Labor Matching Models: Efficiency Properties

January 24, 2020

LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching *TECHNOLOGY***

$$\max_{c_t, v_t, n_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + \gamma v_t = z_t n_t f(h_t) + (1 - n_t) b$$

Fix $h = 1$

$$n_{t+1} = (1 - \rho_x) n_t + m(1 - n_t, v_t)$$

And $n = 1 - u$

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Multipliers

$$c_t + \gamma v_t = z_t n_t f(h_t) + (1 - n_t) b$$ Fix $h = 1$ — λ_t

$$n_{t+1} = (1 - \rho_x) n_t + m(1 - n_t, v_t)$$ And $n = 1 - u$ — μ_t

- **FOCs**

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2(1 - n_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} \left[z_{t+1} - b \right] \right\} + \beta E_t \left\{ \mu_{t+1} \left[(1 - \rho_x) - m_1(1 - n_t, v_t) \right] \right\} = 0$$

↓ **Eliminate multipliers**

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$$\frac{\gamma}{m_2(1-n_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-n_t,v_t)}{m_2(1-n_t,v_t)} + \frac{(1-\rho_x)\gamma}{m_2(1-n_t,v_t)}\right]\right\}$$

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Cobb-Douglas matching

$$m(u,v) = u^{\alpha}v^{1-\alpha}$$

$$m_1(u,v) = \alpha u^{\alpha-1}v^{1-\alpha} = \alpha\theta^{1-\alpha}$$

$$m_2(u,v) = (1-\alpha)u^{\alpha}v^{-\alpha} = (1-\alpha)\theta^{-\alpha}$$

AND

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

AND

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❒ **Social Planning problem**

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Combine and rearrange

AND

$$m_1(u,v) = \alpha k^h(\theta) \qquad m_2(u,v) = (1-\alpha)k^f(\theta)$$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

KEY IDEAS

Taking the pricing kernel as given, the only unknown process here is θ_t

Efficiency in job-postings is governed by efficient market tightness

LABOR-MATCHING EFFICIENCY

- **Socially-efficient vacancy posting characterized by**

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- **Decentralized vacancy posting characterized by**

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

AND

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

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$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - \left(\eta\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\eta)b\right) + \frac{(1-\rho_x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

- **Efficiency in vacancy posting requires $\eta = \alpha$**

Mortensen-Hosios Condition

- **Cobb-Douglas matching technology + Nash bargaining**
  - **Efficient level of job-creation requires $\eta = \alpha$**
  - **Mortensen (1982 *AER*), Hosios (1990 *ReStud*)**

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- **Intuition: search activity generates externalities**
  - **One extra individual (firm) searching for a job (worker) *LOWERS* the probability that *all other* individuals (firms) will find a match…**
  - **…but *RAISES* the probability that *all other* firms (individuals) will find a match**
  - **Congestion externality – search imposes both positive and negative externalities (on opposite sides of the market)**

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- **Nash bargaining: η governs the private returns to search**
  - **Share of total match surplus kept by individual**
- **Cobb-Douglas matching: α governs the social returns to search**
  - **Elasticity of aggregate number of matches with respect to *u***

- **Efficiency requires equating private and social returns: $\eta = \alpha$**

HOSIOS CONDITION

- **Also holds under some more general conditions**
  - **Endogenous search intensity**
  - **Endogenous "vacancy posting intensity" (Pissarides Chapter 5)**
- **Pissarides (2000, p. 198): "..we are not likely to find intuition for it…"**
- **RSW (2005 *JEL* p. 982): "…genuinely surprising result…"**
- **Is the Hosios condition empirically relevant?**
  - **Who knows?…it's a nongeneric parameterization…**
  - **…but valuable because eliminates wage-determination frictions but retains matching technology**
- **Is Nash bargaining empirically relevant?**

How are Wages Determined?

- **Nash bargaining**
  - **Underlying alternating offers bargaining game**
  - **The relevant outside option as bargaining is occurring?**
    - **Value of outside market opportunities?**
    - **Value of continuing negotiations? (Hall and Milgrom 2008 *AER*)**

- **Proportional bargaining**

- **Rigid real wages (completely rigid or partially rigid)**

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- **Bargaining theoretic wages are ex-post of match formation**
- **Seems very different from taking wages as given (ex-ante of match formation)**

- **Competitive search equilibrium**
  - **Moen (1997 *JPE*): basic static partial labor search model**