LABOR MATCHING MODELS: BASIC DSGE IMPLEMENTATION

OCTOBER 15, 2013

FIRM VACANCY-POSTING PROBLEM

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - w_t n_t^f h_t - g(v_t) \right) \right]$$

Discount factor between time 0 and *t* because *dynamic* firm problem; in equilibrium, = household stochastic discount factor

Number of vacancies to post (how many job advertisements)

Desired target *future* firm employment

Total output – sold in perfectly-competitive goods market

Total wage bill depends on both extensive and intensive employment

Total cost of posting *v* vacancies

  - **Subject to (perceived) law of motion for firm's employment stock**

- **Baseline model**
  - **Shut down intensive margin: $h_t = 1$**
  - **Linear posting costs: $g(v) = \gamma v$**
  - **Firm production function: $y_t = z_t * n_t$**
  - **Wage-setting (process) taken as given when posting vacancies**

FIRM VACANCY-POSTING PROBLEM

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f - w_t n_t^f - \gamma v_t \right) \right]$$

$$\text{s.t. } n_{t+1}^f = \underbrace{(1-\rho^x) n_t^f}_{\text{Number of existing jobs that do not end: } \rho^x \text{ exogenous separation rate, but can also endogenize}} + \underbrace{v_t k^f(\theta_t)}_{\text{Each vacancy has probability } k^f(\theta) \text{ of attracting a prospective employee}}$$

Perceived law of motion for evolution of employment stock

Number of existing jobs that do not end: ρ^x exogenous separation rate, but can also endogenize

Each vacancy has probability $k^f(\theta)$ of attracting a prospective employee: depends on a *market* variable, θ, so taken as given

$$\Big\downarrow \text{ FOCs with respect to } v_t,\ n_{t+1}$$

$$-\gamma + \mu_t k^f(\theta_t) = 0$$

$$-\mu_t + E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + (1-\rho^x)\mu_{t+1} \right) \right\} = 0$$

$$\Big\downarrow \text{ Combine}$$

Firm Vacancy-Posting Problem

- **Vacancy posting condition (aka job creation condition)**

$$\gamma = k^f(\theta_t)E_t\left\{\Xi_{t+1|t}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

γ / k^f is capital value of an existing employee – because one *less* worker firm has to find in the future

EMPLOYEES ARE ASSETS

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of attracting a worker) x (expected future benefit of an additional worker)

= marginal output – wage payment + expected asset value of an additional worker

FIRM VACANCY-POSTING PROBLEM

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- Vacancy-posting is a type of investment decision
  - Intertemporal dimension makes discount factor potentially important
    - Makes general equilibrium effects potentially important

- Two prices affect posting decision (aside from intertemporal price)
  - (Future) wage
  - Matching probability (loosely interpret probabilities as prices) which depends on the market variable θ

HOUSEHOLD PROBLEM

- Dynamic household utility-maximization problem
  - A continuum [0, 1] of households (a standard assumption)
  - A continuum [0, 1] of atomistic individuals live in each household
  - Thus representative household has a continuum of "family members"

$$\max_{c_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$\text{s.t.} \quad c_t + a_t = \underbrace{n_t w_t h_t}_{\substack{\text{Measure } n_t \text{ of family members earn}\\ \text{labor income (because they work)}\\ \text{(and recall we've normalized } h = 1)}} + \underbrace{(1-n_t)b}_{\substack{\text{Measure } 1-n_t \text{ of family members receive}\\ \text{unemployment benefits and/or engaged in home}\\ \text{production}}} + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

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KEY: Assuming infinite *family* structure delivers ***full consumption insurance*** – i.e., all employed and unemployed individuals have equal consumption!

Thus individual family members are risk-neutral with respect to their labor-market realization

Analogy with Hansen-Rogerson structure (see Andolfatto 1996 *AER*)

$$\max_{c_t, a_t} \left[E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$\text{s.t. } c_t + a_t = \underbrace{n_t w_t h_t}_{\text{Measure } n_t \text{ of family members earn labor income (because they work) (and recall we've normalized } h = 1)} + \underbrace{(1-n_t)b}_{\text{Measure } 1-n_t \text{ of family members receive unemployment benefits and/or engaged in home production}} + R_t a_{t-1}$$

An (arbitrary) asset to make pricing interest rates explicit

Wage (-setting process) taken as given by household

- Consumption-savings optimality condition: $1 = R_t E_t \underbrace{\left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}}_{\text{Stochastic discount factor}}$

- No labor-supply/part. margin in basic model
  - Each family member either works or is looking for work

WAGE BARGAINING

- (Generalized) Nash Bargaining

$$\max_{w_t} \underbrace{\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)^{\eta}}_{\text{Net payoff to an individual / household of agreeing to wage } w \text{ and beginning production}} \underbrace{\left(\mathbf{J}(w_t)-\mathbf{V}(w_t)\right)^{1-\eta}}_{\text{Net payoff to a firm of agreeing to wage } w \text{ and beginning production}}$$

Bargaining over how to divide the surplus

- Value equations
  - ***W***: value to (representative) household of having one additional member employed
  - ***U***: value to (representative) household of having one additional member unemployed and searching for work
  - ***J***: value to (representative) firm of having one additional employee
  - ***V***: value to (representative) firm of having a vacancy that goes unfilled
    - Free entry in vacancy-posting → *V* = 0

- Define *W* and *U* in terms of household problem
  - i.e., based on envelope conditions of household value function

WAGE BARGAINING

- (Generalized) Nash Bargaining

$$\max_{w_t} \underbrace{\left(\mathbf{W}(w_t)-\mathrm{U}(w_t)\right)^{\eta}}_{\text{Net payoff to an individual / household of agreeing to wage } w \text{ and beginning production}} \underbrace{\left(\mathbf{J}(w_t)-\mathbf{V}(w_t)\right)^{1-\eta}}_{\text{Net payoff to a firm of agreeing to wage } w \text{ and beginning production}}$$

Bargaining over how to divide the surplus

- The Nash surplus-sharing rule

$$\eta\left(\mathbf{W}'(w_t)-\mathbf{U}'(w_t)\right)\mathbf{J}(w_t)=(1-\eta)(-\mathbf{J}'(w_t))\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)$$

(FOC with respect to w_t)

  - Present in any model with Nash bargaining
    - (Most) labor matching models
    - (Most) monetary search models
    - Political bargaining games (Albanesi 2007 *JME*)

- Must specify value equations *W*(.), *U*(.), *J*(.)

VALUE EQUATIONS

- Individual/household value equations (constructed from household problem)

Each searching individual has probability $k^h(\theta)$ of finding a job opening: depends on a *market* variable, θ, so taken as given

$$\mathbf{W}(w_t) = \underbrace{w_t}_{\text{Contemporaneous return is wage}} + E_t\left\{\underbrace{\Xi_{t+1|t}\left[(1-\rho^x)\mathbf{W}(w_{t+1}) + \rho^x\mathbf{U}(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual employed

$$\mathbf{U}(w_t) = \underbrace{b}_{\text{Contemporaneous return is unemployment benefit/home production}} + E_t\left\{\underbrace{\Xi_{t+1|t}\left[k^h(\theta_t)\mathbf{W}(w_{t+1}) + (1-k^h(\theta_t))\mathbf{U}(w_{t+1})\right]}_{\text{Expected future return takes into account transition probabilities}}\right\}$$

Value to household of having the marginal individual unemployed and searching

- Firm value equation

$$\mathbf{J}(w_t) = \underbrace{z_t - w_t}_{\text{Contemporaneous return is marginal output net of wage payment}} + \underbrace{E_t\left\{\Xi_{t+1|t}(1-\rho^x)\mathbf{J}(w_{t+1})\right\}}_{\text{Expected future return takes into account transition probabilities}}$$

Value to firm of the marginal employee

WAGE BARGAINING

❒ The Nash surplus-sharing rule

$$\eta\left(\mathbf{W}'(w_t)-\mathbf{U}'(w_t)\right)\mathbf{J}(w_t)=(1-\eta)(-\mathbf{J}'(w_t))\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)$$

(FOC with respect to w_t)

$\downarrow$ Insert marginal values

$$\eta\mathbf{J}(w_t)=(1-\eta)\left(\mathbf{W}(w_t)-\mathbf{U}(w_t)\right)$$

Firm's surplus *J* a constant fraction of household's surplus *W – U*

NOTE: NOT a general property of Nash bargaining; here due to the linearity of *W, U,* and *J* with respect to wage

$\downarrow$ Using definitions of *W, U,* and *J,* the job-creation condition, and some algebra

$$\boxed{w_t=\eta\underbrace{\left[z_t+\gamma\theta_t\right]}+(1-\eta)b}$$

Contemporaneous marginal output…

…and a term that captures the social savings on future posting costs if match continues

Bargained wage a convex combination of gains from consummating the match and the gains from walking away from the match

NOTE: With CRS matching function,

$\theta = k^h(\theta)\,/\,k^f(\theta)$

LABOR MARKET MATCHING

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

***u_t = 1 – n_t* is measure of individuals searching for work**

Labor Market Matching

- **Aggregate matching function displays CRS**

$$m(u_t, v_t)$$

***$u_t = 1 - n_t$* is measure of individuals searching for work**

- **For any given individual vacancy or individual (partial equilibrium), matching probabilities depend only on *v / u***

NOTE: With CRS matching function,

$\theta = k^h(\theta) / k^f(\theta)$

In matching models, θ is the key driving force of efficiency and therefore optimal policy prescriptions (Hosios 1990 *ReStud* the key reference)

$$\frac{m(u_t, v_t)}{v_t} = m\left(\frac{u_t}{v_t}, 1\right) = m\left(\theta_t^{-1}, 1\right) \equiv k^f(\theta_t)$$

Probability a given vacancy / job posting attracts a worker

$$\frac{m(u_t, v_t)}{u_t} = m\left(1, \frac{v_t}{u_t}\right) = m\left(1, \theta_t\right) \equiv k^h(\theta_t)$$

Probability a given individual finds a job opening

$$\theta_t \equiv \frac{v_t}{u_t}$$

Market tightness: measures relative number of traders on opposite sides of market

- **Market tightness an allocational signal**
  - **Because matching probabilities depend on it**
  - **e.g., the higher (lower) is *v / u*, the easier (harder) it is for a given individual to find a job opening**

LABOR-MARKET EQUILIBRIUM

- **Aggregate law of motion of employment**

$$N_{t+1} = (1-\rho^x)N_t + m(u_t, v_t)$$

- **Flow equilibrium conditions (an accounting identity…)**

$$m(u_t, v_t) = u_t k^h(\theta_t) = v_t k^f(\theta_t)$$

- **Vacancy-posting (aka job-creation) condition**

$$\gamma = k^f(\theta_t) E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **Wage determination (Nash bargaining)**

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Basic labor-theory literature: impose ss on these and analyze, do comparative statics, etc. (exogenous real interest rate)**
  - **Pissarides Chapter 1, RSW 2005 *JEL***

General Equilibrium

- Aggregate law of motion for employment
- Vacancy-posting (aka job-creation) condition
- Wage determination

The labor market equilibrium (*partial* equilibrium from the perspective of the entire environment)

- Consumption-savings optimality condition (endogenizes real interest rate)

$$1 = R_t E_t \left\{ \frac{\beta u'(c_{t+1})}{u'(c_t)} \right\}$$

- Aggregate resource constraint

Often interpreted as the output of a home production sector – only the unemployed produce in the home sector

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$

Vacancy posting costs and "outside option" are real uses of resources

- Exogenous LOMs for any driving processes (TFP, etc)

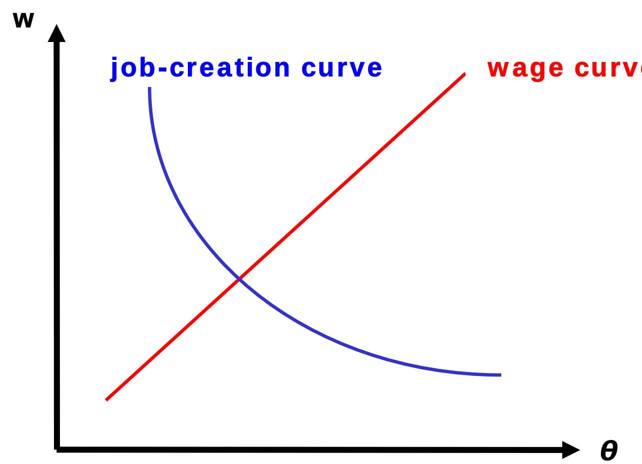
Steady State of Labor Market

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1) $$1-u=(1-\rho^{x})(1-u)+m(u,v)$$ (using $N = 1 - u$)

(2) $$\gamma=\beta k^{f}(\theta)\left(z-w+\frac{(1-\rho^{x})\gamma}{k^{f}(\theta)}\right)$$ w negatively and nonlinearly related to θ (given CRS matching function)

(3) $$w=\eta\left[z+\gamma\theta\right]+(1-\eta)b$$ w positively and linearly related to θ



Pissarides Figure 1.1

"Labor supply curve" and "labor demand curve" replaced by "wage curve" and "job-creation curve"

Steady State of Labor Market

- **Imposing deterministic steady state on labor-market equilibrium conditions**

(1)

$$u = \frac{\rho^x + m(u,v)}{\rho^x}$$

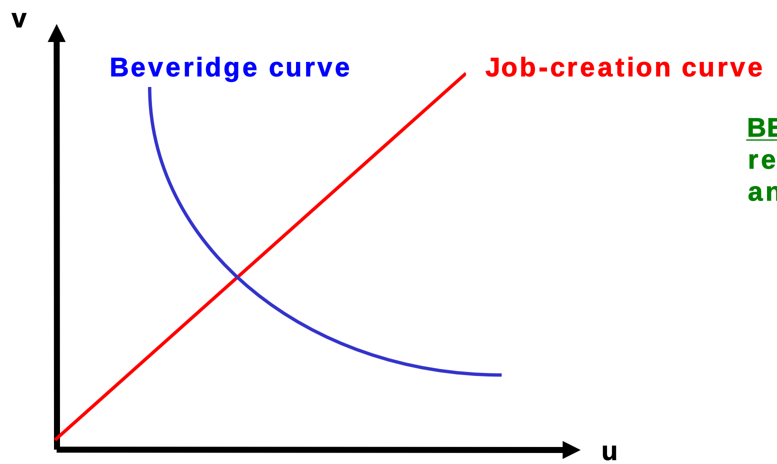
For a given (w,θ), v and u negatively related (given CRS matching function)

(2)

$$\gamma = \beta k^f\left(\frac{v}{u}\right)\left(z - w + \frac{(1-\rho^x)\gamma}{k^f\left(\frac{v}{u}\right)}\right)$$

For a given (w,θ), v and u positively related (given CRS matching function)

Pissarides Figure 1.2



BEVERIDGE CURVE: Empirical relationship in both long run and short run (i.e., cyclical)

Steady State of Labor Market

- **Labor-market equilibrium is (w, u, θ) satisfying (1), (2), (3)**

- **Comparative statics**
  - **A rise in b…**
    - **…raises w**
    - **…lowers θ**
    - **…lowers v and raises u**

    Higher value (ue benefit) of unemployment requires a higher wage to induce individuals to work, which reduces firm incentives to create jobs

  - **A fall in β (or a rise in ρ^x)…**
    - **…lowers w**
    - **…lowers θ**
    - **…raises u**
    - **…ambiguous effect on v**

    Higher real rate and/or faster job separations (i.e., "faster depreciation of employment stock") makes posting vacancies (FOR FIXED u) less attractive for firms (both erode firm profits)

- **See Pissarides Chapter 1 and RSW (2005 *JEL*) for more**

- **Next: dynamic stochastic partial equilibrium (Shimer 2005 *AER*, Hall 2005 *AER*, Hagedorn and Manovskii 2008 *AER*)**