

NOMINAL WAGE RIGIDITY

FEBRUARY 19, 2020

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- ❑ **Nominal rigidities originally discussed in terms of wages, not prices**
- ❑ **But somehow dismissed as modern macro literature evolved**
 - ❑ **Basic reason (roughly): are wage payments allocative?**
 - ❑ **Goodfriend and King (2001): "...potential allocative inefficiencies from infrequent setting of nominal wages are likely to be offset in the context of long-term employment relationships.." and "...unlikely to influence recommendations for policy."**

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 - ❑ **House and Basu (2016, *Handbook of Macroeconomics*): "Modern monetary business-cycle models rely heavily on price and wage rigidity. While there is substantial evidence that prices do not adjust frequently, there is much less evidence on whether wage rigidity is an important feature of real world labor markets....the wages for newly-hired workers [...] respond strongly to identified monetary policy innovations....[and] a model with implicit contracts between workers and firms and a flexible allocative wage replicates these patterns well."**

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- ❑ **EHL 2000: introduce sticky nominal wages in DSGE NK model**
 - ❑ **Adapt Calvo framework**
 - ❑ **Main results**
 - ❑ **Sticky nominal wages introduce an endogenous shifter in the price-Phillips curve (i.e., a source of the "cost-push" shock u_t)**
 - ❑ **Optimal policy**
 - ❑ **Stabilize a weighted average of output gap, nominal price inflation, and nominal wage inflation**
 - ❑ **Complete price inflation stability no longer optimal**
- ❑ **EHL formulation of sticky wages has become a standard in "medium-scale" DSGE models**
 - ❑ **Christiano, Eichenbaum, and Evans (2005 JPE), Smets and Wouters (2007 AER)**

NOMINAL WAGE RIGIDITY

- **Dixit-Stiglitz sticky-price structure**
 - **Continuum of differentiated goods**
 - **Each goods producer has monopoly power**
 - **Each goods producer faces exogenous probability of re-setting nominal price**
 - “Retailer” { □ **Final goods producer “packages” intermediate goods and sells composite final good**

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“Retailer” { □ **Final goods producer “packages” intermediate goods and sells composite final good**

- **Dixit-Stiglitz sticky-*wage* structure (as first formulated by EHL)**

- **Continuum of differentiated *labors***

- **Each *labor supplier* has monopoly power**

- **Each *labor supplier* faces exogenous probability of re-setting *nominal wage***

“Employment agency” { □ **Final *labor* supplier “packages” intermediate *labors* and sells composite final *labor***

- **Dixit-Stiglitz-Calvo machinery adapted to the neoclassical labor market**

EHL MODEL – EMPLOYMENT AGENCY

- **Representative employment agency (perfectly-competitive)**
 - **Aggregates individual households' labors**

$$N_t = \left[\int_0^1 n_{it}^{\frac{\varepsilon^w - 1}{\varepsilon^w}} di \right]^{\frac{\varepsilon^w}{\varepsilon^w - 1}}$$

ε^w the elasticity of substitution between different types of labor

- **Sells N_t to intermediate-goods firms (i.e., composite labor needed for goods production)**

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□ Employment agency profit-maximization problem

$$\max_{n_{it}} W_t N_t - \int_0^1 W_{it} n_{it} di$$

profit maximization

$$n_{it} = \left[\frac{W_{it}}{W_t} \right]^{-\varepsilon^w} N_t$$

Revenues: aggregate wage payments from firms

Costs: individual wage payments to differentiated households

Usual Dixit-Stiglitz demand functions (for each type of labor i)

EHL MODEL – HOUSEHOLDS

Exogenous probability of not being able to (re-)set wage

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s [u(c_{it+s}) + v(n_{it+s})]$$

Full set of state-contingent securities allows each hh i to insure against its own idiosyncratic wage – i.e., full consumption insurance

Households i 's individual labor income

Time t budget constraint

$$P_t c_{it} + B_{it} - R_t B_{it-1} = (1 + \tau^w) W_{it} n_{it} + P_t p r_t + T_t$$

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Proportional subsidy to labor income financed by lump-sum tax corrects long-run distortions due to **BOTH** labor monopoly power **AND** goods monopoly power

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hh i also takes as given demand function for its labor

$$n_{it} = \left[\frac{W_{it}}{W_t} \right]^{-\varepsilon^w} N_t$$



Substitute using demand function

EHL MODEL – HOUSEHOLDS

Exogenous probability of not being able to (re-)set wage

Note time subscripts!

$$\max_{c_{it}, W_{it}, \dots} E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left[u(c_{it+s}) + v \left(\left[\frac{W_{it}}{W_{t+s}} \right]^{-\varepsilon^w} N_{t+s} \right) \right]$$

$= n_{it+s}$

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$\underbrace{\hspace{10em}}_{\text{Demand fct. for } n_{it}}$

- FOCs with respect to
 - c_{it}
 - W_{it}
 - And state-contingent assets

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□ FOC with respect c_{it}

$$u'(c_{it}) - \lambda_{it} \cdot P_t = 0 \quad \Leftrightarrow \quad \lambda_{it} = \frac{u'(c_{it})}{P_t}$$

↑
Multiplier on budget constraint

EHL MODEL – OPTIMAL WAGE-SETTING

FOC wrt W_{it}

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ -\varepsilon^w v'(n_{it+s}) \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon^w} \cdot W_{it}^{-1} N_{t+s} + (1 - \varepsilon^w)(1 + \tau^w) \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon^w} N_{t+s} \cdot \lambda_{it+s} \right\} = 0$$

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↓ Rewrite

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↓ Rewrite: multiply by $1/(\varepsilon^w - 1)$

$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon^w} N_{t+s} \left[-\frac{\varepsilon^w}{\varepsilon^w - 1} v'(n_{it+s}) \cdot W_{it}^{-1} - (1 + \tau^w) \cdot \lambda_{it+s} \right] \right\} = 0$$

$= n_{it+s|t}$

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FOC wrt W_{it}

(copied from bottom of previous page...)

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If wages are completely flexible (i.e., if $\alpha_w = 0$):

$$-\left(\frac{\varepsilon^w}{\varepsilon^w - 1} \right) \left(\frac{v'(n_{it})}{u'(c_{it})} \right) = (1 + \tau^w) \left(\frac{W_{it}}{P_t} \right)$$

mrs (between consumption and labor) of hh i

real wage set by hh i

$$\lambda_{it} = \frac{u'(c_{it})}{P_t}$$

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A Dixit-Stiglitz pricing result

$$\left(\frac{\varepsilon^w}{\varepsilon^w - 1} \right) mrs_{it} = \left(\frac{W_{it}}{P_t} \right)$$

Real wage is a markup over mrs

mrs is the household's "marginal cost" of producing/supplying labor

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$$E_t \sum_{s=0}^{\infty} \beta^s \alpha_w^s \left\{ \left(\frac{W_{it}}{W_{t+s}} \right)^{-\varepsilon^w} \overbrace{N_{t+s}}^{= n_{it+s|t}} \left[-\frac{\varepsilon^w}{\varepsilon^w - 1} v'(n_{it+s}) \cdot W_{it}^{-1} - (1 + \tau^w) \cdot \lambda_{it+s} \right] \right\} = 0$$

multiply entire expression by W_{it}

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substitute $\lambda_{it} = \frac{u'(c_{it})}{P_t}$

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**EHL 2000
eqn. 12 and
eqn. 16,
p. 286 – 288**

EHL MODEL – EQUILIBRIUM

- Price Phillips Curve (i.e., FOC of sticky-price firm)
- Household Euler equation
- Aggregate resource constraint
 - **Deadweight loss from both sticky prices and sticky wages**
- Wage Phillips Curve (aka consumption-leisure optimality condition)
- **Law of motion for real wage**
 - **A non-trivial equilibrium condition in models with sticky nominal wages + sticky nominal prices *and/or* explicit money demand**

EHL set τ^w such that no deadweight loss



$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

$$\text{Growth of real wage} = \frac{\text{Nominal wage inflation}}{\text{Nominal price inflation}}$$

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- **Intuition**
 - Sticky nominal price-setting and/or money demand influences price inflation
 - Sticky nominal wage-setting influences wage inflation
 - Technology influences real wage growth
 - **No reason that all three of these are compatible with each other (simple example: $w_t = \text{TFP}$ every period)**

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EHL set τ^w such that no deadweight loss



$$\frac{w_t}{w_{t-1}} = \frac{\pi_t^w}{\pi_t}$$

In sticky-wage economy, this condition is an allocative condition, not simply an identity – hence part of the definition of equilibrium

$$\text{Growth of real wage} = \frac{\text{Nominal wage inflation}}{\text{Nominal price inflation}}$$

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See Chugh (2006 RED p. 691-692)



LINEAR-QUADRATIC APPROACH

FEBRUARY 19, 2020

THE THREE-EQUATION MODEL

□ The basic framework

Phillips Curve/Aggregate Supply $\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$

IS Curve/Aggregate Demand $\hat{y}_t = -\frac{1}{\sigma} \left[\hat{R}_t - E_t \hat{\pi}_{t+1} \right] + E_t \hat{y}_{t+1} + g_t$

Monetary Policy (interest rate) Rule $\hat{R}_t = \delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t$

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$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

Absent a “cost-push” shock, no tradeoff between stabilizing inflation and stabilizing output (Blanchard and Gali 2007 *JMCB*, 2010 *AEJ:Macro* “divine coincidence”)

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- Optimize criterion function subject to Phillips Curve and IS curve

□ What is the correct/natural/interesting/relevant criterion function?

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- **Typical NK criterion: expected lifetime utility of the representative agent expressed as a second-order approximation → yields a central bank loss function quadratic in inflation gaps and output gaps**

DERIVING CENTRAL BANK OBJECTIVE

- **Lifetime household utility**

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(n_t)]$$

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$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(n_t)]$$

Expand around **PARETO-EFFICIENT** steady-state (c, n)

Pareto-efficient steady-state "achieved" by assuming

- **Long-run inflation = 0**

- **Sufficient fiscal instruments exist to correct long-run distortions due to monopoly power (in particular, a proportional subsidy to labor income to undo the effects of an inefficient real wage, financed with lump-sum tax)**

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Optimal subsidy is

$$1 + \tau^w = \varepsilon / (\varepsilon - 1)$$

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SECOND-order Taylor expansion

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...many many many more steps... (see Woodford and Rotemberg 1998 NBER Appendix)

$$E_0 \sum_{t=0}^{\infty} \beta^t [\hat{\pi}_t^2 + \hat{y}_t^2] \text{ Central Bank Loss Function - depends on only output gaps and inflation gaps}$$

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- Long-run inflation = 0
- Sufficient fiscal instruments exist to correct long-run distortions due to monopoly power (in particular, a proportional subsidy to labor income to undo the effects of an inefficient real wage, financed with lump-sum tax)

SECOND-order Taylor expansion

Some terms drop out due to assumption of long-run Pareto efficiency

...many many many more steps... (see Woodford and Rotemberg 1998 NBER Appendix)

$$E_0 \sum_{t=0}^{\infty} \beta^t [\hat{\pi}_t^2 + \hat{y}_t^2] \quad \text{Central Bank Loss Function - depends on only output gaps and inflation gaps}$$

Introduce labor income subsidy at rate $\tau^w =$ in hh budget constraint.

Optimal subsidy is

$$1 + \tau^w = \varepsilon / (\varepsilon - 1)$$

Unconstrained optimum clearly involves setting all inflation gaps and output gaps to zero: achieves zero loss

OPTIMAL POLICY PROBLEM

□ Minimize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 \right]$$

subject to implementation as a (sticky-price) private-sector equilibrium

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t + u_t$$

$$\hat{y}_t = -\frac{1}{\sigma} \left[\hat{R}_t - E_t \hat{\pi}_{t+1} \right] + E_t \hat{y}_{t+1} + g_t$$

A LINEAR-QUADRATIC
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Rule

$$\hat{R}_t = \delta_{\pi} \hat{\pi}_t + \delta_y \hat{y}_t$$

**A LINEAR-QUADRATIC
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- **Commitment**
 - **Timeless perspective**

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A LINEAR-QUADRATIC PROBLEM

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- **Timeless perspective**

- **Choose δ_{π} and δ_y to minimize monetary-authority loss function**

- **For given processes for the u_t and g_t shocks**
- **Analytical results sometimes feasible – because LQ structure**
- **Bigger models rely on computational methods**

OPTIMAL POLICY: FURTHER ISSUES

□ Richer interest-rate rules

Data suggests high persistence of monetary-policy interest rates – thus allow for (optimal) “interest-rate smoothing”

$$\hat{R}_t = \delta_r \hat{R}_{t-1} + (1 - \delta_r) [\delta_\pi \hat{\pi}_t + \delta_y \hat{y}_t]$$

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□ Relevant “long-run targets?” – i.e., how to define \hat{x}_t ?

- Target a constant (steady-state)?
- Target the “flexible-price” **time-varying** level of inflation/output?
 - Underlying RBC model provides natural benchmark time-varying inflation/output – then define gaps relative to these

EHL MODEL – OPTIMAL POLICY

- **Optimal policy problem – maximize representative household's lifetime utility subject to all private-sector equilibrium conditions**

- **In LQ form (see EHL Appendix B), minimize**

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\hat{\pi}_t^2 + \hat{y}_t^2 + \theta \hat{\pi}_t^{w2} \right]$$

Relative importance of stabilizing wage inflation versus stabilizing price inflation depends on relative stickiness of prices and wages – NO divine coincidence here

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 - EHL p. 298
- Main Result
 - Fully stabilizing price inflation NOT optimal
 - Stabilize a weighted average of price inflation and wage inflation