LABOR SEARCH MODELS: INTENSIVE MARGIN, ENDOGENOUS SEPARATION, AND GE DYNAMICS

APRIL 9, 2013
FULL BUSINESS CYCLE MODEL: SOME ISSUES

- Embed labor-search framework in standard RBC model

- Assume perfect capital markets
  - (Optimal) capital purchased by firm instantaneously on spot market after knowing how many workers it has found
  - Standard condition emerges: $r_t = MPK_t$

- Full consumption insurance
  - Achieved by assumption of “large household”
    - All family members (employed and unemployed) enjoy same $c_t$

- But what about utility from leisure/work?
  - Ex-post, the unemployed are better off! – just as in Rogerson (1988) and Hansen (1985)
  - Doesn’t this miss the main “cost” of unemployment and recessions?...

- Andolfatto (1996) shows formal insurance market – equivalent to Hansen/Rogerson “lotteries”
MODEL DETAILS

- Household-level (not individual-level) utility from leisure

- Solves Social Planner problem
  - Can be decentralized with the Hosios Condition (worker Nash bargaining power = elasticity of workers in matching function) in place
    - Hosios Condition critical for efficiency in search markets

- Household search “effort” $e$
  - Higher $e \rightarrow$ higher probability a searching individual locates a match
  - But fixed search effort, so doesn’t do much – just calibration
  - Can endogenize

- Endogenous intensive margin (average hours per employee)
  - Determined (implicitly) through Nash bargaining
    - Nash bargaining simultaneously over $w_t$ and $h_t$ yields privately-efficient outcome for $h_t$ (see Pissarides p. 175-178)
  - Other mechanisms: allow firm or household to unilaterally choose $h_t$
INTENSIVE MARGIN

- Dynamic firm profit-maximization problem
  \[
  \max_{v_t, n_{t+1}^f} \left[ \sum_{t=0}^{\infty} \Xi_{t|0} \left( z_t n_t^f h_t - w_t n_t^f h_t - \gamma v_t \right) \right]
  \text{ s.t. } n_{t+1}^f = (1 - \rho^x) n_t^f + v_t k^f (\theta_t)
  \]

- Total output produced by all employees = zn_h

- Vacancy posting condition
  \[
  \frac{\gamma}{k^f (\theta_t)} = E_t \left\{ \Xi_{t+1|t} \left( z_{t+1} h_{t+1} - w_{t+1} h_{t+1} + \frac{(1 - \rho^x) \gamma}{k^f (\theta_{t+1})} \right) \right\}
  \]

- How is h determined?
**INTENSIVE MARGIN**

- **How is \( h \) determined?**
- **Two common setups**
  - Firm unilaterally chooses \( h \) for each worker (“right to manage”)
  - Simultaneous Nash bargaining over \( w \) and \( h \)

\[
\max_{w_t, h_t} \left( W_t - U_t \right)^\eta J_t^{1-\eta}
\]

**Value equations**

\[
W_t = w_t h_t - \frac{e(h_t)}{u'(c_t)} + E_t \left\{ \Xi_{t+1|t} \left[ (1 - \rho^x) W_{t+1} + \rho^x U_{t+1} \right] \right\}
\]

\[
U_t = b + E_t \left\{ \Xi_{t+1|t} \left[ k^h (\theta_t) W_{t+1} + (1 - k^h (\theta_t)) U_{t+1} \right] \right\}
\]

\[
J_t = z_t h_t - w_t h_t + E_t \left\{ \Xi_{t+1|t} (1 - \rho^x) J_{t+1} \right\}
\]

**HH level utility function now includes “effort” disutility (aka disutility of \( h \)).**
INTENSIVE MARGIN

- Compute FOCs wrt $w$ and $h$
- FOC wrt $w$ yields
  \[ w_t h_t = \eta [z_t h_t + \gamma \theta_t] + (1 - \eta) b \]
- FOC wrt $h$ yields
  \[ \eta J_t \left( \frac{\partial W_t}{\partial h_t} - \frac{\partial U_t}{\partial h_t} \right) = (1 - \eta)(-1) \left( W_t - U_t \right) \frac{\partial J_t}{\partial h_t} \]
  (VERIFY THE DERIVATION)
  Insert marginal values and rearrange
  (a key observation is that....)
  \[ \frac{e'(h_t)}{u'(c_t)} = z_t \]

- Interpret more broadly: $mrs_t = mpn_t$ for each given worker
  - Private bilateral efficiency on the hours margin
  - Whether or not Hosios efficiency holds on extensive margin
MODEL RESULTS

TFP shocks – standard business cycle statistics

Table 1—Cyclical Properties: U.S. Economy and Model Economies

<table>
<thead>
<tr>
<th>Variable (x)</th>
<th>U.S. economy $\sigma(y) = 1.58$</th>
<th>RBC economy $\sigma(y) = 1.22$</th>
<th>Search economy $\sigma(y) = 1.45$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3)</td>
<td>(1) (2) (3)</td>
<td>(1) (2) (3)</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.56 0.74 0</td>
<td>0.34 0.90 0</td>
<td>0.32 0.91 0</td>
</tr>
<tr>
<td>Investment</td>
<td>3.14 0.90 0</td>
<td>3.05 0.99 0</td>
<td>2.98 0.99 0</td>
</tr>
<tr>
<td>Total hours</td>
<td>0.93 0.78 +1</td>
<td>0.36 0.98 0</td>
<td>0.59 0.96 0</td>
</tr>
<tr>
<td>Employment</td>
<td>0.67 0.73 +1</td>
<td>0.00 0.00 0</td>
<td>0.51 0.82 +1</td>
</tr>
<tr>
<td>Hours/worker</td>
<td>0.34 0.66 0</td>
<td>0.36 0.98 0</td>
<td>0.22 0.66 0</td>
</tr>
<tr>
<td>Wage bill</td>
<td>0.97 0.76 +1</td>
<td>1.00 1.00 0</td>
<td>0.94 1.00 0</td>
</tr>
<tr>
<td>Labor’s share</td>
<td>0.68 -0.38 -3</td>
<td>0.00 0.00 0</td>
<td>0.10 -0.62 -1</td>
</tr>
<tr>
<td>Productivity</td>
<td>0.64 0.43 -2</td>
<td>0.64 0.99 0</td>
<td>0.46 0.94 0</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.44 0.04 -4</td>
<td>0.64 0.99 0</td>
<td>0.39 0.95 0</td>
</tr>
</tbody>
</table>

Notes: $\sigma(y)$ is the percentage standard deviation in real per-capita output. Column (1) is $\sigma(x)/\sigma(y)$. Column (2) is the correlation between $x$ and $y$. Column (3) is the phase shift in $x$ relative to $y$: $-j$ or $+j$ corresponds to a lead or lag of $j$ quarters.

Consumption and investment dynamics little altered compared to basic RBC model.
MODEL RESULTS

TFP shocks – cyclical labor-market statistics

Table 3—Cross Correlations of Unemployment with Unemployment and Vacancies

<table>
<thead>
<tr>
<th>Variable (x)</th>
<th>$x(t-4)$</th>
<th>$x(t-3)$</th>
<th>$x(t-2)$</th>
<th>$x(t-1)$</th>
<th>$x(t)$</th>
<th>$x(t+1)$</th>
<th>$x(t+2)$</th>
<th>$x(t+3)$</th>
<th>$x(t+4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. economy:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.23</td>
<td>0.46</td>
<td>0.69</td>
<td>0.89</td>
<td>1.00</td>
<td>0.89</td>
<td>0.69</td>
<td>0.46</td>
<td>0.23</td>
</tr>
<tr>
<td>Vacancies</td>
<td>-0.39</td>
<td>-0.62</td>
<td>-0.82</td>
<td>-0.92</td>
<td>-0.89</td>
<td>-0.72</td>
<td>-0.47</td>
<td>-0.21</td>
<td>0.02</td>
</tr>
<tr>
<td>Search economy:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment</td>
<td>0.20</td>
<td>0.41</td>
<td>0.65</td>
<td>0.87</td>
<td>1.00</td>
<td>0.87</td>
<td>0.65</td>
<td>0.41</td>
<td>0.20</td>
</tr>
<tr>
<td>Vacancies</td>
<td>-0.51</td>
<td>-0.65</td>
<td>-0.73</td>
<td>-0.65</td>
<td>-0.19</td>
<td>0.05</td>
<td>0.17</td>
<td>0.24</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Empirical Beveridge Curve

Vacancies not nearly as volatile as in data (p. 124)

- General equilibrium effects do little to address the partial-equilibrium dynamic shortcoming of labor search model – i.e., Shimer Puzzle survives in a (simple) DSGE model

Also allows a “matching efficiency shock” $m(u_t, v_t) = \chi_t u_t^\alpha v_t^{1-\alpha}$

Can interpret as a type of “technology shock”...but doesn’t do much...
NATURE OF “LABOR?”

- Extensive vs. intensive?
- What if (costly) vacancies were exogenous....
  - i.e., $v_t = \bar{V}$ is fixed
  - ...but intensive margin is operative

- Free entry condition (aka job-creation condition) into matching market does not hold
  - i.e., $V_t \neq 0$

- Implications (by construction...)
  - Hosios parameterization (ex-post wage setting + directed search) does NOT deliver efficiency along the extensive margin
  - CSE (wage-posting + directed search) does NOT deliver efficiency along the extensive margin

- How to determine $w$ and $h$?
- Introduce new equilibrium concept: competitive equilibrium
**COMPETITIVE EQUILIBRIUM**

- Fundamentals of competitive equilibrium
  - “Entry” is exogenous – aka fixed number of market participants
  - Intensive margin is endogenous
  - All agents who are located in a (sub-)market are price takers

- Set $\rho = 1$ for simplicity / tractability
  - Matching function and vacancy posting costs exist
  - But no long-term relationships exist
  - Essentially makes the model static...can think of as steady state
COMPETITIVE EQUILIBRIUM

- Equilibrium concepts
  - Search Equilibrium (undirected search + wage bargaining)
    - DMP model
  - Competitive Search Equilibrium (directed search + wage posting)
    - Moen (1997)
  - Competitive Equilibrium (exogenous search + price taking)
    - Analogous to Lucas and Prescott (1974 JET) “islands” (aka “sub-markets”) model

- Wage \( w \) adjusts competitively in each island / sub-market to equate aggregate hours demanded and aggregate hours supplied

- Intuitively:
  \[
  \bar{n}h_D = \bar{n}h_S \Rightarrow \bar{n}h_D = \int_{0}^{\bar{n}} h_i^D di
  \]

- Each island’s wage is determined competitively
- But allocation of vacancies / searchers across islands is arbitrary, thus generically inefficient
COMPETITIVE EQUILIBRIUM

- If intensive margin is exogenous AND extensive margin is exogenous, how do these concepts compare?

- If intensive margin is exogenous BUT extensive margin is operative, how do these concepts compare?
NATURE OF SEPARATIONS?

- Is endogenous separation an important amplification mechanism for business cycles?

- Mortensen and Pissarides (1994)
  - Aggregate TFP affects the cutoff threshold for endogenous job destruction
  - i.e., $\tilde{a}'(z_t) < 0$
    - $\tilde{a}$, threshold level of idiosyncratic (match-specific) productivity below which that particular match is terminated

- den Haan, Ramey, Watson conjecture
  - Negative aggregate $z_t$ shock $\rightarrow$ lowers $k_t$ in current and future periods (standard RBC mechanism)
  - Because jobs are forward-looking in nature, lower future path of $k_t$ makes it more attractive to destroy a job in $t$ – i.e., additional magnification through endogenous job destruction
MODEL DETAILS

- Each match $i$ produces using capital, aggregate TFP, and idiosyncratic productivity
  \[ y_{it} = z_i a_{it} k_{it}^{\alpha} \]

- $a_{it}$ drawn from iid lognormal distribution with pdf $f(.)$ and cdf $F(.)$

- Baseline model: all decisions (including capital rental decisions) made after both aggregate and idiosyncratic productivity observed

- Bargaining-relevant value equations affected by $a_{it}$
  \[ W(w_{it}) = w_{it} + PDV \]
  \[ U(w_{it}) = b + PDV \]
  \[ J(w_{it}) = z_i a_{it} k_{it}^{\alpha} - w_{it} - r_i k_{it} + PDV \]

- And destruction probability $\rho_{it}$ now endogenous

- Overall destruction probability: $\rho_{it} = \rho^n x + (1 - \rho^n) \rho_{it}^n$
MODEL DETAILS

☐ Match \( i \) is destroyed if total surplus of match (taking into account capital rental decisions made after retention decision) falls below zero
  ☐ i.e., with \( k_{it} \) chosen optimally if match continues,
    \[
    W(w_{it}) - U(w_{it}) + J(w_{it}) = 0
    \]
    defines cutoff productivity \( \bar{a}_{it} \)
  ☐ Destroy match if \( a_{it} \) below threshold, retain if \( a_{it} \) above threshold
  ☐ Efficient job destruction

☐ Threshold determined by
  \[
  \max_{k_{it}} \left[ z_i a_{it} k_{it}^a - r_t k_{it} \right] + PDV = b
  \]
  Calibration of outside benefit \( b \)?
  Not well explained in dRW...

☐ Endogenous job-destruction not present in Andolfatto (1996) and Merz (1995)

☐ Key observation: aggregate state \( z_t \) affects cutoff rule for a given match → potential interaction between aggregate shocks and idiosyncratic shocks
  ☐ Both directly...
  ☐ ..and potentially indirectly through optimal \( k_{it} \) choices (the main dRW hypothesis)
MODEL DETAILS

- Matching function

\[ m(u_t, v_t) = \frac{u_t v_t}{\left[ u_t^\kappa + v_t^\kappa \right]^{1/\kappa}} \]

- Respects [0,1] matching probabilities
- Unlike Cobb-Douglas matching function
- (Urn-ball matching function also respects [0,1] matching probabilities – see RSW 2005 JEL p. 974)

- Other model details virtually the same as Andolfatto (1996) and Merz (1995)
  - Full consumption insurance between individuals (i.e., “large household” assumption)
  - No labor-force participation choice
  - Value $b$ of outside option exogenous
  - But the first to solve for the decentralized equilibrium of a DSGE search model (Andolfatto and Merz solved planner problems)
ENDOGENOUS DESTRUCTION

- A alternative (but equivalent) formulation to dRW implementation
  - Based on (but not identical to) Krause and Lubik (2007 JME)

- Representative “large firm” (if focusing on symmetric general equilibrium)

\[
\max_{v_t, n_t} E_0 \left[ \sum_{t=0}^{\infty} \Xi_{t|0} \left( y_t - \Omega_t, n_t^f - \gamma v_t \right) \right]
\]

\[
\text{s.t. } n_t^f = (1 - \rho_t)(n_{t-1}^f + \nu_t k^f(\theta_t))
\]

- Total production depends on aggregate TFP and conditional mean productivity of job matches that are not destroyed

\[
y_t = z_t n_t^f \int_{\tilde{a}_t}^{\infty} a f(a) \frac{1}{1 - F(\tilde{a}_t)} da \equiv z_t n_t^f H(\tilde{a}_t)
\]

\[
f(.) \text{ the pdf of idiosyncratic productivity, } F(.) \text{ the cdf}
\]

\[
(\text{could pull denominator out of integral...does not depend on index } a)
\]

- \( \Omega_t \) is average wage bill of firm, \( \Omega_t = \int_{\tilde{a}_t}^{\infty} w(a) f(a) \frac{1}{1 - F(\tilde{a}_t)} da \)
**ENDOGENOUS DESTRUCTION**

- **Representative “large firm”**

\[
\max_{v_t, n_t^f} E_0 \left[ \sum_{t=0}^{\infty} \Xi_{t|0} \left( z_t n_t^f H(\bar{a}_t) - \Omega_t n_t^f - \gamma v_t \right) \right] \\
\text{s.t. } n_t^f = (1 - \rho_t)(n_{t-1}^f + v_t k_t^f(\theta_t))
\]
**ENDOGENOUS DESTRUCTION**

- **Representative “large firm”**

\[
\max_{v_t, n_t^f} E_0 \left[ \sum_{t=0}^{\infty} \Xi_t \left( z_t n_t^f H(\tilde{a}_t) - \Omega_t n_t^f - \gamma v_t \right) \right]
\]

s.t. \[n_t^f = (1 - \rho(\tilde{a}_t))(n_{t-1}^f + v_t k^f(\theta_t))\]

By construction/definition

\[
\rho_t^n = F(\tilde{a}_t) \left( \frac{\tilde{a}_t}{\int_0^\infty f(a) da} \right)
\]

\[
\rho_t = \rho^x + (1 - \rho^x)\rho_t^n
\]
ENDOGENOUS DESTRUCTION

- Representative “large firm”

\[
\max_{n_t^f, v_t} E_0 \left[ \sum_{t=0}^{\infty} \Xi_{t|0} \left( z_t n_t^f H(\tilde{\alpha}_t) - \Omega_t n_t^f - \gamma v_t \right) \right]
\]

s.t. \( n_t^f = (1 - \rho(\tilde{\alpha}_t))(n_{t-1}^f + v_t k^f(\theta_t)) \)

- FOCs with respect to \( n_t \) and \( v_t \) yield job-creation condition

\[
\frac{\gamma}{k^f(\theta_t)} = E_t \left[ \Xi_{t+1|t} (1 - \rho(\tilde{\alpha}_{t+1})) \left( z_{t+1} H(\tilde{\alpha}_{t+1}) - \Omega_{t+1} + \frac{\gamma}{k^f(\theta_{t+1})} \right) \right]
\]

- Vacancy-creation decision in \( t \) depends on expectations about future endogenous separation rate and (effective conditional) productivity

By construction/definition

\[
\rho^n_t = F(\tilde{\alpha}_t) \left( \int_0^a f(a) da \right)
\]

\[
\rho_t = \rho^n_t + (1 - \rho^n) \rho^n_t
\]
ENDOGENOUS DESTRUCTION

Bargaining-relevant value equations for match with realized $a_t$

$$W(a_t) = w(a_t) + E_t \left\{ \Xi_{t+1|t} \left[ (1 - \rho_{t+1}) \int_{\tilde{a}_{t+1}}^\infty W(a) \frac{f(a)}{1 - F(\tilde{a}_{t+1})} da + \rho_{t+1} U(a_{t+1}) \right] \right\}$$

$$U(a_t) = b + E_t \left\{ \Xi_{t+1|t} \left[ k^h(\theta_t)(1 - \rho_{t+1}) \int_{\tilde{a}_{t+1}}^\infty W(a) \frac{f(a)}{1 - F(\tilde{a}_{t+1})} da + (1 - k^h(\theta_t)(1 - \rho_{t+1}))U(a_{t+1}) \right] \right\}$$

$$J(a_t) = z_t a_t - w(a_t) + E_t \left\{ \Xi_{t+1|t} (1 - \rho_{t+1}) \int_{\tilde{a}_{t+1}}^\infty J(a) \frac{f(a)}{1 - F(\tilde{a}_{t+1})} da \right\}$$

Insert in usual Nash sharing rule $\eta(W(a_t) - U(a_t)) = (1 - \eta)J(a_t)$

$$w(a_t) = \eta [z_t a_t + \gamma \theta_t] + (1 - \eta)b$$

For an individual job with idiosyncratic productivity $a_t$ and which is not destroyed...a straightforward generalization
**ENDOGENOUS DESTRUCTION**

- **Wage payment in individual job with productivity** $a_t$
  \[ w(a_t) = \eta [z_t a_t + \gamma \theta_t] + (1-\eta)b \]

- **Average (per-employee) wage bill of representative “large firm”**
  - Integrate over all jobs that are not destroyed
  \[ \Omega_t \equiv \int_{\tilde{a}_t}^{\infty} w(a) \frac{f(a)}{1-F(\tilde{a}_t)} da = \eta z_t \int_{\tilde{a}_t}^{\infty} a \frac{f(a)}{1-F(\tilde{a}_t)} da + \eta \gamma \theta_t + (1-\eta)b \equiv H(\tilde{a}_t) \]

- **Pin down threshold** $a$ **from condition** $J(a) = 0$
  - Equivalent to using $W(a) - U(a) = 0$
  - Equivalent to using vacancy-creation condition evaluated at the threshold job
  \[ \tilde{a}_t = \frac{1}{z_t} \left[ b + \frac{1}{1-\eta} \left( \eta \gamma \theta_t - \frac{\gamma}{k_f(\theta_t)} \right) \right] \quad \tilde{a}'(z_t) < 0 \]

- **Aggregate resource constraint**
  \[ c_t + \gamma v_t = z_t H(\tilde{a}_t) + b \]