# Balanced Growth III

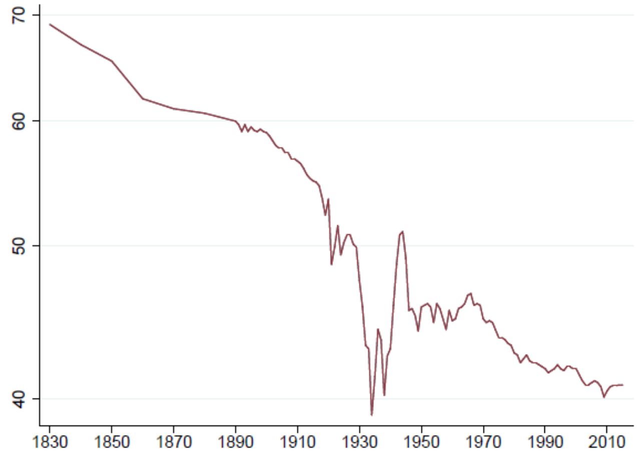
## January 15, 2020

# Calibration of RBC Model

- Must take a stand on three (related) points
  - Which data do we want model to match? (even constructing data is challenging…)
  - Functional forms (utility, production)
  - Parameter values
- Choose functional forms consistent with "Kaldor-plus facts"
  - (K1) Capital income share and labor income share of GDP are stationary
  - (K2) All real quantity variables grow at same rate in the long run ("great ratios")
  - (K3) Real interest rate is stationary
  - (K4) Hours per worker are stationary
  - (K5) (K2) requires trend productivity to be labor-augmenting (Phelps 1966)
- Often start with RBC model that abstracts from long-run growth
- But "true" calibration begins with model featuring *only* long-run growth
  - Puts restrictions on instantaneous utility and production forms
  - Use (K1)-(K5) to obtain these restrictions
- Richer models: more calibration targets and/or treating data differently
  - Monopoly markups (e.g., Dixit-Stiglitz and sticky price models)
  - Probability of finding a job (e.g., labor matching models)
  - Endogenous Growth Models

# Calibration of RBC Model

- **Boppart and Krusell (2020 *JPE*, Figure 2)**




(a) **Weekly U.S. hours worked per worker in nonfarm establishments 1830–2015**

**Notes:** This graph shows an updates series of the data in Greenwood and Vandenbroucke (2008). Regressing the log of hours on a constant and year gives a slope coefficient of -0.00315 in the full sample (and -0.00208 for the years 1970–2015).

# CALIBRATION OF RBC MODEL

- **Boppart and Krusell (2020 *JPE*, Figure 3)**

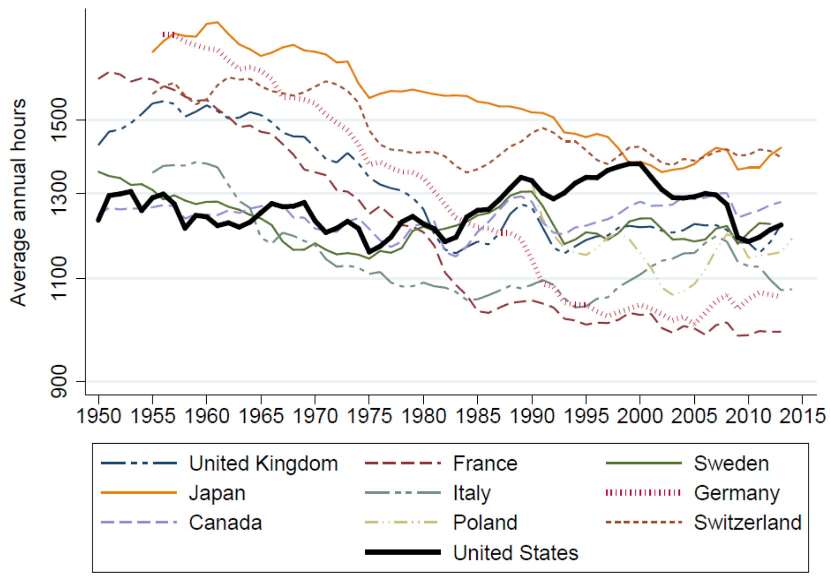



Figure 3: **Selected countries average annual hours per capita aged 15–64, 1950–2015**

**Notes:** Source: GGDC Total Economy Database for total hours worked and OECD for the data on population aged 15–64. The figure is comparable to the ones in Rogerson (2006). Regressing the logarithm of hours worked on time gives a slope coefficient of -0.00393.

# CALIBRATION OF RBC MODEL

- Ramey and Francis (2009 *AEJ: Macro*, Figure 1)

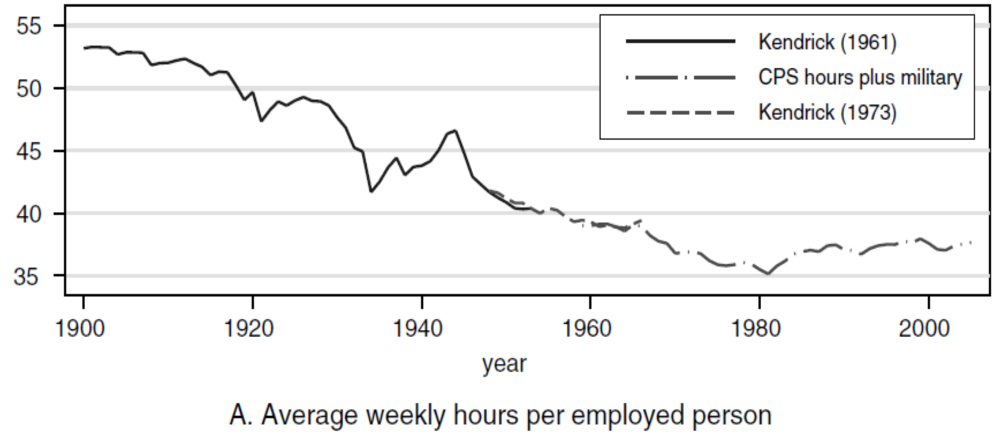



FIGURE 1. MEASURES OF AGGREGATE HOURS

*Sources:* Kendrick (1961, 1973), BLS data, and census data from Ruggles et al. (2004)

# CALIBRATION OF RBC MODEL

- **Boppart and Krusell (2020 *JPE*, Figure 4)**

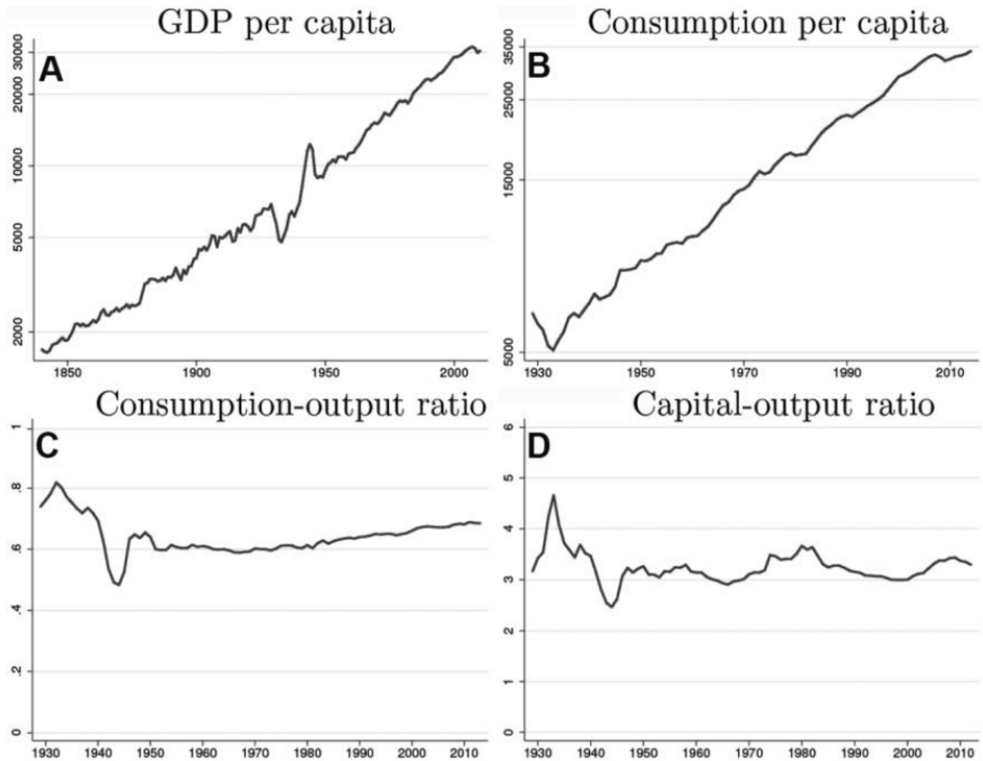



FIG. 4.—Balanced growth. Source: Bureau of Economic Analysis and Maddison Project. A color version of this figure is available online.

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# GENERALIZATION OF KPR

- **Boppart and Krusell (2020 *JPE*, Theorem 1)**

**exogenous parameter**

$\nu < 1$

**governs magnitude of *additional* income effect beyond KPR effect**

$$u(c,n) = \frac{\left(c \cdot v\left(nc^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma} - 1}{1-\sigma} \qquad \text{if } \sigma > 0, \quad \sigma \neq 1$$

$$u(c,n) = \ln c + \ln\left(v\left(nc^{\frac{\nu}{1-\nu}}\right)\right) \qquad \text{if } \sigma = 1$$

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- **$\nu$ = 0 nests KPR**

$$u(c,n) = \begin{cases} \dfrac{[cv(n)]^{1-\sigma} - 1}{1-\sigma} & \text{if } \sigma > 0, \ \sigma \neq 1 \\ \ln c + v(n) & \text{if } \sigma = 1 \end{cases}$$

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- **Intratemporal MRS and intertemporal MRS....**
- **....must allow for a BGP in which *c* and *n* grow at different rates**

$\gamma^{1-\nu}$ $\gamma^{-\nu}$

**Recall** $X_{t+1} = \gamma X_t$

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- Intratemporal MRS and intertemporal MRS....
- ....must allow for a BGP in which *c* and *n* grow at different rates

$c$: $\gamma^{1-\nu}$ $\quad$ $n$: $\gamma^{-\nu}$

Interpretation: "income augmented" *n*

define $x \equiv n \cdot c^{\frac{\nu}{1-\nu}}$

Recall $X_{t+1} = \gamma X_t$

- ***x* grows at rate *γ* (i.e., *x* is stationary)**

# GENERALIZATION OF KPR

- **Utility function**

$$u(c_t,n_t)=\frac{\left(c_t\cdot v\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma}-1}{1-\sigma}$$

- **Marginal utility functions**

$$u_c(c_t,n_t)=\frac{1}{c_t}\cdot\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)}{v\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)}\cdot n_t c_t^{\frac{\nu}{1-\nu}}\right]\cdot\left(c_t\cdot v\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma}$$

$$u_n(c_t,n_t)=\frac{1}{n_t}\cdot\left[\frac{v'\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)}{v\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)}\cdot n_t c_t^{\frac{\nu}{1-\nu}}\right]\cdot\left(c_t\cdot v\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma}$$

# GENERALIZATION OF KPR

- **Utility function**

$$u(c_t, n_t) = \frac{\left(c_t \cdot v\left(n_t c_t^{\frac{v}{1-v}}\right)\right)^{1-\sigma} - 1}{1-\sigma}$$

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definition

$\longrightarrow$

$$x_t \equiv n_t c_t^{\frac{v}{1-v}}$$

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Details

# Generalization of KPR

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$$u(c_t, n_t) = \frac{\left(c_t \cdot v\left(n_t c_t^{\frac{v}{1-v}}\right)\right)^{1-\sigma} - 1}{1-\sigma} \qquad x_t \equiv n_t c_t^{\frac{v}{1-v}}$$

- **Consumption Labor Optimality Condition**

elasticity $\varepsilon_{v,x}$

$$\boxed{\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = \frac{c_t}{n_t} \cdot \frac{\frac{v'(x_t) \cdot x_t}{v(x_t)}}{1 + \frac{v}{1-v} \cdot \frac{v'(x_t) \cdot x_t}{v(x_t)}}}$$

Static MRS

# FUNCTIONAL FORMS

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$\gamma^{1-\nu}$ $\quad$ $\gamma^{-\nu}$

**Recall** $X_{t+1} = \gamma X_t$

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- **Functional form for *v*(*x*)?**

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$$v(x) = \left[1 - a \cdot x^b\right]^{\frac{d}{1-\sigma}}$$

***a*, *b*, and *d* all parametric (p. 137)**

# Functional Forms

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- **MaCurdy (1981 *JPE*)**

# STATIC EXAMPLE

- **MaCurdy (1981 *JPE*)**

$$u(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \psi \cdot \frac{n^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$$

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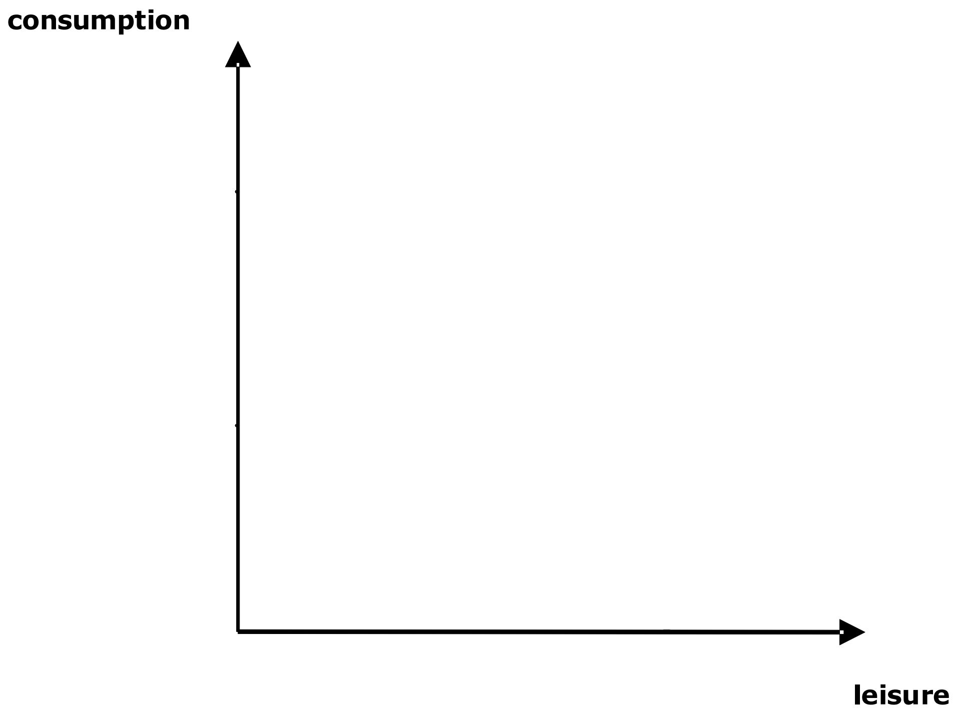
$$u_c = c^{-\sigma}$$

$$u_n = -\psi \cdot n^{\frac{1}{\theta}}$$

$$\Downarrow$$

$$-\frac{u_n}{u_c} = c^{\sigma} \cdot \psi \cdot n^{\frac{1}{\theta}} = w$$



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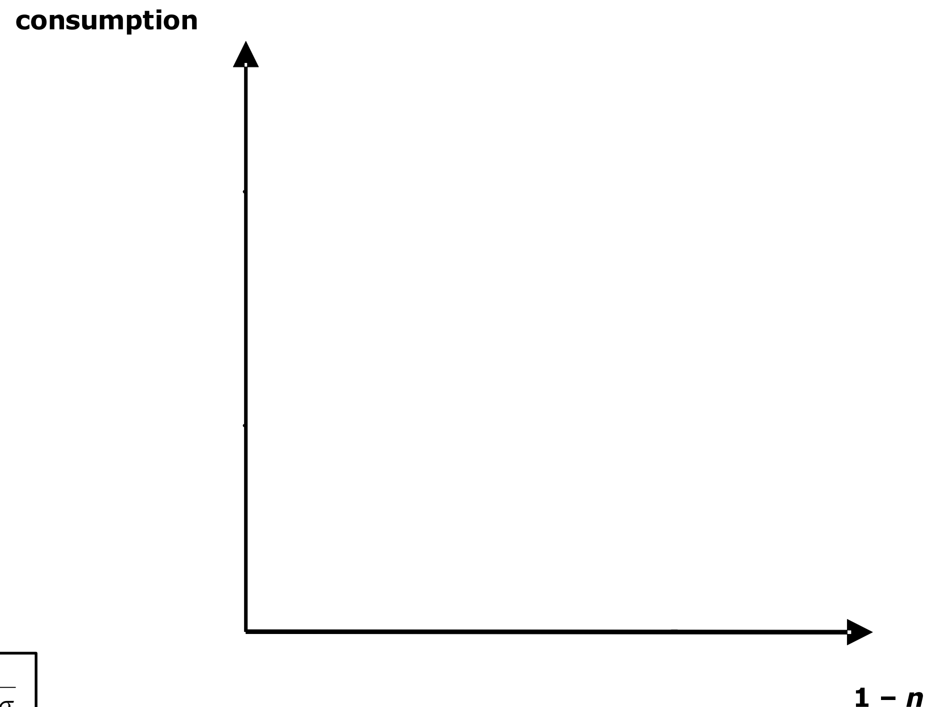
**substitute budget constraint** $\downarrow$ ***c = wn***

$$n^{\sigma} \cdot \psi \cdot n^{\frac{1}{\theta}} = w \quad \Rightarrow \quad \boxed{n^{*} = \left(\frac{w^{1-\sigma}}{\psi}\right)^{\frac{\theta}{1+\theta\cdot\sigma}}}$$



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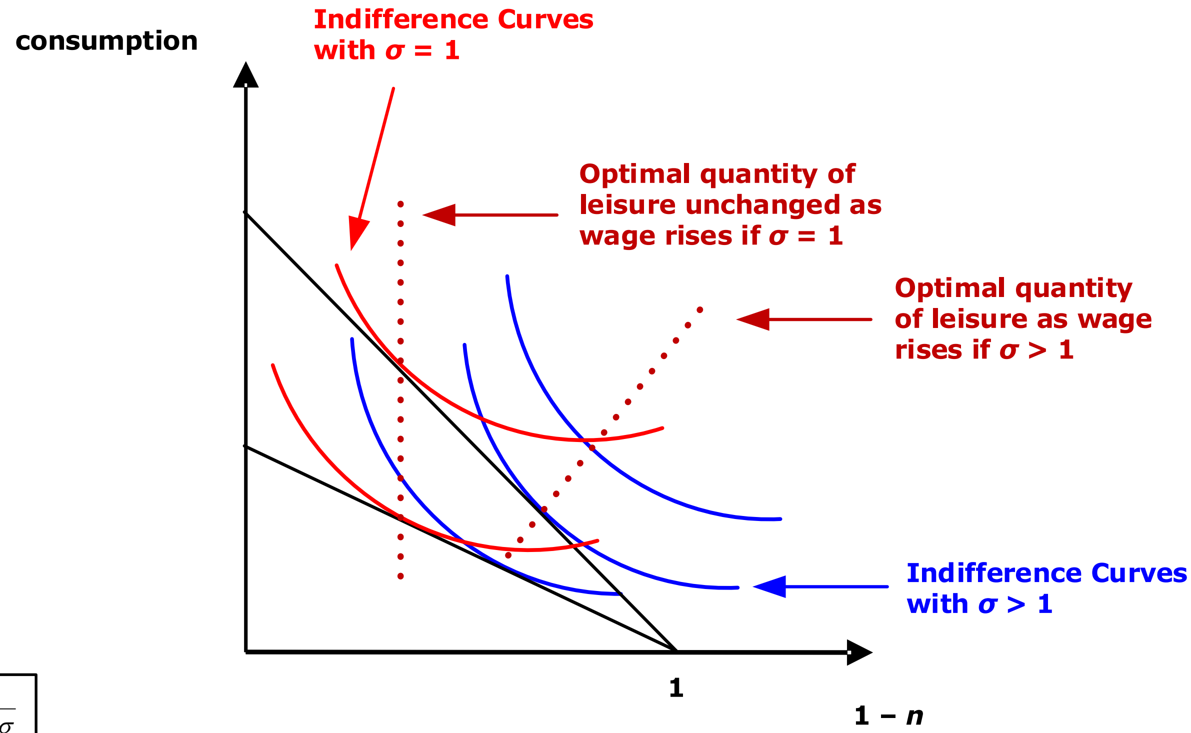

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$$a = \frac{\psi \cdot (1-\sigma)}{1 + \frac{1}{\theta}} \qquad b = 1 + \frac{1}{\theta} \qquad d = 1 \qquad \nu = \frac{\sigma - 1}{\sigma + \frac{1}{\theta}} \qquad \therefore\ \frac{\nu}{1-\nu} = \frac{\theta \cdot (\sigma - 1)}{1+\theta}$$

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# Functional Forms

$$u(c,n)=\frac{c^{1-\sigma}\left[1-\dfrac{\psi\cdot(1-\sigma)}{1+\dfrac{1}{\theta}}\cdot n^{1+\frac{1}{\theta}}\cdot c^{\sigma-1}\right]-1}{1-\sigma}$$

$$\downarrow$$

$$u(c,n)=\frac{c^{1-\sigma}-1-\dfrac{\psi\cdot(1-\sigma)}{1+\dfrac{1}{\theta}}\cdot n^{1+\frac{1}{\theta}}}{1-\sigma}$$

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**BK generalization places MaCurdy on BGP**

# INTERPRETATION

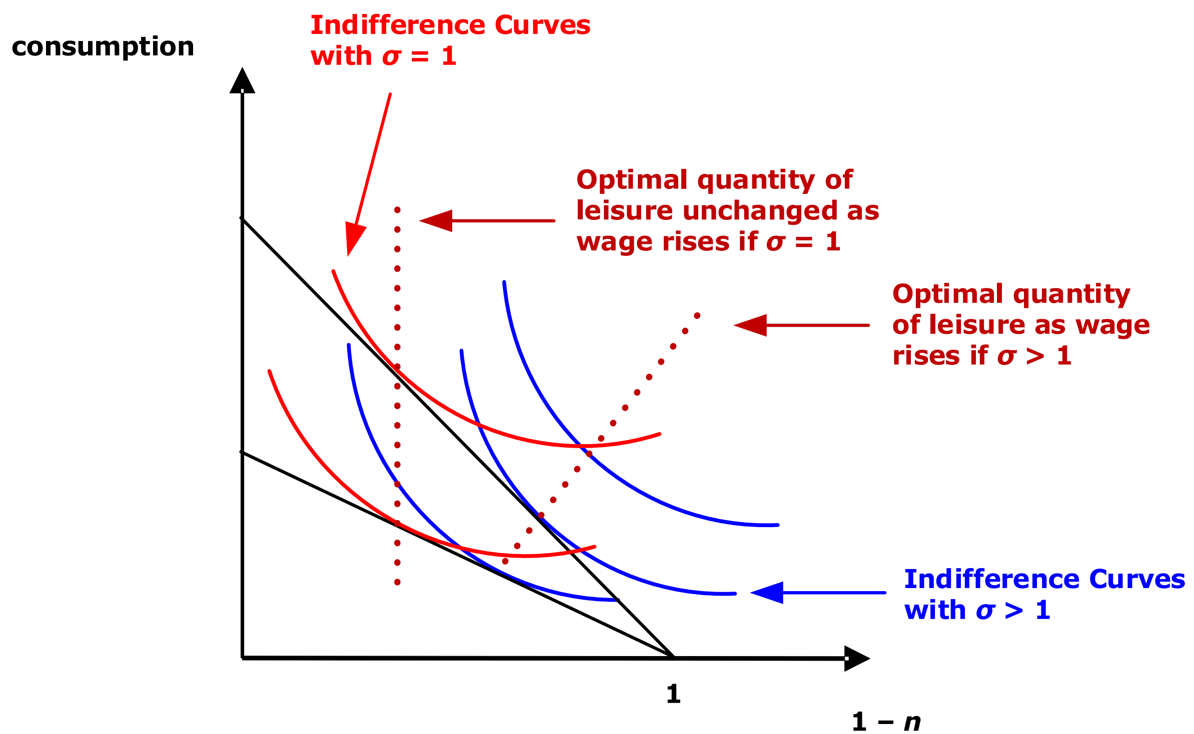
- BK (2020 *JPE*) generalization of MaCurdy (1981 *JPE*)

$$u(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \psi \cdot \frac{n^{1+\frac{1}{\theta}}}{1+\frac{1}{\theta}}$$

**<u>Independent Parameters</u>**

**$1/\sigma$ is intertemporal elasticity of substitution**

**$\theta$ is Frisch elasticity of labor supply**



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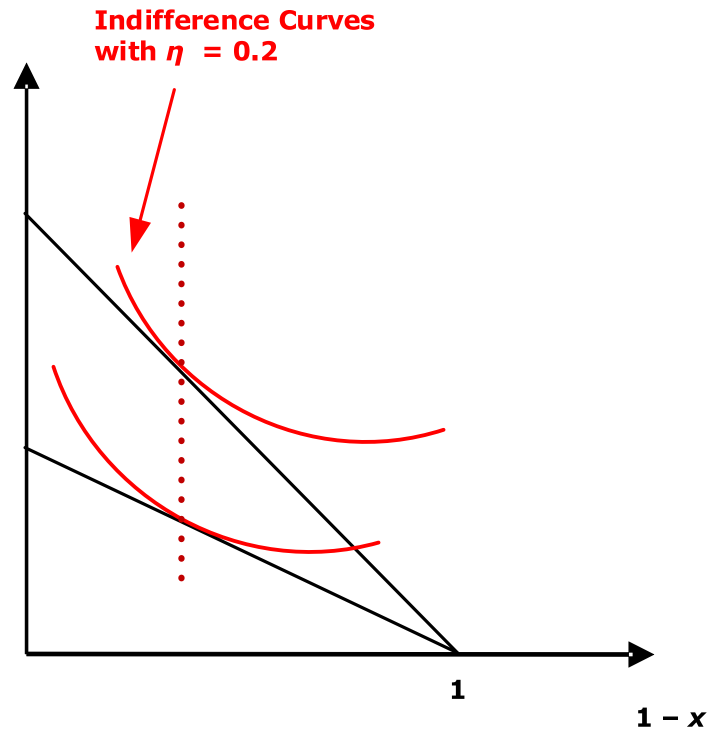
**Independent Parameters**

**1/$\sigma$ is intertemporal elasticity of substitution**

**$\theta$ is Frisch elasticity of labor supply**

**BK model links parameters via**

$$\frac{\nu}{1-\nu} = \frac{\theta \cdot (\sigma - 1)}{1+\theta}$$




$$x \equiv n \cdot c^{\frac{\nu}{1-\nu}}$$

**Define leisure as 1 – $x$**

# Generalization of KPR

- Boppart and Krusell (2020 *JPE*, Assumption 1)
- For any $\lambda > 0$

Both

for any $\lambda > 0$

$$-\frac{u_2\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)}{u_1\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)} = \lambda \cdot MP_{Nt}$$

$$\boxed{\nu < 1}$$

and

$$\frac{u_1\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)}{b \cdot u_1\left(c_{t+1} \cdot \lambda^{1-\nu}\gamma^{1-\nu}, n_{t+1} \cdot \lambda^{-\nu}\gamma^{-\nu}\right)} = R$$

scaling parameter $\lambda^{1-\nu}$
for consumption

scaling parameter $\lambda^{-\nu}$
for labor

must be satisfied

Dynamics

# GENERALIZATION OF KPR

- **Utility function**

$$u(c_t,n_t)=\frac{\left(c_t\cdot v\left(n_t c_t^{\frac{v}{1-v}}\right)\right)^{1-\sigma}-1}{1-\sigma}$$

- **Marginal utility functions**

**simplify notation**

$$u_c(c_t,n_t)=\frac{1}{c_t}\cdot\left[1+\frac{v}{1-v}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot v(x_t)\right)^{1-\sigma}$$

$$u_n(c_t,n_t)=\frac{1}{n_t}\cdot\left[\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot v(x_t)\right)^{1-\sigma}$$

**Observation:**
**elasticity of *v*(.) wrt *x***

# GENERALIZATION OF KPR

- **Utility function**

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**simplify notation** $\longrightarrow$

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$$u_n(c_t, n_t) = \frac{1}{n_t} \cdot \left[\frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot v(x_t)\right)^{1-\sigma}$$

**static MRS** $\longrightarrow$

$$\boxed{\frac{u_n(c_t, n_t)}{u_c(c_t, n_t)} = \frac{c_t}{n_t} \cdot \frac{\frac{v'(x_t)}{v(x_t)} \cdot x_t}{1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t}}$$

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# GENERALIZATION OF KPR

- Does static MRS satisfy Assumption 1?

$$\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)}=\frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

**Assumption 1**

$$-\frac{u_2\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}{u_1\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}=\lambda\cdot MP_n$$

# Generalization of KPR

- Does static MRS satisfy Assumption 1?

$$\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = \frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

**Assumption 1**

$$-\frac{u_2\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}{u_1\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}=\lambda\cdot MP_n$$

**scale *c* by $\lambda^{1-\nu}$**

**scale *n* by $\lambda^{-\nu}$**

$$=\frac{\lambda^{1-\nu}\cdot c_t}{\lambda^{-\nu}\cdot n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

$$=\lambda\cdot\frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

# Generalization of KPR

- Does static MRS satisfy Assumption 1?

$$\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)}=\frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

$$\Big\downarrow$$

**scale *c* by $\boldsymbol{\lambda^{1-\nu}}$**

**scale *n* by $\boldsymbol{\lambda^{-\nu}}$**

$$=\frac{\lambda^{1-\nu}\cdot c_t}{\lambda^{-\nu}\cdot n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

$$=\lambda\cdot\frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

**Assumption 1**

$$-\frac{u_2\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}{u_1\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}=\lambda\cdot MP_n$$

**Note:**

$$x_t=\left(n_t\lambda^{-\nu}\right)\left(c_t\lambda^{1-\nu}\right)^{\frac{\nu}{1-\nu}}=\lambda^{-\nu}n_t c_t^{\frac{\nu}{1-\nu}}\lambda^{\nu}$$

$$=x_t$$

# GENERALIZATION OF KPR

- **Does static MRS satisfy Assumption 1?**

$$\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)} = \frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

**scale *c* by $\lambda^{1-\nu}$**

**scale *n* by $\lambda^{-\nu}$**

$$=\frac{\lambda^{1-\nu}\cdot c_t}{\lambda^{-\nu}\cdot n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}$$

$$\boxed{\cancel{\lambda}\cdot\frac{c_t}{n_t}\cdot\frac{\frac{v'(x_t)}{v(x_t)}\cdot x_t}{1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t}=\cancel{\lambda}\cdot MP_n}$$

**YES**

**Assumption 1**

$$-\frac{u_2\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}{u_1\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}=\lambda\cdot MP_n$$

**Note:**

$$x_t=\left(n_t\lambda^{-\nu}\right)\left(c_t\lambda^{1-\nu}\right)^{\frac{\nu}{1-\nu}}=\lambda^{-\nu}n_tc_t^{\frac{\nu}{1-\nu}}\lambda^{\nu}$$

$$=x_t$$

**Intertemporal MRS?**

# GENERALIZATION OF KPR

- Does intertemporal MRS satisfy Assumption 1?

**Assumption 1**

$$\frac{u_1\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)}{b \cdot u_1\left(c_{t+1} \cdot \lambda^{1-\nu}\gamma^{1-\nu}, n_{t+1} \cdot \lambda^{-\nu}\gamma^{-\nu}\right)} = R$$

# GENERALIZATION OF KPR

- **Does intertemporal MRS satisfy Assumption 1?**

**Assumption 1**

$$\frac{u_1\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)}{b \cdot u_1\left(c_{t+1} \cdot \lambda^{1-\nu}\gamma^{1-\nu}, n_{t+1} \cdot \lambda^{-\nu}\gamma^{-\nu}\right)} = R$$

$\Big\downarrow$ **use $u_c$**

$$\frac{u_c(c_t, n_t)}{b \cdot u_c(c_{t+1}, n_{t+1})} = \frac{\frac{1}{c_t} \cdot \left[1 + \frac{\nu}{1-\nu} \cdot \frac{\nu'(x_t)}{\nu(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot \nu(x_t)\right)^{1-\sigma}}{b \cdot \frac{1}{c_{t+1}} \cdot \left[1 + \frac{\nu}{1-\nu} \cdot \frac{\nu'(x_{t+1})}{\nu(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot \nu(x_{t+1})\right)^{1-\sigma}} = \frac{1}{b} \cdot \frac{c_{t+1}}{c_t} \cdot \frac{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{\nu'(x_t)}{\nu(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot \nu(x_t)\right)^{1-\sigma}}{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{\nu'(x_{t+1})}{\nu(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot \nu(x_{t+1})\right)^{1-\sigma}}$$

# GENERALIZATION OF KPR

- Does intertemporal MRS satisfy Assumption 1?

**Assumption 1**

$$\frac{u_1\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)}{b \cdot u_1\left(c_{t+1} \cdot \lambda^{1-\nu}\gamma^{1-\nu}, n_{t+1} \cdot \lambda^{-\nu}\gamma^{-\nu}\right)} = R$$

**use $u_c$**

$$\frac{u_c(c_t, n_t)}{b \cdot u_c(c_{t+1}, n_{t+1})} = \frac{\frac{1}{c_t} \cdot \left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot v(x_t)\right)^{1-\sigma}}{b \cdot \frac{1}{c_{t+1}} \cdot \left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_{t+1})}{v(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot v(x_{t+1})\right)^{1-\sigma}} = \frac{1}{b} \cdot \frac{c_{t+1}}{c_t} \cdot \frac{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot v(x_t)\right)^{1-\sigma}}{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_{t+1})}{v(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot v(x_{t+1})\right)^{1-\sigma}}$$

**scale $c_t$ by $\lambda^{1-\nu}$ and $c_{t+1}$ by $\lambda^{1-\nu}\gamma^{1-\nu}$**

$$= \frac{1}{b} \cdot \frac{c_{t+1} \cdot \lambda^{1-\nu}\gamma^{1-\nu}}{c_t \cdot \lambda^{1-\nu}} \cdot \frac{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot \lambda^{1-\nu} \cdot v(x_t)\right)^{1-\sigma}}{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_{t+1})}{v(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot \lambda^{1-\nu}\gamma^{1-\nu} \cdot v(x_{t+1})\right)^{1-\sigma}}$$

**collect terms** $\longrightarrow$

# GENERALIZATION OF KPR

- Does intertemporal MRS satisfy Assumption 1?

**Assumption 1**

$$\frac{u_1\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}{b\cdot u_1\left(c_{t+1}\cdot\lambda^{1-\nu}\gamma^{1-\nu},n_{t+1}\cdot\lambda^{-\nu}\gamma^{-\nu}\right)}=R$$

$$\ldots=\frac{1}{b}\cdot\frac{c_{t+1}\cdot\lambda^{1-\nu}\gamma^{1-\nu}}{c_t\cdot\lambda^{1-\nu}}\cdot\frac{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot\lambda^{1-\nu}\cdot v(x_t)\right)^{1-\sigma}}{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_{t+1})}{v(x_{t+1})}\cdot x_{t+1}\right]\cdot\left(c_{t+1}\cdot\lambda^{1-\nu}\gamma^{1-\nu}\cdot v(x_{t+1})\right)^{1-\sigma}}$$

$$=\frac{1}{b}\cdot\frac{c_{t+1}\cdot\gamma^{1-\nu}}{c_t}\cdot\frac{\left(\lambda^{1-\nu}\right)^{1-\sigma}}{\left(\lambda^{1-\nu}\right)^{1-\sigma}\left(\gamma^{1-\nu}\right)^{1-\sigma}}\cdot\frac{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot v(x_t)\right)^{1-\sigma}}{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_{t+1})}{v(x_{t+1})}\cdot x_{t+1}\right]\cdot\left(c_{t+1}\cdot v(x_{t+1})\right)^{1-\sigma}}$$

$$=\frac{1}{b}\cdot\gamma^{\sigma(1-\nu)}\cdot\frac{c_{t+1}}{c_t}\cdot\frac{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot v(x_t)\right)^{1-\sigma}}{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_{t+1})}{v(x_{t+1})}\cdot x_{t+1}\right]\cdot\left(c_{t+1}\cdot v(x_{t+1})\right)^{1-\sigma}}$$

# GENERALIZATION OF KPR

- Does intertemporal MRS satisfy Assumption 1?

**Assumption 1**

$$\frac{u_1\left(c_t\cdot\lambda^{1-\nu},n_t\cdot\lambda^{-\nu}\right)}{b\cdot u_1\left(c_{t+1}\cdot\lambda^{1-\nu}\gamma^{1-\nu},n_{t+1}\cdot\lambda^{-\nu}\gamma^{-\nu}\right)}=R$$

$$\ldots=\frac{1}{b}\cdot\frac{c_{t+1}\cdot\lambda^{1-\nu}\gamma^{1-\nu}}{c_t\cdot\lambda^{1-\nu}}\cdot\frac{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot\lambda^{1-\nu}\cdot v(x_t)\right)^{1-\sigma}}{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_{t+1})}{v(x_{t+1})}\cdot x_{t+1}\right]\cdot\left(c_{t+1}\cdot\lambda^{1-\nu}\gamma^{1-\nu}\cdot v(x_{t+1})\right)^{1-\sigma}}$$

$$=\frac{1}{b}\cdot\frac{c_{t+1}\cdot\gamma^{1-\nu}}{c_t}\cdot\frac{\left(\lambda^{1-\nu}\right)^{1-\sigma}}{\left(\lambda^{1-\nu}\right)^{1-\sigma}\left(\gamma^{1-\nu}\right)^{1-\sigma}}\cdot\frac{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot v(x_t)\right)^{1-\sigma}}{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_{t+1})}{v(x_{t+1})}\cdot x_{t+1}\right]\cdot\left(c_{t+1}\cdot v(x_{t+1})\right)^{1-\sigma}}$$

**$R\equiv\gamma^{\sigma(1-\nu)}$** **(p. 21)**

$$=\frac{1}{b}\cdot\gamma^{\sigma(1-\nu)}\cdot\frac{c_{t+1}}{c_t}\cdot\frac{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_t)}{v(x_t)}\cdot x_t\right]\cdot\left(c_t\cdot v(x_t)\right)^{1-\sigma}}{\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'(x_{t+1})}{v(x_{t+1})}\cdot x_{t+1}\right]\cdot\left(c_{t+1}\cdot v(x_{t+1})\right)^{1-\sigma}}$$

# Generalization of KPR

- Does intertemporal MRS satisfy Assumption 1?

$$\ldots = \frac{1}{b} \cdot \frac{c_{t+1} \cdot \lambda^{1-\nu} \gamma^{1-\nu}}{c_t \cdot \lambda^{1-\nu}} \cdot \frac{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot \lambda^{1-\nu} \cdot v(x_t)\right)^{1-\sigma}}{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_{t+1})}{v(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot \lambda^{1-\nu} \gamma^{1-\nu} \cdot v(x_{t+1})\right)^{1-\sigma}}$$

$$= \frac{1}{b} \cdot \frac{c_{t+1} \cdot \gamma^{1-\nu}}{c_t} \cdot \frac{\left(\lambda^{1-\nu}\right)^{1-\sigma}}{\left(\lambda^{1-\nu}\right)^{1-\sigma} \left(\gamma^{1-\nu}\right)^{1-\sigma}} \cdot \frac{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot v(x_t)\right)^{1-\sigma}}{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_{t+1})}{v(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot v(x_{t+1})\right)^{1-\sigma}}$$

$$\boxed{\frac{1}{b} \cdot \gamma^{\sigma(1-\nu)} \cdot \frac{c_{t+1}}{c_t} \cdot \frac{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_t)}{v(x_t)} \cdot x_t\right] \cdot \left(c_t \cdot v(x_t)\right)^{1-\sigma}}{\left[1 + \frac{\nu}{1-\nu} \cdot \frac{v'(x_{t+1})}{v(x_{t+1})} \cdot x_{t+1}\right] \cdot \left(c_{t+1} \cdot v(x_{t+1})\right)^{1-\sigma}} = R \quad \left(= \gamma^{\sigma(1-\nu)}\right)}$$

**Assumption 1**

$$\frac{u_1\left(c_t \cdot \lambda^{1-\nu}, n_t \cdot \lambda^{-\nu}\right)}{b \cdot u_1\left(c_{t+1} \cdot \lambda^{1-\nu} \gamma^{1-\nu}, n_{t+1} \cdot \lambda^{-\nu} \gamma^{-\nu}\right)} = R$$

**YES**

***R* ≡ $\gamma^{\sigma(1-\nu)}$**
**(p. 21)**

# Marginal Rate of Substitution

- **Utility function**

$$u(c_t, n_t) = \frac{\left(c_t \cdot v\left(n_t c_t^{\frac{\nu}{1-\nu}}\right)\right)^{1-\sigma} - 1}{1-\sigma}$$

- **Marginal utility of $c_t$**

$$u_c(c_t, n_t) = \Big(c_t \cdot v\Big(\underbrace{n_t c_t^{\frac{\nu}{1-\nu}}}\Big)\Big)^{-\sigma} \cdot v\Big(\underbrace{n_t c_t^{\frac{\nu}{1-\nu}}}\Big) + \Big(c_t \cdot v\Big(\underbrace{n_t c_t^{\frac{\nu}{1-\nu}}}\Big)\Big)^{-\sigma} \cdot c_t \cdot v'\Big(\underbrace{n_t c_t^{\frac{\nu}{1-\nu}}}\Big) \cdot \left(\frac{\nu}{1-\nu}\right) \cdot \underbrace{n_t c_t^{\frac{\nu}{1-\nu}}} \cdot c_t^{-1}$$

$$x_t \equiv n_t c_t^{\frac{\nu}{1-\nu}}$$

**use $x_t$ notation (now that FOC has been computed)**

$$u_c(c_t, n_t) = \left(c_t \cdot v(x_t)\right)^{-\sigma} \cdot v(x_t) + \left(c_t \cdot v(x_t)\right)^{-\sigma} \cdot \cancel{c_t} \cdot v'(x_t) \cdot \left(\frac{\nu}{1-\nu}\right) \cdot x_t \cdot \cancel{c_t^{-1}}$$

**simplify**

# Marginal Rate of Substitution

$$u_c(c_t,n_t)=\left(c_t\cdot v\left(x_t\right)\right)^{-\sigma}\cdot\left[v(x_t)+\left(\frac{\nu}{1-\nu}\right)\cdot v'\left(x_t\right)\cdot x_t\right]$$

**multiply and divide by *cv*(*x*)**

$$u_c(c_t,n_t)=\frac{\left(c_t\cdot v\left(x_t\right)\right)^{1-\sigma}\cdot\left[v(x_t)+\left(\frac{\nu}{1-\nu}\right)\cdot v'\left(x_t\right)\cdot x_t\right]}{c_t\cdot v(x_t)}$$

**rearrange**

$$\boxed{u_c(c_t,n_t)=\frac{1}{c_t}\cdot\left[1+\frac{\nu}{1-\nu}\cdot\frac{v'\left(x_t\right)\cdot x_t}{v\left(x_t\right)}\right]\cdot\left(c_t\cdot v\left(x_t\right)\right)^{1-\sigma}}$$

# Marginal Rate of Substitution

- **Utility function**

$$u(c_t,n_t)=\frac{\left(c_t\cdot v\left(n_t c_t^{\frac{v}{1-v}}\right)\right)^{1-\sigma}-1}{1-\sigma}$$

- **Marginal utility of $n_t$**

$$u_n(c_t,n_t)=\left(c_t\cdot v\left(\underbrace{n_t c_t^{\frac{v}{1-v}}}\right)\right)^{-\sigma}\cdot c_t\cdot v'\left(\underbrace{n_t c_t^{\frac{v}{1-v}}}\right)\cdot c_t^{\frac{v}{1-v}}$$

**use $x_t$ notation (now that FOC has been computed)**

$$x_t\equiv n_t c_t^{\frac{v}{1-v}}$$

**multiply and divide by *n***

$$u_n(c_t,n_t)=\frac{c_t}{n_t}\cdot\left(c_t\cdot v\left(x_t\right)\right)^{-\sigma}\cdot v'\left(x_t\right)\cdot \underbrace{n_t c_t^{\frac{v}{1-v}}}$$

**multiply and divide by *cv*(*x*)**

# Marginal Rate of Substitution

$$u_n(c_t,n_t)=\frac{\left(c_t\cdot v\left(x_t\right)\right)^{1-\sigma}\frac{\cancel{c_t}}{n_t}\cdot v'\left(x_t\right)\cdot x_t}{\cancel{c_t}v(x_t)}$$

$$\boxed{u_n(c_t,n_t)=\frac{1}{n_t}\cdot\left[\frac{v'\left(x_t\right)\cdot x_t}{v\left(x_t\right)}\right]\cdot\left(c_t\cdot v\left(x_t\right)\right)^{1-\sigma}}$$

**elasticity $\varepsilon_{v,x}$**

$$\boxed{\frac{u_n(c_t,n_t)}{u_c(c_t,n_t)}=\frac{c_t}{n_t}\cdot\frac{\frac{v'\left(x_t\right)\cdot x_t}{v\left(x_t\right)}}{1+\frac{\nu}{1-\nu}\cdot\frac{v'\left(x_t\right)\cdot x_t}{v\left(x_t\right)}}}$$

**Static MRS**

Main slide