Endogenous Job Separation

February 12, 2020

ENDOGENOUS DESTRUCTION

- **Representative "large firm"**

$$\max_{v_t, n_t^f} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - \Omega_t n_t^f - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^f = (1-\rho_t)(n_{t-1}^f + v_{t-1} k^f(\theta_{t-1}))$$

Endogenous destruction fraction ρ_t. And note timing of employment…

- **Total production depends on aggregate TFP *and conditional mean productivity of job matches that are not destroyed***

$$y_t = z_t n_t^f \int_{\tilde{a}_t}^{\infty} a \frac{f(a)}{1-F(\tilde{a}_t)} da \equiv z_t n_t^f H(\tilde{a}_t)$$

f(.) the pdf of idiosyncratic productivity, F(.) the cdf

(could pull denominator out of integral…does not depend on index *a*)

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- ***Ω_t* is average wage bill of firm,** $\Omega_t = \int_{\tilde{a}_t}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a}_t)} da$

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$$\text{s.t. } n_t^f = (1-\rho(\tilde{a}_t))(n_{t-1}^f + v_{t-1}k^f(\theta_{t-1}))$$

By construction/definition

$$\rho_t^n = F(\tilde{a}_t)\left(= \int_0^{\tilde{a}_t} af(a)da\right)$$

$$\rho_t = \rho_x + (1-\rho_x)\rho_t^n$$

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- **FOCs with respect to n_t and v_t yield job-creation condition**

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left[\Xi_{t+1|t}(1-\rho(\tilde{a}_{t+1}))\left(z_{t+1}H(\tilde{a}_{t+1})-\Omega_{t+1}+\frac{\gamma}{k^f(\theta_{t+1})}\right)\right]$$

- **Vacancy-creation in t depends on expectations about future endogenous separation rate and (effective conditional) productivity**

ENDOGENOUS DESTRUCTION

- **Bargaining-relevant value equations for match with realized a_t**

$$W(a_t) = w(a_t) + E_t\left\{\Xi_{t+1|t}\left[(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\infty} W(a)\frac{f(a)}{1-F(\tilde{a}_{t+1})}da + \rho_{t+1}U(a_{t+1})\right]\right\}$$

$$U(a_t) = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\infty} W(a)\frac{f(a)}{1-F(\tilde{a}_{t+1})}da + (1-k^h(\theta_t)(1-\rho_{t+1}))U(a_{t+1})\right]\right\}$$

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$$J(a_t) = z_t a_t - w(a_t) + E_t\left\{\Xi_{t+1|t}(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\infty} J(a)\frac{f(a)}{1-F(\tilde{a}_{t+1})}da\right\}$$

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Insert in usual Nash sharing rule $\eta(W(a_t)-U(a_t)) = (1-\eta)J(a_t)$

$$w(a_t) = \eta\left[z_t a_t + \gamma\theta_t\right] + (1-\eta)b$$

For an individual job with idiosyncratic productivity a_t and which is *not* destroyed…a straightforward generalization

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- **Wage payment in individual job with productivity a_t**

$$w(a_t) = \eta\left[z_t a_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Average (per-employee) wage bill of representative "large firm"**
  - **Integrate over all jobs that are not destroyed**

$$\Omega_t \equiv \int_{\tilde{a}_t}^{\infty} w(a)\frac{f(a)}{1-F(\tilde{a}_t)}da = \eta z_t \underbrace{\int_{\tilde{a}_t}^{\infty} a\frac{f(a)}{1-F(\tilde{a}_t)}da}_{\equiv H(\tilde{a}_t)} + \eta\gamma\theta_t + (1-\eta)b$$

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- **Pin down threshold *a* from condition *J*(*a*) = 0**
  - **Equivalent to using *W*(*a*) – *U*(*a*) = 0**
  - **Equivalent to using vacancy-creation condition evaluated at the threshold job**

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- **Aggregate resource constraint** $c_t + \gamma v_t = z_t H(\tilde{a}_t)$