Labor Matching Models: Further Equilibrium Concepts

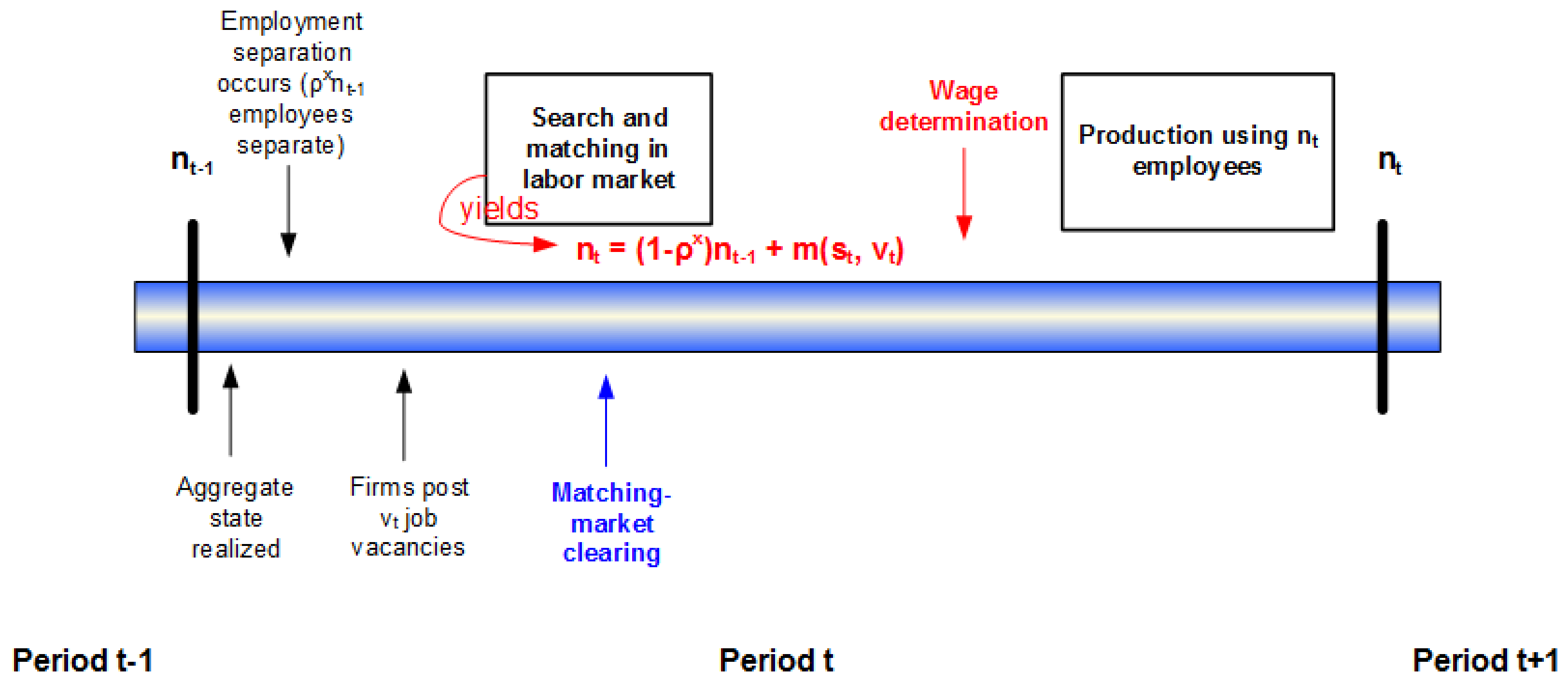
January 24, 2020

Generalize Matching Model

- Equilibrium concepts
- Search Equilibrium (undirected search + post-match wage determination)
  - Extensive margin generically inefficient

Search Equilibrium

- **Matching market clearing …**
- **… then wage determination**

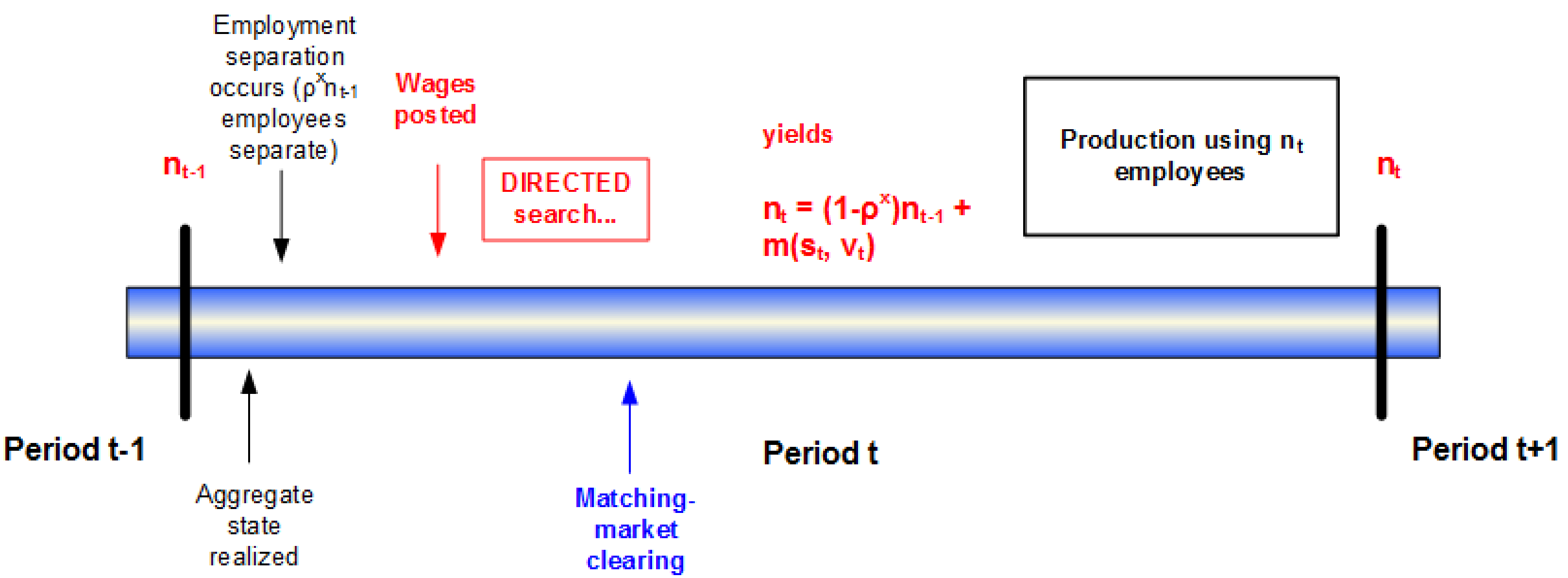


GENERALIZE MATCHING MODEL

- Equilibrium concepts
- Search Equilibrium (undirected search + post-match wage determination)
  - Extensive margin generically inefficient
- Competitive Search Equilibrium (wage posting + directed search)
  - Extensive margin efficient

Competitive Search Equilibrium

- Wage determination …
- … then matching market clearing

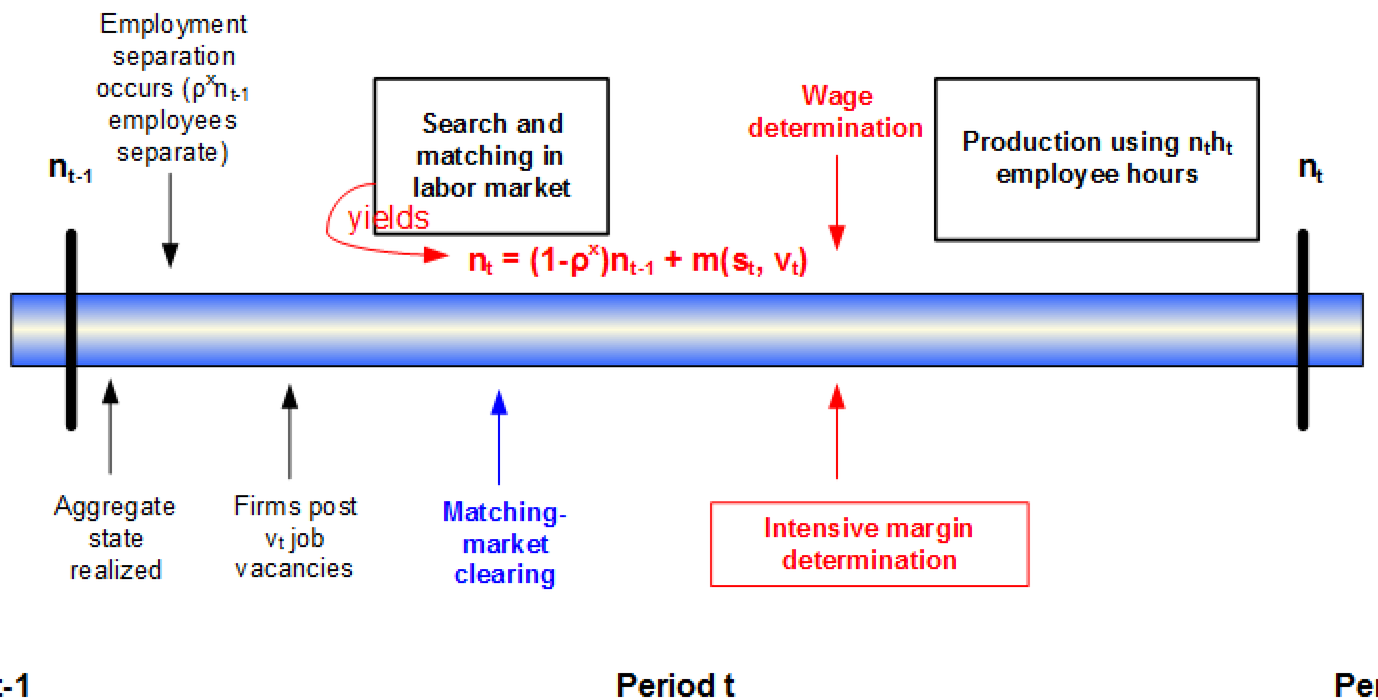


Generalize Matching Model

- Equilibrium concepts
- Search Equilibrium (undirected search + post-match wage determination)
  - Extensive margin generically inefficient
- Competitive Search Equilibrium (wage posting + directed search)
  - Extensive margin efficient
- Competitive Equilibrium
  - Requires intensive margin

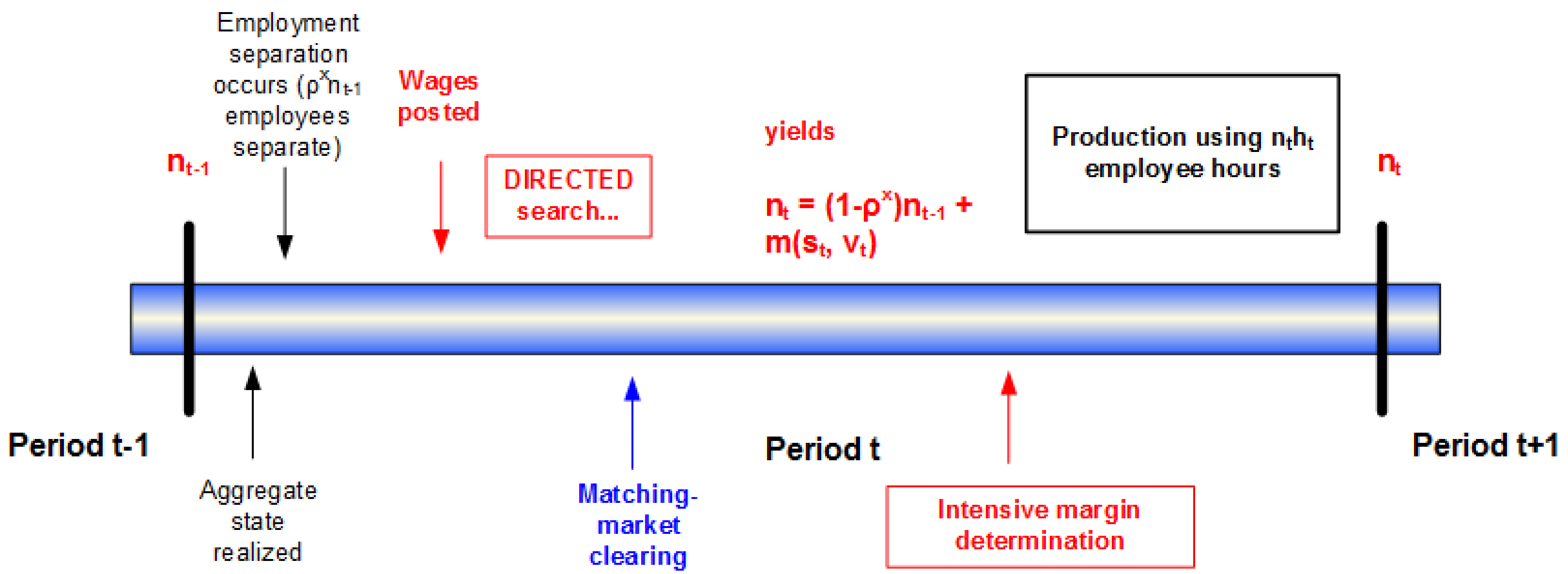
Competitive Equilibrium

- Matching market clearing …
- … then wage determination
- Intensive margin determination



COMPETITIVE SEARCH EQUILIBRIUM

- Wage determination …
- … then matching market clearing
- Intensive margin determination



Generalize Matching Model

- **Equilibrium concepts**
- **Search Equilibrium (undirected search + post-match wage determination)**
  - **Extensive margin generically inefficient**
- **Competitive Search Equilibrium (wage posting + directed search)**
  - **Extensive margin efficient**
- **Competitive Equilibrium**
  - **Requires intensive margin**
  - **Intensive margin determined ex-post of match**
  - **Whether or not extensive margin is efficient**

Generalize Matching Model

- Equilibrium concepts
- Search Equilibrium (undirected search + post-match wage determination)
  - Extensive margin generically inefficient
- Competitive Search Equilibrium (wage posting + directed search)
  - Extensive margin efficient
- Competitive Equilibrium
  - Requires intensive margin
  - Intensive margin determined ex-post of match
  - Whether or not extensive margin is efficient
- Matching Market Equilibrium

INTENSIVE MARGIN

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_t^f} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f f(h_t) - w_t n_t^f h_t - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^f = (1-\rho_x) n_{t-1}^f + v_t k^f(\theta_t)$$

- **Total output produced by all employees = $z \cdot n \cdot f(h)$**

INTENSIVE MARGIN

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_t^f} E_0\left[\sum_{t=0}^{\infty} \Xi_{t|0}\left(z_t n_t^f f(h_t) - w_t n_t^f h_t - \gamma v_t\right)\right]$$

$$\text{s.t. } n_t^f = (1-\rho_x) n_{t-1}^f + v_t k^f(\theta_t)$$

- **Total output produced by all employees = *z · n · f(h)***

- **Vacancy posting condition**

$$\frac{\gamma}{k^f(\theta_t)} = z_t f(h_t) - w_t h_t + (1-\rho_x) E_t\left\{\Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})}\right\}$$

- **How is *h* determined?**

Intensive Margin

- **Dynamic household utility maximization problem**

$$\max_{c_t} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-h\left(n_t^h\cdot e(h_t)\right)\right)\right]$$

$$\text{s.t. } c_t = w_t n_t^h h_t + (1-k^h(\theta_t))\cdot(1-n_t^h)\cdot b$$

$$n_t^h = (1-\rho_x)n_{t-1}^h + (1-n_t^h)k^h(\theta_t)$$

INTENSIVE MARGIN

- **Dynamic household utility maximization problem**

$$\max_{c_t} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-h\left(n_t^h\cdot e(h_t)\right)\right)\right]$$

$$\text{s.t. } c_t = w_t n_t^h h_t + (1-k^h(\theta_t))\cdot(1-n_t^h)\cdot b$$

$$n_t^h = (1-\rho_x)n_{t-1}^h + (1-n_t^h)k^h(\theta_t)$$

- **Suppose household *h*(.) is quasi-linear**

INTENSIVE MARGIN

- Dynamic household utility maximization problem

$$\max_{c_t} E_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_t)-n_t^h\cdot e(h_t)\right)\right]$$

$$\text{s.t. } c_t = w_t n_t^h h_t + (1-k^h(\theta_t))\cdot(1-n_t^h)\cdot b$$

$$n_t^h = (1-\rho_x)n_{t-1}^h + (1-n_t^h)k^h(\theta_t)$$

- Suppose **household** ***h***(.) is quasi-linear

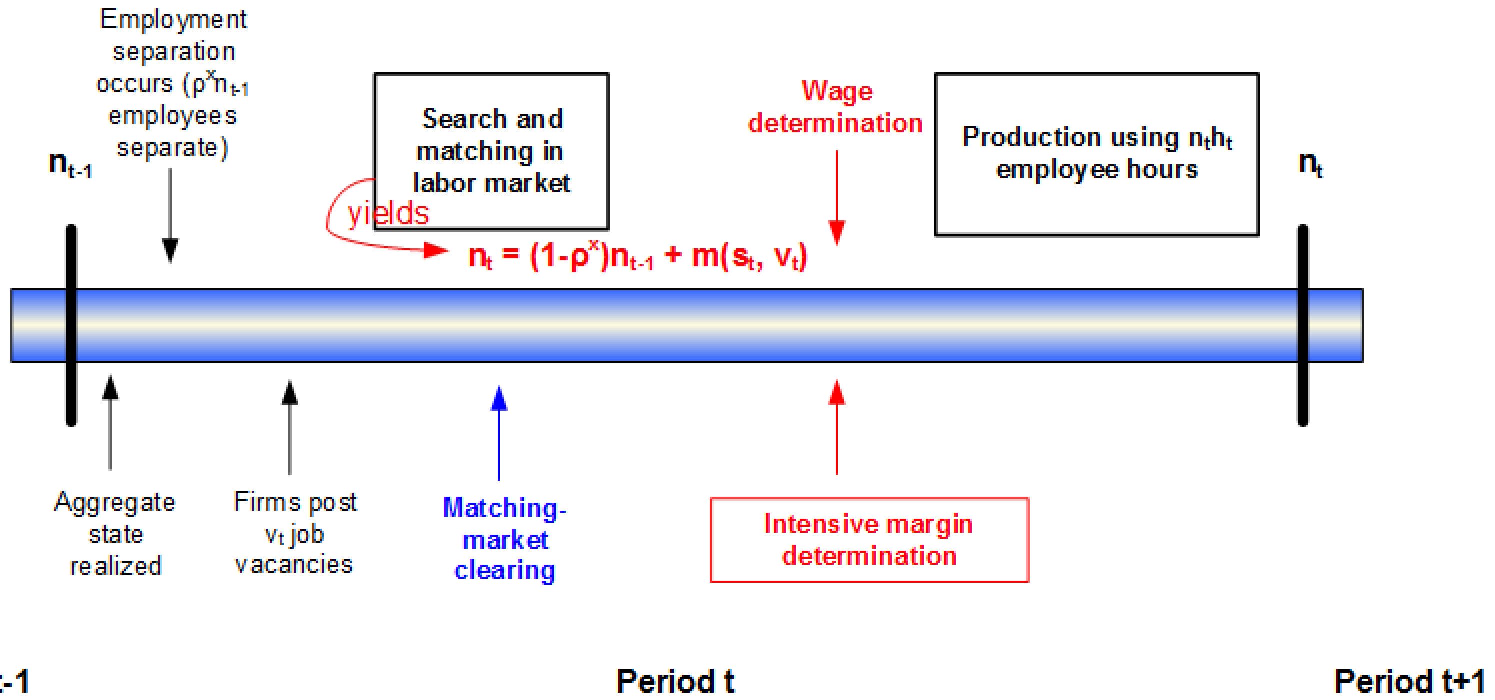
- How is ***h*** determined?

Intensive Margin

- How is *h* determined?
  - Could depend on when wage determination occurs (pre-match? post-match?)

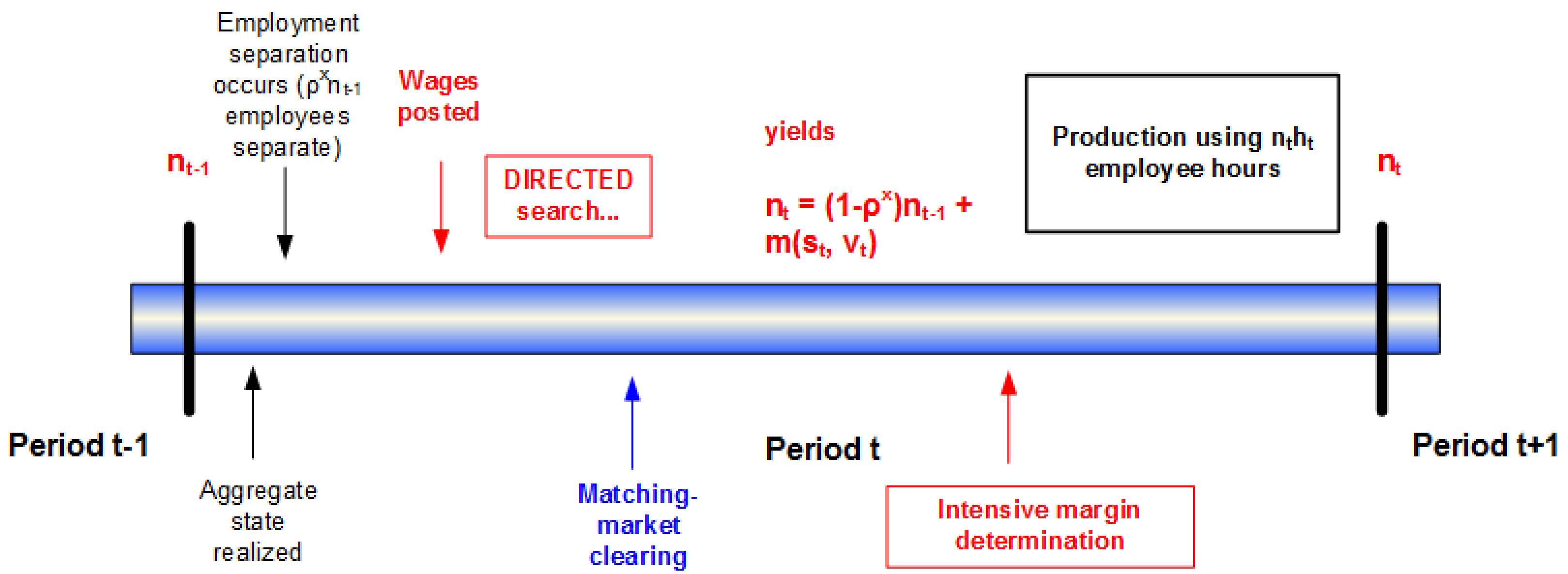
INTENSIVE MARGIN

- Matching market clearing …
- … then wage determination
- Intensive margin determination



INTENSIVE MARGIN

- Wage determination …
- … then matching market clearing
- Intensive margin determination



INTENSIVE MARGIN

- How is *h* determined?
  - Could depend on when wage determination occurs (pre-match? post-match?)

- Common setup for post-match wages
  - Simultaneous Nash bargaining over *w* and *h*

$$\max_{w_t, h_t} \left(\mathbf{W}_t - \mathbf{U}_t \right)^{\eta} \mathbf{J}_t^{1-\eta}$$

INTENSIVE MARGIN

- **How is *h* determined?**
  - **Could depend on when wage determination occurs (pre-match? post-match?)**

- **Common setup for post-match wages**
  - **Simultaneous Nash bargaining over *w* and *h***

$$\max_{w_t, h_t} \left(\mathbf{W}_t - \mathbf{U}_t \right)^{\eta} \mathbf{J}_t^{1-\eta}$$

- **Value expressions for an atomistic individual's potential new job/potential new employee**

$$\mathbf{W}_t = w_t h_t - \frac{e(h_t)}{u'(c_t)} + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho_x)\mathbf{W}_{t+1} + \rho_x \mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{U}_t = b + E_t \left\{ \Xi_{t+1|t} \left[k^h(\theta_{t+1})\mathbf{W}_{t+1} + (1-k^h(\theta_{t+1}))\mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{J}_t = z_t f(h_t) - w_t h_t + E_t \left\{ \Xi_{t+1|t} (1-\rho_x) \mathbf{J}_{t+1} \right\}$$

INTENSIVE MARGIN

- **Compute FOCs wrt *w* and *h***
- **FOC wrt *w* yields**

$$w_t h_t = \eta\left[z_t f(h_t) + \gamma\theta_t\right] + (1-\eta)b$$

Identical algebra to the *h* = 1 case

INTENSIVE MARGIN

- Compute FOCs wrt *w* and *h*
- FOC wrt *w* yields

$$w_t h_t = \eta\left[z_t f(h_t) + \gamma\theta_t\right] + (1-\eta)b$$

Identical algebra to the *h* = 1 case

- FOC wrt *h* yields

$$\eta \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial h_t} - \frac{\partial \mathbf{U}_t}{\partial h_t} \right) = (1-\eta)(-1)\left(\mathbf{W}_t - \mathbf{U}_t\right)\frac{\partial \mathbf{J}_t}{\partial h_t}$$

(VERIFY THE DERIVATION)

Insert marginal values and rearrange
(a key observation is that….)

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

INTENSIVE MARGIN

- Compute FOCs wrt *w* and *h*
- FOC wrt *w* yields

$$w_t h_t = h\left[z_t f(h_t) + \gamma q_t\right] + (1 - h)b$$

Identical algebra to the *h* = 1 case

- FOC wrt *h* yields

$$\eta \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial h_t} - \frac{\partial \mathbf{U}_t}{\partial h_t} \right) = (1-\eta)(-1)\left(\mathbf{W}_t - \mathbf{U}_t\right)\frac{\partial \mathbf{J}_t}{\partial h_t}$$

(VERIFY THE DERIVATION)

↓ Insert marginal values and rearrange (a key observation is that….)

PRIVATE BILATERAL EFFICIENCY

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

(…given simultaneous bargaining over *w* and *h*….)

- Interpretation: $mrs_t = mpn_t$ for each given worker
  - Private bilateral efficiency on the hours margin
  - Whether or not Hosios efficiency holds on extensive margin

COMPETITIVE EQUILIBRIUM

- **Competitive Equilibrium**

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

- **Intensive margin is (bilaterally) efficient**
- **Whether or not extensive margin is efficient**

COMPETITIVE EQUILIBRIUM

- **Competitive Equilibrium**

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

- **Intensive margin is (bilaterally) efficient**
- **Whether or not extensive margin is efficient**

- **Where?**
  - **In a particular industry…**
  - **In a particular submarket…**
  - **In a particular "island"…**

COMPETITIVE EQUILIBRIUM

- **Competitive Equilibrium**

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

- **Intensive margin is (bilaterally) efficient**
- **Whether or not extensive margin is efficient**

- **Where?**
  - **In a particular industry…**
  - **In a particular submarket…**
  - **In a particular "island"…**

- **For whom?…**

Competitive Equilibrium

- **Competitive Equilibrium**

$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

- **Intensive margin is (bilaterally) efficient**
- **Whether or not extensive margin is efficient**

- **Where?**
  - **In a particular industry…**
  - **In a particular submarket…**
  - **In a particular "island"…**

- **For whom?…**
- **Those who have already entered the submarket**
- **But entering a submarket / finding a match could be (temporarily?) inefficient due to congestion externalities in market *ij***
  - **Inefficient θ_{ij}**

FREE-ENTRY CONDITIONS

- Set $h = 1$
- Free entry into matching market for firms?
- Job-creation condition

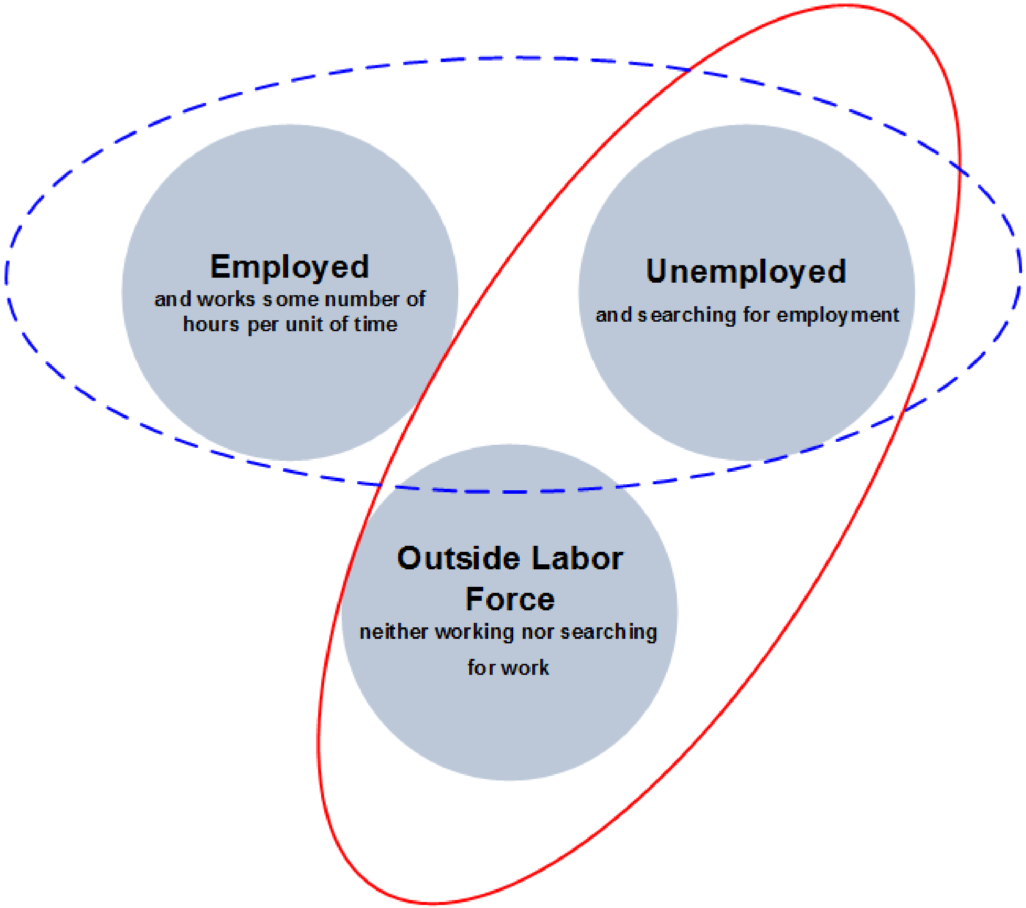
FREE-ENTRY CONDITIONS

- **Set *h* = 1**
- **Free entry into matching market for firms?**
- **Job-creation condition (now with diminishing marg. product in *n*)**

$$\frac{\gamma}{k^f(\theta_t)} = z_t f'(n_t) - w_t n_t + (1-\rho_x) E_t \left\{ \Xi_{t+1|t} \frac{\gamma}{k^f(\theta_{t+1})} \right\}$$

- **Free entry into matching market for workers?**
- **Endogenous labor force participation (aka "labor supply")**

Labor Force Participation



Allows for optimal search and optimal "rest" (outside the LFP, aka "leisure")