is characterized by

$$\frac{h'(1-s_t-n_t) - u'(c_t)b}{u'(c_t)} = (1-\tau_t^n)\varrho'(v_t)\theta_t \frac{\xi}{1-\xi}. \tag{41}$$

Although the financing friction $p_t$ does affect wage determination (recall the Nash wage rule (20)), it does not create a direct wedge between MRS and the search-based MRT, unlike what occurs in a neoclassical model of the labor market. It may be surprising that the financing friction embodied in $p_t$ only appears indirectly through optimal decision functions in (41). Nevertheless, the nature of wage determination matters for this aspect of the equilibrium.[18] Financing frictions do affect the labor market through general-equilibrium effects, as the next section quantitatively shows.

# 5 Quantitative Analysis

## 5.1 Parameterization

Table 1 lists the functional forms used in the quantitative experiments, which, with the exception of the link between the mean of the idiosyncratic productivity distribution and aggregate TFP, are

[18] Note in the Nash wage rule (20) that the term $\frac{z_t F_n(k_t,n_t)}{p_t}$ does arise, as the upper bound of the wage payment the firm would be willing to make to the worker.

| Functional Form | Description |
|---|---|
| $u(c) = \ln c$ | Consumption subutility |
| $h(x) = \psi \ln x$ | Leisure subutility |
| $F(k, n) = k^{\alpha} n^{1-\alpha}$ | Production technology |
| $m(s, v) = \theta s^{\xi} v^{1-\xi}$ | Matching technology |
| $\varrho(v) = \bar{\varrho} v^{\chi}$ | Vacancy creation costs |
| $\ln \omega_{mt} = \nu \ln z_t$ | Link between TFP and mean of idiosyncratic productivity |

Table 1: Functional forms for quantitative analysis.

all typical in business cycle models. We follow Faia and Monacelli (2007) in our specification of the linkage between aggregate and idiosyncratic technology to correct a well-known shortcoming of the agency cost model, namely that it delivers a procyclical external finance premium. This prediction of this class of models is at odds with most empirical evidence that the external finance premium is countercyclical with respect to GDP as well as aggregate TFP, as documented in Gomes, Yaron, and Zhang (2003), Levin, Natalucci, and Zakrajsek (2004), and De Graeve (2008). By setting $\nu > 0$, so that the average idiosyncratic productivity of each firm, $\omega_{mt}$, improves when aggregate productivity improves, the model can instead deliver a countercyclical external finance premium, the intuition for which was described in Section 3. As shown in Figure 2, with $\nu > 0$, aggregate productivity disturbances shift the distribution of idiosyncratic outcomes in the same direction, without affecting its variance.[19]

Table 2 lists our baseline parameter settings. The preference and production parameters are standard in business cycle models. The value for entrepreneurs' discount factor, $\gamma = 0.947$, is taken from Carlstrom and Fuerst (1998), as are the financial market and agency cost parameter values. The labor market parameters are also all standard values in the recent DSGE labor search literature; in particular, although we allow in general for increasing or decreasing marginal costs of vacancy creation, our baseline setting $\chi = 1$ means that marginal costs of vacancy creation are constant, in line with virtually all of the recent quantitative search literature. We explore sensitivity with respect to $\chi$ below. We set a time-invariant labor income tax of 20 percent for our baseline experiments; another key dimension along which we assess the performance of the model below is for alternative tax, higher, tax rates. To avoid distorting the model economy further, we set the capital income tax rate to $\tau_t^k = 0 \ \forall t$. Thus, the single driving process of the model economy is aggregate (log) TFP, shocks to which have quarterly persistence $\rho_z = 0.95$, which is a standard

[19]These two assumptions together of course imply that the coefficient of variation of idiosyncratic shocks declines when aggregate TFP improves.

| Parameter Value | Description/Notes |
|---|---|
| | Preferences |
| $\beta = 0.99$ | Households' quarterly subjective discount factor |
| $\gamma = 0.947$ | Entrepreneurs' (additional) subjective discount factor |
| $\psi = 0.36$ | Leisure calibrating parameter (calibrated in baseline model) |
| | Production Technology |
| $\alpha = 0.36$ | Capital's share in production function |
| $\delta = 0.02$ | Depreciation rate of capital |
| | Financial Markets and Agency Costs |
| $\mu = 0.25$ | Per-unit monitoring cost |
| $\omega_m = 1$ | Long-run mean of idiosyncratic productivity |
| $\sigma_\omega = 0.207$ | Standard deviation of distribution of $\ln \omega$ |
| $\nu = 2$ | Elasticity of mean of idiosyncratic productivity with respect to aggregate TFP ($\ln \omega_{wt} = \nu \ln z_t$ ) |
| | Labor Markets |
| $\xi = 0.40$ | Share of unemployed in aggregate matching |
| $\theta = 0.30$ | Matching function calibrating parameter |
| $\chi = 1$ | Curvature of vacancy creation cost function |
| $\bar{\varrho} =$ | Vacancy cost calibrating parameter |
| $\rho = 0.10$ | Quarterly job separation rate |
| $\eta = 0.40$ | Nash bargaining power of workers (set to deliver Hosios efficiency) |
| $b = 1$ | Delivers unemployment benefits of 50 percent of after-tax wages |
| $\bar{l} = 0.66$ | Labor force participation rate (if exogenous) |
| | Tax Policy |
| $\tau^n = 0.20$ | Labor income tax rate (fixed) |
| $\tau^k = 0$ | Capital income tax rate (fixed) |
| | Exogenous Process |
| $\rho_z = 0.95$ | Quarterly persistence of log TFP process, $\ln z_{t+1} = \rho_z \ln z_t + \epsilon_t^z$, $\epsilon_t^z \sim N(0, \sigma_z)$ |
| $\sigma_z = 0.0023$ | Standard deviation of log TFP shock, calibrated to deliver GDP volatility of 1.6 percent |

Table 2: Parameter values for baseline quantitative analysis.

value in RBC models.

Given these parameter values, the remaining two parameters, the standard deviation $\sigma_z$ of the shocks to log-TFP and the elasticity parameter $\nu$, are selected via simulated method of moments (SMM). Using simulations based on first-order accurate decision rules, $\sigma_z$ and $\nu$ are selected to match a cyclical standard deviation of GDP of 1.7 percent, in line with the evidence presented in King and Rebelo (1999), and a contemporaneous correlation between the external finance premium and aggregate TFP of -0.50, in line with the evidence of Gomes, Yaron, and Zhang (2003) and De Graeve (2008).[20,21] The resulting values are $\nu = 2$, which matches that used by Faia and Monacelli (2007), and $\sigma_z = 0.0023$, which is much lower than the standard RBC calibration of $\sigma_z = 0.007$. The low calibrated value of shocks to TFP itself indicates that, given the rest of the parameterization, the model delivers much more amplification of TFP shocks than a baseline RBC model or a baseline DSGE labor search model.[22]

## 5.2 Steady-State Analysis

We compute the long-run deterministic (steady-state) equilibrium numerically using a nonlinear equation solver. We briefly consider two comparative static exercises with respect to financial-market parameters.[23] Figures 3 and 4 plot the long-run (steady-state) equilibria as we vary either the per-unit monitoring cost, $\mu$, or the standard deviation of the idiosyncratic productivity distribution, $\sigma_\omega$, which are two key long-run financial-market parameters of the model. Note that varying the elasticity $\nu$ has no effect on the steady state because long-run TFP is normalized to unity. All other parameters are held fixed at those shown in Table 2.

In Figure 3, the long-run responses are all quite intuitive. For example, the equilibrium loan size ($M - nw$) falls as monitoring becomes more costly, which causes total production to fall and the external finance premium to rise. Associated with the fall in borrowing and production is a decline in labor market activity, as unemployment rises, vacancy creation declines, employment declines, and wages decline. Quantitatively, however, the effects of variation in $\mu$ on long-run real

[20] The simulated method of moments procedure, as well as the dynamic results reported below, is conducted through numerical approximation of our model by linearization in levels around the deterministic steady state. The numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004).

[21] Depending on the exact measure of the external finance premium used, Gomes, Yaron, and Zhang (2003) show that the contemporaneous correlation between the external finance premium and aggregate TFP is between -0.40 and -0.70 (see their Figures 5 and 6). De Graeve (2008) reports correlations in the same range. Our SMM target is thus within empirical ranges.

[22] For example, in DSGE labor search models with perfect financial markets, Andolfatto (1996) requires $\sigma_z = 0.007$ in order to obtain GDP volatility of 1.5 percent (Andolfatto (1996, p. 121-123)), and Merz (1995) uses $\sigma_z = 0.007$ to obtain GDP volatility of 1.1 percent (Merz (1995, p. 281)).

[23] Appendix F considers one more comparative static exercise with respect to Nash bargaining power.

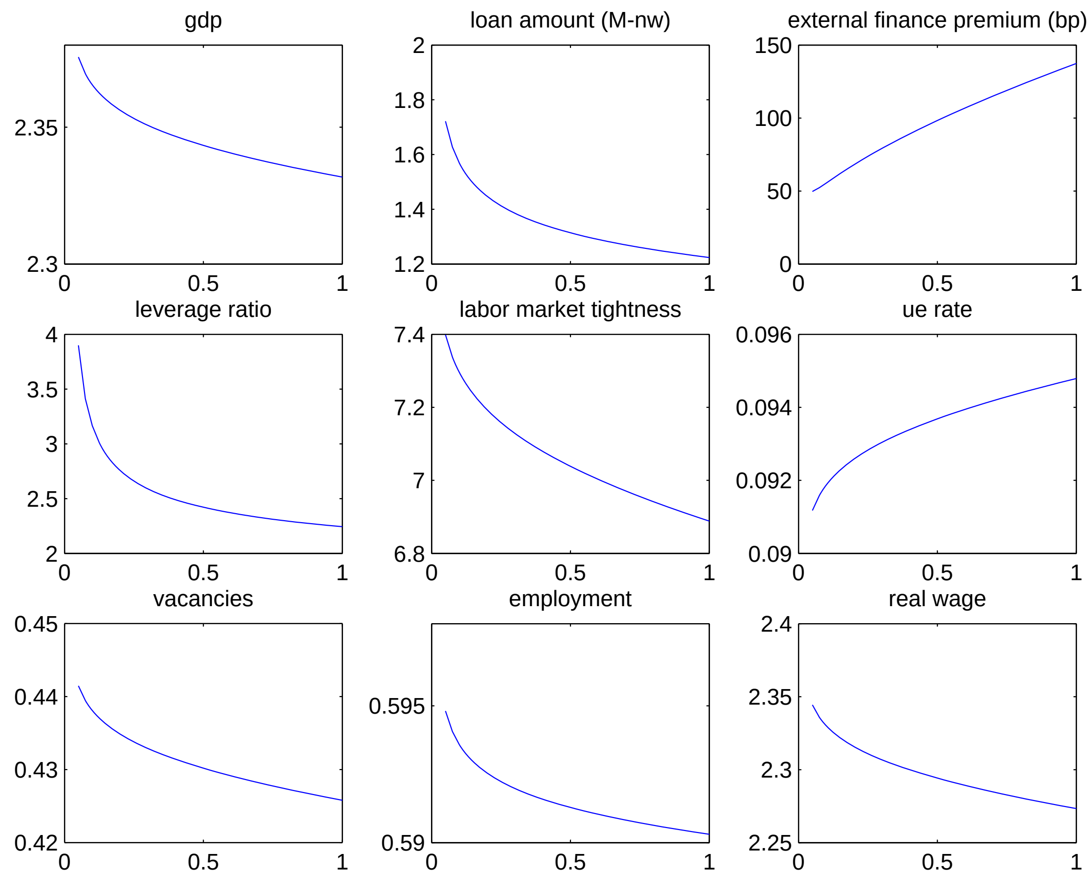


Figure 3: Long-run equilibrium as per-unit monitoring cost, $\mu$, varies; $\mu$ plotted on horizontal axis. All other parameters held fixed at their baseline values from Table 2. External finance premium reported in basis points.

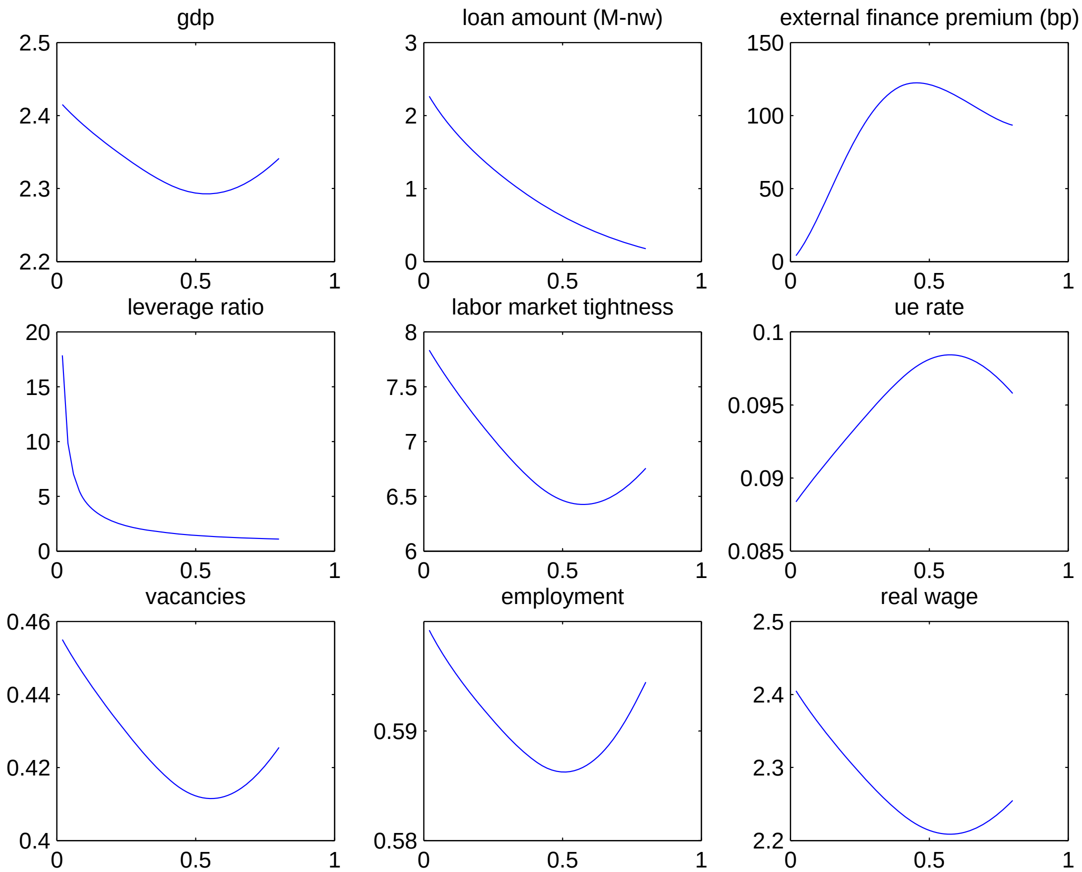


Figure 4: Long-run equilibrium as standard deviation of idiosyncratic productivity distribution, $\sigma_\omega$, varies; $\sigma_\omega$ plotted on horizontal axis. All other parameters held fixed at their baseline values from Table 2. External finance premium reported in basis points.

activity is not very large.

Figure 4 shows that the long-run response of the economy to changes in $\sigma_\omega$ is non-monotonic. For low dispersion of idiosyncratic productivity, GDP falls as dispersion rises, but for high dispersion, the comparative static result reverses. The nonmonotonicity is reflected in most of the other long-run responses shown in Figure 4, as well. This effect is not due to any nonmonotonicity of the contract terms, as the loan amount (upper middle panel) is strictly decreasing in $\sigma_\omega$, and the bankruptcy threshold $\bar{\omega}$ and hence bankruptcies (not shown) are strictly increasing in $\sigma_\omega$. Our parameterized value is $\sigma_\omega = 0.207$, hence our model's fluctuations will not cover the inflection point Figure 4 reveals; we leave to future investigation the nature of the nonmonotonicity.

## 5.3 Basic Business Cycle Dynamics

We begin assessing the dynamics of the model by first examining some standard business cycle statistics.[24] The baseline calibration described in Tables 1 and 2 is intended to broadly match salient aggregate quantity, financial, and labor market facts for the U.S. business cycle during periods of normal economic expansions and contractions. We interpret this broadly to mean the post-WWII period through mid-2007, before the onset of the current economic and financial downturn.

Table 3 presents basic business cycle statistics of the model.[25] The cyclical standard deviation of GDP is 1.7 percent, which was a target of the SMM procedure. Gross investment is over twice as volatile as GDP, also in line with, although a bit lower than, post-war U.S. business cycles. The correlation of investment with GDP is extremely high, similar to an RBC model. Consumption is slightly more than half as volatile as GDP, also in line with basic business cycle facts. Thus, along the dimension of second moments of the main macroeconomic aggregates, the baseline model performs comparably to the simple RBC model when driven by the single aggregate TFP shock. Recall, however, that the model delivers much more amplification because the calibrated shock volatility is a very low $\sigma_z = 0.0023$.

A dimension along which the RBC model is typically not assessed is the labor-force participation rate, perhaps because "labor supply" in the RBC model is not clearly differentiated into the participation margin, the extensive margin of employment, and the intensive margin of employment.[26] Our model, however, has clear notions of participation ($n + s$) and the extensive margin of employment ($n$). Business cycle labor-search models typically ignore the participation margin. Indeed, most of the empirical evidence suggests very small fluctuations along the participation margin at business-cycle timeframes. Elsby, Michaels, and Solon (2009) and many others argue that because cyclical adjustment of participation is so small, assuming no cyclical fluctuations in participation is a useful approximation in order to focus on other labor-market transitions. Empirical arguments such as this are often used in theoretical work to justify modeling the size of the labor force as fixed over the business cycle.

Arseneau and Chugh (2012), however, recently show that this modeling assumption may not be innocuous, at least for some business cycle issues. They compute a cyclical standard deviation of

[24] These business cycle statistics are generated by simulating the model 1000 times around the deterministic steady state equilibrium, with each simulation 1000 periods in length, and then computing the medians across simulations of standard deviations, correlations, etc.

[25] The first-order-accurate decision rules were used to simulate time-paths of the equilibrium in the face of draws of log-TFP, the shocks to which were drawn according to the parameters of the laws of motion described above. 500 simulations were conducted, each 200 periods long. For each simulation, we computed first and second moments and report the medians of these moments across the 500 simulations. The approximated decision rules are used

[26] See Hall (2009) for a recent effort along this line, with application to accounting for the cyclical movements of the labor wedge.

the U.S. participation rate of 0.32 percent.[27] While small, they demonstrate that a search model that matches this cyclical volatility of participation leads to dramatically different results in their particular application (studying optimal fiscal policy) than if the participation rate were assumed constant over the business cycle. In our baseline LFP model, the cyclical volatility of the participation rate, shown in the first row of Table 3, is 0.22 percent, in line with the data. Combined with the results of Arseneau and Chugh (2012), who also show that fluctuations of participation implied by a search model are in line with data, we interpret this result as support for building search models that accommodate a participation margin. A shortcoming of the participation dynamics of our model, however, is the strong countercyclicality with respect to GDP; the model's contemporaneous correlation is -0.99, whereas empirically, the cyclical correlation is 0.30. It is likely that adding other sources of shocks, in particular demand shocks, would improve this dimension of the model's predictions.

A clear Beveridge Curve, the negative relationship between unemployment ($s$) and vacancies ($v$), arises in the model: the contemporaneous correlation between $s$ and $v$ is -0.84, in line with the empirical evidence documented by Shimer (2005) and many others. The second row of Table 3 shows that the persistence of vacancies, at 0.84, is also in line with the data. However, in terms of an impulse response (which we present in Section 6), vacancies do not display any sluggishness in our model. This is in contrast to the empirical evidence presented by Fujita and Ramey (2007), who show that the peak empirical response of vacancies occurs several periods after a technology shock. The lack of a sluggish response of vacancies is also in contrast to the prediction of the model of Petrosky-Nadeau (2009). As Petrosky-Nadeau (2009) describes it, the sluggish response of vacancies in his model is a direct consequence of the sluggish response of the shadow value of external funds.

We conjecture that the reason for the contrast between our result and Petrosky-Nadeau's (2009) result along the vacancy sluggishness dimension lies in the difference between the output-model and the investment-model specifications of agency costs documented and described by Carlstrom and Fuerst (1998). One of the main findings of Carlstrom and Fuerst (1998) is that the propagation (in the sense of delayed peak impulse responses) of the output model is not as strong as that of the investment model (for instance, see their Figure 1). In our model, upfront financing needs afflict all of firms' costs. In Petrosky-Nadeau (2009), upfront financing needs afflict only vacancy posting costs, which constitutes investment spending on new job matches. Thus, it may not be surprising that the propagation properties identified by Carlstrom and Fuerst (1998) show through

[27] Tripier (2003, Table 2) computes a cyclical standard deviation of 0.32 percent for the *non*--participation rate in the U.S. economy over the period 1965-2000. Because the size of the participant and non-participant pools are not the same, Tripier's (2003) and Arseneau and Chugh's (2012) evidence does not have to reconcile exactly, but they are suggestively similar and, at the least, are on the same order of magnitude.

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| Std. dev. (%) | | 0.1484 | 0.1472 | 0.2787 | 0.0022 | 0.0081 | 0.0393 | 0.0172 |
| Auto. corr. | | 0.9684 | 0.8425 | 0.9259 | 0.9766 | 0.9976 | 0.9729 | 0.9647 |
| | $s$ | 1 | -0.8444 | -0.9840 | 0.9748 | -0.4349 | -0.9950 | -0.9703 |
| | $v$ | | 1 | 0.8936 | -0.7891 | 0.2655 | 0.8257 | 0.8172 |
| | $\theta$ | | | 1 | -0.9421 | 0.3770 | 0.9717 | 0.9547 |
| Corr. matrix | $lfp$ | | | | 1 | -0.6119 | -0.9559 | -0.9921 |
| | $c$ | | | | | 1 | 0.3709 | 0.6224 |
| | $i$ | | | | | | 1 | 0.9439 |
| | $gdp$ | | | | | | | 1 |

Table 3: Simulation-based business cycle statistics, baseline calibration.

to models with labor-search frictions, too. We have chosen to start with the apriori view, supported by empirical evidence, that a majority of firms' costs require upfront financing, which led us, as we discussed at the outset, to adopt the output-model specification, rather than choose a model based on the ex-post knowledge of how it will perform.

Finally, for reference, we note that the baseline calibration implies a long-run unemployment rate ($s/(s+n)$) of 9.2 percent. This value is not a target of the calibration procedure, rather simply the value that results from our parameterization. While higher than typical estimates of the natural rate of unemployment for the U.S. economy, if one adopts a broader view of unemployment to also include individuals loosely attached to the labor force and thus not strictly *outside* the labor force, this figure was in fact roughly 9 percent in the U.S. over the period 1994 to 2007.[28] In any case, in our subsequent experiments, we will be most interested in the change in the unemployment rate, rather than its level per se.

### 5.4 Cyclicality of the Finance Premium and Labor Market Dynamics

The central result of the paper is that the model does remarkably well in matching several cyclical labor market facts that have spawned much attention in the DSGE search literature since Shimer (2005).[29] In particular, the cyclical standard deviations of search unemployment, vacancies, and labor market tightness, at 15 percent, 15 percent, and 28 percent, respectively, are extremely similar to the empirical evidence familiar in the literature since Shimer (2005). In terms of volatility relative to GDP (which is 1.7 percent), these values are, respectively, roughly 8.5, 8.5, and 16, again very

[28] See Alvarez and Shimer (2011), in particular the data they cite in their footnote 18.

[29] This literature, which has by now become familiar to virtually all macroeconomists, is far too vast to list here.

much in line with data.[30] Thus, the model does well along the labor market dimension even though it is not directly calibrated to do so.

The crucial element of the model that allows it to perform so well along this dimension is $\nu$, which is the elasticity of the mean of idiosyncratic productivity to aggregate TFP. As described above, the SMM target for this parameter was a contemporaneous correlation of -0.5 between the external finance premium and GDP; the SMM target was *not* any aspect of labor market dynamics. This procedure led to $\nu = 2$, which means that a one-percent rise in aggregate TFP leads to a two-percent rise in the mean of the distribution of firms' idiosyncratic productivity.

To understand how the model works, suppose instead of the calibrated positive technology spillover that there is no spillover at all — that is, consider $\nu = 0$. Table 4 reports business cycle statistics of the model using this alternative setting for $\nu$, with all other parameters held at their baseline values. The overall volatility of the model, as measured by the volatility of GDP, falls by roughly half. This decline is due to a direct productivity channel of amplification, discussed more fully immediately below.[31]

However, the cyclical volatilities of $s$, $v$, and $\theta$ all fall by a much larger factor of five compared to the results in Table 3. This decline reflects changes in the cyclical properties of financial conditions, also discussed more fully immediately below. We can thus conclude that the difference in results between Table 2 and Table 3 is attributable entirely to the positive spillover from aggregate TFP to idiosyncratic productivity, rather than to any of the many existing proposed resolutions of the Shimer puzzle.[32]

In terms of quantitatively parsing the results into the two effects — the direct productivity effect and the financial conditions effect — 60 percent of the model's amplification operates through the financing effect, and 40 percent operates through the productivity effect. I conclude this because the volatility of the overall economy (as measured by the volatility of GDP) doubles in moving from $\nu = 0$ to $\nu = 2$. This by itself (due to the linear approximation) would double the volatility of variables such as $s$, $v$, and $\theta$. The remaining 60 percent of the amplification effect on these variables is thus attributable to changes in financial conditions.

I now discuss the two effects that the positive spillover from aggregate TFP to firm-specific productivity induce: the direct productivity effect and the effect on financial conditions. Consider first the direct productivity effect. Firm $i$'s productivity is $\omega_{it} z_t$, which is obviously composed

[30] See, among others, Elsby, Michaels, and Solon (2009) for the most recent empirical evidence on the dynamics of labor-market fluctuations, as well as Gertler and Trigari (2009), Petrosky-Nadeau (2009), Barnichon (2009), Fujita and Ramey (2007), or any of a vast number of other recent studies.

[31] This can be easily corrected by simply doubling $\sigma_z$, which, recall, was very low in the baseline calibration.

[32] For instance, government-provided unemployment benefits are about 50 percent of after-tax wages under both calibrations, rather than a Hagedorn and Manovskii (2008)-type of calibration that features a much higher replacement rate and is one of the most widely-understood means for addressing the Shimer puzzle.

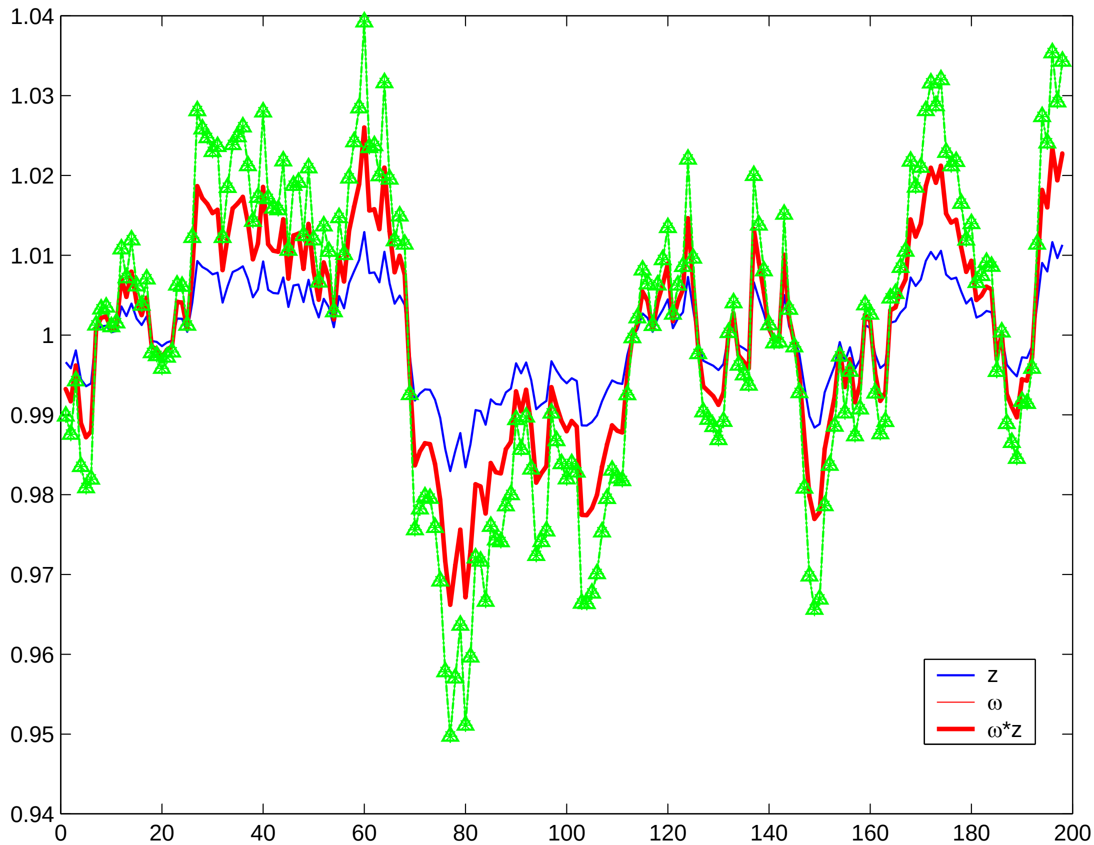


Figure 5: Dynamics of aggregate TFP ($z$), average idiosyncratic productivity ($\bar{\omega}$), and total average productivity ($\bar{\omega}z$), with positive spillover from aggregate TFP to idiosyncratic productivity ($\nu = 2$). Scale is gross percentage deviations from steady state.

of both the aggregate component and an idiosyncratic component. An increase in $z$ that also induces an increase in $\omega$ thus causes a direct productivity effect, leading to a stronger response of firm activity, including hiring activity, than for the same size increase in $z$ absent the positive productivity correlation. Part of the difference in results — 40 percent of the difference, as noted above — between Table 3 and Table 4 is thus simply because effective productivity $\omega_{it}z_t$, which is what is relevant for aggregate production, fluctuates more sharply under $\nu = 2$ than $\nu = 0$. Figure 5 plots the different concepts of productivity from a representative simulation of the model.

However, if the productivity effect were the only channel, the standard deviations of all endogenous variables would scale by the same factor as the scaling of the standard deviation of GDP.[33]

[33]This is both because it is well-known that the RBC model is very nearly linear in its state variables — making it reasonable to conjecture that our model also would behave fairly linearly — and because of the linear solution method we use to approximate the model.

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| Std. dev. (%) | | 0.0296 | 0.0270 | 0.0555 | 0.0005 | 0.0058 | 0.0217 | 0.0094 |
| Auto. corr. | | 0.9649 | 0.8426 | 0.9203 | 0.9659 | 0.9863 | 0.9478 | 0.9524 |
| Corr. matrix | $s$ | 1 | -0.9257 | -0.9833 | 0.8952 | -0.6679 | -0.9962 | -0.9402 |
| | $v$ | | 1 | 0.9776 | -0.7868 | 0.5557 | 0.9527 | 0.8755 |
| | $\theta$ | | | 1 | -0.8599 | 0.6262 | 0.9946 | 0.9274 |
| | $lfp$ | | | | 1 | -0.9220 | -0.8931 | -0.9836 |
| | $c$ | | | | | 1 | 0.6699 | 0.8692 |
| | $i$ | | | | | | 1 | 0.9468 |
| | $gdp$ | | | | | | | 1 |

Table 4: Simulation-based business cycle statistics, no spillover between aggregate TFP and idiosyncratic productivity.

As noted above and as Tables 3 and 4 make clear, the volatilities of the central labor-market variables $s$, $v$, and $\theta$ all scale by a factor of five, not the factor of two by which the volatility of GDP scales. This additional amplification is due to the effect of the positive productivity correlation on financing conditions. In particular, the technology spillover causes the external finance premium to behave countercyclically, which is the source of the additional amplification.

The intuition behind why a sufficiently positive spillover from aggregate TFP to the mean of idiosyncratic productivity can cause a decline in the external finance premium is as discussed in Section 3. Holding constant the contractually-specified bankruptcy threshold $\bar{\omega}_t$, if the distribution $\Phi(\omega, z_t)$ shifts to the right (holding constant its variance), each firm has a greater chance of drawing idiosyncratic productivity $\omega_{it} > \bar{\omega}_t$, hence has a lower probability of going bankrupt. A lower probability of bankruptcy manifests itself as a lower external finance premium because of reduced monitoring costs and the competitive forces at work on lenders' side of financial markets.[34] This effect seems similar to what Petrosky-Nadeau (2009) terms the "cost channel" of vacancy postings, although we parse this mechanism further below. In our model, with $\nu = 0$, neither the productivity effect nor the financial conditions effect are present and, as Table 4 shows, the model displays the Shimer puzzle.

More than just failing to reproduce labor-market fluctuations, though, $\nu = 0$ leads to a contemporaneous cyclical correlation in the model between the finance premium and GDP of 0.70. This value, which is opposite the empirical evidence, is in line with, though a bit higher than, that calculated by Gomes, Yaron, and Zhang (2003, Figure 5 and Figure 6) for a prototypical agency

[34]Recall the background assumption, following Carlstrom and Fuerst (1997, 1998) and the subsequent literature, that lending is a perfectly-competitive activity with costless entry.

cost model.[35] Indeed, a shortcoming of the standard agency-cost model well-known since Bernanke, Gertler, and Gilchrist (1999) and Gomes, Yaron, and Zhang (2003) is its counterfactual prediction of a procyclical finance premium. A countercyclical finance premium is instead, in the words of Bernanke, Gertler, and Gilchrist (1999, p. 1371), "the essence of the financial accelerator."

In this regard, our model's mechanism also seems similar to that of Petrosky-Nadeau (2009), who shows that if vacancy creation costs — in his model, only vacancy creation costs — are subject to financing frictions, the effects of technology shocks on labor market dynamics are amplified through a countercyclical finance premium. The intuition in Petrosky-Nadeau's (2009) model is that a countercyclical finance premium makes the direct costs of vacancy creation ($\frac{\varrho'(v_t)p_t}{p^f(\theta_t)}$ in our notation) countercyclical, which leads to a sharper rise in vacancy creation in good times than if financing frictions did not impinge directly on creation costs. Indeed, in our baseline model with $\nu = 2$, the contemporaneous correlation of $p_t$, which is another indicator of the tightness of financing conditions, with GDP is -0.98, whereas with $\nu = 0$ this correlation is 0.22. Thus, in the sense that the countercyclicality of the tightness of financing conditions is important for firms' incentives to create job openings, our model mechanism is similar to Petrosky-Nadeau (2009).

## 5.5 The Scope of Working Capital Requirements

However, unlike Petrosky-Nadeau (2009), working capital requirements that directly affect vacancy costs are not crucial for the amplification mechanism. What is crucial is the countercyclical finance premium induced by the positive spillover from aggregate TFP to idiosyncratic productivity. To highlight this, we take an alternative view that it is only ongoing operational costs — wage costs and physical capital rental costs — that are subject to working capital requirements, and assume that firm's recruitment costs can be paid for out of current revenue. That is, suppose now that it is only the operational expenses

$$M_{it}^{O} = w_{it}n_{it} + r_t k_{it} \tag{42}$$

that are subject to agency frictions, rather than $M_{it} = M_{it}^{O} + \varrho(v_{it})$ as in (8). It is straightforward to show that the only equilibrium conditions that are affected are the contractually-specified loan size

$$M_t^{O} = \frac{nw_t}{1 - p_t g\left(\bar{\omega}_t\right)}, \tag{43}$$

which shows vacancy costs no longer require loans; the vacancy creation condition, which now reads

$$\frac{\varrho'(v_t)}{p^f(\theta_t)} = \omega_{mt} z_t F_n(k_t, n_t) - p_t w_t + (1-\rho) E_t \left\{ \Xi_{t+1|t} \frac{\varrho'(v_{t+1})}{p^f(\theta_{t+1})} \right\}; \tag{44}$$

[35] Note that Gomes, Yaron, and Zhang (2003) use the "investment model" of Carlstrom and Fuerst (1997), while, for reasons discussed in the introduction, we use the "output model" of Carlstrom and Fuerst (1998), which may also drive some of the difference.

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| Std. dev. (%) | | 0.1229 | 0.1208 | 0.2322 | 0.0012 | 0.0079 | 0.0387 | 0.0161 |
| Auto. corr. | | 0.9695 | 0.8307 | 0.9230 | 0.9783 | 0.9972 | 0.9704 | 0.9633 |
| | $s$ | 1 | -0.8640 | -0.9810 | 0.8812 | -0.4055 | -0.9969 | -0.9651 |
| | $v$ | | 1 | 0.9218 | -0.6307 | 0.2424 | 0.8582 | 0.8377 |
| | $\theta$ | | | 1 | -0.7983 | 0.3433 | 0.9763 | 0.9496 |
| Corr. matrix | $lfp$ | | | | 1 | -0.7274 | -0.8563 | -0.9279 |
| | $c$ | | | | | 1 | 0.3421 | 0.6077 |
| | $i$ | | | | | | 1 | 0.9419 |
| | $gdp$ | | | | | | | 1 |

Table 5: Simulation-based business cycle statistics, baseline calibration, with vacancy creation costs *not* subject to financing constraints.

and the Nash wage outcome, which modifies to

$$\begin{aligned} w_t = \eta \frac{\omega_{mt} z_t F_n(k_t, n_t)}{p_t} + (1-\eta) \frac{h'(1 - s_t^h - n_t^h)}{u'(c_t)(1-\tau_t^n)} \\ - \quad \eta \frac{1-\rho}{1-\tau_t^n} \left( \Xi_{t+1|t} \frac{1-\tau_{t+1}^n}{p_{t+1}} \frac{\varrho'(v_{t+1})}{p^f(\theta_{t+1})} \right) + \frac{\eta(1-\rho)}{p_t} \left( \Xi_{t+1|t} \frac{\varrho'(v_{t+1})}{p^f(\theta_{t+1})} \right). \end{aligned} \tag{45}$$

These should be compared with conditions (10), (17) and (20), respectively. In the latter two, the difference with respect to the previous equilibrium expressions is simply that marginal vacancy creation costs $\varrho'(v)$ are *not* multiplied by $p$ if vacancy costs are not subject to financing frictions. This separation removes any direct effect of financing conditions on vacancy creation costs.

Table 5 presents simulation-based business cycle statistics for this version of the model economy, retaining the parameter $\nu = 2$ that governs the spillover between aggregate and idiosyncratic productivity. Comparing the results in Table 5 with the baseline results in Table 3 shows that the volatilities of $s$, $v$, and $\theta$ are all slightly lower if vacancy costs are not subject to financing frictions; however, they are still clearly quite volatile and similar to the empirical dynamics of U.S. labor markets. Thus, what Petrosky-Nadeau (2009) terms the unit-cost-channel of amplification seems to not be the necessary part of the transmission mechanism of technology shocks to labor markets as amplified through credit market frictions.

While such a channel does impart volatility to labor-market quantities, the bulk of the amplification arises though credit frictions on firms' costs in total and the positive link between aggregate TFP and idiosyncratic productivity. When we repeat the experiment underlying Table 5 except with $\nu = 0$, we find dynamics virtually identical to those in Table 4.

| | Baseline ($\nu = 2$) | No spillover ($\nu = 0$) | Vacancy costs not subject to financing |
|---|---|---|---|
| Std. dev. (%) | 0.0057 | 0.0091 | 0.0101 |
| Auto. corr. | 0.9738 | 0.9773 | 0.9720 |
| Corr($w, y$) | 0.9365 | 0.9038 | 0.9402 |
| Corr($w, z$) | 0.7910 | 0.8471 | 0.8417 |

Table 6: Wage dynamics in baseline model.

### 5.6 Wage Dynamics

As mentioned above and as the recent DSGE matching literature has emphasized (see Rogerson and Shimer (2011) for a comprehensive summary), real wage dynamics play an important role in dynamics. This role also arises in our model. In the baseline economy with technology spillovers, the volatility of real wages is less than two-thirds the volatility of real wages in the economy with spillovers. The first row of Table 6 documents this. Thus, associated with the model's magnification of TFP shocks to labor market dynamics is a type of wage rigidity, the first proposed explanation of the Shimer (2005) puzzle offered by Hall (2005).

However, wage rigidity is not the only source of the magnification in our model. Comparing the third column of Table 6 with the first column of Table 6 shows this. In the economy with technology spillovers but in which vacancy creation is not subject to financing frictions, the volatility of real wages is nearly twice as high as in the baseline economy. Nonetheless, as Table 5 showed, this economy nonetheless features volatile labor market fluctuations.

## 6 A Large Adverse Shock

With the basic business cycle properties of the model established, we now turn to an experiment in which a large adverse shock hits the economy. These experiments are conducted using the baseline model in which aggregate TFP shifts the idiosyncratic productivity distribution. The large adverse shock is a 4-standard-deviation one-time negative shock to aggregate TFP; this size shock is chosen because it causes GDP in the model to reach a peak negative response of 2 percent below the steady state, which is about the size of the decline in quarterly GDP the U.S. economy experienced between the third and fourth quarters of 2008 and again between the fourth quarter of 2008 and first quarter of 2009.[36] Our main interest is in assessing whether the model, when hit with this large adverse shock, does a reasonable quantitative job in describing some of the U.S. economy's

[36]Because our model is quarterly, we take quarter-to-quarter changes in GDP as the right metric to which to calibrate the large adverse shock.

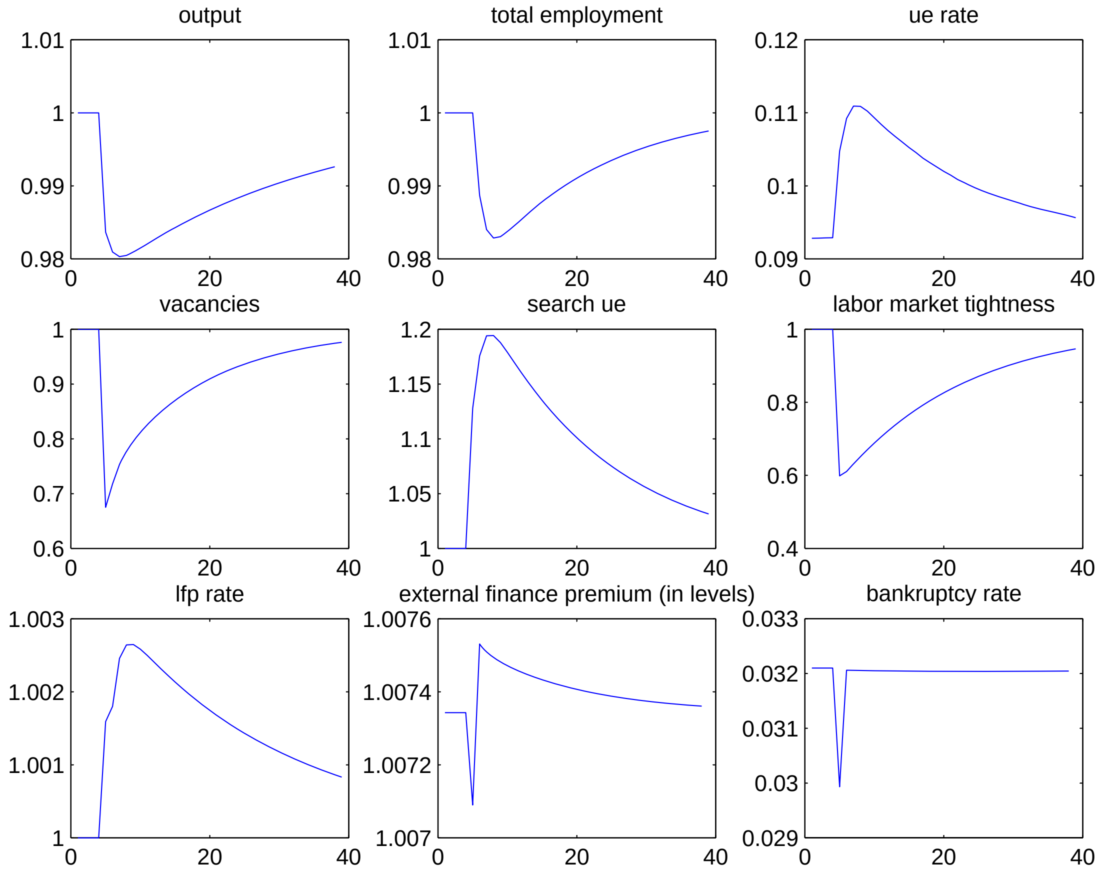


Figure 6: Impulse response of selected aggregate, labor-market, and financial-market variables in response to a one-time negative shock to aggregate TFP. Except where noted, scale is gross percentage point deviation from steady state.

ensuing labor market dynamics.

Figure 6 plots the responses of GDP, labor-market variables, and financial-market variables. The peak response of total employment (upper middle panel) is a roughly 1.75 percent decline, and the peak response of the unemployment rate (upper right panel) is a nearly 2 percentage point rise, from 9.2 percent to 11.1 percent. Comparing these model results to data, between July 2008 and October 2008, total nonfarm employment in the U.S. fell by 0.65 percent; between October 2008 and January 2009, it fell by 1.5 percent; and between January 2009 and April 2009, it fell by 1.4 percent. In the model, the peak response of total employment occurs two periods after the shock. Cumulating the one-quarter empirical changes over two-quarter windows shows that our model's peak employment decline of 1.75 percent is a bit below the roughly 2 to 3 percent decline observed in the U.S. between the fall of 2008 and spring of 2009. A natural mechanism to introduce into our model to improve this dimension of the model's fit would be endogenous employment separations,

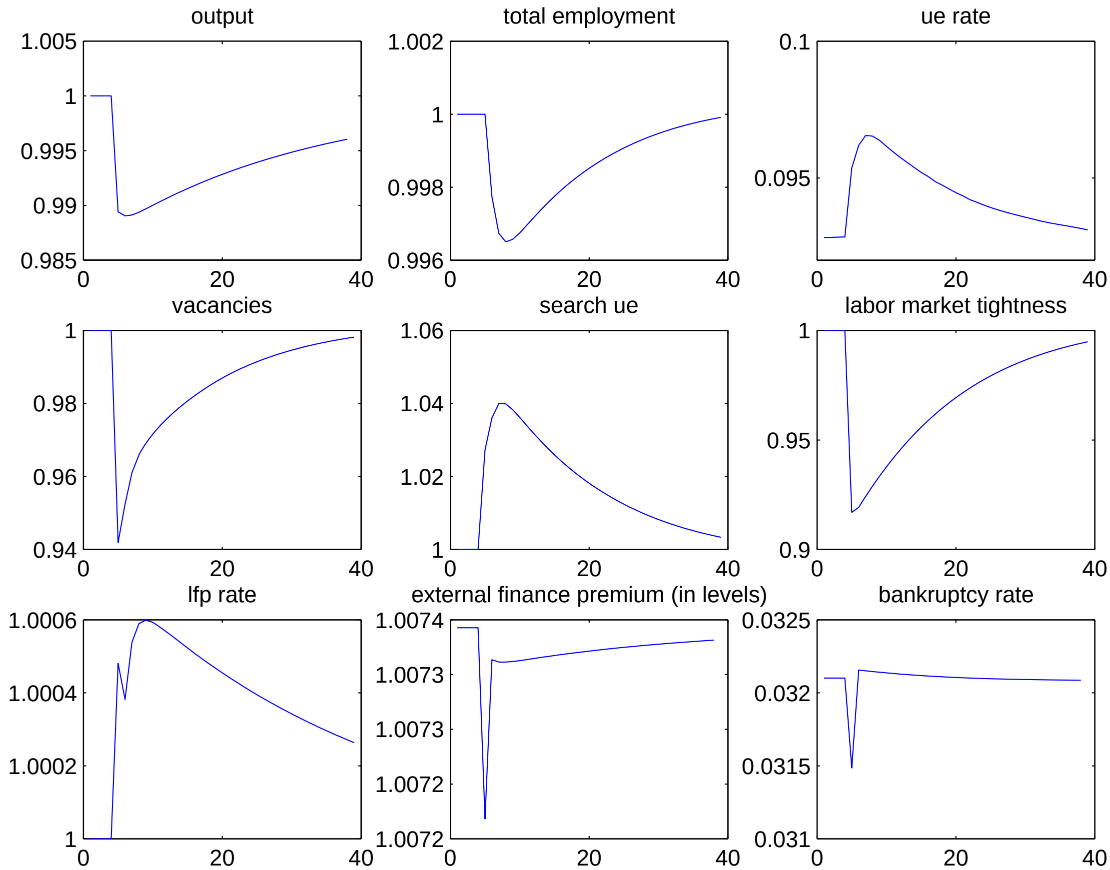


Figure 7: Impulse response of selected aggregate, labor-market, and financial-market variables in response to a one-time negative shock to aggregate TFP, no link between aggregate TFP and distribution of idiosyncratic productivity. Except where noted, scale is gross percentage point deviation from steady state.

along the lines of Mortensen and Pissarides (1994). We know from such studies that a negative aggregate TFP shock would cause a burst in endogenous separations.

In terms of comparing the model's predicted rise in the unemployment rate with the data, the shortfall is even larger. The U.S. unemployment rate rose from 6.2 percent in September 2008 to 8.9 percent in April 2009, and further still to 9.4 percent in May 2009. The roughly 3 percentage point increase in the unemployment rate is thus larger than the model's prediction of a 2 percentage point increase in the unemployment rate. As with the employment response, this dimension of the model's predictions would also be improved by introducing endogenous job separations.

For comparison, we also plot in Figure 7 the same impulse responses under the alternative parameterization of no spillovers between aggregate TFP and idiosyncratic productivity, $\nu = 0$. The impulse responses are all muted compared to the $\nu = 2$ case, which is consistent with the simulation results presented in Tables 3 and 4. Comparing the middle bottom panels of Figures 6 and 7 again makes clear the importance of a counter- versus pro-cyclical finance premium for amplification of aggregate TFP shocks to labor-market quantities.

# 7 Further Quantitative Experiments

In this section, we conduct several robustness experiments of interest. We return to the baseline model specification, which features both a positive effect of aggregate TFP on the distribution of idiosyncratic productivity as well as vacancy creation costs that are subject to financing frictions. In the following three sets of experiments, we vary one feature of the baseline model at at a time.

## 7.1 A "European Experiment:" High Labor Income Taxes

An issue currently under discussion by policy-makers and the press is whether the credit-market-mediated response of labor markets to shocks may be more violent in high-tax economies than in low-tax economies. For example, the steady-state distortion implied by a high average tax rate in Europe is thought by some to be important in the larger labor market fluctuations experienced in European economies during the financial market crisis of 2007-2009 than in the U.S. In this subsection, we offer an assessment of this transmission mechanism by setting a high labor income tax rate in our baseline model.

Specifically, instead of the 20 percent tax rate of our baseline model, consider a "European experiment" in which the labor income tax is constant at 45 percent, which is on the low end of the range of estimates provided by Prescott (2004, Table 2).[37] Table 7 presents simulated second moments for this high-tax version of the model. Comparing the results to those in Table 3 shows that high average tax rates in and of themselves do lead to significant amplification of shocks into labor-market quantities. The standard deviations of $s$, $v$, and $\theta$ all rise by about 50 percent compared to the low-tax economy in Table 3, while the volatility of GDP increases only slightly, from 1.7 percent to 2 percent.

For brevity, we do not report wage dynamics in the table and instead only briefly discuss them. In the high-tax economy, the standard deviation of the real wage is 1.9 percent, its autocorrelation is 0.99, its contemporaneous correlation is a very low 0.10, and its contemporaneous correlation with aggregate TFP is also a very low 0.17. Compared to the results reported in Table 6, wage dynamics are very different than in the baseline low-tax economy; in particular, wages are much

[37] Prescott's (2004, Table 2) estimate of "taxes" is really a computation of "labor wedges," which includes distortions due to taxation as well as any structural, institutional, or other policy-induced distortions. Because we explicitly model some structural and institutional frictions — labor market frictions and credit market frictions — and mean for $\tau^n$ to represent only taxation, we use the lower end of his range of estimates.

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| Std. dev. (%) | | 0.2275 | 0.2543 | 0.4370 | 0.0152 | 0.0083 | 0.0542 | 0.0202 |
| Auto. corr. | | 0.9547 | 0.8229 | 0.9041 | 0.9494 | 0.9972 | 0.9740 | 0.9585 |
| | $s$ | 1 | -0.7876 | -0.9841 | 0.9670 | 0.1092 | -0.8043 | -0.7531 |
| | $v$ | | 1 | 0.8314 | -0.7845 | -0.1143 | 0.5735 | 0.5844 |
| | $\theta$ | | | 1 | -0.9647 | -0.1252 | 0.7577 | 0.7353 |
| Corr. matrix | $lfp$ | | | | 1 | 0.3459 | -0.6294 | -0.5700 |
| | $c$ | | | | | 1 | 0.4714 | 0.5546 |
| | $i$ | | | | | | 1 | 0.9812 |
| | $gdp$ | | | | | | | 1 |

Table 7: Simulation-based business cycle statistics, baseline calibration with high labor income tax rate, $\tau^n = 0.45$.

more volatile, as measured by its standard deviation. The result that labor-market quantities are nonetheless highly volatile further demonstrates that wage dynamics are not the core mechanism by which the model's transmission mechanism works.

Impulse response analysis shows that a negative TFP shock that causes a peak decline of GDP of 3 percent causes a gradual rise in the unemployment rate from a long-run average of 13 percent to 18 percent. High taxes thus lead to both higher long-run unemployment, which is intuitive and squares with the results of studies such as Prescott (2004) and Ljungqvist and Sargent (2004); but also to amplified responses of labor-market quantities to shocks. This indicates that there may be something to the sense amongst some policy-makers that labor-markets are more elastic with respect to shocks in high-tax environments than in low-tax environments.

While we believe the direction of the change in results compared to the baseline economy is correct, because of the extremely violent responses of labor markets to business cycle shocks in the high-tax economy, the numerical results in the high-tax economy should be viewed as illustrative, rather than as precise quantitative statements. The extreme response of the labor market suggests a higher-order numerical solution is called for in order to further assess the model's dynamics.[38] We leave a richer quantitative investigation of the model for future work.

[38] The results of our baseline analysis, in which the volatility of labor-market quantities was on the order of 20 percent, may also suggest something richer than a linear approximation would be useful. The numerical results in the high-tax economy make this need even sharper, though they nonetheless illustrate the model's amplification mechanism.

### 7.2 Non-Constant Returns in Recruiting Technology

In the quantitative analysis so far, we have assumed a fixed per-unit cost of vacancy creation, in keeping with virtually all of the labor-search literature. Here, we briefly explore how the baseline results are affected by the presence of curvature in the vacancy creation cost function.

Suppose now vacancy posting costs, $\varrho(v) = \bar{\varrho}v^{\chi}$, are nonlinear. Consider first the case of $\chi = 2$. Convex creation costs, which reflect a recruiting technology that displays diminishing marginal product, mean that marginal creation costs are increasing in vacancies. In turn, this should dampen fluctuations in vacancy creation and hence labor market tightness. The results in the upper panel of Table 8, in which all parameters, including the technology spillover parameter $\nu = 2$, are held fixed from the baseline model, confirm this intuition.

In the lower panel of Table 8, we consider the alternative case of concave vacancy creation costs, which reflects an increasing-returns-to-scale recruiting technology. Concave creation costs mean that marginal creation costs are decreasing, leading to the natural conjecture that vacancy fluctuations, and hence fluctuations in market-tightness, will be amplified. Indeed, this is one of the main points of Pissarides (2009), as well, who argues that "wage rigidity" per se is likely not the right strategy to pursue in addressing the Shimer puzzle. The results in Table 8 confirm this intuition. We limit the concavity to $\chi = 0.75$ because for smaller values we found incredibly large variations in labor-market quantities, even larger than those reported in the table. Because our numerical method is a linearization and thus cannot accurately portray highly volatile model dynamics, there is little reason to consider such extreme parameterizations without moving to higher-order approximations. This issue arises in the subsequent analysis, as well.

The experiments in this section illustrate the point that the convexity of firms' recruitment technology is important for the quantitative degree of amplification of TFP shocks to labor-market quantities. The importance of the shape of firms' average cost functions for posting vacancies is indeed one of the main points of Pissarides (2009), but we have chosen to focus our main analysis on the fixed-average- cost case in order to highlight the role of financing frictions and because it is directly comparable to the vast majority of existing work in the search and matching literature.

### 7.3 Fixed Labor Force Participation

All of the results presented thus far in this paper have been for a specification of the model in which participation is always optimal over the business cycle. The participation rate can instead be fixed to assess the consequences of the more common assumption in business cycle search models of an exogenous and constant participation rate. As shown in Appendix A, the setup can easily be modified to handle a fixed participation rate.

Table 9 presents results of simulations of the model with fixed participation. The volatilities

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| | | | | Convex vacancy creation costs ($\chi = 2$) | | | | |
| Std. dev. (%) | | 0.0577 | 0.0562 | 0.1101 | 0.0028 | 0.0075 | 0.0385 | 0.0135 |
| Auto. corr. | | 0.9820 | 0.8821 | 0.9548 | 0.9464 | 0.9968 | 0.9553 | 0.9673 |
| | $s$ | 1 | -0.8805 | -0.9728 | -0.9341 | -0.4685 | -0.9624 | -0.9532 |
| | $v$ | | 1 | 0.9609 | 0.9647 | 0.2830 | 0.9617 | 0.8923 |
| | $\theta$ | | | 1 | 0.9815 | 0.3890 | 0.9949 | 0.9551 |
| Corr. matrix | $lfp$ | | | | 1 | 0.2144 | 0.9952 | 0.8826 |
| | $c$ | | | | | 1 | 0.2994 | 0.6367 |
| | $i$ | | | | | | 1 | 0.9208 |
| | $gdp$ | | | | | | | 1 |
| | | | | Concave vacancy creation costs ($\chi = 0.75$) | | | | |
| Std. dev. (%) | | 0.2739 | 0.3287 | 0.5160 | 0.0095 | 0.0092 | 0.0433 | 0.0233 |
| Auto. corr. | | 0.9617 | 0.7804 | 0.8947 | 0.9683 | 0.9979 | 0.9363 | 0.9515 |
| | $s$ | 1 | -0.6624 | -0.9757 | 0.9858 | -0.4567 | -0.9614 | -0.9752 |
| | $v$ | | 1 | 0.7267 | -0.5916 | 0.2277 | 0.5365 | 0.6600 |
| | $\theta$ | | | 1 | -0.9257 | 0.4024 | 0.8790 | 0.9612 |
| Corr. matrix | $lfp$ | | | | 1 | -0.5052 | -0.9910 | -0.9596 |
| | $c$ | | | | | 1 | 0.4519 | 0.6193 |
| | $i$ | | | | | | 1 | 0.9150 |
| | $gdp$ | | | | | | | 1 |

Table 8: Simulation-based business cycle statistics, baseline calibration with nonlinear vacancy creation costs. Upper panel shows results for convex ($\chi = 2$) vacancy creation costs, and lower panel shows results for concave ($\chi = 0.75$) vacancy creation costs.

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| Std. dev. (%) | | 0.1068 | 0.1155 | 0.2125 | 0 | 0.0094 | 0.0438 | 0.0179 |
| Auto. corr. | | 0.9698 | 0.8203 | 0.9136 | — | 0.9976 | 0.9738 | 0.9678 |
| | $s$ | 1 | -0.8641 | -0.9729 | — | -0.4901 | -0.9990 | -0.9521 |
| | $v$ | | 1 | 0.9400 | — | 0.3181 | 0.8477 | 0.8147 |
| | $\theta$ | | | 1 | — | 0.4201 | 0.9635 | 0.9228 |
| Corr. matrix | $lfp$ | | | | 1 | — | — | — |
| | $c$ | | | | | 1 | 0.4795 | 0.7200 |
| | $i$ | | | | | | 1 | 0.9445 |
| | $gdp$ | | | | | | | 1 |

Table 9: Simulation-based business cycle statistics, baseline calibration with constant labor force participation rate.

of $s$, $v$, and $\theta$ are all slightly lower than in the case of endogenous participation (compare again to Table 3), demonstrating that the small fluctuations in the participation rate that occur in the baseline model do in fact contribute to the volatility of the labor market. Labor-market volatility in the fixed-participation model is similar in quantitative magnitude to that in the baseline model in which vacancy costs were not subject to financing costs (compare Table 9 with Table 5). Overall, then, endogenous fluctuations along the participation margin in our model have similar quantitative effects as the inclusion of vacancy costs in financing constraints, although this comparative result is surely a calibration-specific one.

### 7.4 Bargaining Power

The quantitative results so far have been for the case in which workers' bargaining power, $\eta$, is equal to their elasticity in the aggregate matching function, $\xi$, as shown in Table 2. This baseline parameterization satisfies the Hosios (1990) condition for search efficiency, which is well understood in search and matching models. Search efficiency obtains when the societal contribution of searching individuals to the aggregate job-formation process equals workers' share of the match surplus, thus internalizing all search externalities; the parameter setting $\eta = \xi$ achieves exactly this, as Hosios (1990) showed.

For the interested reader, Table 10 displays the model's results for alternative values of bargaining power. Holding fixed $\xi$, the upper panel in Table 10 presents results for a parameterization in which worker bargaining power is inefficiently low ($\eta = 0.20$), and the lower panel in which worker bargaining power is inefficiently high ($\eta = 0.70$).

| | | $s$ | $v$ | $\theta$ | $lfp$ | $c$ | $i$ | $gdp$ |
|---|---|---|---|---|---|---|---|---|
| | | | | Low worker bargaining power ($\eta = 0.20$) | | | | |
| Std. dev. (%) | | 0.1071 | 0.0998 | 0.1987 | 0.0022 | 0.0063 | 0.0324 | 0.0139 |
| Auto. corr. | | 0.9696 | 0.8334 | 0.9267 | 0.9207 | 0.9879 | 0.9702 | 0.9587 |
| | $s$ | 1 | -0.8795 | -0.9827 | -0.9508 | -0.1993 | -0.9953 | -0.9694 |
| | $v$ | | 1 | 0.9338 | 0.9288 | 0.0529 | 0.8703 | 0.8682 |
| | $\theta$ | | | 1 | 0.9797 | 0.1360 | 0.9756 | 0.9612 |
| Corr. matrix | $lfp$ | | | | 1 | -0.0523 | 0.9610 | 0.8892 |
| | $c$ | | | | | 1 | 0.1185 | 0.3920 |
| | $i$ | | | | | | 1 | 0.9414 |
| | $gdp$ | | | | | | | 1 |
| | | | | High worker bargaining power($\eta = 0.70$) | | | | |
| Std. dev. (%) | | 0.1377 | 0.1585 | 0.2810 | 0.0112 | 0.0084 | 0.0584 | 0.0197 |
| Auto. corr. | | 0.9638 | 0.8242 | 0.9061 | 0.9606 | 0.9966 | 0.9731 | 0.9653 |
| | $s$ | 1 | -0.8537 | -0.9730 | 0.9573 | 0.0138 | -0.8527 | -0.7423 |
| | $v$ | | 1 | 0.9256 | -0.8499 | -0.0613 | 0.6831 | 0.6213 |
| | $\theta$ | | | 1 | -0.9498 | -0.0416 | 0.8046 | 0.7153 |
| Corr. matrix | $lfp$ | | | | 1 | 0.2893 | -0.6685 | -0.5275 |
| | $c$ | | | | | 1 | 0.4945 | 0.6441 |
| | $i$ | | | | | | 1 | 0.9775 |
| | $gdp$ | | | | | | | 1 |

Table 10: Simulation-based business cycle statistics, baseline calibration with nonlinear vacancy creation costs. Upper panel shows results for low worker bargaining power ($\eta = 0.20$), and lower panel shows results for high worker bargaining power ($\eta = 0.70$).

### 7.5 Varying Other Financial Market Parameters

We close our robustness analysis by describing, how the baseline model's results change as we vary several other ostensibly key parameters of the model. For brevity, we refrain from showing detailed quantitative results for the following experiments. In agency-cost models ala Carlstrom and Fuerst (1998), the fundamental sources of credit-market frictions are private information on the part of borrowers, monitoring costs on the part of lenders, and some type of impatience or finite lifetimes that rules out self-financing as an equilibrium outcome. In the discussion of the subsequent experiments, we compare the results to the dynamics of $s$, $v$, and $\theta$ and the cyclical Beveridge Curve as presented in Table 3 (that is, the reference model is that in Table 3).

First, we vary the dispersion of idiosyncratic productivity $\sigma_\omega$ between 0.07 (one-third the idiosyncratic dispersion in our baseline model) and 0.8 (four times as large as our baseline parameter). For values of $\sigma_\omega$ below our baseline calibration, none of the results reported in Table 3 changes in any important way (that is, all of the second moments are very close to those in baseline case). For values of $\sigma_\omega$ above our baseline calibration, the amplification the model predicts is strengthened.

Next, we vary the monitoring cost parameter $\mu$ between 0.05, about half the value estimated by Levin, Natalucci, and Zakrajsek (2004) (which itself is only one-third as large as the parameter value used in Carlstrom and Fuerst (1998)) and 1 (about four times higher than our baseline parameterization). Again, none of the results reported in Table 3 is affected in any important way.

Finally, we make firm financing needs even more acute by lowering the degree of entrepreneurs' patience, parameterized by $\gamma$. For $\gamma = 0.70$, the volatility of GDP rises only modestly (from 1.7 percent to 2.0 percent), whereas the cyclical volatilities of $s$, $v$, and $\theta$ are, respectively, 21 percent, 23 percent, and 40 percent, a much sharper rise compared to volatilities presented in Table 3 for the baseline model.

## 8 Conclusion

We have developed a model in which shocks to aggregate TFP lead to large fluctuations in labor markets, and the amplification is mediated through financial conditions. The model performs well in quantitatively accounting for the volatility of vacancies, unemployment, and labor-market tightness, the quantities identified by Shimer (2005) as failing to be explained by standard search models. The key to the amplification is a countercyclical external finance premium driven by a positive spillover from aggregate TFP to firms' idiosyncratic productivity. Because of firms' financing needs, a countercyclical finance premium makes input costs cheaper during booms and more costly during recessions. This leads to much sharper swings in firm recruiting efforts over the business cycle, and thus much sharper swings in labor markets, than in standard search models of the labor market. Cyclical movements in financial conditions are responsible for 60 percent of the model's amplification, with the direct productivity effect of the spillover responsible for the other 40 percent.

The key parameter of the model is the elasticity of the mean of firms' idiosyncratic productivity with respect to aggregate TFP. Firm-level empirical evidence suggests that this elasticity is positive, which is the direction of this relationship needed for the core mechanism of our model to operate. Premised on a positive elasticity, the numerical value for the elasticity is then selected to match the observed countercyclicality of the finance premium with respect to GDP. Thus, the model quantitatively matches labor-market volatility well even though the model is calibrated to the dynamics of financial conditions, not to the dynamics of labor markets.

A number of extensions suggest themselves. One may be to allow for endogenous job separation, which has been known since den Haan, Ramey, and Watson (2000) to potentially matter for many business-cycle issues. Our model performs well on the labor-market amplification dimension despite only exogenous separations, but a mechanism such as den Haan, Ramey, and Watson (2000) or Fujita and Ramey (2007) may improve the model along the propagation dimension. Another extension would be to model credit-market frictions even more deeply, which is sure to be an active area of research in the coming years. Introducing other sources of variation besides just TFP shocks may allow for deeper connection with the rich firm-level evidence offered by studies such as Foster, Haltiwanger, and Syverson (2008) and Petrin, White, and Reiter (2011). Issues of fiscal and monetary policy are also likely to be interesting ones to explore in the type of framework we have developed.

Perhaps most importantly, further empirical resolution on firm-level productivity and its correlation structure with aggregate shocks is needed. In our model, the elasticity of idiosyncratic productivity with respect to aggregate TFP is simply a parameter. Phenomena at the micro level for which it may stand in are, among others, selection effects in firm entry and exit and non-constant returns in production technologies. In a deeper micro-founded model, one would not want to label these phenomena "productivity," even though in our representative-agent model they appear as such. Nonetheless, we view our model and results as suggesting that credit-market frictions may be an important driver of labor market fluctuations.

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# A Derivation of Labor-Force Participation Condition

The household side of the model is constructed in this Appendix in a flexible way that allows for nesting both the fixed-participation case and the endogenous-participation case. In the former case, which corresponds to the benchmark Pissarides (2000) model, the size of the labor force is exogenously fixed at the measure $n_t^h + s_t^h = \bar{l} < 1$, with $\bar{l}$ a parameter of the decentralized economy. The measure $n_t^h$ of individuals are employed in period $t$, and the measure $s_t^h$ of individuals are available for employment in period $t$ should a job be found. With the labor-force participation rate fixed at $\bar{l}$, the fixed measure $1 - \bar{l} = 1 - s_t^h - n_t^h$ of family members thus enjoy leisure in every period. For reasons that will become clear below, we label this Appendix version of the model the "*pseudo-labor-force-participation model*," or pseudo-LFP model for short.

In the endogenous-participation case, which is the benchmark model in the paper, households each period optimally choose the size of $n_t^h + s_t^h$. The labor-force participation rate is thus endogenous in every period $t$. We label this version of the model the *labor-force-participation model* (LFP model). As the discussions below make clear, with the exception of one constraint, the analysis of the household problem is identical in both the pseudo-LFP model and the LFP model. This is the analytical tractability delivered by the formulation of Arseneau and Chugh (2012).

Although the labor-force participation rate is fixed in the pseudo-LFP environment, we formulate the household problem *as if it had free choice over the participation rate*, but impose a constraint that ensures it is always $\bar{l}$. This formulation motivates our label "pseudo" LFP model. We emphasize that while this formulation allows *analysis* of the household problem in a way that is easily comparable with the endogenous LFP model described below and which appears in the main analysis, in no sense do we mean that households actually "make a participation decision" in the pseudo-LFP model. In the pseudo-LFP model, it is always the case that the size of the labor force is $n_t^h + s_t^h = \bar{l}$. The advantage this formulation of the problem delivers is that the shadow value of this size restriction directly enters the analysis.

Thus, we analyze the household as if it maximized its expected lifetime discounted utility

$$\sum_{t=0}^{\infty} \beta^t \left[ u(c_t) + h(1 - s_t^h - n_t^h) \right] \tag{46}$$

subject to a sequence of flow budget constraints

$$c_t + k_{ht+1} + T_t = (1 - \tau_t^n) w_t n_t^h + k_{ht} \left[ 1 + (1 - \tau_t^k)(r_t - \delta) \right] + b s_t^h + \Pi_t, \tag{47}$$

a sequence of perceived laws of motion for the measure of family members that are employed,

$$n_t^h = (1 - \rho) n_{t-1}^h + s_t^h p^h(\theta_t), \tag{48}$$

and the exogenous restriction on the size of the labor force

$$n_t^h + s_t^h = \bar{l}. \tag{49}$$

Constraint (49), if its shadow value is non-zero over the business cycle, ensures that the size of the labor force is fixed over the business cycle. On the other hand, if the shadow value of constraint (49) is zero, as it is in the main results of the text, then the participation rate is endogenous and fluctuates over the business cycle.

Denote by $\{\lambda_t\}$, $\{\mu_t^h\}$, and $\{\iota_t\}$ the sequences of Lagrange multipliers on the sequences of these constraints, respectively. The first-order conditions with respect to $c_t$, $k_{ht+1}$, $s_t^h$ and $n_t^h$ are, respectively,

$$u'(c_t) - \lambda_t = 0, \tag{50}$$

$$-\lambda_t + \beta\lambda_{t+1}\left[1 + (1-\tau_{t+1}^k)(r_{t+1}-\delta)\right] = 0, \tag{51}$$

$$-h'(1 - s_t^h - n_t^h) + \lambda_t b + \mu_t^h p^h(\theta_t) - \iota_t = 0, \tag{52}$$

and

$$-\mu_t^h + \lambda_t(1-\tau_t^n)w_t - h'(1 - s_t^h - n_t^h) - \iota_t + \beta(1-\rho)\mu_{t+1}^h = 0. \tag{53}$$

Conditions (50) and (51) yield a standard capital-supply condition, which is expression (4) in the main text.

To obtain the "pseudo"-labor-force-participation (LFP) condition, start with (52) and (53). Solving (52) for $\mu_t^h$,

$$\mu_t^h = \frac{h'(1 - s_t^h - n_t^h) - u'(c_t)b}{p^h(\theta_t)} + \frac{\iota_t}{p^h(\theta_t)}, \tag{54}$$

in which we have used the result $\lambda_t = u'(c_t)$, which follows from (50). Using this expression and its period $t+1$ analog in (53), we have

$$\frac{h'(1 - s_t^h - n_t^h) - u'(c_t)b}{p^h(\theta_t)} + \frac{\iota_t}{p^h(\theta_t)} = u'(c_t)(1-\tau_t^n)w_t - h'(1-s_t^h-n_t^h) - \iota_t + \beta(1-\rho)E_t\left\{\frac{h'(1 - s_{t+1}^h - n_{t+1}^h) - u'(c_{t+1})b}{p^h(\theta_{t+1})} + \frac{\iota_{t+1}}{p^h(\theta_{t+1})}\right\}, \tag{55}$$

which is a modified form of the LFP condition (5) presented in the main text. This is a "pseudo"-LFP condition because what it pins down is the shadow value $\iota_t$, rather than the participation rate.

Suppose for a moment that $\iota_t = 0$, $\forall t$, which means there is no restriction on the participation rate. With $\iota_t = 0$, condition (55) collapses to the LFP condition (5), which has the interpretation provided in the text: at the optimum, the household each period sends a measure $s_t^h$ of family members to search for jobs until the expected cost of search — the left-hand-side of (55) — is equated to the expected benefit of search — the right-hand-side of (55). The expected cost of search is measured by the marginal utility of leisure (each unit of search involves forgoing one unit of leisure). The expected benefit of search is the marginal utility value of after-tax wage income net of the marginal disutility of work (the first two terms on the right-hand-side of (55)), along with the asset value to the household of having an additional family member engaged in an ongoing employment relationship (the last term on the right hand side of (55)). This asset value reflects the value to the household of sending one fewer family member out to search for a job in the future.

With $\iota_t = 0$, condition (55) (equivalently, equation (5)) naturally has the interpretation of a free-entry condition into the labor force. However, because in the "pseudo"-LFP model the size of the labor force is fixed at $\bar{l} < 1$, the shadow value $\iota_t$ is non-zero. The shadow value $\iota_t$ measures the value to the household of being able to freely adjust its participation rate; as such, it can be interpreted as the price of participation. In equilibrium, this shadow price adjusts so that condition (55) results in *no* net entry into or exit from the labor force. Hence the terminology *pseudo* labor-force participation condition to refer to (55).

We emphasize that in the pseudo-LFP version of the model, the participation rate $n_t^h + s_t^h$ is indeed fixed at $\bar{l}$ every period. Hence, the pseudo-LFP formulation delivers the same household decisions as if we had taken the more standard approach of specifying the household problem as one of choosing only $c_t$ subject to the budget constraint (47), completely dropping constraints (48) and (49), setting $h(.)$ to a constant, and setting, as is common in DGE search models, $n_t^h + s_t^h = \bar{l} = 1$ $\forall t$. It is not the underlying optimal choices of the household that we change by specifying the model in the way that we do, it is only the way in which we analyze the household problem that is different. The important gain our formulation brings to the analysis is being able to measure the shadow value $\iota_t$ and thus being able to nest both cases in one model.

# B Nash Bargaining

The Nash bargaining problem and hence the wage rule does not depend on whether or not labor force participation is fixed or endogenous. The bargaining-relevant value equations are defined using the household-level envelope conditions. A household's state variable is its beginning-of-period-$t$ employment stock, $n_{t-1}^h$. Regardless of whether or not participation is fixed or endogenous, the household perceives that its employment evolves according to

$$n_t^h = (1-\rho)n_{t-1}^h + p^h(\theta_t)s_t^h. \tag{56}$$

Define $\mathbf{V}(n_{t-1}^h)$ as the value function associated with the optimal plan that solves the household problem. The envelope condition is thus $\mathbf{V}'(n_{t-1}^h) = (1-\rho)\mu_t^h$, where $\mu_t^h$ is the value to the household of having one more family member employed in period $t$. In turn, this value is given by

$$\mu_t^h = \lambda_t(1-\tau_t^n)w_t - h'(1-s_t^h-n_t^h) + \beta(1-\rho)\mu_{t+1}^h \tag{57}$$

because an additional employed member brings to the household an after-tax wage (measured in utility — $\lambda_t$ is the marginal utility of wealth), incurs disutility for the household by decreasing its leisure, and has a continuation value from the perspective of the household. Using this value $\mu_t^h$, we can express the envelope condition as

$$\frac{\mathbf{V}'(n_{t-1}^h)}{1-\rho} = \lambda_t(1-\tau_t^n)w_t - h'(1-s_t^h-n_t^h) + \beta(1-\rho)\left(\frac{\mathbf{V}'(n_t^h)}{1-\rho}\right), \tag{58}$$

in which we have normalized by $1-\rho$ due to the timing of events.

To express things in units of goods, define $\mathbf{W_t}$ as

$$\begin{aligned}\mathbf{W_t} \equiv \frac{\mathbf{V}'(n_{t-1}^h)}{\lambda_t(1-\rho)} &= -\frac{h'(1-s_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_t + (1-\rho)\left(\frac{\beta\mathbf{V}'(n_{t-1}^h)}{\lambda_t(1-\rho)}\right)\\ &= -\frac{h'(1-s_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_t + (1-\rho)\left(\Xi_{t+1|t}\mathbf{W_{t+1}}\right).\end{aligned} \tag{59}$$

The second line makes use of the definition of the one-period-ahead stochastic discount factor, $\Xi_{t+1|t} \equiv \beta\lambda_{t+1}/\lambda_t$.

In the pseudo-LFP model, $n_{t-1}^h + s_{t-1}^h = \bar{l}$, which means that, given we have used $n_{t-1}^h$ as the state variable of the household problem, we do not need to also specify $s_{t-1}^h$ as a state. Hence, the value to the household of an unemployed family member is $\mathbf{U_t} \equiv \left(\partial\mathbf{V_t}/\partial s_{t-1}^h\right)/\lambda_t = 0$.

On the firm side, the value of having an additional employee is

$$\mathbf{J_t} = \omega_{mt}z_tF_n(k_t,n_t) - p_tw_t + (1-\rho)\left(\Xi_{t+1|t}\mathbf{J_{t+1}}\right); \tag{60}$$

for use below, note that $\mathbf{J_t} = \frac{\varrho'(v_t)p_t}{p^f(\theta_t)}$.

In generalized Nash bargaining, the parties choose $w_t$ in every period $t$ to maximize

$$(\mathbf{W_t} - \mathbf{U_t})^{\eta}\,\mathbf{J_t}^{1-\eta}. \tag{61}$$

The solution to this problem gives the time-$t$ generalized Nash sharing rule, $\frac{p_t}{1-\tau_t^n}\mathbf{W_t} = \frac{\eta}{1-\eta}\mathbf{J_t}$. Note that both the time-varying labor income tax $\tau_t^n > 0$ and the time-varying financing friction $p_t > 1$ throw a wedge into the standard Nash sharing rule $\mathbf{W_t} = \frac{\eta}{1-\eta}\mathbf{J_t}$.

Now proceed to derive an explicit expression for $w_t$. Inserting the definition of $\mathbf{W_t}$ into the Nash sharing rule,

$$-\frac{h'(1-s_t^h-n_t^h)p_t}{\lambda_t(1-\tau_t^n)} + p_t w_t + \frac{p_t(1-\rho)}{1-\tau_t^n}\left(\Xi_{t+1|t}\mathbf{W_{t+1}}\right) = \frac{\eta}{1-\eta}\mathbf{J_t}, \tag{62}$$

and then using the time-$t+1$ Nash sharing rule,

$$-\frac{h'(1-s_t^h-n_t^h)p_t}{\lambda_t(1-\tau_t^n)} + p_t w_t + \frac{p_t(1-\rho)}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\eta}{1-\eta}\mathbf{J_{t+1}}\right) = \frac{\eta}{1-\eta}\mathbf{J_t}. \tag{63}$$

Make the substitution $\mathbf{J_t} = \frac{\varrho'(v_t)p_t}{p^f(\theta_t)}$, and similarly for $\mathbf{J_{t+1}}$, which yields

$$-\frac{h'(1-s_t^h-n_t^h)p_t}{\lambda_t(1-\tau_t^n)} + p_t w_t + \frac{p_t(1-\rho)}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\eta}{1-\eta}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right) = \frac{\eta}{1-\eta}\frac{\varrho'(v_t)p_t}{p^f(\theta_t)}. \tag{64}$$

Next, use the vacancy-creation condition $\frac{\varrho'(v_t)p_t}{p^f(\theta_t)} = \omega_{mt}z_tF_n(k_t,n_t) - p_t w_t + (1-\rho)\left(\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right)$ to substitute on the right-hand-side, which gives

$$-\frac{h'(1-s_t^h-n_t^h)p_t}{\lambda_t(1-\tau_t^n)} + p_t w_t + \frac{p_t(1-\rho)}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\eta}{1-\eta}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right) = \frac{\eta}{1-\eta}\left[\omega_{mt}z_tF_n(k_t,n_t) - p_t w_t + (1-\rho)\left(\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right)\right]. \tag{65}$$

Grouping terms involving $p_t w_t$,

$$\begin{aligned} p_t w_t\left[1+\frac{\eta}{1-\eta}\right] &= \frac{\eta}{1-\eta}\omega_{mt}z_tF_n(k_t,n_t) + \frac{h'(1-s_t^h-n_t^h)p_t}{\lambda_t(1-\tau_t^n)} \\ &- \frac{\eta}{1-\eta}\frac{p_t(1-\rho)}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right) + \frac{\eta}{1-\eta}(1-\rho)\left(\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right). \end{aligned} \tag{66}$$

Multiplying by $1-\eta$ gives

$$\begin{aligned} p_t w_t &= \eta\omega_{mt}z_tF_n(k_t,n_t) + (1-\eta)\frac{h'(1-s_t^h-n_t^h)p_t}{\lambda_t(1-\tau_t^n)} \\ &- \eta\frac{p_t(1-\rho)}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right) + \eta(1-\rho)\left(\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right), \end{aligned} \tag{67}$$

or, dividing by $p_t$,

$$\begin{aligned} w_t &= \eta\frac{\omega_{mt}z_tF_n(k_t,n_t)}{p_t} + (1-\eta)\frac{h'(1-s_t^h-n_t^h)}{\lambda_t(1-\tau_t^n)} \\ &- \eta\frac{1-\rho}{1-\tau_t^n}\left(\Xi_{t+1|t}\frac{1-\tau_{t+1}^n}{p_{t+1}}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right) + \frac{\eta(1-\rho)}{p_t}\left(\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right), \end{aligned} \tag{68}$$

which is expression (20) in the main text, given that $\lambda_t = u'(c_t)$. Note that if $\tau^n_{t+1} = \tau^n_t = \tau^n$ and $p_{t+1} = p_t = p \;\forall t$, the last two terms cancel with each other and the wage collapses to a simple static split,

$$w_t = \eta \frac{\omega_{mt} z_t F_n(k_t, n_t)}{p} + (1-\eta) \frac{h'(1 - s^h_t - n^h_t)}{\lambda_t (1 - \tau^n)}. \tag{69}$$

# C Elasticity of Market Tightness

Here, we derive expressions for the steady-state elasticity of labor market tightness $\theta$ to the mean of idiosyncratic productivity $\omega_m$ and to the component $p$ of input costs attributable to financing. Start with the steady-state versions of the private-sector job-creation condition (17) and Nash wage outcome (20):

$$[1-\beta(1-\rho)]\varrho'(v)p\theta^{\xi} = \omega_m z_t F_n(k,n) - pw \tag{70}$$

$$w = \eta\frac{\omega_m z_t F_n(k,n)}{p} + (1-\eta)\frac{h'(1-s-n)}{u'(c)(1-\tau^n)}. \tag{71}$$

In (70), we have used the result that, because we assume a Cobb-Douglas matching function, $p^f(\theta) = \theta^{-\xi}$. Inserting (71) into (70), we can define the implicit function

$$G(\theta,\omega_m;.) \equiv [1-\beta(1-\rho)]\varrho'(v)p\theta^{\xi} - (1-\eta)\left[\omega_m z_t F_n(k,n) - \frac{ph'(1-s-n)}{u'(c)(1-\tau^n)}\right] = 0. \tag{72}$$

The objective is to construct $\epsilon_{\theta,\omega_m} \equiv \frac{d\theta}{d\omega_m}\frac{\omega_m}{\theta}$. By the implicit function theorem, $\frac{d\theta}{d\omega_m} = -\frac{G_{\omega_m}}{G_\theta}$. We have

$$G_{\omega_m} = -(1-\eta)z_t F_n(k,n) \tag{73}$$

and

$$G_\theta = \xi[1-\beta(1-\rho)]p\varrho'(v)\theta^{\xi-1}. \tag{74}$$

Constructing the elasticity,

$$\epsilon_{\theta,\omega_m} = \frac{\omega_m}{\theta}\frac{(1-\eta)z_t F_n(k,n)}{\xi[1-\beta(1-\rho)]p\varrho'(v)\theta^{\xi-1}}, \tag{75}$$

which is expression (38) in the main text. This elasticity is increasing in $\omega_m$ and decreasing in $p$. As discussed in Section 3, the former reflects the direct productivity effect of the model, and the latter reflects the financing conditions effect of the model. With appropriate parameterization of the elasticity $\nu$, the two effects can be made to reinforce each other (by making $p$ countercyclical with respect to aggregate TFP and hence with respect to GDP).

We can also construct the elasticity $\epsilon_{\theta,p}$, which requires computing

$$G_p = [1-\beta(1-\rho)]\varrho'(v)\theta^{\xi} + (1-\eta)\frac{h'(1-s-n)}{u'(c)(1-\tau^n)}. \tag{76}$$

Then, $\epsilon_{\theta,p} = \frac{d\theta}{dp}\frac{p}{\theta} = -\frac{G_p}{G_\theta}\frac{p}{\theta}$, so that

$$\epsilon_{\theta,p} = -\frac{p}{\theta}\left[\frac{\theta}{p} + \frac{(1-\eta)h'(1-s-n)}{u'(c)(1-\tau^n)\xi[1-\beta(1-\rho)]p\varrho'(v)\theta^{\xi-1}}\right] \tag{77}$$

$$= -1 - \frac{(1-\eta)h'(1-s-n)}{u'(c)(1-\tau^n)\xi[1-\beta(1-\rho)]\varrho'(v)\theta^{\xi}}, \tag{78}$$

which, is independent of $p$.

# D General Equilibrium Consumption-Leisure Margin

To derive the equilibrium margin between consumption and leisure, start with the pseudo-labor-force participation condition

$$\frac{h'(1-s_t-n_t)-u'(c_t)b}{p^h(\theta_t)}+\frac{\iota_t}{p^h(\theta_t)}=u'(c_t)(1-\tau_t^n)w_t-h'(1-s_t-n_t)$$
$$-\quad \iota_t+\beta(1-\rho)E_t\left\{\frac{h'(1-u_{t+1}-n_{t+1})-u'(c_{t+1})b}{p^h(\theta_{t+1})}+\frac{\iota_{t+1}}{p^h(\theta_{t+1})}\right\}. \tag{79}$$

and the vacancy-creation condition

$$\frac{\varrho'(v_t)p_t}{p^f(\theta_t)}=\omega_{mt}z_tF_n(k_t,n_t)-p_tw_t+(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right\}. \tag{80}$$

Divide (79) by (80) to get

$$\frac{p^f(\theta_t)\left[\frac{h'(1-s_t-n_t)-u'(c_t)b}{p^h(\theta_t)}+\frac{\iota_t}{p^h(\theta_t)}\right]}{\varrho'(v_t)p_t}$$
$$=\quad\frac{u'(c_t)(1-\tau_t^n)w_t-h'(1-s_t-n_t)-\iota_t+\beta(1-\rho)E_t\left\{\frac{h'(1-u_{t+1}-n_{t+1})-u'(c_{t+1})b}{p^h(\theta_{t+1})}+\frac{\iota_{t+1}}{p^h(\theta_{t+1})}\right\}}{\omega_{mt}z_tF_n(k_t,n_t)-p_tw_t+(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right\}} \tag{81}$$

Using the result that $p^f(\theta_t)/p^h(\theta_t)=\theta_t^{-1}$ due to the assumption of Cobb-Douglas matching, we can re-arrange the left-hand-side as

$$\frac{h'(1-s_t-n_t)-u'(c_t)b+\iota_t}{\varrho'(v_t)p_t\theta_t}$$
$$=\quad\frac{u'(c_t)(1-\tau_t^n)w_t-h'(1-s_t-n_t)-\iota_t+\beta(1-\rho)E_t\left\{\frac{h'(1-u_{t+1}-n_{t+1})-u'(c_{t+1})b}{p^h(\theta_{t+1})}+\frac{\iota_{t+1}}{p^h(\theta_{t+1})}\right\}}{\omega_{mt}z_tF_n(k_t,n_t)-p_tw_t+(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\varrho'(v_{t+1})p_{t+1}}{p^f(\theta_{t+1})}\right\}} \tag{82}$$

Next, note from our work in Appendix B that the numerator on the right-hand-side is $u'(c_t)\left[\mathbf{W}_t-\mathbf{U}_t\right]=u'(c_t)\mathbf{W}_t$, the surplus to the household (expressed in terms of utility) of having the marginal member enter into an employment relationship. Also note that the denominator on the right-hand-side of the previous expression is $\mathbf{J}_t$, the surplus to the firm of entering into an employment relationship with one additional worker. Thus, the previous expression can be simplified to

$$\frac{h'(1-s_t-n_t)-u'(c_t)b+\iota_t}{\varrho'(v_t)p_t\theta_t}=\frac{u'(c_t)\mathbf{W}_t}{\mathbf{J}_t}. \tag{83}$$

Using in this expression the private economy's Nash-bargaining outcome $\frac{p_t}{1-\tau_t^n}\mathbf{W}_t=\frac{\eta}{1-\eta}\mathbf{J}_t$, we have

$$\frac{h'(1-s_t-n_t)-u'(c_t)b+\iota_t}{\varrho'(v_t)p_t\theta_t}=u'(c_t)\frac{\eta}{1-\eta}\frac{(1-\tau_t^n)}{p_t}. \tag{84}$$

Rearranging, we have that in the decentralized Nash-bargaining economy with taxes, financing frictions, unemployment benefits, and no ability for individuals to enter or exit the labor force,

$$\frac{h'(1-s_t-n_t)-u'(c_t)b+\iota_t}{u'(c_t)}=(1-\tau_t^n)\varrho'(v_t)\theta_t\frac{\eta}{1-\eta}, \tag{85}$$

in which, recall, $\eta \in (0, 1)$ is the Nash bargaining power of households. This condition characterizes the general equilibrium consumption-leisure margin and appears as condition (41) in the main text. Note in particular that in moving from the next-to-last expression to condition (85), the financing wedge $p_t$ disappears.

## E Competitive Search Equilibrium

While Nash bargaining has become the standard wage-determination mechanism in DSGE search models, many other models of wage determination have been developed and usefully employed in the literature on labor-market theory. One particularly appealing alternative is competitive search equilibrium, which entails decentralized determination of wages which are taken as given by both firms and households in their optimal search behavior. From the point of view of standard DSGE macroeconomic models, this equilibrium concept is appealing because there are no bilateral negotiations whatsoever; wages are always determined in a market-clearing fashion. Because competitive search has not yet found its way to mainstream DSGE macro models, we show here how to implement a competitive search equilibrium. Our treatment adapts Moen's (1997) implementation for a full general-equilibrium environment. As we show, the condition characterizing equilibrium along the consumption-leisure (outside the labor force) margin that arises from competitive search is identical to that which arises under Nash bargaining under the Hosios (1990) parameterization for bargaining power. Because the Hosios parameterization is the case for which our main results are obtained, all of our optimal-policy results are thus identical under competitive search. Indeed, the derivations below can be viewed as extending the equivalence shown by Moen (1997) between competitive search equilibrium and Nash-Hosios bargaining.

To implement competitive search equilibrium, we must first define payoff functions to search for both firms and households. In a particular labor submarket $i$, any firm $j$ that pays the vacancy-posting cost $\varrho'(v_{ijt})p_t$, which includes the up-front financing costs, has expected payoff of matching with a worker

$$p^f(\theta_{ijt})\left[\omega_{mt}z_tF_n(k_{ijt},n_{ijt})-p_tw_{ijt}+(1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\varrho'(v_{it+1})p_{t+1}}{p^f(\theta_{it+1})}\right\}\right], \tag{86}$$

in which it is supposed the tightness of financing is determined by aggregate conditions, and not submarket-specific conditions, hence the absence of a submarket $i$ index on $p_t$. As in Moen (1997), the matching probability in period $t$, $p^f(\theta_{ijt})$, and the wage payment in period $t$, $w_{ijt}$, are firm $ij$-specific. The continuation value captured by $\varrho'(v_{ijt})p_t/p^f(\theta_{it+1})$, however, is a sub-market-specific, but not firm-specific, value, reflecting the replacement value of a given worker at sub-market $i$ prices (because there are no match-specific idiosyncracies regarding employment in our model).

For the representative household, payoff functions, defined in terms of goods, for searching for ("applying to") a job at firm $j$ in labor submarket $i$ are given by

$$p^h(\theta_{ijt})\left[-\frac{h'(1-s_t^h-n_t^h)}{\lambda_t}+(1-\tau_t^n)w_{ijt}+(1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\}\right] \tag{87}$$

if match-formation at firm $ij$ is successful, which occurs with probability $p^h(\theta_{ijt})$, and

$$(1-p^h(\theta_{ijt}))\,[0] \tag{88}$$

if match-formation is unsuccessful, which occurs with probability $1 - p^h(\theta_{ijt})$. Note that these payoffs are from the point of view of the household — i.e., they reflect the gain to the household of having one more household member search for a job at firm $ij$ — and are straightforward generalizations of the payoff functions defined in Appendix B. As derived in Appendix B, there is zero payoff to the household if search is unsuccessful.

For the concept of competitive search equilibrium, *directed* search, rather than random search, is a critical component.[39] The consequence of optimally-directed job search at firm $ij$ is thus that

$$p^h(\theta_{ijt})\mathbf{W_{ijt}} + (1 - p^h(\theta_{ijt}))\,[0] = X, \tag{89}$$

where $X$ is the expected payoff of searching for a job at a firm different from $ij$ (at either a different firm in labor submarket $i$ or in another labor submarket altogether) and hence is independent of firm $ij$ outcomes. In writing (89), we have used $\mathbf{W_{ijt}} = -\frac{h'(1-s_t^h-n_t^h)}{\lambda_t} + (1-\tau_t^n)w_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\mathbf{W_{t+1}}\right\}$ from Appendix B. The competitive-search wage $w_{ijt}$ and tightness $\theta_{ijt}$ are the solutions to maximization of (86) subject to (89).

Denoting by $p_{ijt}$ the multiplier on the constraint, the first-order condition with respect to $w_{ijt}$ yields

$$p_{ijt} = \frac{p^f(\theta_{ijt})p_t}{p^h(\theta_{ijt})}\frac{1}{1-\tau_t^n}. \tag{90}$$

Given constant-returns-to-scale matching, this reduces to

$$p_{ijt} = \frac{p_t}{\theta_{ijt}}\frac{1}{1-\tau_t^n}. \tag{91}$$

Then, using this expression in the first-order condition with respect to $\theta_{ijt}$, we have

$$\frac{\partial p^f(\theta_{ijt})}{\partial \theta_{ijt}}\left[\omega_{mt}z_tF_n(k_{ijt},n_{ijt}) - p_tw_{ijt} + (1-\rho)E_t\left\{\Xi_{t+1|t}\frac{\varrho'(v_{it+1})p_{t+1}}{p^f(\theta_{it+1})}\right\}\right] = -\frac{p_t}{\theta_{ijt}}\frac{1}{1-\tau_t^n}\frac{\partial p^h(\theta_{ijt})}{\partial \theta_{ijt}}\mathbf{W_{ijt}}. \tag{92}$$

We now restrict attention to equilibria that are symmetric across firms in a given submarket and across submarkets, so we drop $ij$ indexes. Given Cobb-Douglas matching $m(u,v) = u^{\xi}v^{1-\xi}$, we have $(\partial p^h(\theta)/\partial\theta)/(\partial p^f(\theta)/\partial\theta) = -\theta(1-\xi)/\xi$. Also, using the symmetric-equilibrium version of the vacancy-creation condition, we can replace the term in brackets on the left hand side of the previous expression with $\varrho'(v_t)p_t/p^f(\theta_t)$; making these substitutions,

$$\frac{\varrho'(v_t)p_t}{p^f(\theta_t)} = \frac{1-\xi}{\xi}\frac{p_t}{1-\tau_t^n}\mathbf{W_t}. \tag{93}$$

Cancel the $p_t$ terms, which shows that, in equilibrium, financing does not create a wedge between the welfare-relevant labor-market MRS and MRT.

[39]See Rogerson, Shimer, and Wright (2005, p. 972-976) for more discussion.

Then, from optimal household labor-force-participation and the definition of $\mathbf{W_t}$ above, this can be written as

$$\varrho'(v_t)\frac{p^h(\theta_t)}{p^f(\theta_t)}\frac{\xi}{1-\xi}(1-\tau_t^n) = \frac{h'(1-s_t-n_t)-u'(c_t)b+\iota_t}{u'(c_t)}. \tag{94}$$

Once again recognizing that $\frac{p^h(\theta)}{p^f(\theta)} = \theta$ because of constant-returns matching, we have that period-$t$ equilibrium outcomes under competitive search are described by

$$\frac{h'(1-s_t-n_t)-u'(c_t)b+\iota_t}{u'(c_t)} = (1-\tau_t^n)\varrho'(v_t)\theta_t\frac{\xi}{1-\xi}. \tag{95}$$

Comparing this with the outcome in the decentralized bargaining economy (under the Hosios parameterization $\eta = \xi$) presented in condition (41) shows that they are identical. Hence, the search-based labor wedge in the competitive search economy is identical to the wedge in the bargaining economy, which necessarily implies that the wage outcome in the competitive search economy is identical to that in the bargaining economy.

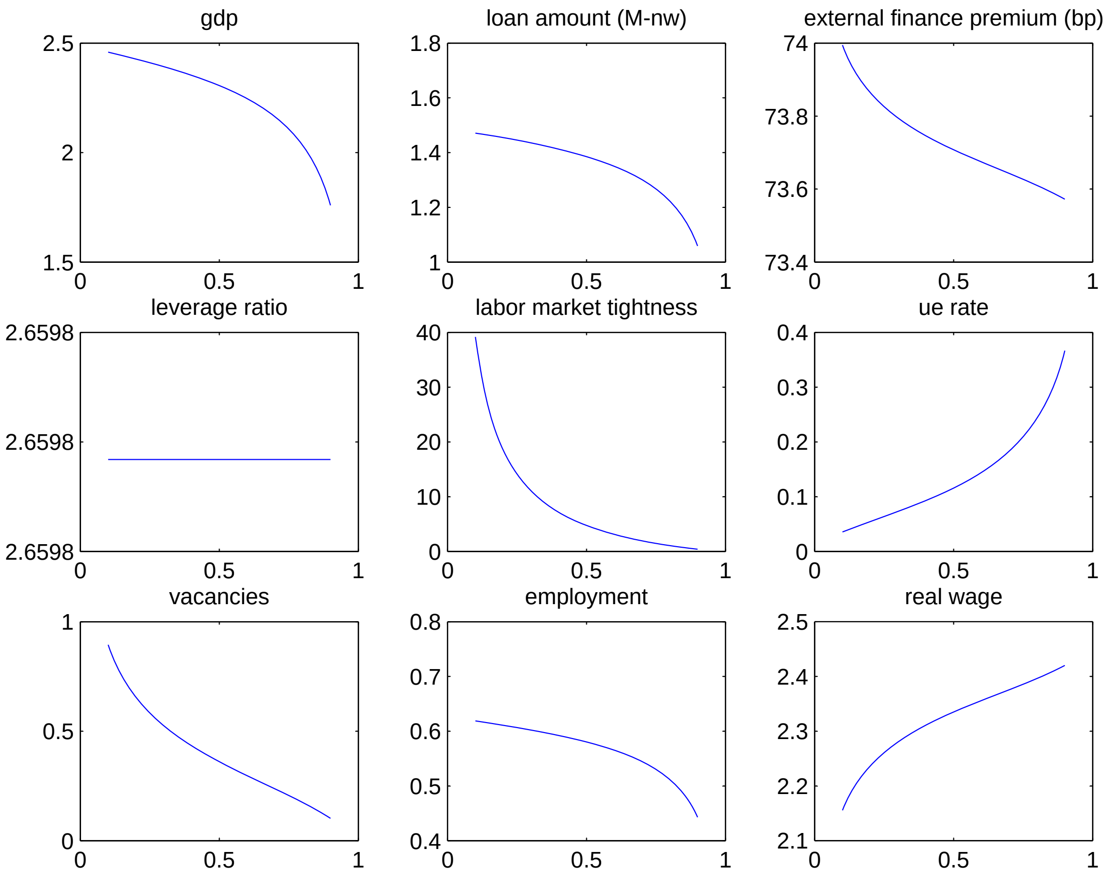


Figure 8: Long-run equilibrium as workers' Nash bargaining power, $\eta$, varies; $\eta$ plotted on horizontal axis. All other parameters held fixed at their baseline values from Table 2. External finance premium reported in basis points.

# F Steady State as Function of Bargaining Power

As noted in Section 5, we also consider how long run equilibria depend on workers' bargaining power, $\eta$, which are plotted in Figure 8.