



**THE BASELINE RBC MODEL:
THEORY AND COMPUTATION**

JANUARY 8, 2020

STYLIZED MACRO FACTS

- **Foundation of (virtually) all DSGE models (e.g., RBC model) is neoclassical growth model**
- **So want/need/desire business-cycle models to be consistent with basic growth facts**
- **Kaldor's (1957) Stylized Growth Facts (time averages)**
 1. **Output per worker exhibits ~constant growth**
 2. **Capital per worker exhibits ~constant growth**
 3. **Rate of return on capital is ~constant**
 4. **Capital-output ratio is ~constant**
 5. **Factor shares (i.e., payments to capital and payments to labor as fraction of GDP) are ~constant**
- **Business-cycle model**
 - **Interested in fluctuations around long-run growth path**
- **How to construct/extract long-run trend?**
 - **Most common procedure: HP filter**
 - **Eliminates (if data were stationary) fluctuations at frequencies lower than eight years**
 - **An alternative: band pass (BP) filter – allows specifying upper and lower frequencies to be eliminated**

STYLIZED MACRO FACTS

- **Kaldor's Stylized Facts:**
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 2. Capital per worker exhibits ~constant growth
 3. Rate of return on capital is ~constant
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- **Some basic cyclical volatilities – SD% (i.e., time-series SD of HP-filtered component)**

GDP: 1.47% (1974Q1-2009Q4)	C: 1.16% (1974Q1-2009Q4) CNDUR: 1.05% CSERV: 0.74% CDUR: 3.82%
I: 7.03% (1974Q1-2009Q4)	TOTAL HOURS: 1.59% (K&R) AVG HOURS: 0.63% (K&R)
WAGE: 0.76% (K&R)	

WHAT DO WE WANT TO MODEL?

- **Relative volatilities**
- **Persistence (i.e., first-order serial correlation) of various series**
- **Business cycle comovements**
 - **Corr(C, Y)**
 - **Corr(I, Y)**
 - **Corr(HOURS, Y)**
 - **Corr(WAGE, Y)**
 - **Others?...**
- **Labor markets?**
 - **Extensive margin – movements of individuals in and out of employment (i.e., work $n = 0$ hours or $n > 0$ hours)**
 - **Intensive margin – how many hours to work given an individual already works (i.e., work $n = 39$ hours or $n = 40$ hours or $n = 41$ etc...)**
 - **The basic RBC model blurs the difference**
 - **Prescott: “LS elasticity of 3 is right...”**

THE THREE MACRO MARKETS

- **Goods Market(s)**
- **Labor Market(s)**
- **Asset/Savings Market(s)**

- **Consumers**
 - Demand goods
 - Supply labor
 - Supply assets/savings

- **Firms**
 - Produce goods
 - Demand labor
 - Demand assets/savings (capital)

- **Government:** auxiliary in the basic model

BUILDING BLOCKS

- **Consumers**
 - Maximize **lifetime** utility (i.e., a **dynamic** problem)

- **Firms**
 - Maximize profits

- **Prices adjust to clear all markets**
 - Hence a **general equilibrium** model

- **Unpredictable fluctuations in total factor productivity (TFP) are the driving source of business cycles in baseline RBC model**
 - Identify TFP as the Solow residual
 - $y(t) = z(t)*f(k(t), n(t))$

HOUSEHOLDS

- Maximize **lifetime** utility (i.e., a **dynamic** problem) subject to budget constraint

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \quad \text{s.t.} \quad c_t + k_{t+1} - (1 - \delta)k_t = w_t n_t + r_t k_t$$

- Optimality conditions

- Consumption-Leisure Optimality Condition:** MRS between consumption and labor equals real wage

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t$$

- Consumption-Savings Optimality Condition (Euler equation):** MRS between present and future consumption equals real return on savings (**a difference equation**)

$$u_c(c_t, n_t) = \beta E_t \{ u_c(c_{t+1}, n_{t+1})(1 + r_{t+1} - \delta) \}$$

THE REST OF THE MODEL

- **Firms: maximize profits period-by-period** $\max_{n_t, k_t} z_t f(k_t, n_t) - w_t n_t - r_t k_t$
 - **FOCs yield factor-pricing conditions:** $w_t = z_t f_n(k_t, n_t), r_t = z_t f_k(k_t, n_t)$
- **Government: omit (WLOG) from baseline RBC model**
- **Resource constraint:** $c_t + k_{t+1} - (1 - \delta)k_t = z_t f(k_t, n_t)$

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- **Government: omit (WLOG) from baseline RBC model**
- **Resource constraint:** $c_t + k_{t+1} - (1 - \delta)k_t = z_t f(k_t, n_t)$
- **Exogenous process**

$$\ln z_{t+1} = (1 - \rho_z) \ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}^z$$
 - **TFP follows AR(1) (satisfies Markov property), with persistence ρ_z**
 - **Average productivity z^{bar} ; white noise process $\varepsilon \sim N(0, \sigma_z^2)$**
 - **Specification in logs implies fluctuations are in deviations of TFP from the average z^{bar}**

PUTTING THE MODEL TOGETHER: EQUILIBRIUM

INDIVIDUALS' DECISIONS ARE OPTIMAL

- **Consumer decisions:**
 - Taking as given the real wage and the rental price of capital, choices of consumption, investment, and labor solve utility maximization
- **Firm decisions:**
 - Taking as given the real wage and the rental price of capital, choices of labor and capital solve profit maximization

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ALL MARKETS CLEAR

- **Prices in goods, labor, and asset/savings markets adjust**
 - Price of consumption normalized to one in every period
 - Prices (and thus decisions) depend on how TFP (and any other exogenous processes) evolves over time

THE MODEL DETERMINES:

Allocations: consumption, labor, savings/investment

Prices: real wage, rental rate of capital

THE EQUATIONS AND VARIABLES

- **Equilibrium Conditions**

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = w_t$$

$$u_c(c_t, n_t) = \beta E_t \{u_c(c_{t+1}, n_{t+1})(1 + r_{t+1} - \delta)\}$$

$$c_t + k_{t+1} - (1 - \delta)k_t = z_t f(k_t, n_t)$$

$$w_t = z_t f_n(k_t, n_t), \quad r_t = z_t f_k(k_t, n_t)$$

$$\ln z_{t+1} = (1 - \rho_z) \ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}^z$$

- **Exogenous variables (the inputs to the model): process for z**
- **Endogenous variables (the outputs of the model): endogenous time-invariant state-contingent functions for c , n , k' , w , and r**
 - **Easy to express wage and rental rate as functions of z , k , and n**

THE EQUATIONS AND VARIABLES

- Equilibrium Conditions

$$-\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = z_t f_n(k_t, n_t)$$

Consumption-Labor
Efficiency Condition

$$u_c(c_t, n_t) = \beta E_t \{u_c(c_{t+1}, n_{t+1})(1 + z_{t+1} f_k(k_{t+1}, n_{t+1}) - \delta)\}$$

Consumption-
Investment
Efficiency Condition

$$c_t + k_{t+1} - (1 - \delta)k_t = z_t f(k_t, n_t)$$

Resource Constraint

$$\ln z_{t+1} = (1 - \rho_z) \ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}^z$$

Law of motion for
TFP

- Exogenous variables (the inputs to the model): process for z
- Endogenous variables (the outputs of the model): endogenous time-invariant state-contingent functions for c , n , and k'

HOW TO USE THE MODEL?

- **Need to specify/solve for FUNCTIONS (aka “decision rules”) that describe how:**
 - **Consumers make choices based on prices and policies**
 - **Firms make choices based on prices and policies**
 - **Prices depend on state variables (capital, TFP, and all other exogenous variables)**
- **Except for very special cases, must turn to quantitative (i.e., numerical) methods**
 - **Because of the difference (differential) equation in the model:**

Euler equation

APPROXIMATIONS

- Looking for an equilibrium in which endogenous variables are **time-invariant functions of current state** $\mathbf{S}_t \equiv [k_t; z_t]$
 - State describes the dynamic position of the model
 - Need to solve $c(\mathbf{S}_t)$, $n(\mathbf{S}_t)$, $k'(\mathbf{S}_t)$

- Cannot solve difference equations analytically in general
 - These solutions are **unknowable** in general
 - Hence need to approximate – so look for

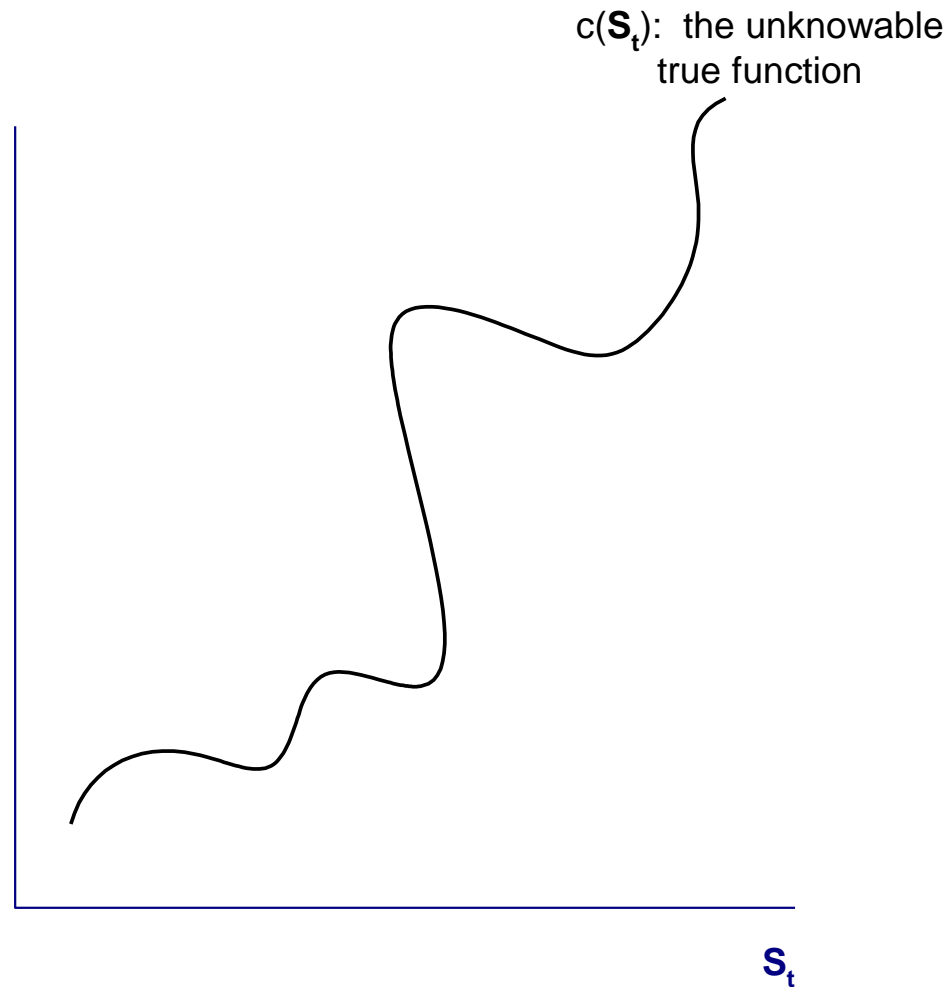
$$c^{\text{approx}}(\mathbf{S}_t), n^{\text{approx}}(\mathbf{S}_t), k'^{\text{approx}}(\mathbf{S}_t)$$

which are **hopefully** near the (unknowable...) truth...

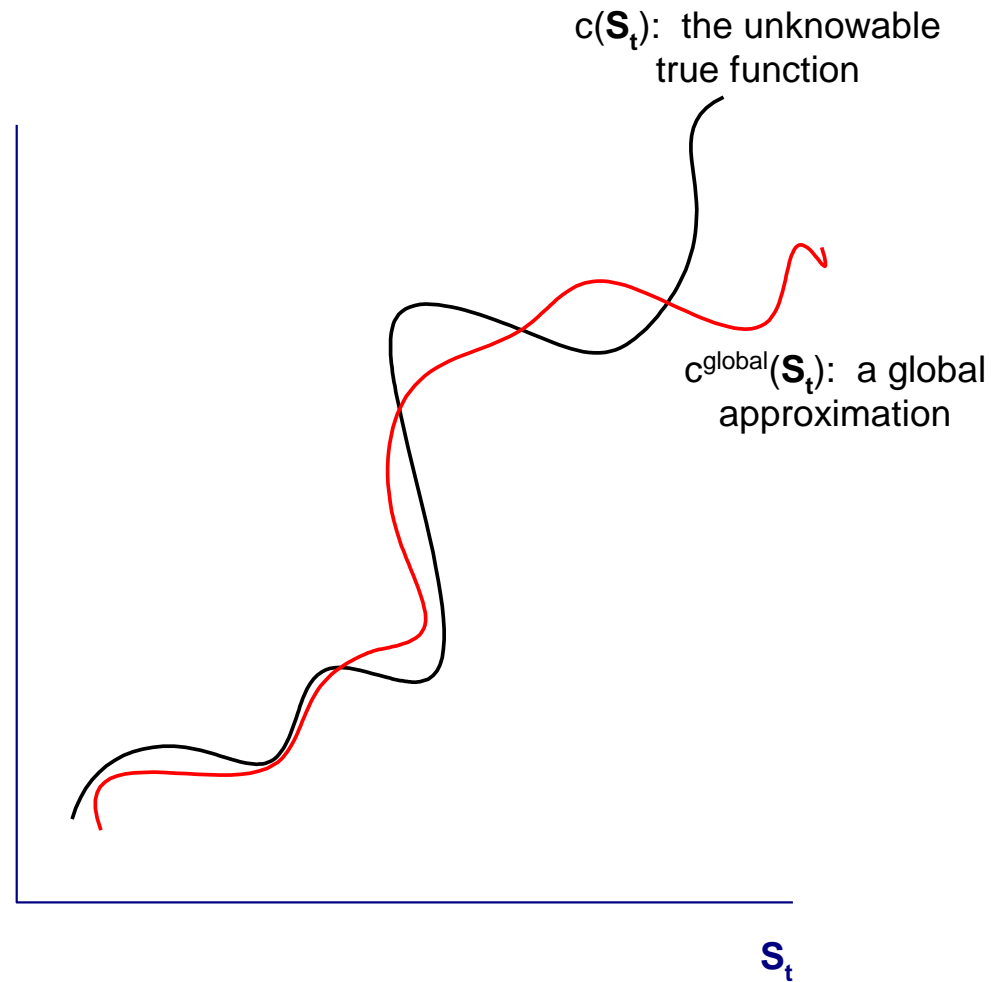
TYPES OF APPROXIMATIONS

- **Global:** approximated functions are close to true functions “everywhere” (over a very broad range of states)
 - Hard to implement for medium- and large-scale models given current hardware capacity
 - A few methods
 - Chebyshev polynomials
 - Finite-element methods
 - Value function iteration
- **Local:** approximated functions are close to true functions only in a relatively small range of the state space
 - Simple implementation
 - Based on Taylor approximations
 - Linear (first-order)
 - Quadratic (second-order)
 - Etc.

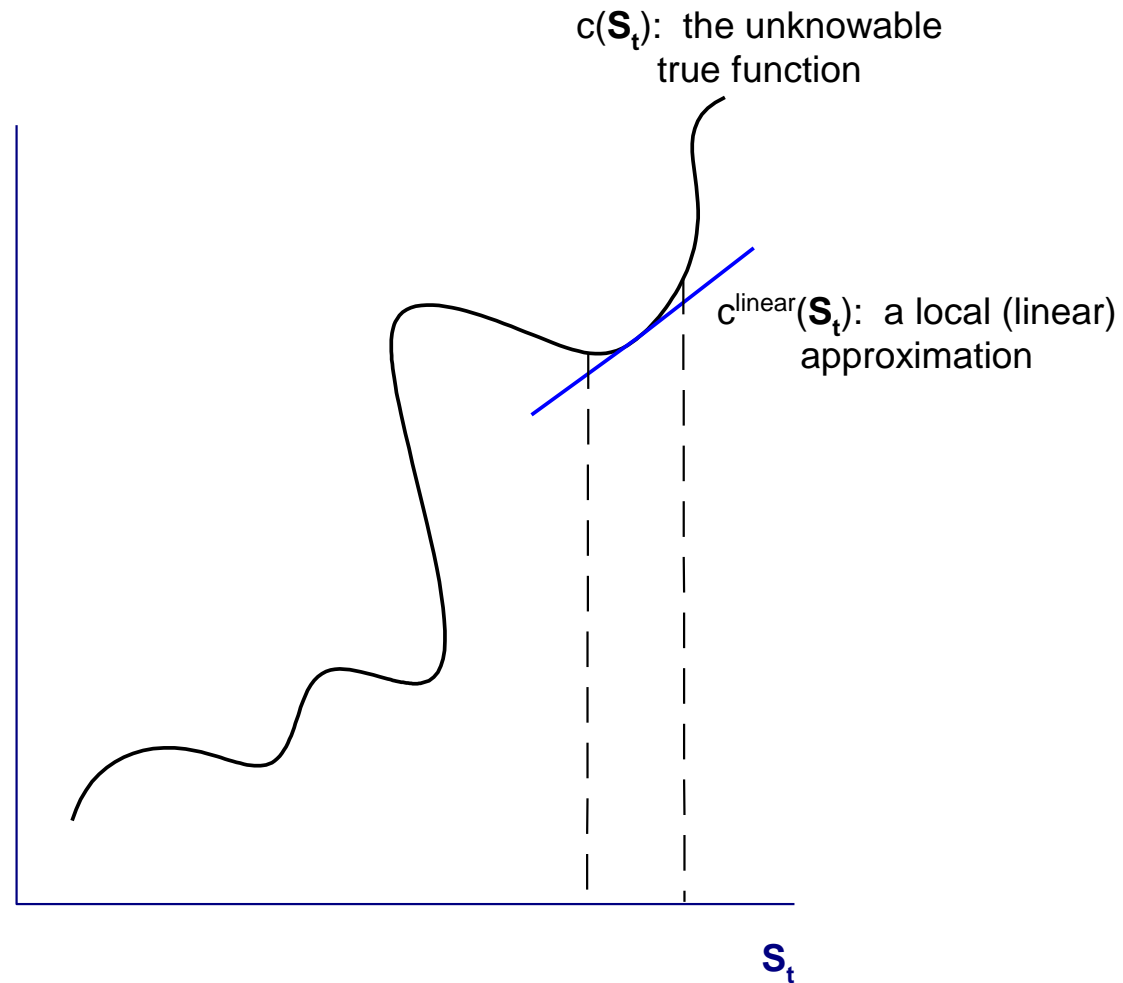
APPROXIMATIONS



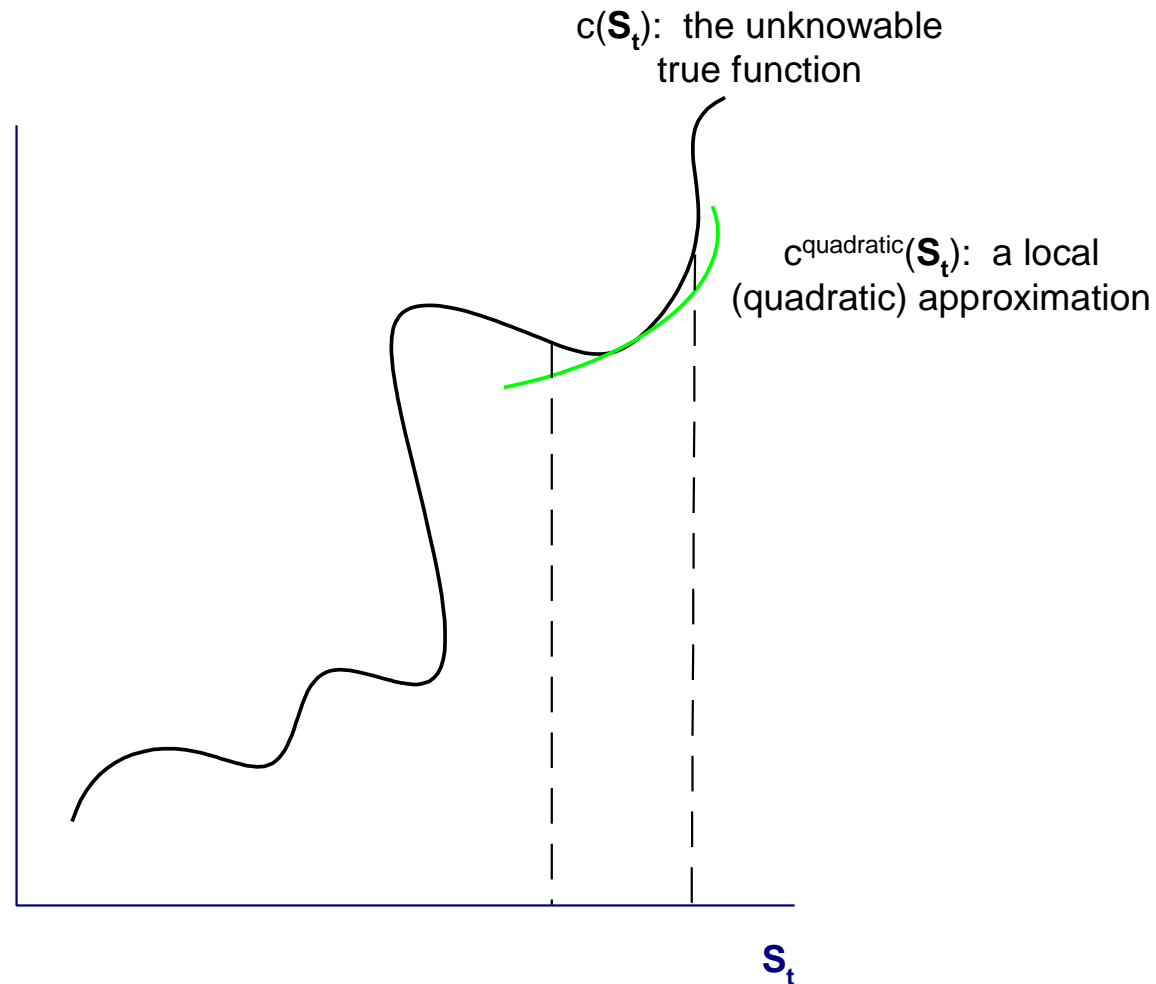
APPROXIMATIONS



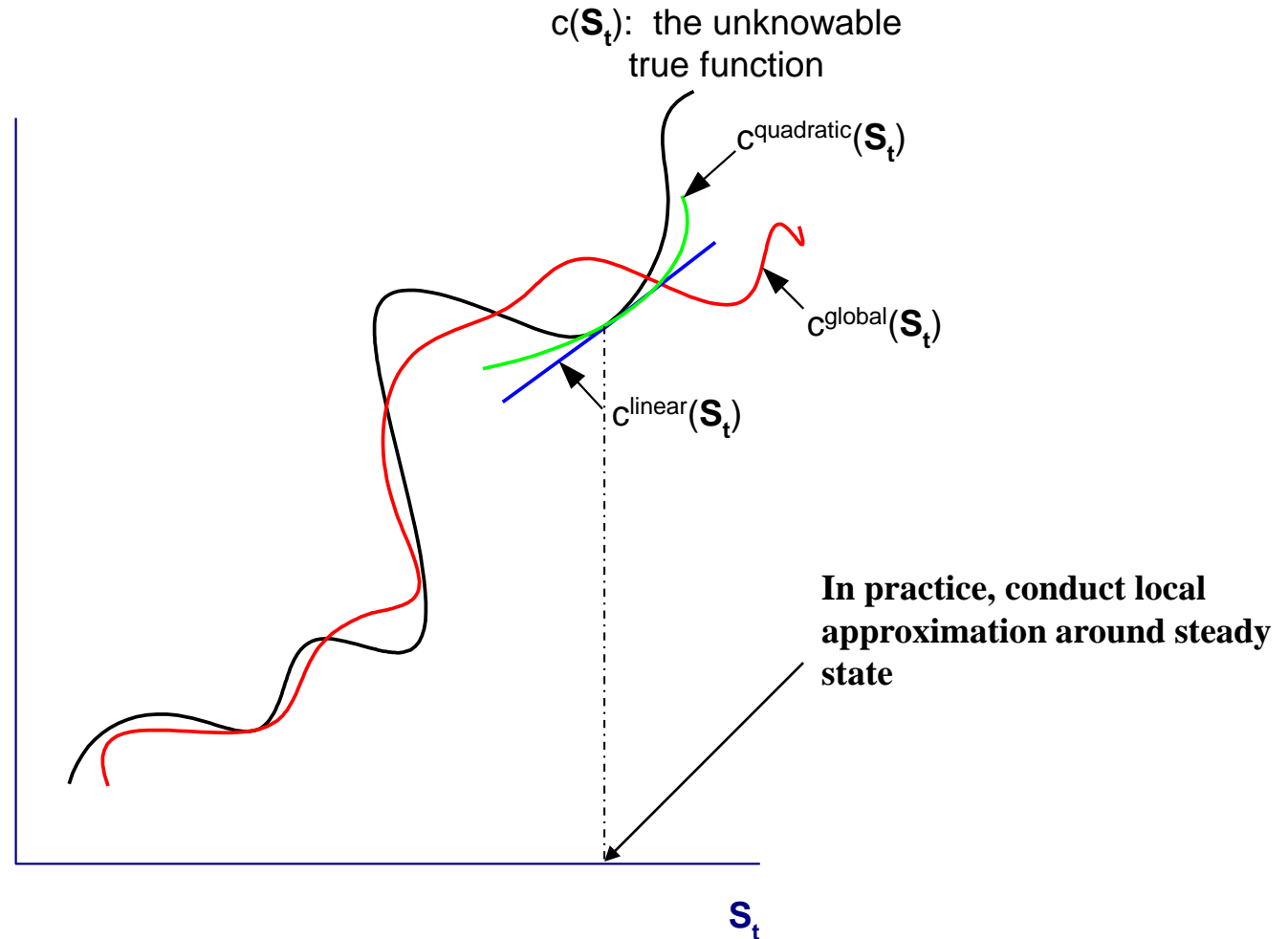
APPROXIMATIONS



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APPROXIMATIONS



STEADY STATE

- Shut down all shocks and set exogenous variables at their means
- Let model economy run for many (infinite) periods
 - Time eventually “doesn’t matter” any more
 - Drop all time indices

$$-\frac{u_n(c, n)}{u_c(c, n)} = \bar{z}f_n(k, n)$$

$$u_c(c, n) = \beta u_c(c, n)(1 + \bar{z}f_k(k, n) - \delta)$$

$$c + \delta k = \bar{z}f(k, n)$$

- (c, n, k) is a triple of **scalars** that are the **deterministic** steady state (aka long run) outcomes of the model economy
- Given functional forms and parameter values, solve for (c, n, k)
 - **Conduct local approximation around this point**