Labor Matching Models: Basic Building Blocks

January 22, 2020

Basic DSGE Issues

- Labor fluctuations at extensive margin (number of people working) larger than at intensive margin (hours worked per employee)

- Labor market structure(s) important to understand/model more deeply
  - Theoretical interest: Many results from existing frameworks point to it
  - Empirical interest: Labor-market outcomes the most important economic aspect of many (most?) people's lives
  - "Labor wedges" – CKM (2007 *EC*), Shimer (2009 *AEJ:Macro*), Karabarbounis (2014 *RED*), ***MANY*** others

- Explosion of DSGE labor matching models the past ten years
  - Sparked in part by Shimer (2005 *AER*) and Hall (2005 *AER*)
    - Although their models were not full GE models
  - Not yet clear what problems incorporating labor matching has helped solve….
  - …but has likely shed insight on some issues (e.g., in cyclical fluctuations and in policy analysis, real wage dynamics matter a lot)

- Rogerson and Shimer (2011 *Handbook of Labor Economics*)

Basic Labor Market Issues

- **How can production resources sit idle even when there is "high aggregate demand?"**

- **Coordination frictions in labor markets**
  - **Finding a job or an employee takes time and/or resources**
  - **Not articulated in basic neoclassical/Walrasian framework**

- **Are labor market transactions "spot" transactions?**
  - **Or do they occur in the context of ongoing relationships?**
  - **The answer implies quite different roles for prices (wages)**

- **"Structural" vs. "frictional" unemployment**
  - **Structural: unemployment induced by fundamental changes in technology, etc – dislocations due to insufficient job training, changing technical/educational needs of workforce, etc.**

  - **Frictional: temporarily unemployed as workers and jobs shuffle from one partner to another**

Basic Building Blocks

- Aggregate matching function
- Law of motion for employment
- Vacancy posting costs
- Some wage determination mechanism (Nash or many others…)
- Intensive (aka "hours") margin?
  - Often absent…
- Endogenous consumption/labor "supply" decision?
  - Typically absent…
  - Can consider it implicitly in the background (might depend on the wage determination mechanism…)
  - …or consider it explicitly by introducing a third activity for individuals ("outside the labor force")

Basic Building Blocks

- **Aggregate matching function**

$$m(u_t, v_t)$$

Typically assumed to be Cobb-Douglas (see Petrongolo and Pissarides 2001 *JEL*)

  - **Brings together individuals looking for work (*u*) and employers looking for workers (*v*)**
  - **A technology from the perspective of the economy (just like aggregate production function)**
  - **Black box that describes all the possible coordination, matching, informational, temporal, geographic, etc. frictions in finding workers and jobs**

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- **Employment is a state variable (one specific timing)**

Churning of jobs; a job is not an absorbing state

$$n_{t+1} = (1-\rho_x)n_t + m(u_t, v_t)$$

Aggregate law of motion of employment

Number of existing jobs that end: ρ_x exogenous separation rate, but can also endogenize

Number of new jobs (matches) that form in *t* and will become active in *t*+1

ANALOGY: $k_{t+1} = (1-\delta)k_t + i_t$

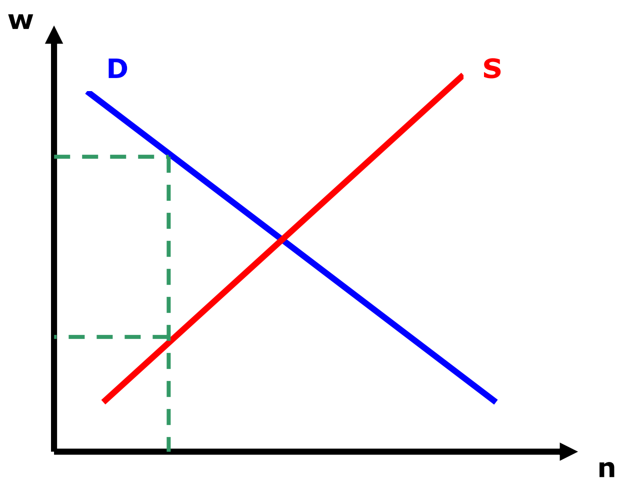
Basic Building Blocks

- **Vacancy posting costs**
  - **Each new job opening incurs a cost**
  - **A primitive cost**
- **Suppose total vacancy posting costs = γv_t**
- **→ marginal cost of vacancy posting = ...?...**
- **→ average cost of vacancy posting = ...?...**
- **(Typical assumption in literature)**
- **Realistic for recruiting departments?**
- **If not, suppose convex (concave?) costs of posting vacancies**
- **Total vacancy posting costs = $\gamma g(v_t)$**
- **Does marginal cost = average cost ?....**

BASIC BUILDING BLOCKS

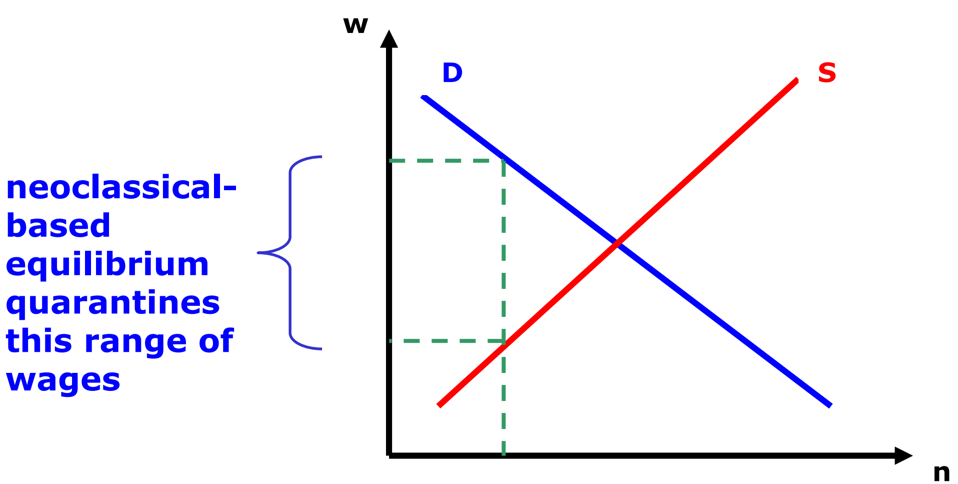
- **Wage determination**
  - **Labor transactions not neoclassical(-based), so no simple supply-and-demand based pricing**

Walrasian "wedge" between $MRS_{C,L}$ and MP_N



BASIC BUILDING BLOCKS

- **Wage determination**
  - **Labor transactions not neoclassical(-based), so no simple supply-and-demand based pricing**
  - **Local (bilateral, *not* market-based) monopolies (local rents) exist between each worker-employer pair**
    - **Exist due to the matching friction and ex-ante costs of hiring**
    - **Allows a wide range (too wide?) of wage-determination schemes – one of the points of Hall (2005 *AER*)**



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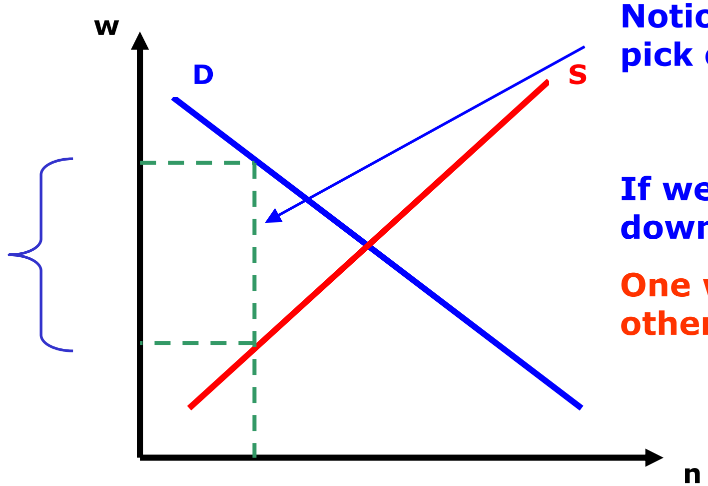
neoclassical-based equilibrium quarantines this range of wages



Notion of *matching equilibrium* can pick out these *w*'s…

If we have a systematic way of pinning down a particular *w*

One wage model: Nash bargaining (many others…)

IMPORTANT: **wage plays a very different role than in neoclassical(-based) labor market – *not* purely allocative, now also plays a distributive role**

BASIC BUILDING BLOCKS

- (Generalized) Nash Bargaining

Bargaining powers *η* and 1-*η* measure "strength" of each party in negotiations

$$\max_{w_t} \left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)^{h} \left(\mathbf{J}(w_t) - \mathbf{V}(w_t)\right)^{1-h}$$

Net payoff to an individual of agreeing to wage *w* and beginning production

Net payoff to a firm of agreeing to wage *w* and beginning production

Original Nash 1950 was *η* = 0.5

- The unique problem whose solution satisfies three axioms (Nash 1950)
  - Pareto optimality
  - Scale invariance
  - Independence of irrelevant alternatives

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- Given an extensive-form foundation by Binmore (1980) and Binmore, Rubinstein, Wolinksy (1986)
  - Nash solution the limiting solution of a Rubinstein alternating-offers game (as time interval between successive offers → zero)
  - In which (η, 1- η) measure discount factors of each party between successive offers

Analysis of Model

- ❒ **Study firm vacancy posting decision**
  - ❒ **Representative firm chooses desired number of workers to hire**
    - ❒ **Typical setup in DSGE labor matching models…**
    - ❒ **…in contrast to partial equilibrium labor matching models (one firm = one job) – but equivalent if sufficient linearity**

- ❒ **Study household/worker decision(s)**
  - ❒ **No labor-force participation decision in baseline model**
  - ❒ **Full consumption insurance the norm in DSGE matching models**
    - ❒ **All individuals live in a "large" (infinite) household, so full risk-sharing**

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- How do matching markets clear?

- How are wages determined?

Labor Matching Models: Basic DSGE Implementation

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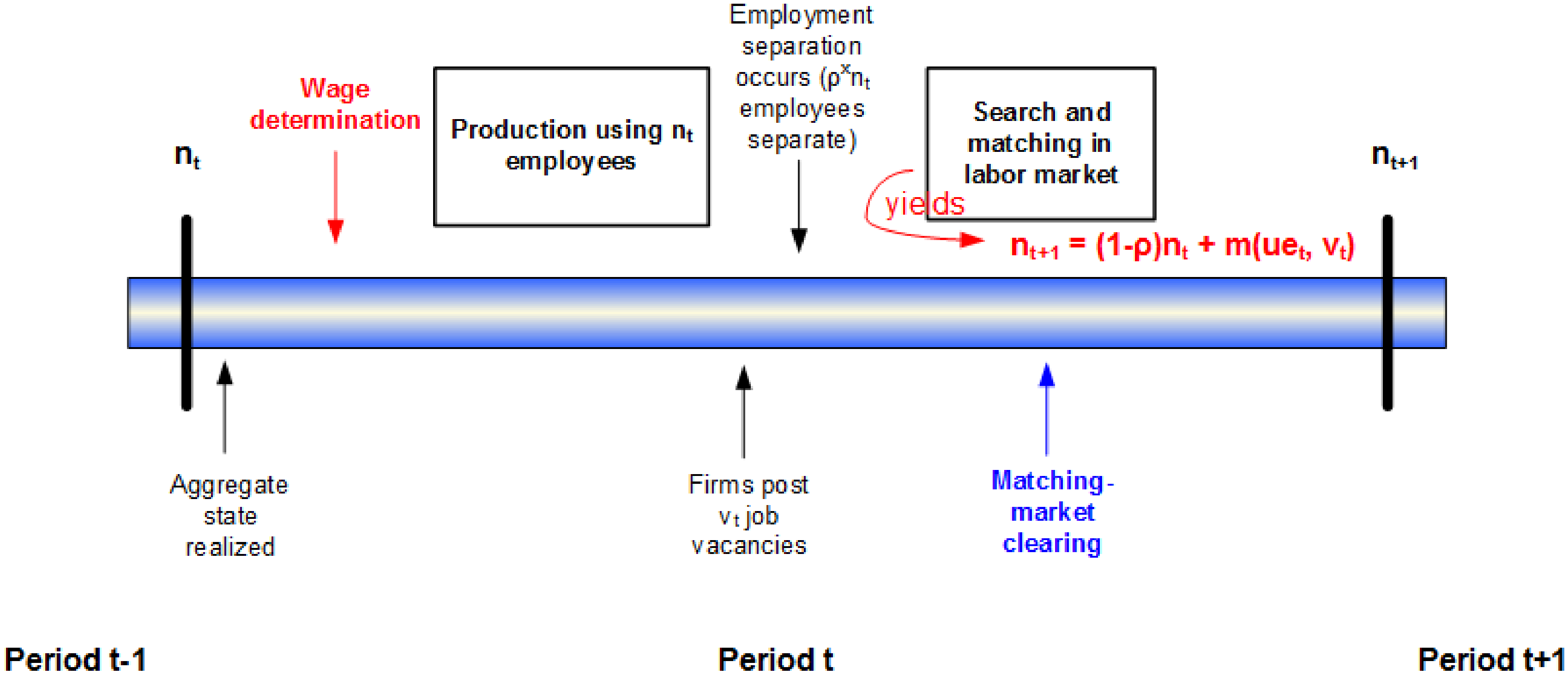
TIMELINE



("Lagged production" timing – use for now…)

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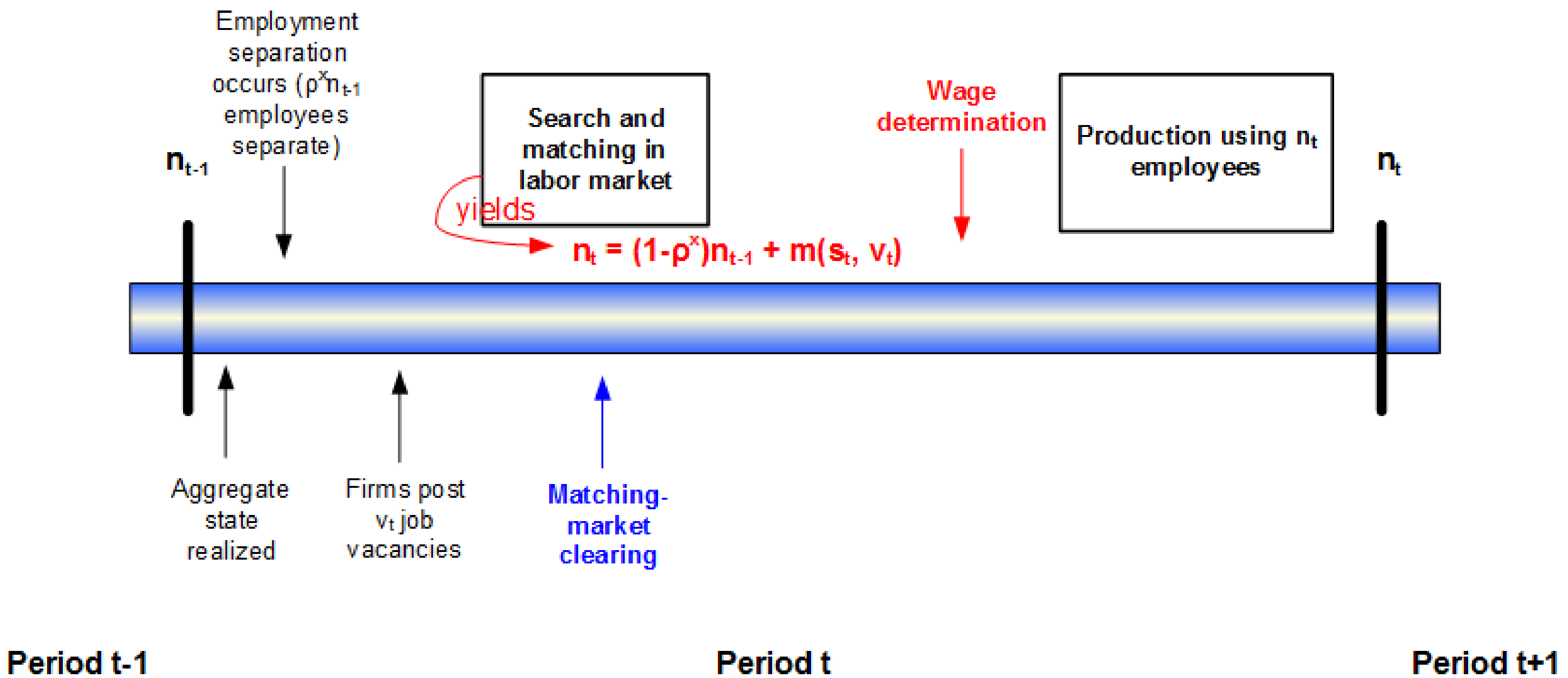

("Instantaneous production" timing…)

FIRM VACANCY-POSTING PROBLEM

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n^f_{t+1}} \left[E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n^f_t f(h_t) - w_t n^f_t h_t - \gamma g(v_t) \right) \right]$$

Discount factor between time 0 and *t* because *dynamic* firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many job advertisements)

Desired target *future* firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting *v* vacancies

- **Subject to (perceived) law of motion for firm's employment stock**

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- Basic model
  - Shut down intensive margin: $h_t = 1$
  - Linear posting costs: $g(v) = v$
  - Firm production function: $y_t = z_t n_t$

Firm Vacancy-Posting Problem

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f - w_t n_t^f - \gamma v_t \right) \right]$$

- **Two "market-determined" prices taken as given**
- **Wage-setting (process) taken as given**
  - **(IF goods production function is constant returns)**

  - **Subject to (perceived) law of motion for firm's employment stock**

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$$\text{s.t. } n_{t+1}^f = \underbrace{(1-\rho_x) n_t^f}_{} + v_t k^f(\theta_t)$$

Perceived law of motion for evolution of employment stock

Number of existing jobs that remain intact: ρ_x exogenous separation rate, but can also endogenize

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Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a *market* variable, θ, so taken as given

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FOCs with respect to v_t, n_{t+1}

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho_x)\mu_{t+1} \right) \right\} = 0$$

Combine

FIRM VACANCY-POSTING PROBLEM

- Vacancy posting condition (aka job creation condition)

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of attracting a worker) x (expected future benefit of an additional worker)

= marginal output – wage payment + expected asset value of an additional worker

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- **Vacancy-posting is a type of investment decision**
  - **Intertemporal dimension makes discount factor potentially important**
    - **Makes general equilibrium effects potentially important**

- **Two prices affect posting decision (aside from intertemporal price)**
  - **Wage**
  - **Matching probability k^f (which depends on the market variable θ)**

HOUSEHOLD PROBLEM

- Dynamic household utility-maximization problem
  - A continuum [0, 1] of households (a standard assumption)
  - A continuum [0, 1] of atomistic individuals live in each household
  - Representative household has continuum of "family members"

$$\max_{c_t, n_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - A \cdot n_t \right) \right]$$

$$\text{s.t.} \quad c_t + a_t = \underbrace{n_t w_t h_t}_{} + \underbrace{(1-n_t) b}_{} + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

Measure n_t of family members earn labor income (because they work) (and recall we've normalized h = 1)

Measure 1-n_t of family members receive unemployment benefits and/or engaged in home production

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KEY: Assuming infinite *family* structure delivers *full consumption insurance* – i.e., all employed and unemployed individuals have equal consumption

Individual family members are risk-neutral with respect to their labor-market realization

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- Consumption-savings optimality condition: $1 = R_t E_t \left\{ \underbrace{\frac{\beta u'(c_{t+1})}{u'(c_t)}}_{} \right\}$

Stochastic discount factor

- Start with exogenous LFP
  - Each family member either works or is looking for work

Wage Bargaining

- **(Generalized) Nash Bargaining**

$$\max_{w_t} \left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)^{\eta} \left(\mathbf{J}(w_t) - \mathbf{V}(w_t)\right)^{1-\eta}$$

Bargaining over how to divide the surplus

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- **Value equations**
  - ***W***: value to (representative) household of having one additional member employed
  - ***U***: value to (representative) household of having one additional member unemployed and searching for work
  - ***J***: value to (representative) firm of having one additional employee
  - ***V***: value to (representative) firm of having a vacancy that goes unfilled
    - Free entry in vacancy-posting → ***V*** = 0

- **Define *W* and *U* in terms of household problem**
  - i.e., based on household value function

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- Nash surplus-sharing rule

$$\eta\left(\mathbf{W}'(w_t) - \mathbf{U}'(w_t)\right)\mathbf{J}(w_t) = (1-\eta)(-\mathbf{J}'(w_t))\left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)$$

(FOC with respect to w_t)

- Must specify value equations *W*(.), *U*(.), *J*(.)

VALUE EQUATIONS

- **Individual/household value equations (constructed from household problem)**

$$\mathbf{W}(w_t) = \underbrace{w_t}_{\text{Contemporaneous return is wage}} + E_t\left\{\underbrace{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{t+1}) + \rho_x\mathbf{U}(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual employed

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Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

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Value to household of having the marginal individual employed

Contemporaneous return is wage

Expected future return takes into account transition probabilities

$$\mathbf{U}(w_t) = b + E_t\left\{\Xi_{t+1|t}\left[k^h(q_t)\mathbf{W}(w_{t+1}) + (1-k^h(q_t))\mathbf{U}(w_{t+1})\right]\right\}$$

Value to household of having the marginal individual unemployed and searching

Contemporaneous return is unemployment benefit/home production

Expected future return takes into account transition probabilities

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Value to household of having the marginal individual employed

$$\mathbf{U}(w_t) = \underbrace{b}_{\text{Contemporaneous return is unemployment benefit/home production}} + E_t \underbrace{\left\{\Xi_{t+1|t}\left[k^h(\theta_t)\mathbf{W}(w_{t+1}) + (1-k^h(\theta_t))\mathbf{U}(w_{t+1})\right]\right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to household of having the marginal individual unemployed and searching

- **Firm value equation**

$$\mathbf{J}(w_t) = \underbrace{z_t - w_t}_{\text{Contemporaneous return is marginal output net of wage payment}} + E_t \underbrace{\left\{\Xi_{t+1|t}(1-\rho_x)\mathbf{J}(w_{t+1})\right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to firm of the marginal employee

Wage Bargaining

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(FOC with respect to w_t)

Insert marginal values

$$\eta\mathbf{J}(w_t) = (1-\eta)\left(\mathbf{W}(w_t) - \mathbf{U}(w_t)\right)$$

Firm's surplus *J* a constant fraction of household's surplus *W – U*

NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

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↓ **Using definitions of *W, U,* and *J,* the job-creation condition, and some algebra**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

WAGE BARGAINING

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NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

↓ **Using definitions of *W, U,* and *J,* the job-creation condition, and some algebra**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

Contemporaneous marginal output…

…plus term that captures savings on future posting costs if match continues

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

NOTE: With CRTS matching function,

$\theta = k^h(\theta)/k^f(\theta)$

LABOR MARKET MATCHING

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

$u_t = 1 - n_t$ is measure of individuals searching for work

- **For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on *v/u***

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: **measures relative number of traders on opposite sides of market**

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In matching models, θ is key driving force of efficiency and thus optimal policy prescriptions

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<u>Market tightness:</u> measures relative number of traders on opposite sides of market

- Market tightness an allocational signal
  - Because matching probabilities depend on it
  - e.g., the higher (lower) is *v/u,* the easier (harder) it is for a given individual to find a job opening

Labor Market Equilibrium

- **Aggregate law of motion of employment**

$$n_{t+1} = (1-\rho_x)n_t + m(u_t, v_t)$$

- **Matching-market equilibrium**

$$m(u_t, v_t) = u_t \cdot k^h(\theta_t) = v_t \cdot k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **Wage model (Nash bargaining)**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

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- **Wage model (Nash bargaining)**

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- **Basic labor-theory literature: impose ss, comparative static exercises, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

GENERAL EQUILIBRIUM

- Aggregate law of motion for employment
- Vacancy-posting (aka job-creation) condition
- Wage determination

The labor market equilibrium (*partial* equilibrium)

- Consumption-savings optimality condition (endogenizes real interest rate)

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- Aggregate resource constraint

$$c_t + g_t + \gamma v_t = z_t n_t h_t + (1 - n_t) b$$

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

Vacancy posting costs and "outside option" are real uses of resources

- Exogenous LOMs for any driving processes (TFP, etc)

Steady State of Labor Market

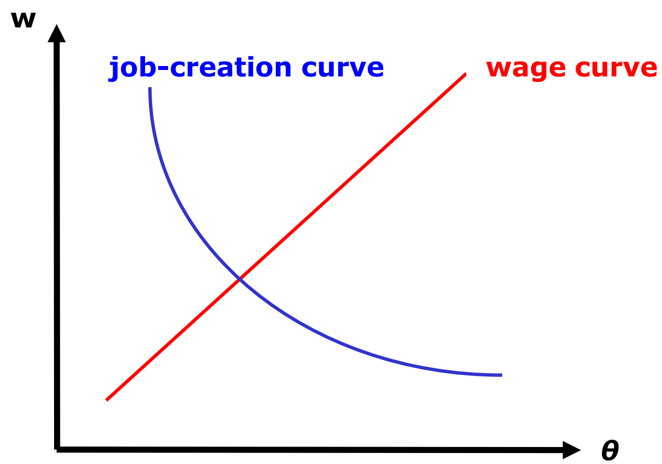
- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1-u=(1-\rho_x)(1-u)+m(u,v)$$ (using $n = 1 - u$)

(2) $$\gamma=\beta k^f(\theta)\left(z-w+\frac{(1-\rho_x)\gamma}{k^f(\theta)}\right)$$ w negatively and nonlinearly related to θ (given CRS matching function)

(3) $$w=\eta\left[z+\gamma\theta\right]+(1-\eta)b$$ w positively and linearly related to θ

Pissarides 2000, Figure 1.1



NOTE: wage function entirely due to ASSUMPTION of Nash-bargained wage model

STEADY STATE OF LABOR MARKET

- Imposing deterministic steady state on labor-market equilibrium conditions

(1)
$$u = \frac{\rho_x - m(u,v)}{\rho_x}$$

For a given (w,θ), v and u negatively related (given CRS matching function)

(2)
$$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho_x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)

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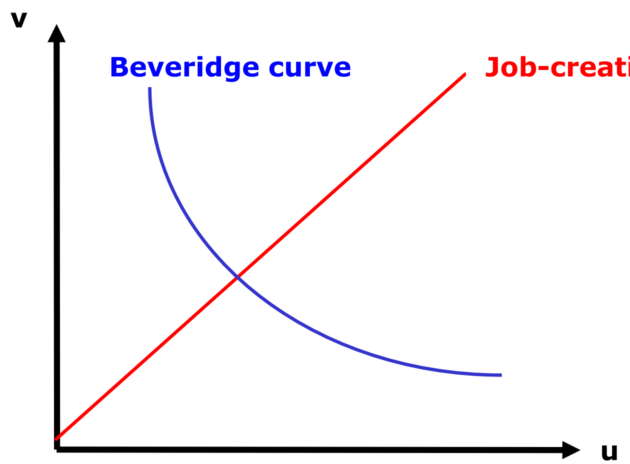

BEVERIDGE CURVE: Empirical relationship in both long run and short run (i.e., cyclical)

Pissarides 2000, Figure 1.2

Steady State of Labor Market

- **Labor-market equilibrium is (*w, u, θ*) satisfying (1), (2), (3)**

- **Comparative statics**
  - **A rise in *b*...**
    - **...raises *w***
    - **...lowers *θ***
    - **...lowers *v* and raises *u***

Higher value (outside option) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

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  - **A fall in *β* (or a rise in ρ_x)…**
    - **…lowers *w***
    - **…lowers *θ***
    - **…raises *u***
    - **…ambiguous effect on *v***

    Higher real rate and/or faster job separations (i.e., "faster depreciation of employment stock") makes posting vacancies (FOR FIXED *u*) less attractive for firms (both erode firm profits)

- **See Pissarides Chapter 1 and RSW (2005 *JEL*) for more**

- **Dynamic stochastic partial equilibrium (Shimer 2005 *AER*, Hall 2005 *AER*, Hagedorn and Manovskii 2008 *AER*)**