Department of Economics Boston College

Economics 202 (Section 05)
Macroeconomic Theory
Problem Set 2
Professor Sanjay Chugh
Fall 2013
Due: Tuesday, December 10, 2013

Instructions: Written (typed is strongly preferred, but not required) solutions must be submitted no later than 1:30pm on the date listed above.

You must submit your own independently-written solutions. You are permitted (in fact, encouraged) to work in groups **no larger than three members** to think through issues and ideas, but you must submit your own independently-written solutions. **Under no circumstances will multiple verbatim identical solutions be considered acceptable.**

If you do work with others, each member of the group must state the other members of the group with whom he/she worked. A person can *only* work in one group.

Your solutions, which likely require some combination of mathematical derivations, economic reasoning, graphical analysis, and pure logic, should be **clearly, logically, and thoroughly presented;** they should not leave the reader (i.e., your grading assistant and I) guessing about what you actually meant. Your method of argument(s) and approach to problems is as important as, if not more important than, your "final answer." Throughout, your analysis should be based on the frameworks, concepts, and methods developed in class.

Problem 1. Technology Over the Past Century (20 points). In any given time period *t*, the representative firm uses the Cobb-Douglas production technology

$$A_t \cdot f(k_t, n_t) = A_t \cdot k_t^{\alpha_t} n_t^{1-\alpha_t}$$

in producing its output of goods and services. As standard, the exponent $\alpha_t \in (0,1)$ in every period t **– but note here that the exponent could be different in different time periods.** The rest of the notation is identical to that used in class.

In the early 20th century, U.S. firms used less capital in their production process than they did in the early 21st century.

For simplicity, suppose that total factor productivity did not change at all during the century. And further suppose that neither real wages nor real interest rates changed at all during the century.

If the representative firm (which, as per usual economic analysis, maximizes its economic profits) uses **a larger RATIO of capital to labor (that is, a larger profit-maximizing RATIO *k*/*n*)** in the early 21st century compared to the early 20th century, what change(s) must have occurred?

Base the analysis on the *given* production function. Provide brief yet complete mathematical justification, brief economic interpretation, **and** a simple, qualitative, and clearly-labeled pair of graphs that depicts what occurred over the course of the century: one for the demand side of the labor market and one for the demand side of the capital market.

Problem 2. Fiscal Policy and Monetary Policy Interactions (20 points). Suppose that at the beginning of period t, $M_{t-1} = 100$, and the government has to repay 10 *nominal* units in government bonds (our usual one-period, FV = 1 bonds). In period t, the fiscal authority (Congress) decides to spend 190 *nominal* units in government spending, collect 180 *nominal* units in taxes, and instructs the Treasury to raise 20 *nominal* units by issuing new (one-period, FV = 1) bonds (that is, the Treasury is ordered to raise 20 nominal units by selling bonds, not ordered to sell 20 bonds).

a. **(10 points)** Under this scenario, can the monetary authority decide to expand the money supply **during period *t*?** (That is, can it choose $M_t > M_{t-1}$)? Briefly explain why or why not, or, if it is not possible to determine, explain why it cannot be determined.

b. **(10 points)** Under the scenario described, is the monetary authority active or passive? Briefly explain.

Problem 3: The Term Structure and Financial Market Regulation (60 points). In this problem, you will study a version of the accelerator framework we studied in class. As in our basic analysis, we continue to use the two-period theory of firm profit maximization as our vehicle for studying the effects of financial-market developments on macroeconomic activity. However, rather than supposing it is just "stock" that is the financial asset at firms' disposal for facilitating physical capital purchases, we will now suppose that **both "stock" and "bonds" are at firms' disposal for facilitating physical capital purchases.**

Before describing more precisely the analysis you are to conduct, a deeper understanding of "bond markets" is required. In "normal – aka conventional – economic conditions" (i.e, in or near a "steady state," in the sense we first discussed in Chapter 8), it is usually sufficient to think of all bonds of various maturity lengths in a highly simplified way: by supposing that they are all simply one-period face-value = 1 bonds with the same nominal interest rate. Recall, in fact, that this is how our basic discussion of monetary policy proceeded. In "unusual" (i.e., far away from steady state) financial market conditions, however, it can become important to distinguish between different types of bonds and hence different types of nominal interest rates on those bonds.

We have discussed in class that the U.S. Federal Reserve has over the past few years been "purchasing bonds" as one part of how it conducts its "quantitative easing" policy. Viewed through the conventional lens of how open-market operations work, this policy is hard to understand because in the standard view, central banks **already do** buy (and sell) "bonds" as the mechanism by which they conduct open-market operations.

A difference that becomes important to understand during unusual financial market conditions is that open-market operations are conducted using the **shortest-maturity** "bonds" that the Treasury sells, of duration one month or shorter. In the lingo of finance, this type of "bond" is called a "Treasury bill." The term "Treasury bond" is usually used to refer to **longer-maturity** Treasury securities – those that have maturities of one, two, five, or more years. These longer-maturity Treasury "bonds" have typically **not** been assets that the Federal Reserve buys and sells as regular practice; buying such longer-maturity bonds is/has not been the usual way of conducting monetary policy. But it has become common practice over the past few years.

In the ensuing analysis, part of the goal will be to understand/explain why policy-makers have chosen this option. Before beginning this analysis, though, there is more to understand.

Problem 3 continued

In private-market borrower/lender relationships, longer-maturity Treasury bonds ("bonds") are typically allowed to be used just like stocks in financing firms' physical capital purchases.[1] We can capture this idea by enriching the financing constraint in our financial accelerator framework to read:

$$P_1 \cdot (k_2 - k_1) = R^S \cdot S_1 \cdot a_1 + R^B \cdot P_1^b \cdot B_1 .$$

The left hand side of this richer financing constraint is the same as the left hand side of the financing constraint we considered in our basic theory (and the notation is identical, as well – refer to your notes for the notational definitions).

The right hand side of the financing constraint is richer than in our basic theory, however. The market value of "stock," $S_1 a_1$, still affects how much physical investment firms can do, scaled by the government regulation R^S. **In addition, now the market value of a firm's "bond-holdings" (which, again, means long-maturity government bonds) also affects how much physical investment firms can do,** scaled by the government regulation R^B. The notation here is that B_1 is a firm's holdings of nominal bonds ("long-maturity") at the end of period 1, and P_1^b is the nominal price of that bond during period 1. Note that R^B and R^S need not be equal to each other.

In the context of the two-period framework, the firm's two-period discounted profit function now reads:

$$P_1 f(k_1, n_1) + P_1 k_1 + (S_1 + D_1) a_0 + B_0 - P_1 w_1 n_1 - P_1 k_2 - S_1 a_1 - P_1^b B_1$$
$$+ \frac{P_2 f(k_2, n_2)}{1+i} + \frac{P_2 k_2}{1+i} + \frac{(S_2 + D_2) a_1}{1+i} + \frac{B_1}{1+i} - \frac{P_2 w_2 n_2}{1+i} - \frac{P_2 k_3}{1+i} - \frac{S_2 a_2}{1+i} - \frac{P_2^b B_2}{1+i}$$

The new notation compared to our study of the basic accelerator mechanism is the following: B_0 is the firm's holdings of nominal bonds (**which have face value = 1)** at the start of period one, B_1 is the firm's holdings of nominal bonds (**which have face value = 1**) at the end of period one, and B_2 is the firm's holdings of nominal bonds (**which have face value = 1)** at the end of period two.

Note that period-2 profits are being discounted by the nominal interest rate i: in this problem, we will consider i to be the "Treasury bill" interest rate (as opposed to the "Treasury bond" interest rate). The Treasury-**bill** interest rate is the one the Federal Reserve usually (i.e., in "normal times") controls. We can **define** the nominal interest rate on Treasury **bonds** as

$$i^{BOND} = \frac{1}{P_1^b} - 1 \qquad \left(\Leftrightarrow P_1^b = \frac{1}{1 + i^{BOND}} \right)$$

Thus, note that i^{BOND} and i need not equal each other.

---

[1] Whereas, for various institutional and regulatory reasons, very short-term Treasury assets ("T-bills") are typically not allowed to be used in financing firms' physical capital purchases.

Problem 3 continued

The rest of the notation above is just as in our study of the basic financial accelerator framework. Finally, because the economy ends at the end of period 2, we can conclude (as usual) that $k_3 = 0$, $a_2 = 0$, and $B_2 = 0$.

With this background in place, you are to analyze a number of issues.

a. **(10 points)** Using λ as your notation for the Lagrange multiplier on the financing constraint, construct the Lagrangian for the representative firm's (two-period) profit-maximization problem.

b. **(10 points**) Based on this Lagrangian, compute the first-order condition with respect to nominal bond holdings at the end of period 1 (i.e., compute the FOC with respect to B_1). **(Note:** This FOC is critical for much of the analysis that follows, so you should make sure that your work here is absolutely correct! If your FOC here is incorrect, we will **not** necessarily "carry through the error" all the way through the remainder of your analysis when reviewing solutions.)

c. **(10 points**) Recall that in this enriched version of the accelerator framework, the nominal interest rate on "Treasury bills," i, and the nominal interest rate on "Treasury bonds," i^{BOND}, are potentially different from each other. If financing constraints do NOT at all affect firms' investment in physical capital, how does i^{BOND} compare to i? Specifically, is i^{BOND} equal to i, is i^{BOND} smaller than i, is i^{BOND} larger than i, or is it impossible to determine? Be as thorough in your analysis and conclusions as possible (i.e., tell us as much about this issue as you can!). Your analysis here should be based on the FOC on B_1 computed in part b above. (**Hint:** if financing constraints "don't matter," what is the value of the Lagrange multiplier λ?)

d. **(10 points)** If financing constraints DO affect firms' investment in physical capital, how does i^{BOND} compare to i? Specifically, is i^{BOND} equal to i, is i^{BOND} smaller than i, is i^{BOND} larger than i, or is it impossible to determine? Furthermore, if possible, use your solution here as a basis for justifying whether or not it is appropriate in "normal economic conditions" to consider both "Treasury bills" and "Treasury bonds" as the "same" asset. Be as thorough in your analysis and conclusions as possible (i.e., tell us as much about this issue as you can!). Once again, your analysis here should be based on the FOC on B_1 computed in part b above. **(Note:** the government regulatory variables R^S and R^B are both strictly positive – that is, neither can be zero or less than zero).

The above analysis was framed in terms of nominal interest rates; the remainder of the analysis is framed in terms of real interest rates.

Problem 3 continued

e. **(10 points)** By computing the first-order condition on firms' stock-holdings at the end of period 1, a_1, and following exactly the same algebra as presented in class, we can express the Lagrange multiplier λ as

$$\lambda = \left[\frac{r - r^{STOCK}}{1+r}\right] \cdot \frac{1}{R^S}.$$

Use the first-order condition on B_1 you computed in part b above to derive an analogous expression for λ except in terms of the real interest rate on bonds (i.e., r^{BOND}) and R^B (rather than R^S). (**Hint:** Use the FOC on B_1 you computed in part b above and follow a very similar set of algebraic manipulations as we followed in class.)

f. **(10 points)** Compare the expression you just derived in part e with the expression for lambda above. Suppose $r = r^{STOCK}$. If this is the case, is r^{BOND} equal to r, is r^{BOND} smaller than r, is r^{BOND} larger than r, or is it impossible to determine? Furthermore, in this case, does the financing constraint affect firms' physical investment decisions? Briefly justify your conclusions and provide brief explanation.