Balanced Growth

January 10, 2020

Calibration – Philosophy

- An economic model is a measuring device
- If model makes "believable" predictions along some important dimensions (i.e., "matches some key data")...
- ...then maybe its predictions are "believable" along the novel dimensions of the model
- Getting some "partial derivatives" of the model in known directions correct...
- ...may build credibility that its "partial derivatives" in novel directions are at least not grossly incorrect
- Make model match some data of interest – often long-run (i.e., time-averaged data) growth facts
  - Preferably well-accepted "stylized facts"
  - Solow growth model in the background
  - Natural candidate: Kaldor growth facts

Calibration of Baseline RBC Model

- Must take a stand on three (related) points
  - Which data do we want model to match? (even constructing data is challenging…)
  - Functional forms (utility, production)
  - Parameter values

- Choose functional forms consistent with "Kaldor-plus facts"
  - (K1) Capital income share and labor income share of GDP are stationary
  - (K2) All real quantity variables grow at same rate in the long run ("great ratios")
  - (K3) Real interest rate is stationary
  - (K4) Hours per worker are stationary
  - (K5) (K2) requires trend productivity to be labor-augmenting (Phelps 1966)

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- Often start with RBC model that abstracts from long-run growth

- But "true" calibration begins with model featuring *only* long-run growth
  - Puts restrictions on instantaneous utility and production forms
  - Use (K1)-(K5) to obtain these restrictions

- Richer models: more calibration targets and/or treating data differently
  - Monopoly markups (e.g., Dixit-Stiglitz and sticky price models)
  - Probability of finding a job (e.g., labor matching models)
  - Endogenous Growth Models

RBC MODEL WITH GROWTH

- **Absent shocks, TFP grows at deterministic rate** γ

- **Planner problem/perfect competition**

$$\max E_0 \sum_{t=0}^{\infty} b^t u(C_t, n_t) \qquad \textbf{subject to}$$

Trend productivity is labor-augmenting (Harrod-neutral) (Makes use of fact (K5))

$$C_t + K_{t+1} - (1-\delta)K_t = z_t F(K_t, n_t X_t)$$

Flow resource constraint

Red indicates variables or parameters that will be modified when detrending the model

$$X_{t+1} = \gamma X_t, \qquad \gamma \geq 1$$

Evolution of deterministic component of productivity

given stochastic process for evolution of z_t **and** (K_{-1}, z_0, X_0)

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Evolution of deterministic component of productivity

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given stochastic process for evolution of z_t and (K_{-1}, z_0, X_0)

- Suppose $z_t = 1$ always, so only deterministic growth
- Deterministic dynamics of $(C_t, K_{t+1}, n_t, X_{t+1})$ governed by

(1) $$-\frac{u_n(C_t, n_t)}{u_C(C_t, n_t)} = X_t F_2(K_t, n_t X_t)$$ Labor supply function (aka consumption-labor optimality)

(2) $$\frac{u_C(C_t, n_t)}{b u_C(C_{t+1}, n_{t+1})} = F_1(K_{t+1}, n_{t+1} X_{t+1}) + 1 - \delta$$ Capital supply function (aka consumption-savings optimality)

(3) $$C_t + K_{t+1} - (1-\delta)K_t = F(K_t, n_t X_t)$$

(4) $$X_{t+1} = \gamma X_t$$ Normalize $X_0 = 1$

RESTRICTIONS ON FUNCTIONAL FORMS

- **Deterministic dynamics of (C_t, K_{t+1}, n_t, X_t) governed by**

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- **(K1) Capital income share and labor income share of GDP are stationary**
  And viewing economic profits as zero

$$\Rightarrow F(K,nX) = K^{\alpha}(nX)^{1-\alpha} \qquad (\alpha \approx 0.4)$$

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$$\Rightarrow y_t\equiv\frac{Y_t}{X_t}=\bar{y},\quad k_t\equiv\frac{K_t}{X_t}=\bar{k},\quad c_t\equiv\frac{C_t}{X_t}=\bar{c},\quad\forall t$$

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Note: constant, but *possibly different*, rates depending on model (see LS 2018 p. 634)

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- Scale (3) by X_t for stationarity

Note long run growth rate affects capital accumulation even in stationary representation

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- **(K4) Hours per worker are stationary**

$$\Rightarrow n_t = \bar{n}$$

along deterministic path.

BUT $\bar{n}$ is endogenous…

Restrictions on Functional Forms

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- **Observations**
  - **Optimal choice of labor ($\bar{n}$) must be independent of X_t (from (1))**
    - **Requires offsetting income and substitution effects of wages (long-run productivity) on labor supply**
  - **IMRS can only depend on C_{t+1}/C_t (from (2)), which in turn = γ**

Require *CONSTANT* elasticity of intertemporal substitution (see LS 2018 p. 635)

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- **Two requirements together imply**

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King, Plosser, Rebelo (1988 *JME*)

STEADY STATE

- **Steady state $\left(\bar{c}, \bar{n}, \bar{k}\right)$ solves (1), (2), (3)**
- **A dynamic phenomenon!**
  - **Not static!**
  - **Economy is moving exactly along its long-run (i.e., deterministic) growth path**
  - **Balanced growth path**

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- **Scale of absolute quantity outcomes within model is meaningless**
  - **What does, e.g., $\bar{c} = 1.56$ mean?**
- **Relative quantity outcomes are interpretable**
  - **Provide calibration targets**
  - **e.g., $\bar{c} / \bar{y} = 0.70, \quad \bar{k} / \bar{y} = 2.5$ (if annual measurement)**

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- **Time use and intertemporal price outcomes within model are interpretable**
  - **Provide calibration targets**
  - **$\bar{n}$ is fraction of time spent in paid market work**
    - **Empirical: *n* ≈ 0.30**
  - **Return on capital $\alpha\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha-1} + 1 - \delta$**
- **Use ss calibration targets to set parameter values, given functional forms**

RBC Model Without Growth

- Often start instead with

$$u(c_t,n_t)=\begin{cases}\dfrac{[c_t v(n_t)]^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma>0,\ \sigma\neq 1\\ \ln c_t+v(n_t) & \text{if } \sigma=1\end{cases} \quad \text{and} \quad \sum_{t=0}^{\infty}\beta^t u(c_t,n_t)$$

$c_t = C_t/X_t$

- For this transformed model to deliver same steady state (relative quantities, time use, and *r*), require
  - Resource constraint $c_t+\gamma k_{t+1}-(1-\delta)k_t = z_t k_t^{\alpha} n_t^{1-\alpha}$
  - Subjective discount factor $\beta \equiv b\gamma^{1-\sigma}$ (King and Rebelo, p. 945)
- Typical assumption $\gamma = 1$ omits growth altogether
- What if trend growth rate fluctuates, γ_t?
  - Typical representation cannot accommodate trend shocks because $\gamma = 1$
  - Trend shocks have become increasingly common in DSGE models
  - Affects discount factor and capital accumulation equation

BASELINE RBC MODEL

- Assuming γ = 1…
- …complete calibration?

- Data: long-run labor income share of GDP ≈ 0.60
  - Cobb-Douglas *F*(.) implies

$$\frac{wn}{F(k,n)} = \frac{(1-\alpha)k^{\alpha}n^{1-\alpha}}{k^{\alpha}n^{1-\alpha}} = 1-\alpha \qquad \Rightarrow \qquad \alpha = 0.40$$

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$$\frac{k-(1-\delta)k}{k} = \frac{\delta k}{k} = 0.07 \qquad \Rightarrow \qquad \delta = 0.07 \ \text{(annual) or } 0.018 \ \text{(quarterly)}$$

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- **Data: long-run ratio of (annual) output to capital stock ≈ 0.4**
  - **Steady-state Euler equation**

$$1 = \beta\left[\frac{\alpha k^{\alpha}n^{1-\alpha}}{k} + 1 - \delta\right] = \beta\left[\frac{\alpha F(k,n)}{k} + 1 - \delta\right] \qquad \Rightarrow \qquad \beta = 0.95 \text{ (annual) or } 0.99 \text{ (quarterly)}$$

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OR **Data: avg. net real return on capital ≈ 5% per year (e.g, return on S&P500)**
  - Steady-state Euler equation

$$f_k(k,n) = \frac{1}{\beta} - 1 + \delta \qquad \Rightarrow \qquad \beta = 0.96 \ \text{(annual) or } 0.99 \ \text{(quarterly)}$$

BASELINE RBC MODEL

- **Utility parameters**

$$u(c_t, n_t) = \begin{cases} \dfrac{\left[c_t v(n_t)\right]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0,\ \sigma \neq 1 \\ \ln c_t + v(n_t) & \text{if } \sigma = 1 \end{cases}$$

- **Data: IES is around unity(?) or lower**
  - **Implies $\sigma > 1$**
  - **(Recall: IES = $1/\sigma$ for time-separable CRRA utility)**
  - **$\sigma = 1$ a conventional value**

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- **Labor subutility**
  - **Common form**

$$v(n) = -\frac{\psi}{1+1/\eta} n^{1+1/\eta}$$

  - **η measures Frisch elasticity of labor supply (use C-L optimality condition)**
  - **Calibrate Ψ to hit $\bar{n} \approx 0.3$**
  - **Empirical evidence on Frisch elasticity?**

BASELINE RBC MODEL

- Labor supply elasticity "controversial"

- Micro evidence: very low – η (substantially) smaller than one
- Macro evidence: very high – η (substantially) larger than one

- "Tension" between macro and micro evidence not useful way to frame the "controversy"

- Micro studies pick up intensive margin of labor supply
- Macro studies pick up (mostly) extensive margin of labor supply
  - And other frictions in allocation of workers to jobs…

- Common in DSGE models: $\eta \geq 1$

BASELINE RBC MODEL

- **Exogenous process for TFP (deviations from long-run trend productivity)**

$$\ln z_{t+1} = (1-\rho_z)\ln \bar{z} + \rho_z \ln z_t + \varepsilon_{t+1}$$

$$\varepsilon_{t+1} \text{ distibuted iid } N(0,\sigma_z^2)$$

- **Normalize** $\bar{z} = 1$
  - **Only governs absolute scale of model, which is arbitrary**
  - **What does, e.g.,** $\bar{c} = 1.56$ **mean?**

- **Construct time-series for z_t using**
  - **Data on labor, (detrended) capital, and (detrended) output**

- **AR(1) estimation**
  - **Quarterly frequency**

$$\Rightarrow \rho_z = 0.95 \quad \text{and} \quad \sigma_z = 0.007$$

USING THE RBC (OR ANY DSGE) MODEL

1. Dream up/construct/write fully-articulated model
   - ❒ Ideally to answer questions motivated by data and with hypotheses

2. Define equilibrium

3. Choose parameter values
   - ❒ Perhaps extremely rigorously, if goal is to match certain empirical facts very precisely
   - ❒ Perhaps adopting generally-accepted values, if goal is to illustrate some insight

4. Solve for deterministic steady state (balanced growth path)

5. Solve for dynamic decision rules (e.g., linear approximation, second-order approximation, global approximation)

6. Conduct informative battery of experiments (impulse responses, simulations, etc.) to try to falsify hypotheses

7. Tabulate results, write a (good!) paper, get it published