LABOR SEARCH MODELS: GENERAL-EQUILIBRIUM DYNAMICS

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FULL BUSINESS CYCLE MODEL: SOME ISSUES

- Embed labor-search framework in standard RBC model
- Assume perfect capital markets
  - (Optimal) capital purchased by firm instantaneously on spot market after knowing how many workers it has found
  - Standard condition emerges: $r_t = MPK_t$
- Full consumption insurance
  - Achieved by assumption of "large household"
    - All family members (employed and unemployed) enjoy same c_t
- But what about utility from leisure/work?
  - Ex-post, the unemployed are better off! – just as in Rogerson (1988) and Hansen (1985)
  - Doesn't this miss the main "cost" of unemployment and recessions?...

Krusell et al (2010) and Nakajima (2012) try relaxing this

- Andolfatto (1996) shows formal insurance market – equivalent to Hansen/Rogerson "lotteries"

Model Details

- Household-level (not individual-level) utility from leisure

- Solves Social Planner problem
  - Can be decentralized with the Hosios Condition (worker Nash bargaining power = elasticity of workers in matching function) in place
    - Hosios Condition critical for efficiency in search markets

- Household search "effort" *e*
  - Higher *e* → higher probability a searching individual locates a match
  - But fixed search effort, so doesn't do much – just calibration
  - Can endogenize – e.g., Krause and Lubik (2007)

- Endogenous intensive margin (average hours per employee)
  - Determined (implicitly) through Nash bargaining
    - Nash bargaining simultaneously over w_t and h_t yields privately-efficient outcome for h_t (see Pissarides p. 175-178)
  - Other mechanisms: allow household or firm to unilaterally choose h_t

INTENSIVE MARGIN

- **Dynamic firm profit-maximization problem**

$$\max_{v_t, n_{t+1}^f} \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f f(h_t) - w_t n_t^f h_t - \gamma v_t \right) \right]$$
$$\text{s.t. } n_{t+1}^f = (1-\rho^x) n_t^f + v_t k^f(\theta_t)$$

- **Total output produced by all employees = *znf(h)***

- **Vacancy posting condition**

$$\frac{\gamma}{k^f(\theta_t)} = E_t \left\{ \Xi_{t+1|t} \left(z_{t+1} f(h_{t+1}) - w_{t+1} h_{t+1} + \frac{(1-\rho^x)\gamma}{k^f(\theta_{t+1})} \right) \right\}$$

- **How is *h* determined?**

Intensive Margin

- How is *h* determined?
- Two common setups
  - Firm unilaterally chooses *h* for each worker ("right to manage")
  - Simultaneous Nash bargaining over *w* and *h*

$$\max_{w_t, h_t} \left(\mathbf{W}_t - \mathbf{U}_t \right)^{\eta} \mathbf{J}_t^{1-\eta}$$

INTENSIVE MARGIN

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HH level utility function now includes "effort" disutility (aka disutility of *h*)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - n_t e(h_t) \right]$$

- Value equations

$$\mathbf{W}_t = w_t h_t - \frac{e(h_t)}{u'(c_t)} + E_t \left\{ \Xi_{t+1|t} \left[(1-\rho^x) \mathbf{W}_{t+1} + \rho^x \mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{U}_t = b + E_t \left\{ \Xi_{t+1|t} \left[k^h(\theta_t) \mathbf{W}_{t+1} + (1 - k^h(\theta_t)) \mathbf{U}_{t+1} \right] \right\}$$

$$\mathbf{J}_t = z_t f(h_t) - w_t h_t + E_t \left\{ \Xi_{t+1|t} (1-\rho^x) \mathbf{J}_{t+1} \right\}$$

INTENSIVE MARGIN

- **Compute FOCs wrt *w* and *h***
- **FOC wrt *w* yields**

$$w_t h_t = \eta\left[z_t f(h_t) + \gamma\theta_t\right] + (1-\eta)b$$

Identical algebra to the *h* = 1 case

- **FOC wrt *h* yields**

$$\eta \mathbf{J}_t \left(\frac{\partial \mathbf{W}_t}{\partial h_t} - \frac{\partial \mathbf{U}_t}{\partial h_t} \right) = (1-\eta)(-1)\left(\mathbf{W}_t - \mathbf{U}_t\right)\frac{\partial \mathbf{J}_t}{\partial h_t}$$

(VERIFY THE DERIVATION)

Insert marginal values and rearrange
(a key observation is that....)

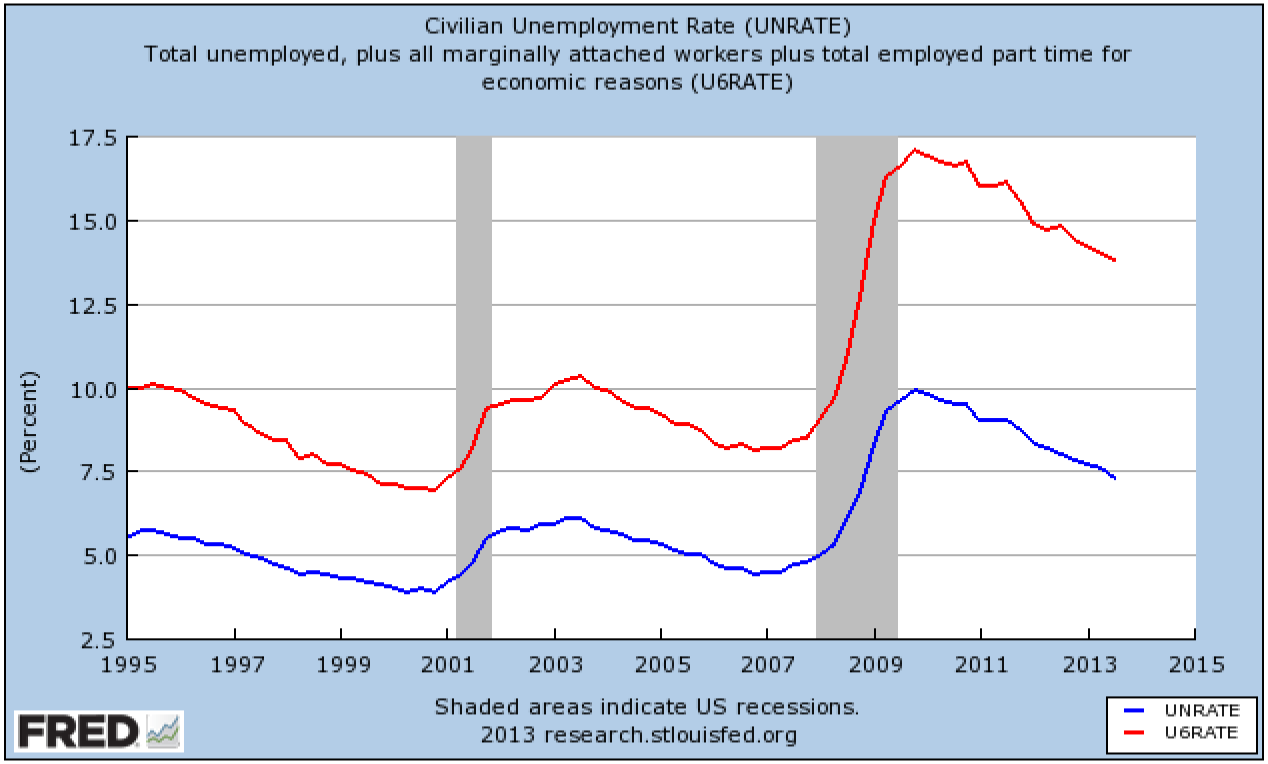
$$\frac{e'(h_t)}{u'(c_t)} = z_t f'(h_t)$$

PRIVATE BILATERAL EFFICIENCY

- **Interpretation: $mrs_t = mpn_t$ for each given worker**
  - **Private bilateral efficiency on the hours margin**
  - **Whether or not Hosios efficiency holds on extensive margin**

NATURE OF "UNEMPLOYMENT?"

- "Search unemployment"
- "Rest unemployment"
- Part-time employment



NATURE OF "UNEMPLOYMENT?"

- ❒ **"Search unemployment"**
- ❒ **"Rest unemployment"**
- ❒ **Part-time employment**

Marginally attached workers (Current Population Survey)

Persons not in the labor force who want and are available for work, and who have looked for a job sometime in the prior 12 months (or since the end of their last job if they held one within the past 12 months), but were not counted as unemployed because they had not searched for work in the 4 weeks preceding the survey. Discouraged workers are a subset of the marginally attached. (See Discouraged workers.)

Discouraged workers (Current Population Survey)

Persons not in the labor force who want and are available for a job and who have looked for work sometime in the past 12 months (or since the end of their last job if they held one within the past 12 months), but who are not currently looking because they believe there are no jobs available or there are none for which they would qualify.

NATURE OF "LABOR?"

- Extensive vs. intensive?
- What if (costly) vacancies were exogenous….
  - i.e., $v_t = \bar{v}$ is fixed
- …but intensive margin is operative

- Free entry condition (aka job-creation condition) into matching market does not hold
  - i.e., $\mathrm{V}_t \neq 0$

- Implications (by construction…)
  - Hosios parameterization (ex-post wage setting + directed search) does NOT deliver efficiency along the extensive margin
  - CSE (wage-posting + directed search) does NOT deliver efficiency along the extensive margin

- How to determine *w* and *h*?
- Introduce new equilibrium concept: competitive equilibrium

Competitive Equilibrium

- Equilibrium concepts
  - Search Equilibrium (undirected search + wage bargaining)
    - DMP model
  - Competitive Search Equilibrium (directed search + wage posting)
    - Moen (1997)
  - Competitive Equilibrium (directed search + spot-market price taking)
    - Analogous to Lucas and Prescott (1974 *JET*) "islands" (aka "sub-markets") model

- Wage w adjusts competitively in each island / sub-market to equate aggregate hours demanded and aggregate hours supplied
- Intuitively

$$\bar{n}h^D = \bar{n}h^S \Rightarrow \bar{n}h^D = \int_0^{\bar{n}} h_i^D di$$

- Each island's wage is determined competitively
- But allocation of vacancies / searchers across islands is arbitrary, thus generically inefficient

Model Results

- **TFP shocks – standard business cycle statistics**

Extensive margin fluctuates more than intensive margin

Table 1—Cyclical Properties: U.S. Economy and Model Economies

	U.S. economy $\sigma(y) = 1.58$			RBC economy $\sigma(y) = 1.22$			Search economy $\sigma(y) = 1.45$		
Variable (x)	(1)	(2)	(3)	(1)	(2)	(3)	(1)	(2)	(3)
Consumption	0.56	0.74	0	0.34	0.90	0	0.32	0.91	0
Investment	3.14	0.90	0	3.05	0.99	0	2.98	0.99	0
Total hours	0.93	0.78	+1	0.36	0.98	0	0.59	0.96	0
Employment	0.67	0.73	+1	0.00	0.00	0	0.51	0.82	+1
Hours/worker	0.34	0.66	0	0.36	0.98	0	0.22	0.66	0
Wage bill	0.97	0.76	+1	1.00	1.00	0	0.94	1.00	0
Labor's share	0.68	−0.38	−3	0.00	0.00	0	0.10	−0.62	−1
Productivity	0.64	0.43	−2	0.64	0.99	0	0.46	0.94	0
Real wage	0.44	0.04	−4	0.64	0.99	0	0.39	0.95	0

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between x and y. Column (3) is the phase shift in x relative to y: $-j$ or $+j$ corresponds to a lead or lag of j quarters.

Productivity fluctuates more than real wage

Consumption and investment dynamics little altered compared to basic RBC model

MODEL RESULTS

- **TFP shocks – cyclical labor-market statistics**

Empirical Beveridge Curve

TABLE 3—CROSS CORRELATIONS OF UNEMPLOYMENT WITH UNEMPLOYMENT AND VACANCIES

Variable (x)	$x(t-4)$	$x(t-3)$	$x(t-2)$	$x(t-1)$	$x(t)$	$x(t+1)$	$x(t+2)$	$x(t+3)$	$x(t+4)$
U.S. economy:									
Unemployment	0.23	0.46	0.69	0.89	1.00	0.89	0.69	0.46	0.23
Vacancies	−0.39	−0.62	−0.82	−0.92	−0.89	−0.72	−0.47	−0.21	0.02
Search economy:									
Unemployment	0.20	0.41	0.65	0.87	1.00	0.87	0.65	0.41	0.20
Vacancies	−0.51	−0.65	−0.73	−0.65	−0.19	0.05	0.17	0.24	0.27

Qualitatively reproduced by model

- **Vacancies not nearly as volatile as in data (p. 124)**
  - **General equilibrium effects do little to address the *partial-equilibrium* dynamic shortcoming of labor search model – i.e., Shimer Puzzle survives in a (simple) DSGE model**

Allow to be stochastic

- **Also allows a "matching efficiency shock"** $m(u_t, v_t) = \chi_t u_t^{\alpha} v_t^{1-\alpha}$
  - **Can interpret as a type of "technology shock"…but doesn't do much…**

NATURE OF SEPARATIONS?

- Is endogenous separation an important amplification mechanism for business cycles?
  - Andolfatto (1996 *AER*), Merz (1995 *JME*): exogenous separations, ala Pissarides (1985)
  - den Haan, Ramey, Watson (2000 *AER*): endogenous separations, ala Mortensen and Pissarides (1994)

- Mortensen and Pissarides (1994)
  - Aggregate TFP affects the cutoff threshold for endogenous job destruction
  - i.e., $\tilde{a}'(z_t) < 0$
    - $\tilde{a}_t$ threshold level of idiosyncratic (match-specific) productivity below which that particular match is terminated

- den Haan, Ramey, Watson conjecture
  - Negative aggregate z_t shock → lowers k_t in current *and* future periods (standard RBC mechanism)
  - Because jobs are forward-looking in nature, lower future path of k_t makes it more attractive to destroy a job in t – i.e., additional magnification through endogenous job destruction

MODEL DETAILS

- **Each match *i* produces using capital, aggregate TFP, and idiosyncratic productivity**

$$y_{it} = z_t a_{it} k_{it}^{\alpha}$$

- a_{it} **drawn from *iid* lognormal distribution with pdf *f*(.) and cdf *F*(.)**

- **Baseline model: all decisions (including capital rental decisions) made after both aggregate and idiosyncratic productivity observed**

- **Bargaining-relevant value equations affected by** a_{it}

$$W(w_{it}) = w_{it} + PDV$$

$$U(w_{it}) = b + PDV$$

$$J(w_{it}) = z_t a_{it} k_{it}^{\alpha} - w_{it} - r_t k_{it} + PDV$$

- **And destruction probability** ρ_{it} **now endogenous**

- **Overall destruction probability:** $\rho_{it} = \rho^x + (1-\rho^x)\rho_{it}^n$

Model Details

- Match *i* is destroyed if total surplus of match (taking into account capital rental decisions made after retention decision) falls below zero
  - i.e., with k_{it} chosen optimally if match continues,

    $$W(w_{it}) - U(w_{it}) + J(w_{it}) = 0$$

    defines cutoff productivity $\tilde{a}_{it}$
  - Destroy match if a_{it} below threshold, retain if a_{it} above threshold
  - Efficient job destruction

- Threshold determined by

$$\max_{k_{it}} \left[z_t a_{it} k_{it}^{\alpha} - r_t k_{it} \right] + PDV = b$$

- Endogenous job-destruction not present in Andolfatto (1996) and Merz (1995)

- Key observation: aggregate state z_t affects cutoff rule for a given match → potential interaction between aggregate shocks and idiosyncratic shocks
  - Both directly…
  - ..and potentially indirectly through optimal k_{it} choices (the main dRW hypothesis)

MODEL DETAILS

- **Matching function**

$$m(u_t, v_t) = \frac{u_t v_t}{\left[u_t^{\kappa} + v_t^{\kappa}\right]^{1/\kappa}}$$

  - **Respects [0,1] matching probabilities**
  - **Unlike Cobb-Douglas matching function**

- **Urn-ball matching function also respects [0,1] matching probabilities (see RSW 2005 *JEL* p. 974)**

$$m(u_t, v_t) = 1 - e^{-u_t/v_t}$$

Model Details

- **Matching function**

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  - **(Urn-ball matching function also respects [0,1] matching probabilities – see RSW 2005 *JEL* p. 974)**

- **Other model details virtually the same as Andolfatto (1996) and Merz (1995)**
  - **Full consumption insurance between individuals (i.e., “large household” assumption)**
  - **No labor-force participation choice**
  - **Value *b* of outside option exogenous**
  - **But the first to solve for the decentralized equilibrium of a DSGE search model (Andolfatto and Merz solved planner problems)**

MODEL RESULTS

- Model decision rules approximated using parameterized expectations approach (Christiano and Fisher 2000 *JEDC*)

- Metrics used
  - **Impact magnification:** ratio of movement in GDP to exogenous shock to TFP in the period of the impulse
  - **Total magnification:** ratio of SD(GDP) to SD(TFP) across all time periods (obtained from simulations)
  - The difference: how quickly or slowly endogenous variables return to their steady state levels compared to speed with which TFP returns to its steady state level

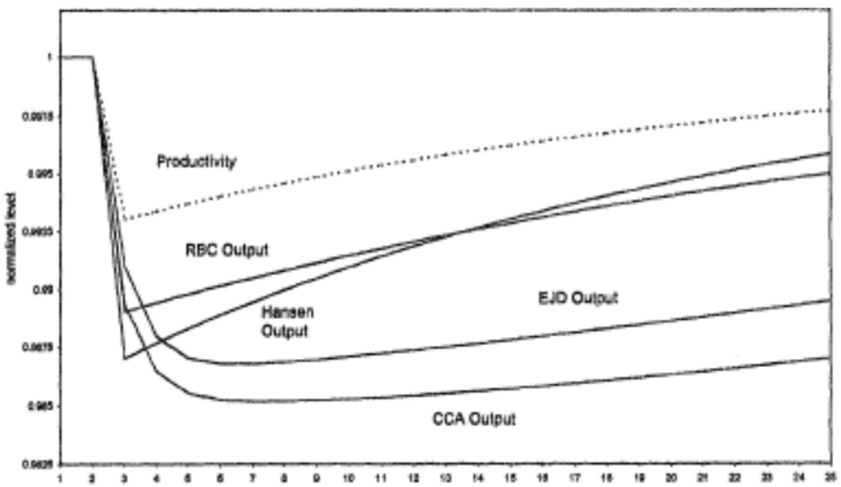
TABLE 5—IMPACT AND TOTAL MAGNIFICATION

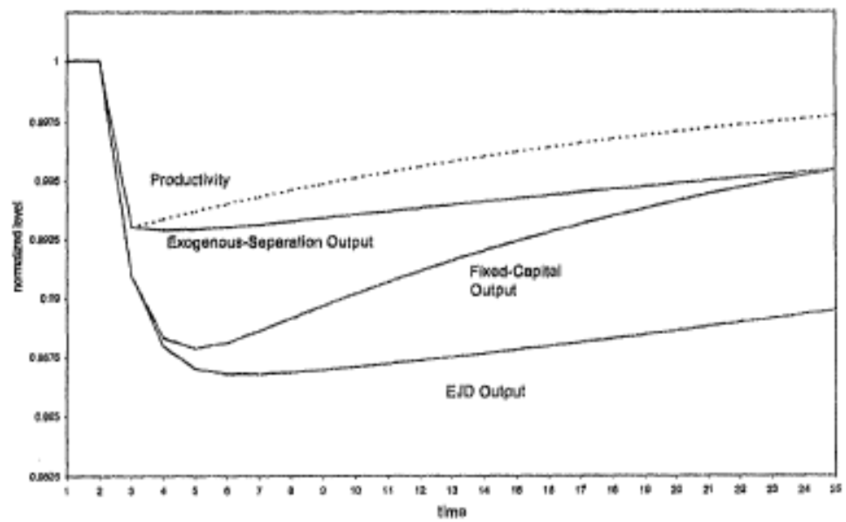
	EJD	RBC	Hansen	Fixed capital	Exogenous separation	CCA
Impact magnification	1.28	1.57	1.86	1.30	1.00	1.52
Total magnification	2.45	1.55	1.86	1.85	1.25	2.85

Baseline model

MODEL RESULTS

- **Impact magnification vs. total magnification**
  - **The difference: how quickly or slowly endogenous variables return to their steady state levels compared to speed with which TFP returns to its steady state level**





- **Baseline RBC model and Hansen-Rogerson RBC model**
  - **Output response dies out at same rate as TFP impulse → total mag = impact mag**
- **Search model with endogenous separation**
  - **Output response dies out more slowly than TFP impulse → total mag > impact mag**
- **Search model with exogenous separation**
  - **Output response dies out more slowly than TFP impulse → total mag > impact mag**
  - **But both measures of magnification smaller than with endogenous separation**

MODEL RESULTS

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Impact magnification	1.28	1.57	1.86	1.30	1.00	1.52
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- **Other robustness exercises**
  - **Fixed capital – shut down capital adjustment**
  - **"Costly capital adjustment" – capital rental decisions made *before* observation of idiosyncratic productivity**
    - **i.e., k_{it} NOT a function of a_{it}**
  - **Persistent component of idiosyncratic productivity**
    - **Total magnification: 2.43, similar to with pure iid idiosyncratic shocks**
    - **Not many details provided…**

ENDOGENOUS DESTRUCTION

- **A alternative (but equivalent) formulation to dRW implementation**
  - **Based on (but not identical to) Krause and Lubik (2007 *JME*)**

- **Representative "large firm" (if focusing on symmetric general equilibrium)**

$$\max_{v_t, n_t^f} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(y_t - \Omega_t n_t^f - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^f = (1-\rho_t)(n_{t-1}^f + v_t k^f(\theta_t))$$

Endogenous destruction fraction ρ_t.
And note timing of employment…

- **Total production depends on aggregate TFP *and conditional mean productivity of job matches that are not destroyed***

$$y_t = z_t n_t^f \int_{\tilde{a}_t}^{\infty} a \frac{f(a)}{1-F(\tilde{a}_t)} da \equiv z_t n_t^f H(\tilde{a}_t)$$

***f*(.) the pdf of idiosyncratic productivity, *F*(.) the cdf**

(could pull denominator out of integral…does not depend on index *a*)

- **Ω_t is average wage bill of firm,** $\Omega_t = \int_{\tilde{a}_t}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a}_t)} da$

ENDOGENOUS DESTRUCTION

- **Representative "large firm"**

$$\max_{v_t, n_t^f} E_0 \left[\sum_{t=0}^{\infty} \Xi_{t|0} \left(z_t n_t^f H(\tilde{a}_t) - \Omega_t n_t^f - \gamma v_t \right) \right]$$

$$\text{s.t. } n_t^f = (1-\rho_t)(n_{t-1}^f + v_t k^f(\theta_t))$$

Endogenous Destruction

- **Representative "large firm"**

$$\max_{v_t, n_t^f} E_0\left[\sum_{t=0}^{\infty}\Xi_{t|0}\left(z_t n_t^f H(\tilde{a}_t) - \Omega_t n_t^f - \gamma v_t\right)\right]$$

$$\text{s.t. } n_t^f = (1-\rho(\tilde{a}_t))(n_{t-1}^f + v_t k^f(\theta_t))$$

By construction / definition

$$\rho_t^n = F(\tilde{a}_t)\left(= \int_0^{\tilde{a}_t} af(a)da\right)$$

$$\rho_t = \rho^x + (1-\rho^x)\rho_t^n$$

- **FOCs with respect to** n_t **and** v_t **yield job-creation condition**

$$\frac{\gamma}{k^f(\theta_t)} = E_t\left[\Xi_{t+1|t}(1-\rho(\tilde{a}_{t+1}))\left(z_{t+1}H(\tilde{a}_{t+1}) - \Omega_{t+1} + \frac{\gamma}{k^f(\theta_{t+1})}\right)\right]$$

- **Vacancy-creation decision in *t* depends on expectations about future endogenous separation rate and (effective conditional) productivity**

ENDOGENOUS DESTRUCTION

- **Bargaining-relevant value equations for match with realized a_t**

$$W(a_t) = w(a_t) + E_t\left\{\Xi_{t+1|t}\left[(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\infty} W(a)\frac{f(a)}{1-F(\tilde{a}_{t+1})}da + \rho_{t+1}U(a_{t+1})\right]\right\}$$

$$U(a_t) = b + E_t\left\{\Xi_{t+1|t}\left[k^h(\theta_t)(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\infty} W(a)\frac{f(a)}{1-F(\tilde{a}_{t+1})}da + (1-k^h(\theta_t)(1-\rho_{t+1}))U(a_{t+1})\right]\right\}$$

$$J(a_t) = z_t a_t - w(a_t) + E_t\left\{\Xi_{t+1|t}(1-\rho_{t+1})\int_{\tilde{a}_{t+1}}^{\infty} J(a)\frac{f(a)}{1-F(\tilde{a}_{t+1})}da\right\}$$

Insert in usual Nash sharing rule $\eta(W(a_t)-U(a_t)) = (1-\eta)J(a_t)$

$$w(a_t) = \eta\left[z_t a_t + \gamma\theta_t\right] + (1-\eta)b$$

For an individual job with idiosyncratic productivity a_t and which is *not* destroyed…a straightforward generalization

ENDOGENOUS DESTRUCTION

- **Wage payment in individual job with productivity a_t**

$$w(a_t) = \eta\left[z_t a_t + \gamma\theta_t\right] + (1-\eta)b$$

- **Average (per-employee) wage bill of representative "large firm"**
  - **Integrate over all jobs that are not destroyed**

$$\Omega_t \equiv \int_{\tilde{a}_t}^{\infty} w(a)\frac{f(a)}{1-F(\tilde{a}_t)}da = \eta z_t \underbrace{\int_{\tilde{a}_t}^{\infty} a\frac{f(a)}{1-F(\tilde{a}_t)}da}_{\equiv H(\tilde{a}_t)} + \eta\gamma\theta_t + (1-\eta)b$$

- **Pin down threshold *a* from condition *J*(*a*) = 0**
  - **Equivalent to using *W*(*a*) – *U*(*a*) = 0**
  - **Equivalent to using vacancy-creation condition evaluated at the threshold job**

$$\tilde{a}_t = \frac{1}{z_t}\left[b + \frac{1}{1-\eta}\left(\eta\gamma\theta_t - \frac{\gamma}{k^f(\theta_t)}\right)\right] \qquad \tilde{a}'(z_t) < 0$$

- **Aggregate resource constraint** $c_t + \gamma v_t = z_t H(\tilde{a}_t) + b$