Balanced Growth II

January 10, 2020

RESTRICTIONS ON FUNCTIONAL FORMS

- **Deterministic dynamics of C_t governed by**

(1) $$-\frac{u_n(C_t,\bar{n})}{u_C(C_t,\bar{n})} = (1-\alpha)X_t\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

(3) $$\bar{c} + \gamma\bar{k} - (1-\delta)\bar{k} = \bar{k}^{\alpha}\bar{n}^{1-\alpha}$$

(2) $$\frac{u_C(C_t,\bar{n})}{bu_C(C_{t+1},\bar{n})} = \alpha\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha-1} + 1 - \delta$$

- **Final step – functional form for utility?**
- **Observations**
  - **Optimal choice of labor ($\bar{n}$) must be independent of X_t (from (1))**
    - **Requires offsetting income and substitution effects of wages (long-run productivity) on labor supply**

$$u(C_t,\bar{n}) = \ln C_t - \frac{\psi}{1+1/\eta}\cdot\bar{n}^{1+1/\eta}$$

$$-u_n(C_t,\bar{n}) = \psi\cdot\bar{n}^{1/\eta}$$

$$u_C(C_t,\bar{n}) = 1/C_t$$

$$-\frac{u_n(C_t,\bar{n})}{u_C(C_t,\bar{n})} = \frac{\psi\cdot\bar{n}^{1/\eta}}{1/C_t} = (1-\alpha)X_t\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

↓ **divide by X_t**

$$\frac{\psi\cdot\bar{n}^{1/\eta}}{X_t/C_t} = (1-\alpha)\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

→ $c_t \equiv C_t / X_t = \bar{c}$

$$\bar{c}\cdot\psi\cdot\bar{n}^{1/\eta} = (1-\alpha)\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

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$$\Rightarrow\quad -\frac{u_n(C_t,\bar{n})}{u_C(C_t,\bar{n})}=\psi\cdot\bar{n}^{1/\eta}=(1-\alpha)X_t\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

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GHH utility function does NOT satisfy BGP

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RESTRICTIONS ON FUNCTIONAL FORMS

$$x_{JR,t} = C_t^{\omega} \cdot x_{JR,t-1}^{1-\omega}$$

$$\frac{x_{JR,t}}{X_t} = \left(\frac{C_t}{X_t}\right)^{\omega} \left(\frac{x_{JR,t-1}}{X_{t-1}} \frac{X_{t-1}}{X_t}\right)^{1-\omega}$$

$$\overline{x}_{JR} = \overline{c}^{\omega} \cdot \overline{x}_{JR}^{1-\omega} \cdot \gamma^{\omega-1}$$

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$$\bar{x}_{JR} = \bar{c}^{\omega}\cdot\bar{x}_{JR}^{1-\omega}\cdot\gamma^{\omega-1}$$

JR utility function DOES satisfy BGP

divide by X_t

$$\frac{x_{JR,t}}{X_t}\cdot\psi\cdot\bar{n}^{1/\eta} = (1-\alpha)\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha} \longrightarrow \bar{x}_{JR}\cdot\psi\cdot\bar{n}^{1/\eta} = (1-\alpha)\left(\frac{\bar{k}}{\bar{n}}\right)^{\alpha}$$

$$\tilde{x}_{JR,t} \equiv x_{JR,t}/X_t = \bar{x}_{JR}$$

DYNAMICS

- **Jaimovich and Rebelo (2009 *AER*)**

parameter ω governs *short-run* income effect on labor supply

As ω ranges from 1 to 0, short-run income effect declines

$$u(c_t, n_t) = \ln\left(c_t - x_{JR,t} \cdot \frac{\psi}{1+\nu} n_t^{1+\nu} \right)$$

$$x_{JR,t} = c_t^{\omega} \cdot x_{JR,t-1}^{1-\omega}$$

i.e., short-run

parameter ω governs how quickly n responds to (temporary) TFP shock

As $\omega \rightarrow 0$, GHH preferences

If $\omega = 1$, KPR preferences

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$$u_c(c_t, n_t) = \frac{1}{c_t - x_{JR,t} \cdot \frac{\psi}{1+\nu} n_t^{1+\nu}}$$

$$u_n(c_t, n_t) = -\frac{x_{JR,t} \cdot \psi \cdot n_t^{\nu}}{\left(c_t - x_{JR,t} \cdot \frac{\psi}{1+\nu} n_t^{1+\nu} \right)}$$

$$MRS_{c_t, n_t} = -\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = \boxed{x_{JR,t} \cdot \psi \cdot n_t^{\nu} = w_t}$$

Short-Run Labor Supply

$$x_{JR,t} = c_t^{\omega} \cdot x_{JR,t-1}^{1-\omega}$$

$$x_{JR,t} \cdot \psi \cdot n_t^{\nu} = w_t \xrightarrow[\text{period-t LOM}]{\text{substitute}} c_t^{\omega} \cdot x_{JR,t-1}^{1-\omega} \cdot \psi \cdot n_t^{\nu} = w_t \xrightarrow[\text{period-(t-1) LOM}]{\text{substitute}} c_t^{\omega} \cdot \left(c_{t-1}^{\omega} \cdot x_{JR,t-2}^{1-\omega}\right)^{1-\omega} \cdot \psi \cdot n_t^{\nu} = w_t$$

$$\xrightarrow{\text{rewrite}} c_t^{\omega} \cdot c_{t-1}^{\omega(1-\omega)} \cdot x_{JR,t-2}^{(1-\omega)(1-\omega)} \cdot \psi \cdot n_t^{\nu} = w_t \xrightarrow[\text{period-(t-2) LOM}]{\text{substitute}} c_t^{\omega} \cdot c_{t-1}^{\omega(1-\omega)} \cdot \left(c_{t-2}^{\omega} \cdot x_{JR,t-3}^{1-\omega}\right)^{(1-\omega)(1-\omega)} \psi \cdot n_t^{\nu} = w_t$$

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$$\longrightarrow \boxed{\prod_{j=0}^{\infty} \left[c_{t-j}^{\omega(1-\omega)^j} \right] \cdot \psi \cdot n_t^{\nu} = w_t}$$

Short-run labor supply function

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$$\prod_{j=0}^{\infty}\left[\bar{c}^{\,\omega(1-\omega)^j}\right] \cdot \psi \cdot \bar{n}^{\nu} = w \xrightarrow[\text{geometric series}]{\text{infinite}} \bar{c} \cdot \psi \cdot \bar{n}^{\nu} = w$$

Long-run labor supply function

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$\omega = 1$ →

$$c_t \cdot \psi \cdot n_t^{\nu} = w_t$$

$$u(c_t, n_t) = \ln c_t - \frac{\psi}{1+\nu} n_t^{1+\nu}$$

$\omega = 0$ ↓

$$\psi \cdot n_t^{\nu} = w_t$$

$$u(c_t, n_t) = \ln\left(c_t - \frac{\psi}{1+\nu} n_t^{1+\nu}\right)$$

***ZERO* income effect on labor in GHH**

Smaller ω implies slower return of cyclical n_t to steady state n