Labor Matching Models: Efficiency Properties

October 17, 2013

Labor-Matching Efficiency

- **Social Planning problem**
  - **Social Planner also subject to matching "technology"**

$$\max_{c_t, v_t, N_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$

Fix $h = 1$

$$N_{t+1} = (1 - \rho^x) N_t + m(u_t, v_t)$$

And $N = 1 - u$

LABOR-MATCHING EFFICIENCY

- **Social Planning problem**
  - **Social Planner also subject to matching "technology"**

$$\max_{c_t, v_t, N_{t+1}} \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \right]$$

Multipliers

$$c_t + g_t + \gamma v_t = z_t N_t h_t + (1 - N_t) b$$ **Fix $h = 1$** — λ_t

$$N_{t+1} = (1 - \rho^x) N_t + m(1 - N_t, v_t)$$ **And $N = 1 - u$** — μ_t

- **FOCs**

$$u'(c_t) - \lambda_t = 0$$

$$-\lambda_t \gamma + \mu_t m_2 (1 - N_t, v_t) = 0$$

$$-\mu_t + \beta E_t \left\{ \lambda_{t+1} \left[z_{t+1} - b \right] \right\} + \beta E_t \left\{ \mu_{t+1} \left[(1 - \rho^x) - m_1 (1 - N_t, v_t) \right] \right\} = 0$$

↓ **Eliminate multipliers**

LABOR-MATCHING EFFICIENCY

- **Social Planning problem**

$$\frac{\gamma}{m_2(1-N_t,v_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - b - \frac{\gamma m_1(1-N_t,v_t)}{m_2(1-N_t,v_t)} + \frac{(1-\rho^x)\gamma}{m_2(1-N_t,v_t)}\right]\right\}$$

Cobb-Douglas matching

$$m(u,v) = u^{\alpha}v^{1-\alpha}$$

$$m_1(u,v) = \alpha u^{\alpha-1}v^{1-\alpha} = \alpha\theta^{1-\alpha}$$

$$m_2(u,v) = (1-\alpha)u^{\alpha}v^{-\alpha} = (1-\alpha)\theta^{-\alpha}$$

AND

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

Combine and rearrange

AND $m_1(u,v) = \alpha k^h(\theta)$ $\quad m_2(u,v) = (1-\alpha)k^f(\theta)$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

KEY IDEAS

Taking the pricing kernel as given, the only unknown process here is θ_t!

Efficiency in job-postings is governed by "getting market tightness right!"

LABOR-MATCHING EFFICIENCY

- Socially-efficient vacancy posting described by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left[z_{t+1} - \left(\alpha\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\alpha)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right]\right\}$$

- Recall decentralized vacancy posting described by

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - w_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

AND

$$w_t = \eta\left[z_t + \gamma\theta_t\right] + (1-\eta)b$$

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left\{\frac{\beta u'(c_{t+1})}{u'(c_t)}\left(z_{t+1} - \left(\eta\left[z_{t+1} + \gamma\theta_{t+1}\right] + (1-\eta)b\right) + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})}\right)\right\}$$

- **Efficiency in vacancy posting requires $\eta = \alpha$!**

MORTENSEN-HOSIOS CONDITION

- Cobb-Douglas matching technology + Nash bargaining
  - Efficient level of job-creation requires $\eta = \alpha$
  - Mortensen (1982 *AER*), Hosios (1990 *ReStud*)

- Intuition: search activity generates externalities
  - One extra individual (firm) searching for a job (worker) lowers the probability that *all other* individuals (firms) will find a match…
  - …but raises the probability that *all other* firms (individuals) will find a match
  - Congestion externality – search imposes both positive and negative externalities (on opposite sides of the market)

- Nash bargaining: η governs the private returns to search
  - Share of total match surplus kept by individual
- Cobb-Douglas matching: α governs the social returns to search
  - Elasticity of aggregate number of matches with respect to u

- Efficiency requires equating private and social returns: $\eta = \alpha$

HOSIOS CONDITION

- Also holds under some more general conditions
  - Endogenous search intensity
  - Endogenous "vacancy posting intensity" (Pissarides Chapter 5)

- Pissarides (2000, p. 198): "..we are not likely to find intuition for it…"

- RSW (2005 *JEL* p. 982): "…genuinely surprising result…"

- Is the Hosios condition empirically relevant?
  - Who knows?…it's a nongeneric parameterization…
  - …but valuable because eliminates wage-determination frictions but retains matching frictions

- Hosios efficiency emerges endogenously in competitive search equilibrium (CSE) concept
  - Moen (1997 *JPE*): basic static partial labor search model
  - A well-understood concept in labor theory, but little incorporation into DSGE models

Labor Matching Models: Competitive Search Equilibrium

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Competitive Search Equilibrium (CSE)

- **Question: can a "competitive" notion of wage-setting be entertained in a search and matching model?**
  - **Would get away from the non-genericity of the Hosios bargaining parameterization**
  - **May be apriori an appealing way of describing labor markets**
    - **Locating a firm or a worker is costly and time-consuming…**
    - **…but once matched, wages are more or less determined by "market forces," perhaps with little / no room for "bargaining"**

- **Moen (1997 *JPE*) and Shimer (1996) the original implementations of CSE**
  - **Static partial equilibrium labor matching models**

- **Will implement in the context of DSGE labor matching model**
  - **Only recently have started to become incorporated into DSGE matching models….**
  - **…but goods matching models, not labor matching (Arseneau and Chugh (2007)), Gourio and Rudanko (2009) (Menzio and Shi (2010 *JET*) a labor matching application)**

CSE – Basics of Environment

- Need "many markets" and "many firms"
  - To rationalize "competition," so can operationalize decentralized wage-formation process
- Index continuum of labor "submarkets" by *j* – e.g., local labor markets
- Within a submarket *j*, many firms looking to hire workers
  - Even within a "local" labor market, coordination frictions in finding workers may exist
  - Index by *i*
- Unemployed individuals ***direct*** their job search ("send an application") to a particular submarket
  - Based on wages announced by firms in that submarket, and on likelihood of getting a job in that submarket
  - Not random search – directed search is key for concept of CSE
  - Once search is directed, random matching process governs whether an individual gets a job – match formation is still subject to frictions
- Wages determined before search, not after search
  - All parties direct search according to "posted" wages

CSE – Basics of Environment

- **Wages determined before search, not after search**
  - All parties direct search according to "posted" wages

- **Several equivalent ways to implement**
  - Perfectly-competitive "market-maker" sector
  - Individuals announce wages before firms search for workers
  - Firms announce wages before individuals search for jobs
    - The implementation we will pursue
  - See RSW 2005 *JEL* survey for alternative implementations

- **Idea of firm wage-posting / wage-announcement implementation**
  - Define (expected) payoff function to firm *ij* of finding an additional worker
  - Define (expected) payoff function to individual searching for / applying to a job at firm *ij*
  - Firm *ij* maximizes its payoff subject to the reaction function defined by the individual's payoff function
    - i.e., firm *internalizes* the effect of wages on the other side of the market…
    - …can already see how congestion *externality* issues will be taken care of…

- **Internalizing congestion externalities would also be achieved by…**
  - Individuals announcing wages taking into account reactions by firms
  - "Market maker" calling out wages taking account reactions by both sides of market

CSE – Implementation

- Firm *ij* payoff function described by vacancy-posting decision!

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{\Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})}\right)\right\}\right]$$

Cost of posting a vacancy

Expected benefit of posting a vacancy

= (probability of matching with a worker) x (contemporaneous payoff + continuation payoff)

Note *ij* subscripts:

Matching probability depends on tightness of "applications" at firm *ij*...

...but future asset value of employee depends on market *j* conditions (i.e., replacement value depends on (sub-)market conditions)

- Value equations for an individual searching for a match at firm *ij*

$$W(w_{ijt}) = w_{ijt} + E_t\left\{\Xi_{t+1|t}\left[(1-\rho^x)W(w_{ijt+1}) + \rho^x U_{t+1}\right]\right\}$$

With probability $k^h(\theta_{ijt})$, individual gets this payoff

$$U_t = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)W(w_{t+1}) + (1-k^h(\theta_t))U_{t+1}\right]\right\}$$

With probability $1-k^h(\theta_{ijt})$, individual gets this payoff

- With individuals (households) optimally directing their search, the expected payoff of searching for / applying to a job at firm *ij* is

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

Payoff of searching at another firm or another submarket independent of *ij*

CSE – IMPLEMENTATION

- **Firm *ij* maximizes**

$$\gamma = k^f(\theta_{ijt})\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]$$

taking as constraint

$$k^h(\theta_{ijt})W(w_{ijt}) + (1-k^h(\theta_{ijt}))U_t = X$$

- **Choice variables: w_{ijt} and θ_{ijt} (isomorphic to choosing v_{ijt} for a given number of searchers u_{ijt})**

- **First-order conditions**

1)
$$-k^f(\theta_{ijt}) - \varphi_{ijt}k^h(\theta_{ijt})W'(w_{ijt}) = 0$$

2)
$$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}}\left[z_t - w_{ijt} + (1-\rho^x)E_t\left\{ \Xi_{t+1|t}\left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right] - \varphi_{ijt}\frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}}\left[W(w_{ijt}) - U_t \right] = 0$$

Taking into account how matching probabilities are affected by tightness is the central idea

CSE – IMPLEMENTATION

- First-order conditions

1) $$-k^f(\theta_{ijt}) - \varphi_{ijt} k^h(\theta_{ijt}) W'(w_{ijt}) = 0 \xrightarrow{W'(.)=1} \varphi_{ijt} = -\frac{k^f(\theta_{ijt})}{k^h(\theta_{ijt})}$$

2) $$\frac{\partial k^f(\theta_{ijt})}{\theta_{ijt}} \underbrace{\left[z_t - w_{ijt} + (1-\rho^x) E_t \left\{ \Xi_{t+1|t} \left(\frac{\gamma}{k^f(\theta_{jt+1})} \right) \right\} \right]}_{= J_{ijt} \text{ if firms are optimizing}} - \varphi_{ijt} \frac{\partial k^h(\theta_{ijt})}{\theta_{ijt}} \left[W(w_{ijt}) - U_t \right] = 0$$

Cobb-Douglas matching

$$m(u,v) = u^\alpha v^{1-\alpha}$$

$$k^h(\theta) = \frac{m(u,v)}{u} = m(1,\theta) = \theta^{1-\alpha}$$

$$k^f(\theta) = \frac{m(u,v)}{v} = m(\theta^{-1},1) = \theta^{-\alpha}$$

AND

$$\frac{\partial k^h(\theta)}{\partial \theta} = (1-\alpha)\theta^{-\alpha}$$

$$\frac{\partial k^f(\theta)}{\partial \theta} = -\alpha\theta^{-\alpha-1}$$

Combine and rearrange

(Competition within submarket *j* and symmetry across submarkets: drop *ij* indices)

$$(1-\alpha)\big(W(w_t) - U_t\big) = \alpha J(w_t)$$

Exactly the Nash-bargaining sharing rule with endogenous emergence of Hosios condition ($\eta = \alpha$)!!!

Inserting value equations and solving explicitly for wage obviously gives same outcome as Nash-bargained wage with $\eta = \alpha$…

CSE – INTERPRETATIONS

- Mortensen and Pissarides (1999 *Handbook Chapter* p. 2589-2592)
  - "Price of time" priced efficiently by markets in CSE
  - "Price of time" generically mispriced in bargaining equilibrium
  - ("Price of time" = matching probabilities, which reflect congestion externalities)
  - Bargaining equilibrium features a particular type of market incompleteness: workers and firms cannot contract on efficient surplus sharing before meeting
  - CSE effectively fills in this missing market…
  - …provided we're willing to assume/believe the strong degree of commitment built into CSE model
    - (i.e., each side of a job-match would have an incentive to try to "renegotiate" the "posted" wage once they actually meet)
    - An open question in search theory
- CSE in principle an alternative equilibrium concept in search models
  - But turns out to be equivalent to bargaining equilibrium with Hosios condition
  - (At least in simple environments….will equivalence hold in richer environments?…)
- Little explored in DSGE contexts
  - Question: Would some types of market frictions, tax issues, etc break the equivalence between CSE and Nash-Hosios bargaining?…

Relevance of Hosios Condition in DSGE

- Optimal policy (monetary and/or fiscal) will depend on whether or not $\eta = \alpha$
  - Yet another distortion (if $\eta = \alpha$ not satisfied) for policy to respond to
  - Deviation from Friedman Rule can be used to correct search externalities (Cooley and Quadrini (2004 *JET*), Arseneau and Chugh (2008 *JME)*, Arseneau, Chahrour, Chugh, and Finkelstein-Shapiro (*JMCB* revision in progress)), Faia (2008 *JEDC*))

- Model dynamics can depend (noticeably) on whether or not $\eta = \alpha$
  - Positive analysis: Walsh (2005 *RED*) the first to demonstrate this, many others since
  - Optimal policy analysis: Arseneau and Chugh (2012 *JPE*)

- Hosios issues arise in any DGE model with any type of search market
  - Monetary search models
    - Rocheteau and Wright (2005 *Econometrica*)
    - Aruoba and Chugh (2010 *JET*)
  - Product search models (Hall (2007), Arseneau and Chugh (2007))

DSGE (Labor) Search Models

- Search models articulate trading frictions – cannot instantaneously/costlessly find trading partners
  - An appealing description of labor markets
  - Maybe of other markets also
- Tractable to incorporate in DSGE models because of assumption of aggregate matching function
- Too ad-hoc or "reduced-form" because of assumption of (black box) aggregate matching friction?
- The Shimer Puzzle and attempted answers continue(?)…
- …as do New Keynesian modelers' incorporation of labor matching structure
  - Perhaps enables talking meaningfully about the tradeoffs between inflation and unemployment…
  - …i.e., seemingly resuscitates the original Phillips Curve, not the NK Phillips Curve (which links inflation to marginal costs…)