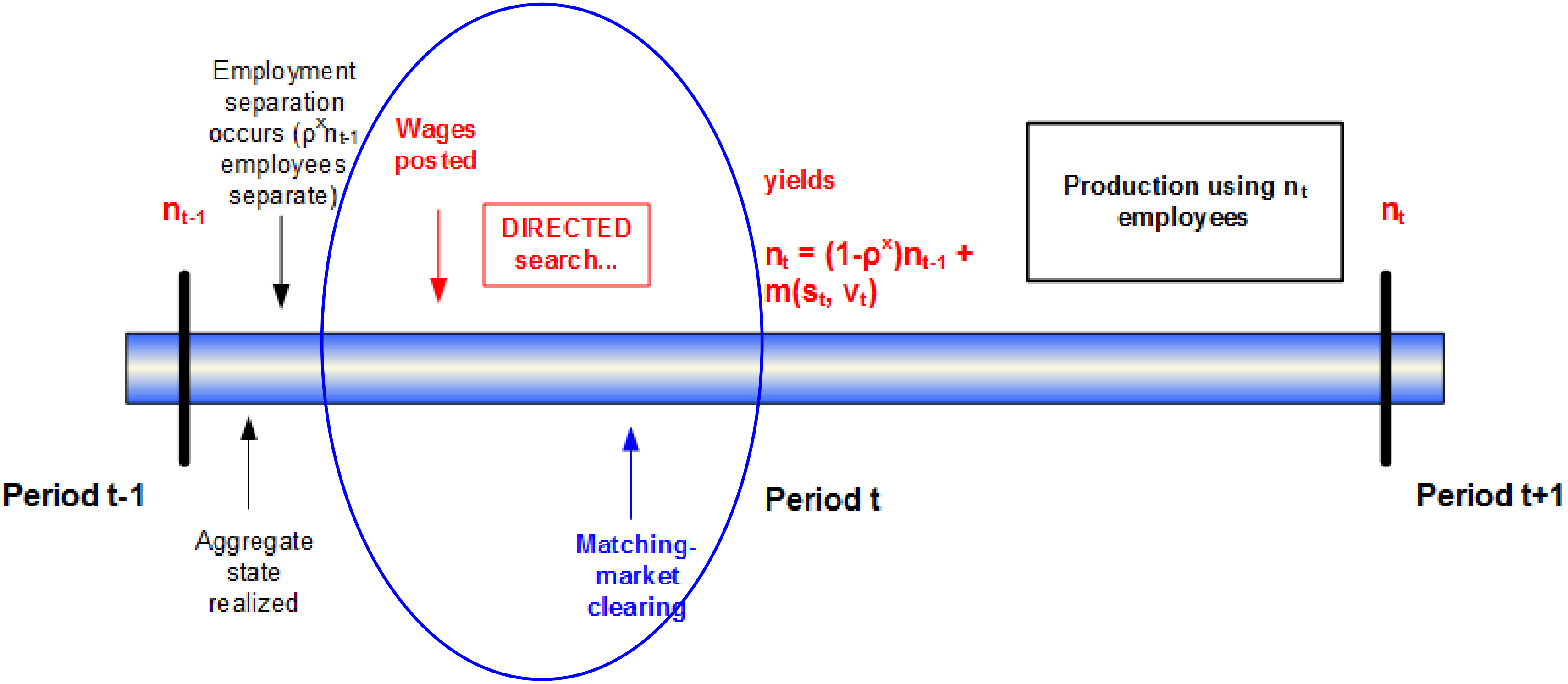
COMPETITIVE SEARCH EQUILIBRIUM (CSE)

JANUARY 24, 2020

COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- **Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?**
  - **Wages playing allocational role in determining meeting process**
  - **In contrast to wage bargaining, which plays small/no allocational role**



Employment separation occurs ($\rho^x n_{t-1}$ employees separate)

Wages posted

DIRECTED search...

yields

$n_t = (1-\rho^x)n_{t-1} + m(s_t, v_t)$

Production using n_t employees

n_{t-1}

n_t

Period t-1

Aggregate state realized

Matching-market clearing

Period t

Period t+1

COMPETITIVE SEARCH EQUILIBRIUM (CSE)

- Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?
  - Wages playing allocational role in determining meeting process
  - In contrast to wage bargaining, which plays small/no allocational role
- May be apriori an appealing way of describing labor markets
  - Locating a firm or a worker is costly and time-consuming…
  - …but once matched, wages are more or less determined by "market forces," perhaps with little/no room for "bargaining"
- Moen (1997 *JPE*) and Shimer (1996) the pioneers of CSE
  - Static partial equilibrium labor matching models
  - "Small firms" (one firm = one job)
- Re-explore CSE framework
- Recent burst of work extending CSE model in various dimensions

CSE – BASICS OF ENVIRONMENT

- Need "many markets" and "many firms"
- To rationalize "competition"

- Index continuum of labor "submarkets" by *ij*
  - Same geographic region
  - Same career, regardless of geography
  - *Many* ways to interpret.....

"submarket" denotes notion of "closeness"

CSE – BASICS OF ENVIRONMENT

- ❒ Need "many markets" and "many firms"
- ❒ To rationalize "competition"

- ❒ Index continuum of labor "submarkets" by *ij*
  - ❒ Same geographic region
  - ❒ Same career, regardless of geography
  - ❒ *Many* ways to interpret.....

"submarket" denotes notion of "closeness"

- ❑ Several equivalent ways to implement ***perfectly*** CSE

- ❑ Firms post wages before individuals search for job opportunities
- ❑ Perfectly-competitive recruiting sector
- ❑ Individuals announce wages before firms direct their vacancies

- ❑ Regardless of implementation, market tightness is efficient

CSE – IMPLEMENTATION I

- Need "many markets" and "many firms"
- To rationalize "competition"

- Index continuum of labor "submarkets" by *ij*
  - Same geographic region
  - Same career, regardless of geography
  - *Many* ways to interpret…..

"submarket" denotes notion of "closeness"

- Several equivalent ways to implement *perfectly* CSE

- Firms post wages before individuals search for job opportunities
- Perfectly-competitive recruiting sector
- Individuals announce wages before firms direct their vacancies

- **Regardless of implementation, market tightness is efficient**

CSE – IMPLEMENTATION I

- **Job seekers *direct* their active search ("send an application") to a particular submarket**
  - **Based on wages announced by goods-producing firms in that submarket**
  - **And on probability of contacting an open vacancy in that submarket**
  - **Directed search a key component of CSE…**
  - **…but match formation still subject to probabilities**

- **Ordering of events**
  - **Wages determined before search…**
  - **…job seekers actively direct search according to posted wages…**
  - **…then probability of landing a match resolved**

CSE – IMPLEMENTATION I

- **Firm *ij* payoff function described by vacancy-posting decision**

$$\gamma = k^{f}(\theta_{ijt})\left[z_t - w_{ijt} \right]$$

Cost of posting a vacancy

CSE – IMPLEMENTATION I

❒ **Firm *ij* payoff function described by vacancy-posting decision**

$$\underset{\uparrow}{\gamma} = \underbrace{k^{f}(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^{f}(\theta_{jt+1})}\right)\right\}\right]}$$

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*…

***…*but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)**

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

CSE – IMPLEMENTATION I

- **Firm *ij* payoff function described by vacancy-posting decision**

$$\gamma = \underbrace{k^{f}(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^{f}(\theta_{jt+1})}\right)\right\}\right]}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

<u>Note *ij* subscripts</u>:

Matching probability depends on tightness of "applications" at firm *ij*…

…but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

- **Value equations for an individual searching for a match at firm *ij***

$$\mathbf{W}(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{jt+1}) + \rho_x\mathbf{U}_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$\mathbf{U}_t = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_{t+1})\mathbf{W}(w_{t+1}) + (1-k^h(\theta_{t+1}))\mathbf{U}_{t+1}\right]\right\}$$

With probability 1-$k^h(\theta_{ijt})$, individual gets this payoff

CSE – IMPLEMENTATION I

- **Firm *ij* payoff function described by vacancy-posting decision**

$$\gamma = \underbrace{k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]}$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

<u>Note *ij* subscripts</u>:

Matching probability depends on tightness of "applications" at firm *ij*…

…but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

- **Value equations for an individual searching for a match at firm *ij***

$$\mathbf{W}(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho_x)\mathbf{W}(w_{jt+1}) + \rho_x\mathbf{U}_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$\mathbf{U}_t = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_{t+1})\mathbf{W}(w_{t+1}) + (1-k^h(\theta_{t+1}))\mathbf{U}_{t+1}\right]\right\}$$

With probability 1-$k^h(\theta_{ijt})$, individual gets this payoff

- **Individuals seeking a job optimally direct their search so that expected payoff of successful contact with a job opening at firm *ij* is**

$$k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t = \underbrace{\mathbf{X}^H}$$

Payoff of searching at another firm or another submarket independent of *ij*

CSE – IMPLEMENTATION I

- **Firm *ij* maximizes**

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]$$

taking as constraint

$$k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t = \mathbf{X}^H$$

- **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

CSE – IMPLEMENTATION I

- **Firm *ij* maximizes**

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]$$

taking as constraint

$$k^h(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1-k^h(\theta_{ijt}))\mathbf{U}_t = \mathbf{X}^H$$

- **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

- **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0$$

2)

$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t\right] = 0$$

Taking into account how matching probabilities are affected by tightness is the central idea

CSE – IMPLEMENTATION I

- **First-order conditions**

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \xrightarrow{W'(.) = 1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2) $$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

CSE – IMPLEMENTATION I

- ☐ **First-order conditions**

1)

$$-k^{f}(\theta_{ijt})-\varphi_{ijt}k^{h}(\theta_{ijt})\mathbf{W}'(w_{ijt})=0 \xrightarrow{W'(.)\,=\,1} \boxed{\varphi_{ijt}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

2)

$$\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\underbrace{\left[z_t-w_{ijt}+(1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^{f}(\theta_{jt+1})}\right)\right\}\right]}_{=\,J_{ijt}}-\varphi_{ijt}\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\left[\mathbf{W}(w_{ijt})-\mathbf{U}_t\right]=0$$

Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$k^{h}(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^{f}(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^{h}(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^{f}(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Combine and rearrange

$$\boxed{\xi\cdot\mathbf{J}(w_t)=(1-\xi)\cdot\left(\mathbf{W}(w_t)-\mathbf{U}_t\right)}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition ($\eta=\xi$)…

CSE – IMPLEMENTATION I

$$m(s,v) = \qquad s^{\xi} \cdot v^{1-\xi}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition ($\eta = \xi$)…

CSE – IMPLEMENTATION I

$$m(s,v) = m^{EFF} \cdot s^{\xi} \cdot v^{1-\xi}$$

Change *m*^{EFF} to ensure probabalitites lie within [0, 1] boundaries

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition ($\eta = \xi$)…

CSE – IMPLEMENTATION I

- ☐ **First-order conditions**

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0 \xrightarrow{\mathbf{W}'(.)\,=\,1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2) $$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

What if different matching function?

$$m(s,v) = \frac{s \cdot v}{\left(s^{\epsilon} + v^{\epsilon}\right)^{1/\epsilon}}$$

denHaan, Ramey, Watson (2000 *AER*)

CSE – IMPLEMENTATION I

- ☐ **First-order conditions**

1) $-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \xrightarrow{W'(.) = 1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$

2)
$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

What if different matching function?

$$m(.) = \frac{s \cdot \theta}{\left(1 + \theta^{\epsilon}\right)^{1/\epsilon}}$$

denHaan, Ramey, Watson (2000 *AER*)

CSE – IMPLEMENTATION I

- **First-order conditions**

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0 \xrightarrow{W'(.)\,=\,1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2) $$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

dHRW matching

$$m(.) = \frac{s\cdot\theta}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \frac{1}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

CSE – IMPLEMENTATION I

- **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt})\mathbf{W}'(w_{ijt}) = 0 \xrightarrow{\mathbf{W'(.) = 1}} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2)

$$\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho_x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]}_{= J_{ijt}} - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t\right] = 0$$

dHRW matching

$$m(.) = \frac{s \cdot \theta}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \frac{1}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$\frac{\partial k^h(\theta)}{\partial \theta} = \frac{1}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}} \cdot \left[1 - \frac{\theta^{\epsilon}}{1+\theta^{\epsilon}}\right] \qquad \frac{\partial k^f(\theta)}{\partial \theta} = -\frac{\theta^{\epsilon}}{\left(1+\theta^{\epsilon}\right)^{\frac{1+\epsilon}{\epsilon}} \cdot \theta}$$

CSE – IMPLEMENTATION I

- **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \xrightarrow{\mathbf{W'(.) = 1}} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2)

$$\underbrace{\left[z_t - w_{ijt} + (1-\rho^x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt} \left(\frac{\partial k^h(\theta_{ijt}) / \partial \theta_{ijt}}{\partial k^f(\theta_{ijt}) / \partial \theta_{ijt}} \right) \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

dHRW matching

$$m(.) = \frac{s \cdot \theta}{\left(1+\theta^\epsilon\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^\epsilon\right]^{1/\epsilon}}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \frac{1}{\left[1+\theta^\epsilon\right]^{1/\epsilon}}$$

$$\frac{\partial k^h(\theta)}{\partial \theta} = \frac{1}{\left(1+\theta^\epsilon\right)^{1/\epsilon}} \cdot \left[1 - \frac{\theta^\epsilon}{1+\theta^\epsilon} \right] \qquad \frac{\partial k^f(\theta)}{\partial \theta} = -\frac{\theta^\epsilon}{\left(1+\theta^\epsilon\right)^{\frac{1+\epsilon}{\epsilon}} \cdot \theta}$$

CSE – IMPLEMENTATION I

- **First-order conditions**

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \xrightarrow{W'(.)\,=\,1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2) $$\underbrace{\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]}_{=\,J_{ijt}} - \varphi_{ijt}\underbrace{\left(\frac{\partial k^h(\theta_{ijt})/\partial\theta_{ijt}}{\partial k^f(\theta_{ijt})/\partial\theta_{ijt}}\right)}_{=\,\theta^{-\epsilon}}\left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t\right] = 0$$

dHRW matching

$$m(.) = \frac{s \cdot \theta}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \frac{1}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$\frac{\partial k^h(\theta)}{\partial\theta} = \frac{1}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}} \cdot \left[1 - \frac{\theta^{\epsilon}}{1+\theta^{\epsilon}}\right] \qquad \frac{\partial k^f(\theta)}{\partial\theta} = -\frac{\theta^{\epsilon}}{\left(1+\theta^{\epsilon}\right)^{\frac{1+\epsilon}{\epsilon}} \cdot \theta}$$

CSE – IMPLEMENTATION I

- ☐ **First-order conditions**

1)

$$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) \mathbf{W}'(w_{ijt}) = 0 \xrightarrow{W'(.)\,=\,1} \boxed{\varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})} = -\theta_{ijt}^{-1}}$$

2)

$$\underbrace{\left[z_t - w_{ijt} + (1-\rho^x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt}} - \varphi_{ijt} \underbrace{\left(\frac{\partial k^h(\theta_{ijt}) / \partial \theta_{ijt}}{\partial k^f(\theta_{ijt}) / \partial \theta_{ijt}} \right)}_{= \theta^{-\epsilon}} \left[\mathbf{W}(w_{ijt}) - \mathbf{U}_t \right] = 0$$

dHRW matching

$$m(.) = \frac{s \cdot \theta}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}}$$

$$k^h(\theta) = \frac{m(s,v)}{s} = m(1,\theta) = \frac{\theta}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$k^f(\theta) = \frac{m(s,v)}{v} = m(\theta^{-1},1) = \frac{1}{\left[1+\theta^{\epsilon}\right]^{1/\epsilon}}$$

$$\frac{\partial k^h(\theta)}{\partial \theta} = \frac{1}{\left(1+\theta^{\epsilon}\right)^{1/\epsilon}} \cdot \left[1 - \frac{\theta^{\epsilon}}{1+\theta^{\epsilon}} \right] \qquad \frac{\partial k^f(\theta)}{\partial \theta} = -\frac{\theta^{\epsilon}}{\left(1+\theta^{\epsilon}\right)^{\frac{1+\epsilon}{\epsilon}} \cdot \theta}$$

(Competition within submarket *j* and symmetry across submarkets: drop *ij* indices)

Combine and rearrange

$$\boxed{\theta_t^{-\epsilon} \cdot \left(\mathbf{W}(w_t) - \mathbf{U}_t \right) = \mathbf{J}(w_t)}$$

CSE surplus-sharing rule for dHRW m(.)

CSE – IMPLEMENTATION II

- Need "many markets" and "many firms"
- To rationalize "competition"

- Index continuum of labor "submarkets" by *ij*
  - Same geographic region
  - Same career, regardless of geography
  - *Many* ways to interpret.....

"submarket" denotes notion of "closeness"

- Several equivalent ways to implement ***perfectly*** CSE

- Firms post wages before individuals search for job opportunities
- Perfectly-competitive recruiting sector
- Individuals announce wages before firms search for workers

- **Regardless of implementation, market tightness is efficient**

CSE – IMPLEMENTATION II

- ❒ **Recruiting agency *ij* total profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m(s_{ijt}, v_{ijt})$$

recruiter *ij* operates matching technology m_{ij}

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* marginal profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* marginal profit function**

$$\left(\rho_{ijt} - mc_{jt}\right) \cdot m_{v_{ijt}}$$

- **Moen (1997)**
  - **ONE firm = ONE job ("small" firms)**
  - **"Marginal" profit term vacuous in ONE-worker firm**
- **Micro origins of search and matching framework: ONE firm = ONE worker**
  - **Pissarides (1985), Mortensen and Pissarides (1994)**
- **Macro: Goods-producing firms that "need" / hire "many" workers**
  - **CRTS f(k,n): lack of IO structure means number of workers can be ANYTHING**

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}}\left(\rho_{ijt}-mc_{jt}\right)\cdot m_{v_{ijt}}$$

subject to

$$\gamma-k^{f}(\theta_{ijt})\cdot\mathbf{J}(w_{ijt})-\mathbf{X}^{F}=0$$

$$k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt})+(1-k^{h}(\theta_{ijt}))\mathbf{U}_{t}-\mathbf{X}^{H}=0$$

m_{sij}

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}}\left(\rho_{ijt}-mc_{jt}\right)\cdot(1-\xi)\cdot k^{f}(\theta_{ijt})$$

subject to

$$\gamma-k^{f}(\theta_{ijt})\cdot\mathbf{J}(w_{ijt})-\mathbf{X}^{F}=0$$

$$k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt})+(1-k^{h}(\theta_{ijt}))\mathbf{U}_{t}-\mathbf{X}^{H}=0$$

Suppose

$m(s,v)=s^{\xi}v^{1-\xi}$

CSE – IMPLEMENTATION II

- ❒ **Recruiting agency *ij* marginal profit function**

Suppose

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$\max_{w_{ijt},\theta_{ijt}}\left(\rho_{ijt}-mc_{jt}\right)\cdot(1-\xi)\cdot k^{f}(\theta_{ijt})$$

subject to

multipliers

$$\gamma-k^{f}(\theta_{ijt})\cdot\mathbf{J}(w_{ijt})-\mathbf{X}^{F}=0$$

1

$$k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt})+(1-k^{h}(\theta_{ijt}))\mathbf{U}_{t}-\mathbf{X}^{H}=0$$

$\boldsymbol{\kappa}^{H}_{ijt}$

(given CRS m(.), only one multiplier needed)

FOCs wrt w_{ijt} and θ_{ijt}

CSE – IMPLEMENTATION II

- ❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)
$$-k^{f}(\theta_{ijt})\cdot\frac{\partial\mathbf{J}(w_{ijt})}{\partial w_{ijt}}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\frac{\partial\mathbf{W}(w_{ijt})}{\partial w_{ijt}}=0$$

2)
$$\underbrace{\left(\rho_{ijt}-mc_{jt}\right)}_{=0}\cdot(1-\xi)\cdot\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}-\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^{H}\cdot\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

CSE – IMPLEMENTATION II

- ❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^{f}(\theta_{ijt})\cdot\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}+\kappa_{ijt}^{H}\cdot k^{h}(\theta_{ijt})\cdot\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}=0$$

2)

$$-\frac{\partial k^{f}(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^{H}\cdot\frac{\partial k^{h}(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

CSE – IMPLEMENTATION II

- FOCs with respect to w_{ijt} and θ_{ijt}

1)
$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa_{ijt}^H\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa_{ijt}^H=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}^{-1}}$$

b/c zero proportional taxation on wage

2)
$$-\frac{\partial k^f(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa_{ijt}^H\cdot\frac{\partial k^h(\theta_{ijt})}{\partial \theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

CSE – IMPLEMENTATION II

❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+\kappa^H_{ijt}\cdot k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta^{-1}_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$-\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\kappa^H_{ijt}\cdot\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$k^h(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^f(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^h(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^f(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Combine and rearrange

Symmetric equilibrium

$$\boxed{(1-\xi)\left(\mathbf{W}(w_t)-\mathbf{U}_t\right)=\xi\mathbf{J}(w_t)}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition (η = ξ)…

CSE – IMPLEMENTATION III

- Need "many markets" and "many firms"
- To rationalize "competition"

- Index continuum of labor "submarkets" by *ij*
  - Same geographic region
  - Same career, regardless of geography
  - *Many* ways to interpret…..

"submarket" denotes notion of "closeness"

- Several equivalent ways to implement ***perfectly*** CSE

- Firms post wages before individuals search for job opportunities
- Perfectly-competitive recruiting sector
- Individuals announce wages before firms direct their vacancies

- **Regardless of implementation, market tightness is efficient**

APPENDIX

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right) \cdot m_{s_{ijt}}$$

subject to

$$\gamma - k^{f}(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

CSE – IMPLEMENTATION II

Suppose

$m(s,v) = s^{\xi} v^{1-\xi}$

❒ **Recruiting agency *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt} \right) \cdot \xi \cdot k^{h}(\theta_{ijt})$$

subject to

$$\gamma - k^{f}(\theta_{ijt}) \cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$$k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

CSE – IMPLEMENTATION II

- **Recruiting agency *ij* marginal profit function**

$$\max_{w_{ijt},\theta_{ijt}} \left(\rho_{ijt} - mc_{jt}\right)\cdot \xi \cdot k^{h}(\theta_{ijt})$$

Suppose

$m(s,v) = s^{\xi}v^{1-\xi}$

subject to

multipliers

$$\gamma - k^{f}(\theta_{ijt})\cdot \mathbf{J}(w_{ijt}) - \mathbf{X}^{F} = 0$$

$\boldsymbol{\kappa^{F}_{ijt}}$

$$k^{h}(\theta_{ijt})\mathbf{W}(w_{ijt}) + (1 - k^{h}(\theta_{ijt}))\mathbf{U}_{t} - \mathbf{X}^{H} = 0$$

1

(given CRS m(.), only one multiplier needed)

$\downarrow$

FOCs wrt w_{ijt} and θ_{ijt}

CSE – IMPLEMENTATION II

❐ **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-\kappa_{ijt}^{F} \cdot k^{f}(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + k^{h}(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0$$

2) $$\underbrace{\left(\rho_{ijt} - mc_{jt}\right)}_{=0} \cdot \xi \cdot \frac{\partial k^{h}(\theta_{ijt})}{\partial \theta_{ijt}} - \kappa_{ijt}^{F} \cdot \frac{\partial k^{f}(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^{h}(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \left(\mathbf{W}(w_{ijt}) - \mathbf{U}_{t}\right) = 0$$

CSE – IMPLEMENTATION II

- ❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1) $$-\kappa_{ijt}^{F} \cdot k^{f}(\theta_{ijt}) \cdot \frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}} + k^{h}(\theta_{ijt}) \cdot \frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}} = 0$$

2) $$-\kappa_{ijt}^{F} \cdot \frac{\partial k^{f}(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \mathbf{J}(w_{ijt}) + \frac{\partial k^{h}(\theta_{ijt})}{\partial \theta_{ijt}} \cdot \left(\mathbf{W}(w_{ijt}) - \mathbf{U}_{t}\right) = 0$$

CSE – IMPLEMENTATION II

❒ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa^F_{ijt}\cdot k^f(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+k^h(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa^H_{ijt}=-\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}=-\theta_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$-\kappa^F_{ijt}\cdot\frac{\partial k^f(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\frac{\partial k^h(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_t\right)=0$$

CSE – IMPLEMENTATION II

❐ **FOCs with respect to w_{ijt} and θ_{ijt}**

1)

$$-\kappa_{ijt}^{F}\cdot k^{f}(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{J}(w_{ijt})}{\partial w_{ijt}}}_{=-1}+k^{h}(\theta_{ijt})\cdot\underbrace{\frac{\partial \mathbf{W}(w_{ijt})}{\partial w_{ijt}}}_{=1}=0 \longrightarrow \boxed{\kappa_{ijt}^{H}=-\frac{k^{f}(\theta_{ijt})}{k^{h}(\theta_{ijt})}=-\theta_{ijt}}$$

b/c zero proportional taxation on wage

2)

$$-\kappa_{ijt}^{F}\cdot\frac{\partial k^{f}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\mathbf{J}(w_{ijt})+\frac{\partial k^{h}(\theta_{ijt})}{\partial\theta_{ijt}}\cdot\left(\mathbf{W}(w_{ijt})-\mathbf{U}_{t}\right)=0$$

Cobb-Douglas matching

$$m(s,v)=s^{\xi}v^{1-\xi}$$

$$k^{h}(\theta)=\frac{m(s,v)}{s}=m(1,\theta)=\theta^{1-\xi}$$

$$k^{f}(\theta)=\frac{m(s,v)}{v}=m(\theta^{-1},1)=\theta^{-\xi}$$

AND

$$\frac{\partial k^{h}(\theta)}{\partial\theta}=(1-\xi)\theta^{-\xi}$$

$$\frac{\partial k^{f}(\theta)}{\partial\theta}=-\xi\theta^{-\xi-1}$$

Combine and rearrange

$$\boxed{(1-\xi)\left(\mathbf{W}(w_{t})-\mathbf{U}_{t}\right)=\xi\mathbf{J}(w_{t})}$$

Exactly the Nash-bargained sharing rule with knife-edge Hosios condition (η = ξ)…