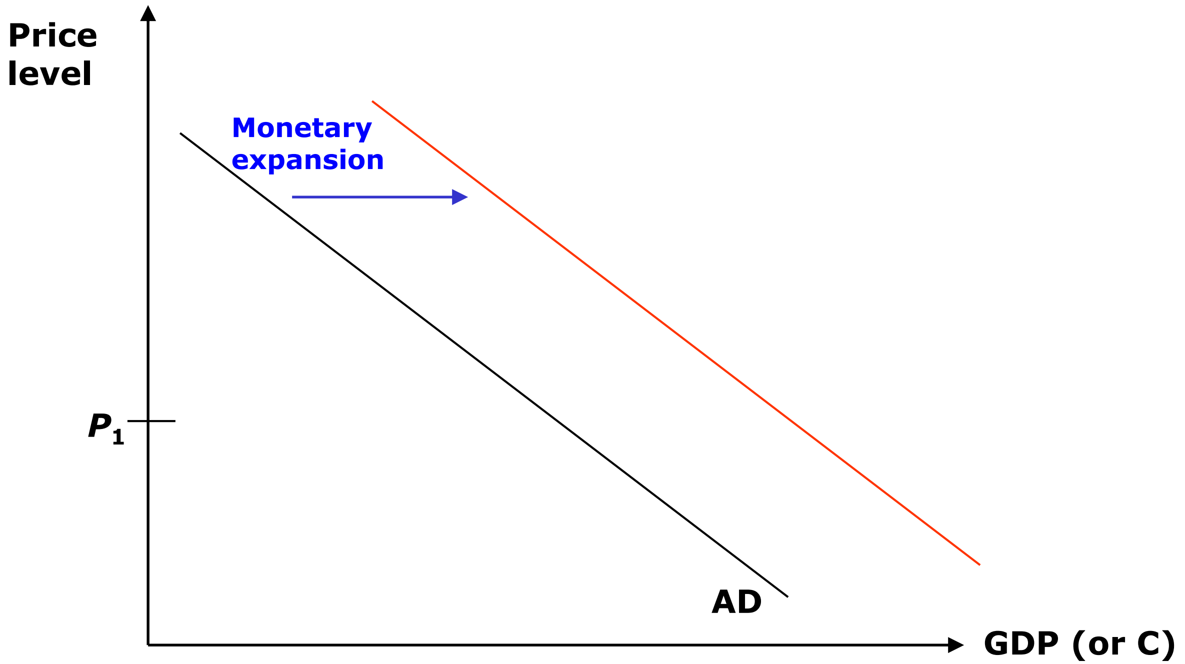
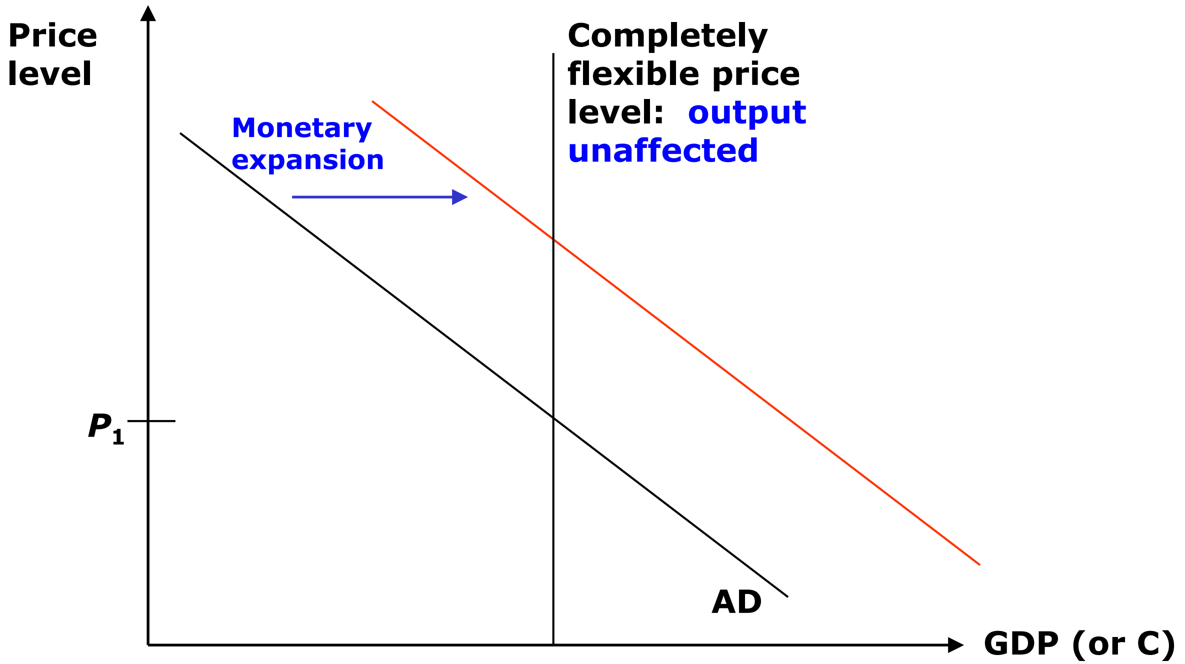
Nominal Price Rigidities in a DSGE Model

February 14, 2020

Cyclical Implications of Monetary Policy

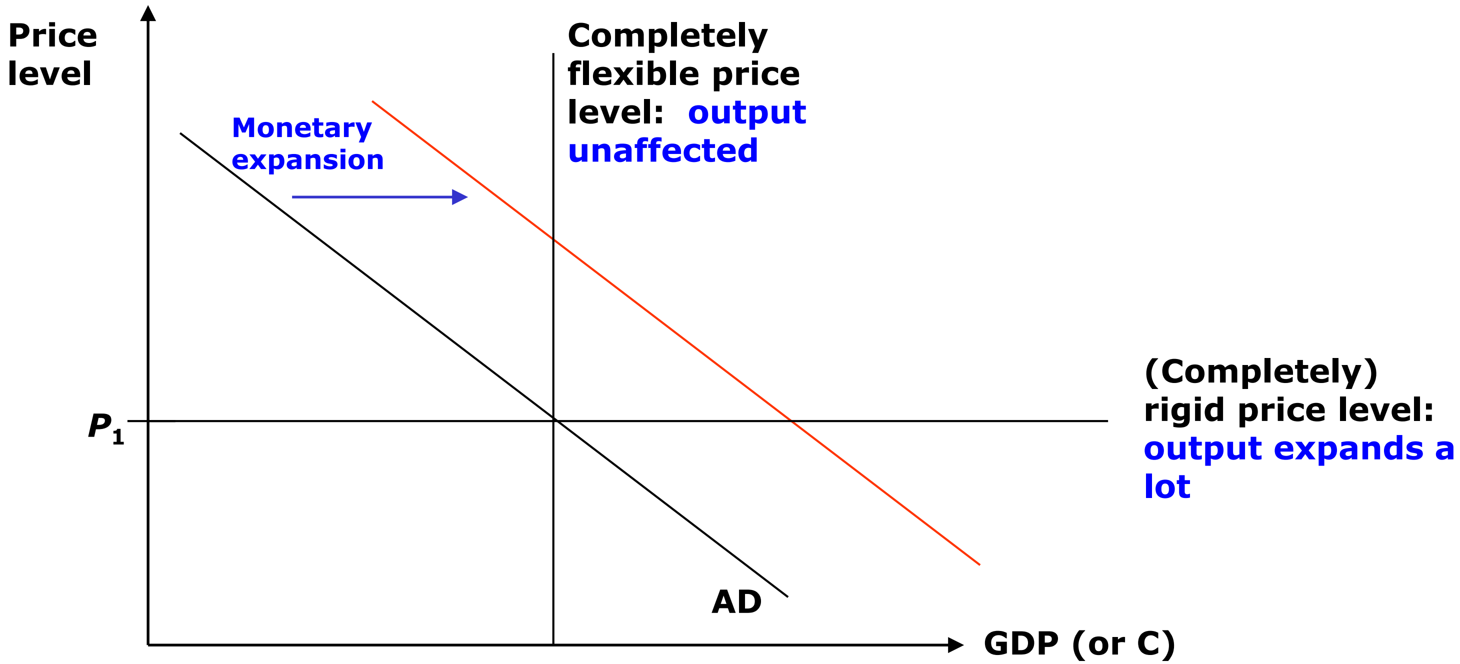


Cyclical Implications of Monetary Policy



Cyclical Implications of Monetary Policy

- **Conventional Keynesian view: nominal rigidities (in price and/or wage level) cause monetary shifts to have real effects**



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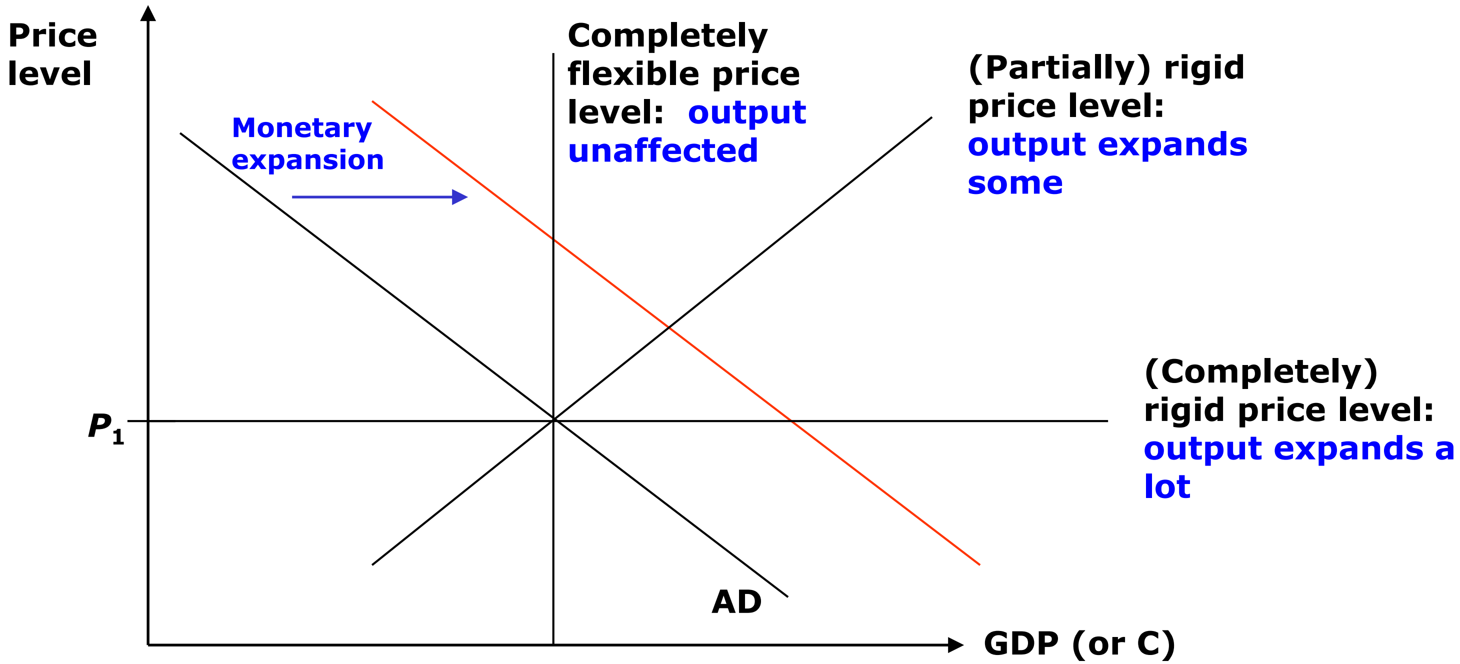

https://en.wikipedia.org/wiki/Nominal_rigidity

General Issues

- **How often do prices change empirically?...**
  - **Wide heterogeneity across goods/categories of goods**
    - **Bils and Klenow (2004 *JPE*), Nakamura and Steinsson (2007), Kehoe and Midrigan (2007), Klenow and Krystov (2007), many others...**
  - **Median ~ 2-3 quarters...**

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  - **Aggregate price level?**
  - **Individual goods/sectors? Thus require a model of heterogeneity in pricing outcomes?**

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- ...and which price changes are most relevant for macro phenomena?
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  - Individual goods/sectors? Thus require a model of heterogeneity in pricing outcomes?

- How to introduce nominal rigidities in basic DSGE model?
  - **Time-dependent or cost-dependent:** firms (re-)set prices according to some exogenous time interval
  - **State-dependent:** firms (re-)set prices according to endogenous (potentially firm-specific) state

More tractable to model

Canonical DSGE Sticky-Price Model

- **Cashless environment**
  - **Woodford (2000, *Interest and Prices*): "…effectiveness of monetary policy does not depend on the ability of the central bank to manipulate significant market distortions…"**

- **Nominal prices (hence nominal price level) move sluggishly**
  - **Due to some "costs" or other "timing impediments"**

- **Built on monopolistic foundation**
  - **Infrequent (re-)setting of price requires imperfect competition**
  - **Dixit-Stiglitz aggregator readily tractable in DSGE environment**
  - **(Other tractable aggregators soon…)**

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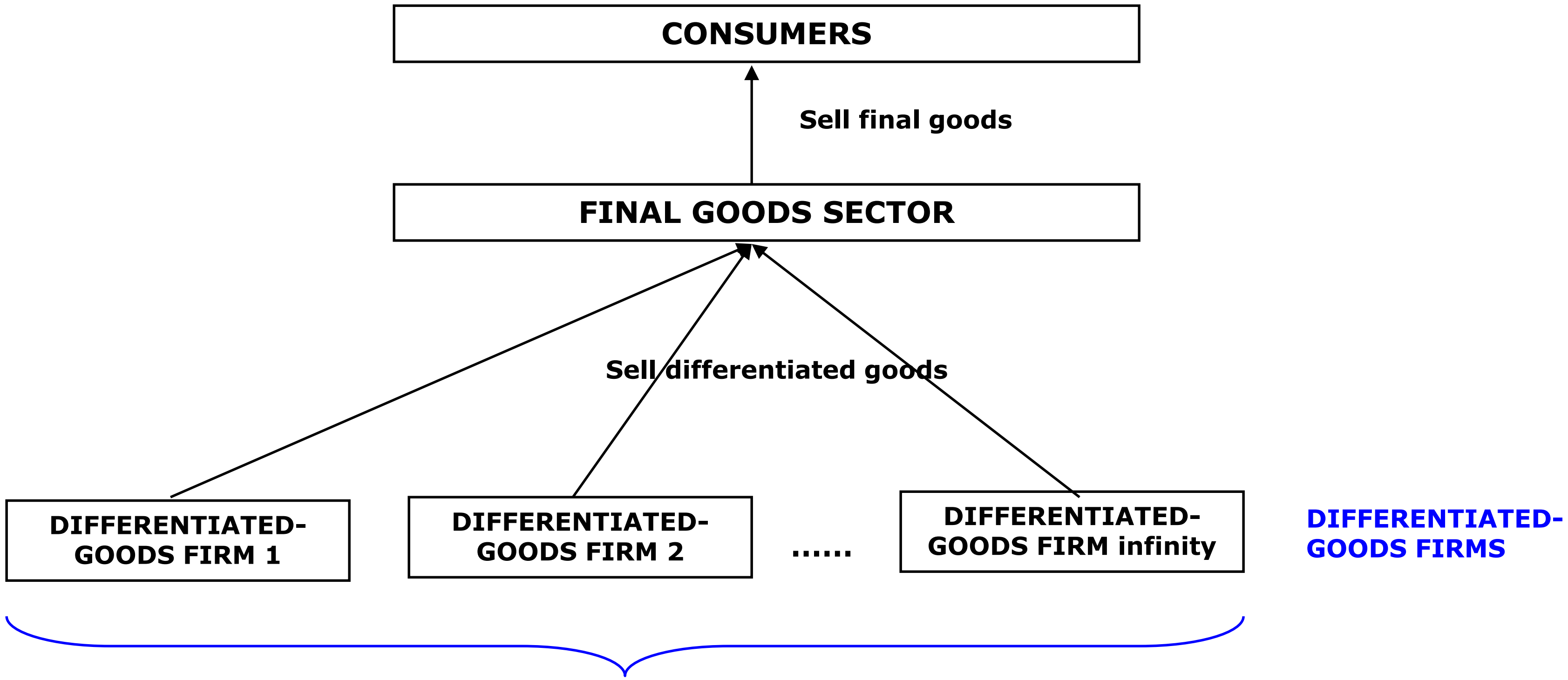
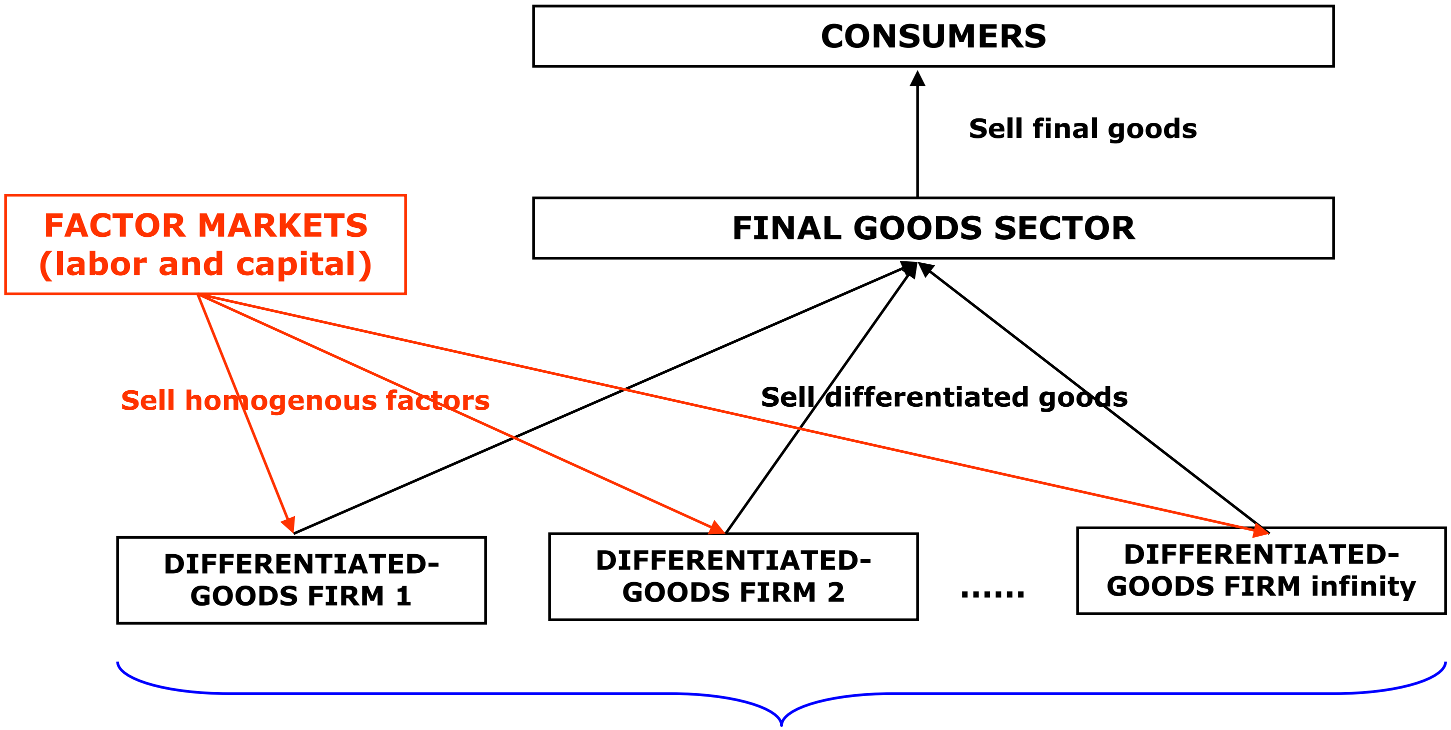
  - **Dixit-Stiglitz aggregator readily tractable in DSGE environment**
  - **(Other tractable aggregators soon…)**
- **Common DSGE sticky-price mechanisms**
  - **Rotemberg: firm can re-optimize nominal price every period, but subject to a quadratic "menu cost"**
  - **Calvo-Yun: firm receives exogenous "signal" to re-optimize nominal price**
  - **Taylor: firm can re-optimize nominal price every *T* periods**

CANONICAL DSGE STICKY-PRICE MODEL



Measure-one continuum of firms, each of which can (re-)set its *nominal* price only infrequently (Calvo and Taylor models) or at some cost (Rotemberg model)

Canonical DSGE Sticky-Price Model



FINAL-GOODS FIRMS DO NOT HIRE OTHER FACTORS

DIFFERENTIATED-GOODS FIRMS

Measure-one continuum of firms, each of which can (re-)set its *nominal* price only infrequently (Calvo and Taylor models) or at some cost (Rotemberg model)

FINAL-GOODS FIRMS

- **Aggregator** $y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$

Final-Goods Firms

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- **Nominal profit-maximization problem**

$$\max_{y_{it}\rangle_{i=0}^{1}} P_t y_t - \int_0^1 P_{it} y_{it} di$$

$$\Big\downarrow \quad \textbf{Substitute in CES final-goods aggregator}$$

$$\max_{y_{it}\rangle_{i=0}^{1}} P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di$$

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P_t* the nominal price of final good; in equilibrium, *P_t* is nominal *price level

$$\max_{y_{it}\rangle_{i=0}^{1}} P_t y_t - \int_0^1 P_{it} y_{it} di$$

P_{it}* the nominal price of differentiated good *i

Substitute in CES final-goods aggregator

$$\max_{y_{it}\rangle_{i=0}^{1}} P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di$$

Final-Goods Firms

- **Production Model**

$$\max_{\langle y_{it} \rangle_{i=0}^{1}} P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di$$

- **FOC with respect to y_{it}**

$$P_t \cancel{\left(\frac{\varepsilon}{\varepsilon-1} \right)} \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}} \cancel{\left(\frac{\varepsilon-1}{\varepsilon} \right)} y_{it}^{-1/\varepsilon} - P_{it} = 0 \longrightarrow P_{it} = P_t y_{it}^{-1/\varepsilon} \cdot \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}}$$

$$\longrightarrow y_{it}^{1/\varepsilon} = \left(\frac{P_t}{P_{it}} \right) \cdot \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{1}{\varepsilon-1}} \longrightarrow y_{it} = \left(\frac{P_t}{P_{it}} \right)^{-\varepsilon} \cdot \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Use D-S aggregator

$$\longrightarrow \boxed{y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t}$$

Demand Function

Dixit-Stiglitz

Final-Goods Firms

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P_t* the nominal price of final good; in equilibrium, *P_t* is nominal *price level

$$\max_{y_{it}\rangle_{i=0}^{1}} P_t y_t - \int_0^1 P_{it} y_{it} di$$

P_{it}* the nominal price of differentiated good *i

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$\downarrow$ **Profit-maximization**

$$y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t$$

TAKEN AS GIVEN BY DIFFERENTIATED FIRM *i*

Each differentiated firm *i* chooses its nominal *P_i* to maximize profit

Relative price of firm *i*'s output

Aggregate output a *shifter* of firm *i*'s demand function

DIFFERENTIATED-GOODS FIRMS

- "Two-stage" optimization problem (Dixit & Stiglitz, 1977, p. 298)
  - Stage 1: Choose optimal nominal P_{it} subject to pricing impediment
  - (Intermediate "stage"): "choose" to produce the y_{it} corresponding to the implied value of P_{it}/P_t
  - Stage 2: Choose factor inputs to produce y_{it} at minimum cost

production y_i pinned down from demand curve

- Differentiated producer *i* production technology

$$y_{it} = \overbrace{z_t f\left(k_{it}, n_{it}\right)}^{\text{Usual CRS}} - \cancel{\Phi}$$

Assume = 0 as before → mc = ac CONSTANT (with respect to quantity)

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- **Implementation of two nominal sticky-price models**
  - **Rotemberg: firm can re-optimize price every period, but subject to a quadratic "menu cost"**
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Differentiated-Goods Firms

- Rotemberg Model: Dynamic profit maximization problem

- Calvo-Yun Model: Dynamic profit-maximization problem

Differentiated-Goods Firms

- **Rotemberg Model: Dynamic profit maximization problem**

Note distinction between *t* and *s* subscripts

$$\max_{P_{is}} E_t \left\{ \sum_{s=t}^{\infty} \Xi_{s|t} \left\{ \left(P_{is} - P_s mc_s \right) y_{is} - \frac{\psi}{2} \left(\frac{P_{is}}{P_{is-1}} - 1 \right)^2 P_s \right\} \right\}$$

***Discount factor* between *t* and *s* because *dynamic* firm problem; in equilibrium, = household stochastic discount factor**

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***Discount factor* between t and s because *dynamic* firm problem; in equilibrium, = household stochastic discount factor**

Rewrite

$$\max_{P_{it}} \left\{ \left(P_{it} - P_t mc_t\right) y_{it} - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 P_t + E_t \Xi_{t+1|t} \left(\left(P_{it+1} - P_{t+1} mc_{t+1}\right) y_{it+1} - \frac{\psi}{2}\left(\frac{P_{it+1}}{P_{it}} - 1\right)^2 P_{t+1} \right) \right\}$$

$$+ \max_{P_{it}} E_t \sum_{s=t+2}^{\infty} \Xi_{s|t+2} \left\{ \left(P_{is} - P_s mc_s\right) y_{is} - \frac{\psi}{2}\left(\frac{P_{is}}{P_{is-1}} - 1\right)^2 P_s \right\}$$

First line contains t and $t+1$ terms

Second line contains $t+2$ and beyond terms

Substitute demand functions

DIFFERENTIATED-GOODS FIRMS

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$$\max_{P_{it}} \left\{ \left(P_{it} - P_t mc_t\right) \cdot \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \cdot y_t - \frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}} - 1\right)^2 P_t \right\}$$

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Periods after t+1 not needed for period-t optimization

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- **FOC wrt P_{it}**

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_{it} - P_t mc_t}{P_{it}}\right)\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi\left(\frac{P_{it}}{P_{it-1}} - 1\right)\frac{P_t}{P_{it-1}} + \psi E_t \left\{ \Xi_{t+1}\left(\frac{P_{it+1}}{P_{it}} - 1\right)\frac{P_{t+1} P_{it+1}}{P_{it} \cdot P_{it}} \right\} = 0$$

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- **If $\psi = 0$**

$$\frac{P_{it}}{P_t} = \underbrace{\left(\frac{\varepsilon}{\varepsilon - 1}\right)}_{\equiv \mu} mc_t$$

Flexible-price Dixit-Stiglitz pricing rule

DIFFERENTIATED-GOODS FIRMS

- **Rotemberg Model: Optimal-pricing condition**

$$\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_{it} - P_t mc_t}{P_{it}}\right)\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi\left(\frac{P_{it}}{P_{it-1}} - 1\right)\frac{P_t}{P_{it-1}} + \psi E_t\left\{\Xi_{t+1}\left(\frac{P_{it+1}}{P_{it}} - 1\right)\frac{P_{t+1}P_{it+1}}{P_{it}\cdot P_{it}}\right\} = 0$$

$$\Big\downarrow \quad \textbf{In symmetric equilibrium}$$

$$\left(\frac{P_t}{P_t}\right)^{-\varepsilon} y_t - \left(\frac{P_t - P_t mc_t}{P_t}\right)\left(\frac{P_t}{P_t}\right)^{-\varepsilon} \varepsilon y_t - \psi\left(\frac{P_t}{P_{t-1}} - 1\right)\frac{P_t}{P_{t-1}} + \psi E_t\left\{\Xi_{t+1|t}\left(\frac{P_{t+1}}{P_t} - 1\right)\cdot\frac{P_{t+1}\cdot P_{t+1}}{P_t\cdot P_t}\right\} = 0$$

Differentiated-Goods Firms

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$\downarrow$ **Cancel terms**

Definition of inflation: $1+\pi_{t+1} = P_{t+1}/P_t$

$\downarrow$ **Combine terms**

Algebra

$\downarrow$

$$[1-\varepsilon+\varepsilon mc_t]y_t - \psi\pi_t(1+\pi_t) + \psi E_t\left\{\Xi_{t+1}\cdot\pi_{t+1}\cdot(1+\pi_{t+1})^2\right\} = 0$$

DIFFERENTIATED-GOODS FIRMS

❒ **Rotemberg Model: Optimal-pricing condition**

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New Keynesian Phillips Curve

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- Calvo-Yun Model: Dynamic profit-maximization problem

Differentiated-Goods Firms

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Note distinction between *t* and *s* subscripts

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left\{ P_{it} y_{is} - P_s mc_s y_{is} \right\} \right\}$$

Exogenous probability **of *not* being able to (re-)set price – the "Calvo fairy"**

Discount factor **between *t* and *s* because *dynamic* firm problem; in equilibrium, = household stochastic discount factor**

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DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Dynamic profit maximization problem**

Price change opportunities arrive according to Poisson process

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Differentiated-Goods Firms

❒ **Calvo-Yun Model: Dynamic profit maximization problem**

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Rewrite

$$\max_{P_{it}} E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left\{ P_{it}^{1-\varepsilon} P_s^{\varepsilon} - P_{it}^{-\varepsilon} P_s^{1+\varepsilon} mc_s \right\} y_s \right\}$$

Differentiated-Goods Firms

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- **FOC wrt P_{it}**

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left[(1-\varepsilon) \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

Continue manipulating

Differentiated-Goods Firms

- **Calvo-Yun Model: Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[(1-\varepsilon)+\varepsilon\left(\frac{P_{it}}{P_s}\right)^{-1}mc_s\right]\right\}=0$$

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Rewrite: multiply by -1/ε

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}-\left(\frac{P_{it}}{P_s}\right)^{-1}mc_s\right]\right\}=0$$

Differentiated-Goods Firms

- **Calvo-Yun Model: Optimal-pricing condition**

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[(1-\varepsilon) + \varepsilon \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

Rewrite: multiply by -1/ε

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon - 1}{\varepsilon} - \left(\frac{P_{it}}{P_s} \right)^{-1} mc_s \right] \right\} = 0$$

Rewrite: multiply each term by P_s/P_s and multiply entire expression by P_{it}

$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon - 1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

Differentiated-Goods Firms

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If prices are completely flexible (i.e., if α = 0)

Differentiated-Goods Firms

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Flexible-price Dixit-Stiglitz pricing rule

If prices are completely flexible (i.e., if α = 0)

$$\frac{P_{it}}{P_t}=\left(\frac{\varepsilon}{\varepsilon-1}\right)mc_t$$

DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

DIFFERENTIATED-GOODS FIRMS

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- **With price impediment, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization**

DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Optimal-pricing condition**

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- **With price impediment, optimal P_i balances current and future marginal revenues against current and future marginal costs until the next (expected) price re-optimization**

- **Differentiated firm *i*'s (and hence the aggregate) markup will be time-varying**
  - **As "initial marginal revenues" > "initial marginal costs" to balance against "later marginal revenues" < "later marginal costs"**
  - **Inflation erodes *relative* price of firm *i***

DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Optimal-pricing condition**

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  - **Inflation erodes *relative* price of firm *i***

- **Conduct full non-linear analysis (around distorted steady state)**
  - **"Textbook" New Keynesian analysis is around efficient steady state**

DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

DIFFERENTIATED-GOODS FIRMS

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$\downarrow$ **Divide entire expression by P_t**

PDV of marginal revenues on LHS, PDV of marginal costs on RHS

$$\frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_s\right\}$$

$$=\frac{P_t}{P_t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_tmc_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_smc_s\right\}$$

Differentiated-Goods Firms

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$$E_t \left\{ \sum_{s=t}^{\infty} \alpha^{s-t} \Xi_{s|t} P_s \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s \left[\frac{\varepsilon - 1}{\varepsilon} \frac{P_{it}}{P_s} - mc_s \right] \right\} = 0$$

Divide entire expression by P_t

PDV of marginal revenues on LHS, PDV of marginal costs on RHS

$$\cancel{\frac{P_t}{P_t}} \left(\frac{P_{it}}{P_t} \right)^{1-\varepsilon} \frac{\varepsilon - 1}{\varepsilon} y_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} \frac{\varepsilon - 1}{\varepsilon} y_s \right\}$$

$$= \cancel{\frac{P_t}{P_t}} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_t mc_t + E_t \left\{ \sum_{s=t+1}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{-\varepsilon} y_s mc_s \right\}$$

Rewrite

DIFFERENTIATED-GOODS FIRMS

❒ **Calvo-Yun Model: Optimal-pricing condition**

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_t}\frac{P_t}{P_s}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_s\right\}$$

$$=\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_t mc_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_t}\frac{P_t}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

Differentiated-Goods Firms

❑ **Calvo-Yun Model: Optimal-pricing condition**

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_t}\frac{P_t}{P_s}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_s\right\}$$

$$=\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_t mc_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_t}\frac{P_t}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

Rewrite

$$\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}y_t+\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}\frac{\varepsilon-1}{\varepsilon}E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_s}{P_t}\right)^{\varepsilon}y_s\right\}$$

$$=\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_t mc_t+\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon}E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_s}{P_t}\right)^{1+\varepsilon}y_s mc_s\right\}$$

Rewrite again

Differentiated-Goods Firms

❒ **Calvo-Yun Model: Optimal-pricing condition**

$$\left(\frac{P_{it}}{P_t}\right)\frac{\varepsilon-1}{\varepsilon}y_t+\left(\frac{P_{it}}{P_t}\right)\frac{\varepsilon-1}{\varepsilon}E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_s}{P_t}\right)^{\varepsilon}y_s\right\}$$

$$=y_t mc_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_s}{P_t}\right)^{1+\varepsilon}y_s mc_s\right\}$$

Rewrite again

$$\frac{P_{it}}{P_t}=\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{y_t mc_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_s}{P_t}\right)^{1+\varepsilon}y_s mc_s\right\}}{y_t+E_t\left\{\sum_{s=t+1}^{\infty}\alpha^{s-t}\Xi_{s|t}\left(\frac{P_s}{P_t}\right)^{\varepsilon}y_s\right\}}\right)$$

Calvo Model: optimal nominal price = PDV mc divided by PDV mr

DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Optimal-pricing condition**

$$E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\left[\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}-mc_s\right]\right\}=0$$

- **Define**

$$P_t x_t^1=E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

PDV of nominal marginal revenues until next price change

$$P_t x_t^2=E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

PDV of nominal marginal costs until next price change

Differentiated-Goods Firms

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PDV of nominal marginal costs until next price change

- **Optimal-pricing condition:** $x_t^1 = x_t^2$
  - **Emphasizes that optimal P_i balances current and future *mr* against current and future *mc***

DIFFERENTIATED-GOODS FIRMS

- **Calvo-Yun Model: Optimal-pricing condition**

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PDV of nominal marginal revenues until next price change

$$P_t x_t^2 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s mc_s\right\}$$

PDV of nominal marginal costs until next price change

- **Optimal-pricing condition:** $x_t^1 = x_t^2$
  - **Emphasizes that optimal P_i balances current and future *mr* against current and future *mc***

- **Write** x_t^1, x_t^2 **recursively**

OPTIMAL-PRICING CONDITION

- **Some notation and definitions**

$$P_{it}$$

Nominal price of good *i* in period *t*

$$p_{it} \equiv \frac{P_{it}}{P_t}$$

Relative price of good *i* in period *t*

$$P_{it+1} = P_{it}$$

Evolution of nominal price if no price change

OPTIMAL-PRICING CONDITION

- **Some notation and definitions**

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Nominal price of good *i* in period *t*

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Relative price of good *i* in period *t*

$$P_{it+1} = P_{it}$$

Evolution of nominal price if no price change

$$\downarrow$$

$$p_{it+1} = \frac{P_{it+1}}{P_{t+1}} = \frac{P_{it}}{P_{t+1}}$$

$$= \frac{P_{it}}{P_t}\frac{P_t}{P_{t+1}}$$

$$= \frac{p_{it}}{\pi_{t+1}}$$

As long as no nominal price change, a firm's relative price erodes at the rate of inflation

($\pi_{t+1} = P_{t+1}/P_t$)

OPTIMAL-PRICING CONDITION

- **Write x_t^1 recursively**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon} y_s\,\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

$$\Big\downarrow \text{ Divide by } P_t\text{; write out first two terms}$$

$$x_t^1 = \frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t\,\frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon} y_{t+1}\,\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\,\frac{\varepsilon-1}{\varepsilon}\right\}$$

OPTIMAL-PRICING CONDITION

- **Write x_t^1 recursively**

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Divide by P_t; write out first two terms

$$x_t^1 = \frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

$P_{it+1} = P_{it}$ while no price change opportunity

$$x_t^1 = \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it+1}}{P_{t+1}}\right)^{1-\varepsilon} y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\} + E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon} y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

Optimal-Pricing Condition

- **Write x_t^1 recursively**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

Divide by P_t; write out first two terms

$$x_t^1 = \frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

$P_{it+1} = P_{it}$ while no price change opportunity

$$x_t^1 = \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it+1}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

Use definitions

$$x_t^1 = p_{it}^{\,1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}p_{it+1}^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

Optimal-Pricing Condition

- **Write x_t^1 recursively**

$$P_t x_t^1 = E_t\left\{\sum_{s=t}^{\infty}\alpha^{s-t}\Xi_{s|t}P_s\left(\frac{P_{it}}{P_s}\right)^{-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\frac{P_{it}}{P_s}\right\}$$

Divide by P_t; write out first two terms

$$x_t^1 = \frac{P_t}{P_t}\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

$P_{it+1} = P_{it}$ while no price change opportunity

$$x_t^1 = \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\frac{P_{t+1}}{P_t}\left(\frac{P_{it+1}}{P_{t+1}}\right)^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

Use definitions

$$x_t^1 = p_{it}^{1-\varepsilon}y_t\frac{\varepsilon-1}{\varepsilon}+\alpha E_t\left\{\Xi_{t+1|t}\pi_{t+1}p_{it+1}^{1-\varepsilon}y_{t+1}\frac{\varepsilon-1}{\varepsilon}\right\}+E_t\left\{\sum_{s=t+2}^{\infty}\alpha^{s-t}\Xi_{s|t}\frac{P_s}{P_t}\left(\frac{P_{it}}{P_s}\right)^{1-\varepsilon}y_s\frac{\varepsilon-1}{\varepsilon}\right\}$$

Use $p_{it+1} = p_{it}/\pi_{t+1}$

OPTIMAL-PRICING CONDITION

- **Write x_t^1 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon - 1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon - 1}{\varepsilon} \right\}$$

OPTIMAL-PRICING CONDITION

❐ **Write x_t^1 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon - 1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon - 1}{\varepsilon} \right\}$$

Rearrange

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} p_{it}^{1-\varepsilon} y_{t+1} \frac{\varepsilon - 1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon - 1}{\varepsilon} \right\}$$

Optimal-Pricing Condition

- **Write x_t^1 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

$\downarrow$ **Rearrange**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} p_{it}^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

$\downarrow$ **Multiply and divide by $p_{it+1}^{1-\varepsilon}$**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} p_{it+1}^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

Optimal-Pricing Condition

- **Write x_t^1 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

Rearrange

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} p_{it}^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}$$

Multiply and divide by $p_{it+1}^{1-\varepsilon}$

$$x_t^1 = \underbrace{p_{it}^{1-\varepsilon} y_t \frac{\varepsilon-1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon}}_{} \underbrace{p_{it+1}^{1-\varepsilon} y_{t+1} \frac{\varepsilon-1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon-1}{\varepsilon} \right\}}_{}$$

Have generated a recursive term

Optimal-Pricing Condition

- **Write x_t^1 recursively**

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1} \left(\frac{p_{it}}{\pi_{t+1}} \right)^{1-\varepsilon} y_{t+1} \frac{\varepsilon - 1}{\varepsilon} \right\} + E_t \left\{ \sum_{s=t+2}^{\infty} \alpha^{s-t} \Xi_{s|t} \frac{P_s}{P_t} \left(\frac{P_{it}}{P_s} \right)^{1-\varepsilon} y_s \frac{\varepsilon - 1}{\varepsilon} \right\}$$

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Have generated a recursive term

Express recursively

$$x_t^1 = p_{it}^{1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} x_{t+1}^1 \right\}$$

***x^1* expressed recursively**

OPTIMAL-PRICING CONDITION

- **Both x_t^1, x_t^2 recursively**

$$x_t^1 = p_{it}^{\;1-\varepsilon} y_t \frac{\varepsilon - 1}{\varepsilon} + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{1-\varepsilon} x_{t+1}^1 \right\}$$

x^1 expressed recursively

$$x_t^2 = p_{it}^{\;-\varepsilon} y_t mc_t + \alpha E_t \left\{ \Xi_{t+1|t} \pi_{t+1}^{1+\varepsilon} \left(\frac{p_{it}}{p_{it+1}} \right)^{-\varepsilon} x_{t+1}^2 \right\}$$

x^2 expressed recursively

- **Optimal-pricing condition expressed compactly**

$$x_t^1 = x_t^2$$

- **Aggregate price index follows from Dixit-Stiglitz aggregation**

OPTIMAL-PRICING CONDITION

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x^1 expressed recursively

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- Start with final goods' producers profit function

$$P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_{it} y_{it} di = 0$$

= 0 due to perfect competition in final goods market

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substitute demand functions

$$P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} = \int_0^1 P_{it} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t \cdot di$$

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- ❒ **Start with final goods' producers profit function**

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$$P_t \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} = \int_0^1 P_{it} \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \cdot y_t \cdot di$$

based on $y_t = \left[\int_0^1 y_{it}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$**, cancel terms**

$$P_t = \int_0^1 P_{it}^{1-\varepsilon} \cdot P_t^{\varepsilon} \cdot di \xrightarrow{\text{rearrange}}$$

AGGREGATE PRICE LEVEL

- **Aggregate price index follows from Dixit-Stiglitz aggregation**

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di$$

Aggregate Price Level

- Aggregate price index follows from Dixit-Stiglitz aggregation

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di = \underbrace{\int_0^{\alpha} P_{it-1}^{1-\varepsilon} di}_{\text{non-adjusters}} + \underbrace{\int_{\alpha}^1 P_{it}^{*1-\varepsilon} di}_{\text{adjusters}}$$

Obtained by substituting demand functions into D-S aggregator

AGGREGATE PRICE LEVEL

- **Aggregate price index follows from Dixit-Stiglitz aggregation**

non-adjusters **adjusters**

Emphasize *optimal* new price

$$P_t^{1-\varepsilon} = \int_0^1 P_{it}^{1-\varepsilon} di = \int_0^{\alpha} P_{it-1}^{1-\varepsilon} di + \int_{\alpha}^1 P_{it}^{*1-\varepsilon} di$$

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Emphasize *optimal* new price

TRACTABILITY DUE TO THE CALVO *RANDOM-ADJUSTMENT* ASSUMPTION

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Obtained by substituting demand functions into D-S aggregator

$$= \alpha P_{t-1}^{1-\varepsilon} + (1-\alpha) P_t^{*1-\varepsilon}$$

<u>KEY:</u> Because adjusters were *randomly* selected, average (aggregate) price of non-adjusters is identical to previous period's average (aggregate) price

Fraction 1 - α re-set price optimally (and symmetrically)

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$$1 = \alpha \pi_t^{\varepsilon-1} + (1-\alpha) p_t^{*1-\varepsilon}$$

EQUILIBRIUM EVOLUTION OF AGGREGATE INFLATION – depends on relative price set by firms currently adjusting nominal price

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Fraction 1 - α re-set price optimally (and symmetrically)

Together form the "aggregate supply" block of New Keynesian sticky-price model

$$1 = \alpha \pi_t^{\varepsilon-1} + (1-\alpha) p_t^{*1-\varepsilon}$$

EQUILIBRIUM EVOLUTION OF AGGREGATE INFLATION – depends on relative price set by firms currently adjusting nominal price

$$x_t^1 = x_t^2$$

Optimal pricing condition

PRICE DISPERSION

- **Calvo model implies dispersion of relative prices**
  - **As does Taylor model …**
  - **…but not Rotemberg model (quadratic cost of nominal price adjustment)**

- **Dispersion often ignored…**
  - **…due to linearization around a zero-inflation steady state (typical simple New Keynesian model soon…)**
  - **With better numerical tools, easier to take account of dispersion**

Price Dispersion

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  - **With better numerical tools, easier to take account of dispersion**
- **Price dispersion the basic source of welfare losses of non-zero inflation**
  - **Because it implies quantity dispersion across intermediate producers…**
  - **…which is inefficient because Dixit-Stiglitz aggregator is symmetric and concave in every good *i***

The basic driving force of optimal policy in any NK model

PRICE DISPERSION

- **For firm *i*,** $y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} y_t = z_t f(k_{it}, n_{it})$

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Symmetric choices of *k/n ratio* across all firms *i*…

$$y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$

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Symmetric choices of *k/n ratio* across all firms *i*…

$$\underbrace{y_t \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di}_{\equiv S_t} = z_t f\left(\frac{k_t}{n_t}, 1\right) \int_0^1 n_{it} di$$

A measure of dispersion: relative price dispersion leads to dispersion of factor usage across differentiated firms, hence dispersion of quantity across differentiated firms

Express s_t recursively

Price Dispersion

$$s_t = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di = \int_\alpha^1 \left(\frac{P_{it}^*}{P_t}\right)^{-\varepsilon} di + \int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$$

non-adjusters

adjusters

Re-setters all choose same price

$$= (1-\alpha)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \int_0^\alpha \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} di$$

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Multiply by $(P_{t-1}/P_{t-1})^{-\varepsilon}$

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P*$_{it}$* **=** ***P****$_{it-1}$* **for firms that cannot re-set price**

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Multiply by (*P*$_{t-1}$/*P*$_{t-1}$)$^{-\varepsilon}$

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NOTE:

***a* = 0: s_t = 1 (no dispersion)**

***a* > 0: s_t > 1 (dispersion)**

Resource Constraint

- **Summarized by *three* conditions**

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + g_t$$

"Usual" resource constraint

And using factor market clearing conditions here

$$k_t = \int_0^1 k_{it}\,di\,,\; n_t = \int_0^1 n_{it}\,di$$

$$y_t = \frac{z_t f(k_t, n_t)}{s_t}$$

Some output is a pure deadweight loss (note s_t < 1 cannot occur)

$$s_t = (1-\alpha)\,p_t^{*-\varepsilon} + \alpha\pi_t^{\varepsilon} s_{t-1}$$

Law of motion for deadweight loss

Resource Constraint

- **Summarized by *three* conditions**

= 0 in Yun model

$$y_t = c_t + k_{t+1} - (1-\delta)k_t + g_t$$

"Usual" resource constraint

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$$s_t = (1-\alpha) p_t^{*-\varepsilon} + \alpha \pi_t^{\varepsilon} s_{t-1}$$

Law of motion for deadweight loss

- **Law of motion for s_t represented using laws of motion for both P_{t-1} and P^*_{t-1}**
  - **See equations (25) and (26)**

OTHER MODEL DETAILS

- Cash/credit to motivate money demand
  - i.e., Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1991)

Could have been cashless...

- (Habit persistence (i.e., time-non-separability) in leisure)
  - As in Kydland and Prescott (1982); unimportant for main results...

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- Exogenous AR(1) TFP process

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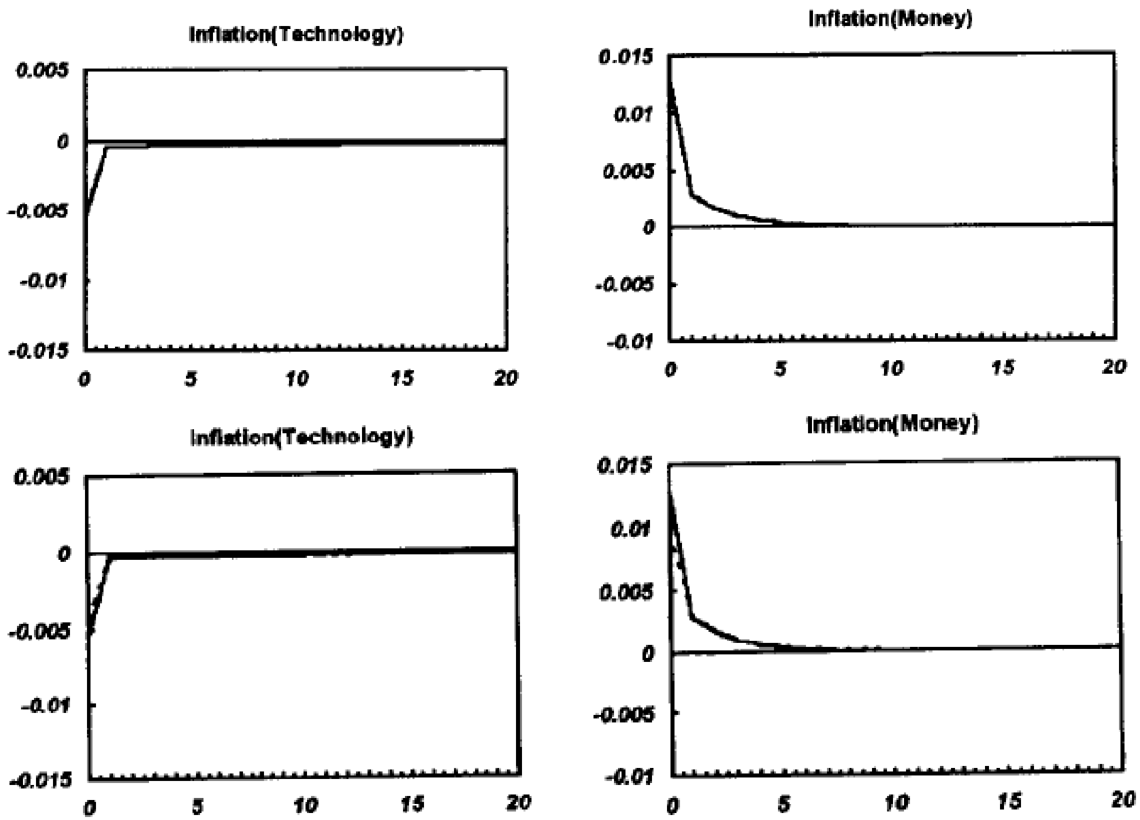
- Exogenous AR(1) money growth process
  - Also "endogenous" money supply process, but not as interesting
- Exogenous AR(1) TFP process
- Indexation of prices to average (i.e., steady-state) inflation
  - For firms not re-setting price, $P_{it} = \pi P_{it-1}$
- Approximated and simulated using linear methods

NOMINAL EFFECTS OF STICKY PRICES

- ❒ **Effects on inflation not very different compared to flex-price case**



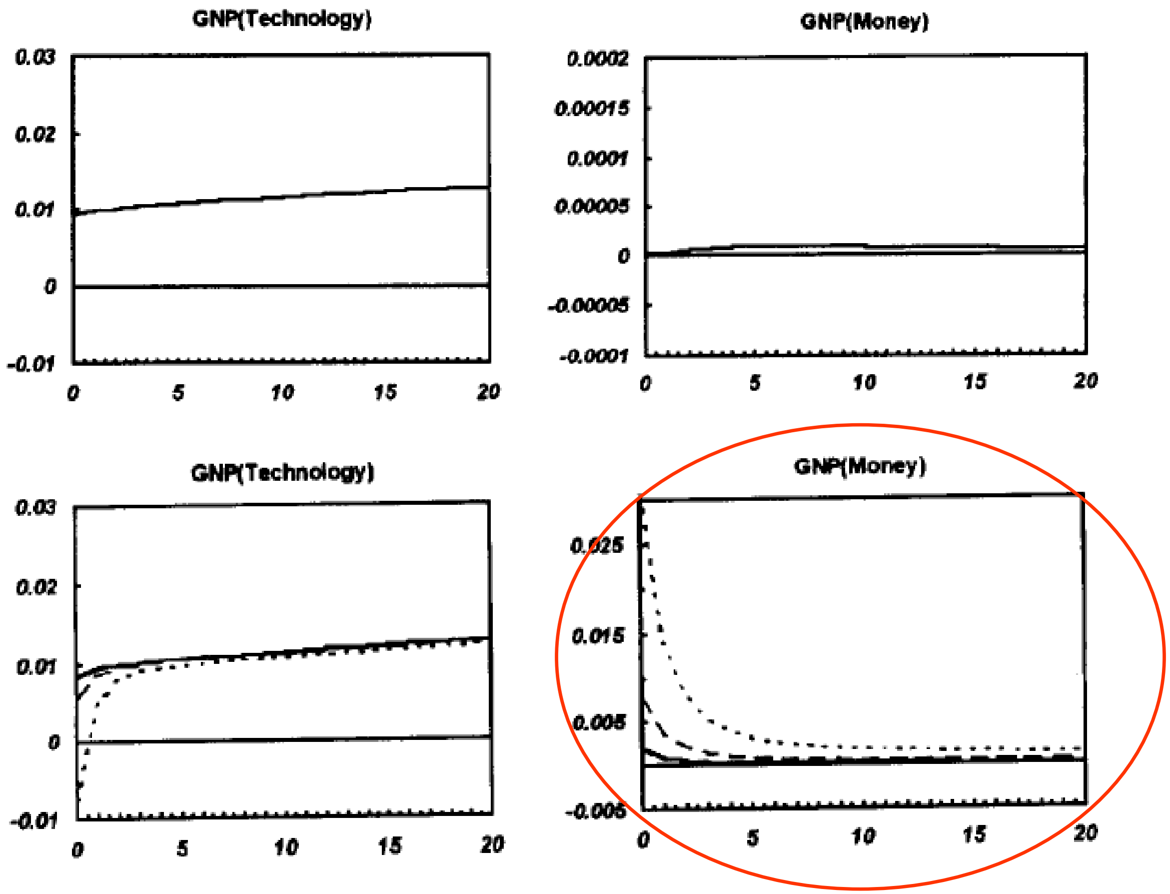
Flexible Prices

Sticky Prices

Lack of inflation persistence in basic Calvo-Yun model (now) well-known –

Real Effects of Sticky Prices

- Effects on GDP much bigger the stickier are prices



Flexible Prices

Sticky Prices

alpha = 0.25

alpha = 0.50

alpha = 0.75

Markup Dynamics

- Money (i.e., nontechnology) shock → aggregate output expands
- With z_t, k_t fixed, output expansion due to increased (equilibrium) employment
  - Downward-sloping product demand curves → individual (differentiated) firms must expand their output (partial equilibrium)
    - Recall aggregate output a shifter of firm *i* demand function

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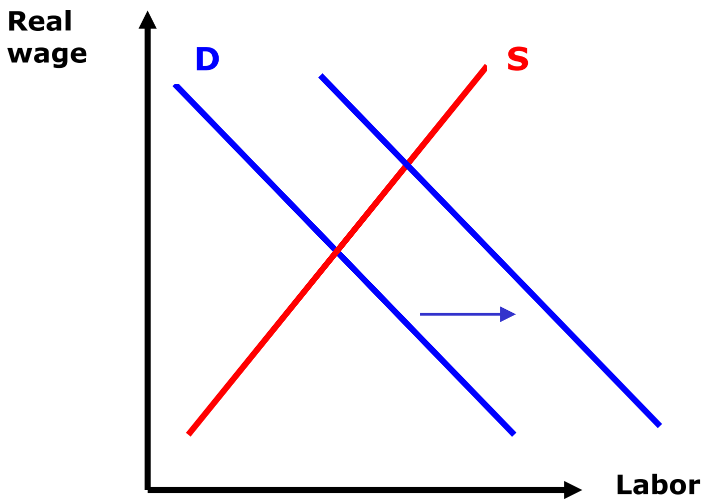
  - **Can only be achieved in short-run if a given firm *i* hires more labor at any real wage**



$$\frac{w_t}{z_t f_n(k_t, n_t)} = \overbrace{\frac{1}{\mu_t}}^{= mc_t}$$

Sticky-price model delivers endogenously-countercyclical markup…

…and endogenous variations in "labor wedge"

DSGE Sticky-Price Models

- **Nominal rigidities embedded in DSGE model**
  - **Monetary shifts → quantitatively "big" effects on output**
  - **(Re-)articulates "old" Keynesian ideas**
  - **Goodfriend and King (1997 *NBER Macroeconomics Annual*): the New-Neoclassical Synthesis**

- **Output effect not very long-lasting (peak response occurs in period of monetary shock, inconsistent with data)**
  - **The "Persistence Puzzle"**
  - **Examined in Chari, Kehoe, and McGrattan (2000 *Econometrica*) and Christiano, Eichenbaum, and Evans (2005 *JPE*)**
    - **CEE (2005 *JPE*) and Smets and Wouters (2007 *AER*) began literature on "medium scale macro models"**

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- Inflation not very persistent
- A Phillips Curve?
- Optimal policy?

Model Interface with Data

- **How often do prices change in Calvo-Yun model?**
  - **1-α the probability a given firm re-optimizes in a given period**
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- **With median price changes every ~ 2-3 quarters, need α between [0.5, 0.67] in a quarterly model**
  - **i.e., between 1/3 and 1/2 of firms can change price in a given period**