Department of Economics Boston College

Economics 8861.01
Assignment 2
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Provide a **recursive definition of consumption-savings optimality** that "differs" from the one presented on the following page. (Note: this "different" definition **must** coincide with the definition on the following page, but there **are** alternative representations of it.)

DETERMINISTIC – RECURSIVE ANALYSIS

- Solution of infinite-horizon consumer problem (starting from date zero)…
- …is a consumption decision rule $c(a_{-1};.)$, asset decision rule $a(a_{-1};.)$, and value function $V(a_{-1};.)$ that satisfies
  - Bellman equation

$$V(a_{-1}, r_0;.) \equiv u(c(a_{-1})) + \lambda_0 \left(y_0 + (1+r_{-1})a_{-1} - c(a_{-1}) - a(a_{-1}) \right) + \beta \cdot V(a(a_{-1}), r_1;.)$$

  - Euler equation

$$u'(c(a_{-1})) = \beta V_1(a(a_{-1}), r_1;.) \quad \xleftrightarrow{\text{by envelope theorem}} \quad u'(c(a_{-1})) = \beta(1+r_0)u'(c(a_0))$$

    - which is the TVC in the limit $t \to \infty$: $\lim_{t\to\infty} \beta^t u'(c(a^*_{t-1})) \cdot a(a^*_{t-1}) = 0$
  - Budget constraint

$$y_0 + (1+r_{-1})a_{-1} - c(a_{-1}) - a(a_{-1}) = 0$$

taking as given $\left(a_{-1}, r_0, r_{-1}\right)$