BASICS OF DYNAMIC STOCHASTIC (GENERAL) EQUILIBRIUM

SEPTEMBER 5, 2013

HETEROGENEITY

- Implementing representative consumer
  - An infinity of consumers, each indexed by a point on the unit interval [0,1]
  - Each individual is identical in preferences and endowments
  - Implies aggregate consumption demand and asset demand

Aggregate consumption demand = One individual's consumption demand x 1

Aggregate savings demand = One individual's savings demand x 1

- Under some particular types of heterogeneity, a representative-consumer foundation of aggregates exists
  - Provided complete set of Arrow-Debreu securities exists…
  - …to allow individuals to diversify away (insure) their idiosyncratic risk

- Consider heterogeneity
  - In income realizations (from Markov process)
  - In initial asset holdings *a*
  - In utility functions (application to CRRA utility)
  - Example: two types of individuals to illustrate

HETEROGENEITY

- **Two types of individuals, *i* ∈ {1,2}, each with population weight 0.5**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u^i(c_t^i) \quad \textbf{subject to} \quad c_t^i + \sum_j R_t^j a_t^{ij} = y_t^i + a_{t-1}^i$$

- **Optimization between period *t* and state *j* in period *t*+1 (conditional on period *t* outcomes)**

$$\frac{u_c^1(c_t^1)}{\beta u_c^1(c_{t+1}^{1j})} = \frac{p_{t+1}^j}{R_t^j} \qquad \frac{p_{t+1}^j}{R_t^j} = \frac{u_c^2(c_t^2)}{\beta u_c^2(c_{t+1}^{2j})}$$

Given all individuals base choices on same prices and probabilities

RISK SHARING

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In all states at all dates

- **PERFECT RISK SHARING**
  - **IMRS, for each state *j*, equated across individuals**
  - **Individuals experiencing idiosyncratic shocks can insure them away (provided complete markets)**

- **Risk sharing about equalizing fluctuations of *u*' (.) across individuals**
  - **Not about equalizing levels of *u*' (.) or consumption over time**

- **If initial conditions, period-zero outcomes, and *u*(.) are identical (e.g., due to identical a_0 and realized y_0), then risk sharing → identical outcomes for all t**

→ A representative consumer

RISK SHARING

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Risk sharing across individuals ≈ consumption smoothing for a given individual

(= if initial conditions, *t*=0 outcomes, and *u*(.) identical)

RISK SHARING

- **Example: CRRA utility, but heterogenous RRA/IES**
  - $\sigma^1 \neq \sigma^2$

$$\left(\frac{c_t^1}{c_{t+1}^{1j}}\right)^{-\sigma^1} = \left(\frac{c_t^2}{c_{t+1}^{2j}}\right)^{-\sigma^2}$$

Perfect risk sharing

  - **IMRS equated across individuals**

- **Growth rates of consumption not equated unless $\sigma^1 = \sigma^2$**

$$\frac{c_{t+1}^{1j}}{c_t^1} = \left(\frac{c_{t+1}^{2j}}{c_t^2}\right)^{\sigma^2/\sigma^1}$$

AGGREGATION

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- **Allocations are Pareto-optimal (implied by First Welfare Theorem)**
  - **All MRS's (across individuals, states, and dates) are equated**
  - **Even though levels of consumption may differ across individuals**
  - **No individual can be made better off without making some agent worse off**
  - **(Pareto welfare concept takes distributions of outcomes as given)**
  - **Due to complete financial markets**

- **Pareto-optimal allocations + heterogeneity of utility functions**
  - **There exists a utility function $u(c)$ in aggregate $c = c^1 + c^2$ that leads to the same aggregates (Constantanides (1982)); if CRRA, $u(.)$ has $\sigma \in (\sigma^1, \sigma^2)$**

AGGREGATION

- Now consider **economy-wide aggregates**

	$c_t = 0.5c_t^1 + 0.5c_t^2$	**Aggregate consumption**
	$y_t = 0.5y_t^1 + 0.5y_t^2$	**Aggregate income (endowment)**
(For each type of asset)	$a_t = 0.5a_t^1 + 0.5a_t^2$	**Aggregate assets?**

AGGREGATION

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- So far have been considering assets as claims (paper!) (partial equilibrium)
- In aggregate, must be some tangible asset(s) backing them (gen. equil.)

- No physical assets in model so far → a_t = ***0 in aggregate*** for all t !

AGGREGATION

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(For each type of asset) $$0 = a_t = 0.5a_t^1 + 0.5a_t^2$$ Aggregate assets = 0 if no ***physical*** assets

- So far have been considering assets as claims (paper!) (partial equilibrium)
- In aggregate, must be some tangible asset(s) backing them (gen. equil.)
- No physical assets in model so far → a_t = ***0 in aggregate*** for all t !
- Heterogeneous individuals creating/buying/selling assets vis-à-vis each other
- Richer models
  - Mediate through "banking" or "insurance" markets, etc.
  - But only meaningful if some friction/imperfections in model of financial markets…
  - …otherwise identical outcomes (in which case "banking" sector is a "veil")

AGGREGATION

- **Economy-wide aggregates**

$$c_t = 0.5c_t^1 + 0.5c_t^2$$ **Aggregate consumption**

$$y_t = 0.5y_t^1 + 0.5y_t^2$$ **Aggregate income (endowment)**

Asset market clearing condition
(for each type of asset)

$$0 = a_t = 0.5a_t^1 + 0.5a_t^2$$

Aggregate assets = 0 if no *physical* assets

- **Aggregate savings = $a_t - a_{t-1}$ = 0 for all *t***

- **Aggregate together two types' budget constraints**

$$c_t^1 + \sum_j R_t^j a_t^{1j} = y_t^1 + a_{t-1}^1 \qquad c_t^2 + \sum_j R_t^j a_t^{2j} = y_t^2 + a_{t-1}^2$$

- **Weight by share of population**
- **Impose asset-market clearing condition(s)**

$$\Rightarrow \quad 0.5(c_t^1 + c_t^2) + \sum_j R_t^j 0.5\underbrace{(a_t^{1j} + a_t^{2j})} = 0.5(y_t^1 + y_t^2) + 0.5\underbrace{(a_{t-1}^1 + a_{t-1}^2)}$$

= 0 8 *j* **= 0**

$$\Rightarrow \quad c_t = y_t$$

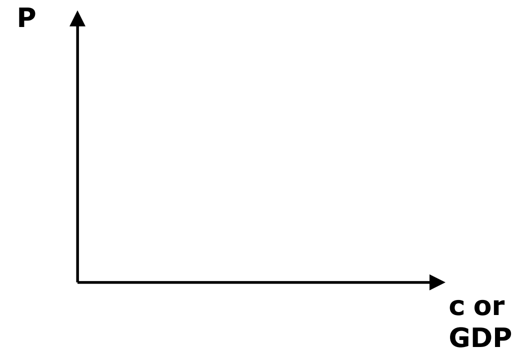
A general procedure for constructing economy-wide resource constraint

goods available = goods used

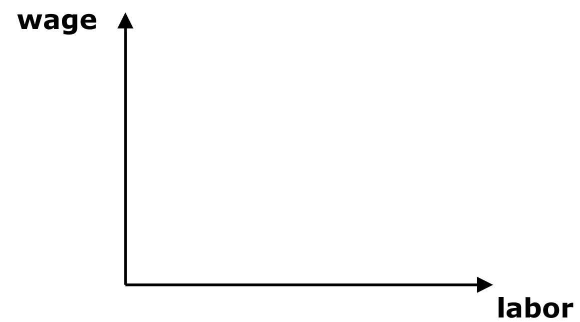
Goods market clearing condition – aka resource constraint

The Three Macro (Aggregate) Markets

- ❑ **Goods Markets**



- ❑ **Labor Markets**



- ❑ **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

- Lifecycle/permanent income consumption model the most basic building block of all macro models
- Dynamic stochastic general equilibrium (DSGE) theory
  - (DGE if deterministic)
  - GE: simultaneous determination of prices and quantities in all markets (macro markets: goods, labor, capital)

- Foundations of baseline DSGE model
  - Representative consumer
  - Representative firm
  - Perfect competition in all markets
  - Rational expectations
  - Perfect AD financial markets

THE REAL BUSINESS CYCLE MODEL

Kydland and Prescott (1982), Long and Plosser (1983), King, Plosser, and Rebelo (1988)

- All modern macro models descend from RBC model – dynamic GE
  - No matter how many market imperfections, heterogeneity, etc, etc.

- Foundations of the RBC model
  - Without optimizing consumers: Solow growth model
  - With optimizing consumers: Ramsey/Cass/Koopmans model

"The Solow growth model"

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

$$y_t + (1 + r_t) a_{t-1}$$

- **Model of non-asset income so far: endowment y_t, possibly stochastic**
- **Now suppose y_t is labor income**

$$y_t = w_t n_t$$

- **Normalize "time available" in each time period to one unit**
  - **Individual decides how to divide between "labor" and "leisure"**
    - **(Basic models: leisure is all "non-labor," but empirical and theoretical work recently studying the importance of finer categorizations of "non-labor time" for macro issues – e.g., search and matching theory)**
  - **Labor = n_t ←→ leisure is l_t = 1-n_t**
  - **Time is now the *endowment!***

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  - **Labor = n_t ←→ leisure is $l_t = 1-n_t$**
  - **Time is now the *endowment!***
- **Assert that individuals care about leisure,** $u(c_t, \ell_t)$
  - $u_{ct} > 0$, $u_{lt} > 0$, $u_{cct} < 0$, $u_{llt} < 0$
  - **Inada conditions on both *c* and *l***
- **Sometimes more convenient to represent as** $u(c_t, n_t)$
  - $u_{ct} > 0$, $u_{nt} < 0$, $u_{cct} < 0$, $u_{nnt} > 0$ **(strictly decreasing and convex in *n*)**

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

- **Intertemporal optimization problem**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^S) \quad \textbf{subject to} \quad c_t + a_t = w_t n_t^S + (1+r_t) a_{t-1}$$

  - **Individual takes as given $\{w_t, r_t\}_{t=0,1,2,\ldots}$ -- price-taker in labor market**
    - **From perspective of individual, (w,r) evolve as Markov**
  - **Notation n^S emphasizes individual's supply of labor**

- **Recursive representation**
  - **State vector in arbitrary period t: $[a_{t-1}; w_t, r_t]$**

Numeraire object: consumption

$$V(a_{t-1}; w_t, r_t) = \max_{\{c_t, n_t^S, a_t\}} \left\{ u(c_t, n_t^s) + \beta E_t V(a_t; w_{t+1}, r_{t+1}) \right\}$$

$$\textbf{subject to} \quad c_t + a_t = w_t n_t^S + (1+r_t) a_{t-1}$$

- **FOCs**

c_t **:**

n^S_t **:**

Labor Supply

- **Intertemporal optimization problem**

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- **FOCs**

$$c_t: \quad u_{ct} - \lambda_t = 0$$

$$n^S_t: \quad u_{nt} + \lambda_t w_t = 0$$

$$-\frac{u_n(c_t, n_t^S)}{u_c(c_t, n_t^S)} = w_t$$

CONSUMPTION-LEISURE OPTIMALITY CONDITION

A *static* condition

LABOR SUPPLY

$$-\frac{u_n(c_t, n_t^S)}{u_c(c_t, n_t^S)} = w_t \qquad \Rightarrow \qquad n_t^S = n^S(w_t; c_t)$$

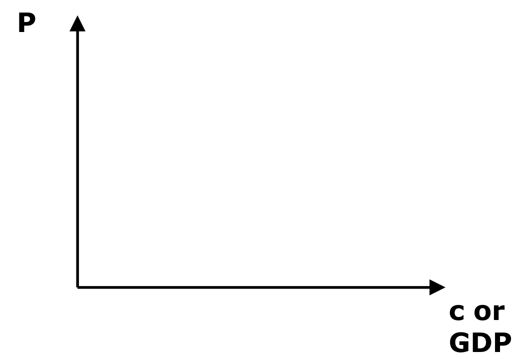
- **Consumption-leisure (aka consumption-labor) optimality condition**
  - **An intratemporal optimality condition**

- **Defines period-*t* labor supply function**
  - **For given individual…**
  - **…but if representative agent, equivalent to aggregate labor supply**
  - **Note: for given *c***

- **Example:** $u(c,n) = \ln c - \frac{\theta}{1+1/\psi} n^{1+1/\psi}$
  - **Compute labor supply function?**
  - **Compute elasticity of n^S_t with respect to w_t?**
    Frisch elasticity of labor supply

The Three Macro (Aggregate) Markets

- **Goods Markets**



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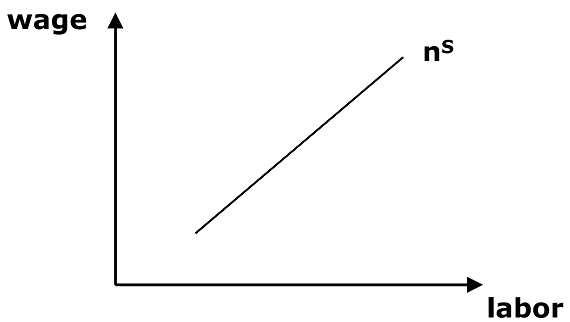

- **Capital/Savings/Funds/Asset Markets (aka Financial Markets)**

PRODUCTION OF GOODS

- **Representative firm** produces the numeraire output good of the economy
- A homogenous output good
- Perfect competition in goods supply
- Inputs
  - Labor
  - Capital
    - E.g., machines, factories, computers, intangibles, …

- Firm produces using a (aggregate) production technology

$$y_t = z_t \cdot f(k_t^D, n_t^D)$$

  - k^D the firm's capital demand
  - n^D the firm's labor demand
  - $f(.)$ often assumed CRS (Cobb-Douglas, in particular)
  - z_t a process that shifts the production function

- Empirically identify z_t as Solow residual
  - Growth theory: z deterministic
  - Business cycle theory: z stochastic (Markov)

PRODUCTION OF GOODS

- **Representative firm profit maximization**
  - **Price taker in capital market, labor market, and output market**
  - **Baseline model(s)**
    - **Firm hires/rents labor and capital each period**
    - **Firm does not "own" any capital or labor (without loss of generality if no financial market imperfections)**

$$\max_{n_t^D, k_t^D} \left(z_t f(k_t^D, n_t^D) - w_t n_t^D - r_t^k k_t^D \right)$$

- **FOCs**

***n^D_t* :** $z_t f_n(k_t^D, n_t^D) - w_t = 0$

***k^D_t* :** $z_t f_k(k_t^D, n_t^D) - r_t^k = 0$

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- **FOCs**

n^D_t **:** $z_t f_n(k_t^D, n_t^D) - w_t = 0$ **DEFINES labor demand function** $n^D(w_t)$

k^D_t **:** $z_t f_k(k_t^D, n_t^D) - r_t^k = 0$ **DEFINES capital demand function** $k^D(r^k_t)$

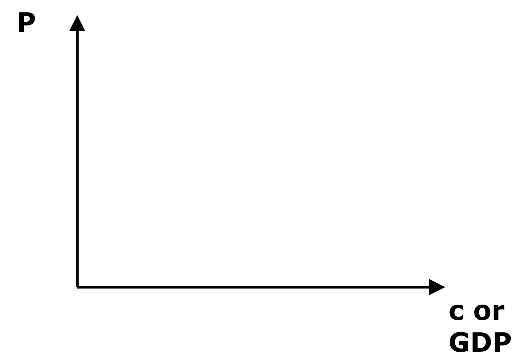
For a given firm

If rep. firm, equivalent to aggregate factor demands

- **Firms entirely static entities in baseline macro model(s)**
  - **Contrast with consumers**
  - **(NK theory and matching theory: firms are dynamic entities)**

THE THREE MACRO (AGGREGATE) MARKETS

- **Goods Markets**



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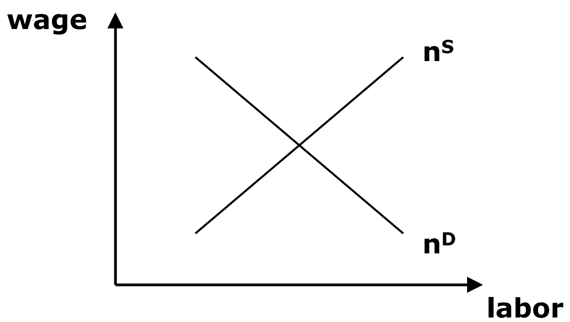

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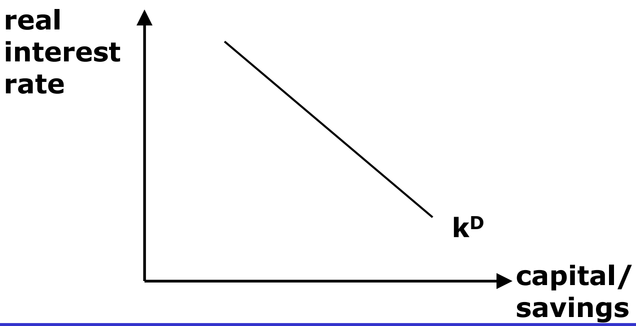

CAPITAL SUPPLY

- **Baseline model(s)**
  - **Physical capital takes "time to build"**
    - **Simplest: one-period lag between building and using capital**
  - **Closed economy**
    - **Aggregate capital demand must be supplied domestically**

- **Consumer intertemporal optimization problem**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^S) \quad \textbf{subject to} \quad c_t + a_t = w_t n_t^S + (1+r_t)a_{t-1}$$

  - **a_{t-1} is a given individual's pre-determined stock of assets**
  - **Representative agent: a_{t-1} is economy's pre-determined stock of assets**

- **Capital-market clearing in each period t**

$$k_t^D = a_{t-1} \left(= k_t^S \right)$$

THE THREE MACRO (AGGREGATE) MARKETS

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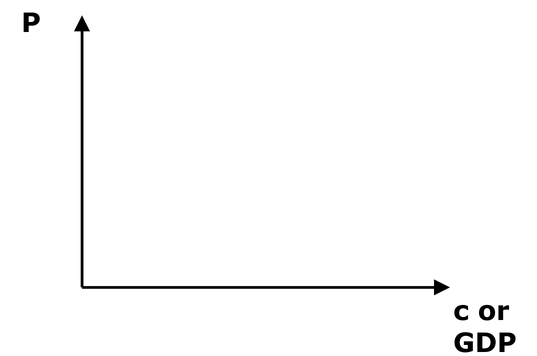

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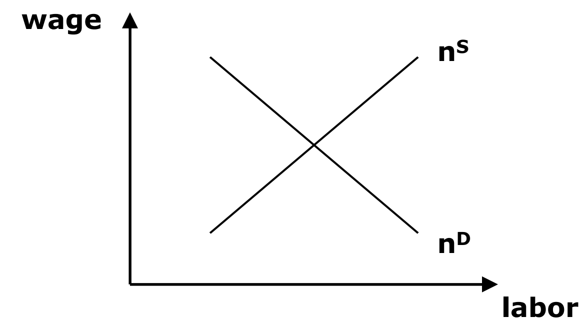

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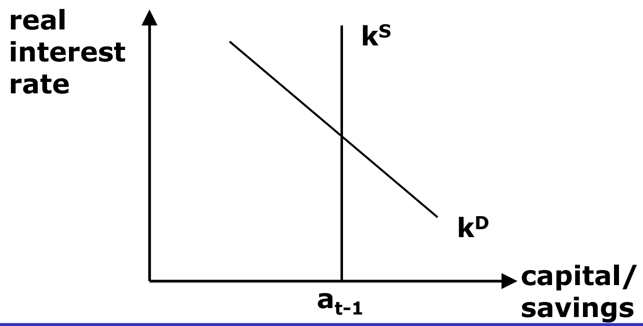

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- Capital-market clearing in each period t

$$k_t^D = a_{t-1} \left(= k_t^S \right)$$

- Capital depreciates at rate δ each period
  - Economic depreciation, due to physical wear and tear of production
  - Not accounting depreciation
  - Compensation reflected in capital-market-clearing price: $r_t = r^k_t - \delta$

CAPITAL SUPPLY

- **Capital depreciates at rate *δ* each period**
  - **Compensation reflected in capital-market-clearing price: $r_t = r^k_t - \delta$**
- **Implies capital supply has to be periodically replenished**
  - **From where?**

- **Consumer intertemporal optimization problem**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t^S) \text{ subject to } c_t + a_t = w_t n_t^S + (1 + r_t) a_{t-1}$$

- **Euler equation**

$$u'(c_t) = \beta E_t \left\{ u'(c_{t+1})(1 + r_{t+1}^k - \delta) \right\}$$

  - **From perspective of single individual: characterizes optimal savings (flow!) decision between *t* and *t*+1**
  - **From perspective of entire economy: characterizes optimal investment (flow!) in capital stock between *t* and *t*+1**

- **Closed economy: domestic savings = domestic investment**

- **Note timing: savings/investment decisions in *t* alter the available capital stock in period *t*+1 ("time to build")**

TOWARDS DYNAMIC GENERAL EQUILIBRIUM

- Round out final details
- Baseline model(s)
  - Consumption goods and capital goods are freely interchangeable
  - i.e., capital good in a given period can be "dismantled" and used for consumption in future periods
  - No irreversibility of investment process
  - Implies relative price (not interest rate…) of capital = 1

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- **CRS production process *f*(k,n), firms earn profits = …?…**
  - **Corollary: factors of production are paid their …?…**

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- **Labor-market clearing**

  n_t **defined as** $n^D_t = n^S_t$, **for all t (with clearing price** w_t**)**

- **Capital-market clearing**

  k_t **defined as** $k^D_t = k^S_t$, **for all t (with clearing price** r^k_t**)**

- **Goods market clearing**

  $c_t + k_{t+1} - (1-\delta)k_t = z_t f(k_t,n_t)$, **for all t (with clearing price = …?…)**

DYNAMIC GENERAL EQUILIBRIUM

- **Economy-wide state vector in period t: (k_t; z_t)**
- **Consider $T \rightarrow$ infinity**

- **<u>Definition:</u> a dynamic stochastic general equilibrium is time-invariant state-contingent price functions $w(k_t; z_t)$, $r^k(k_t; z_t)$ and state-contingent consumption, labor, and (one-period-ahead) capital decision rules $c(k_t; z_t)$, $n(k_t; z_t)$, and $k(k_t; z_t)$ that jointly satisfy the following:**

DYNAMIC GENERAL EQUILIBRIUM

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  1. (Consumer optimality) Given $w(k_t; z_t)$, $r^k(k_t; z_t)$, the functions $c(k_t; z_t)$, $n(k_t; z_t)$, and $k(k_t; z_t)$ solve the Euler equation (replaced by TVC as $T \rightarrow$ infinity), labor supply function, and flow budget constraint of the representative consumer

  2. (Firm optimality) Given $w(k_t; z_t)$, $r^k(k_t; z_t)$, the function $n(k_t; z_t)$ satisfies the labor demand function and k_t satisfies the capital demand function

  3. (Markets clear)
     - Labor-market clearing

       $n(k_t; z_t)$ defined as $n^D{}_t = n^S{}_t$, for all t
     - Capital-market clearing

       k_t defined as $k^D{}_t = k^S{}_t$, for all t
     - Goods market clearing

       $$c(k_t; z_t) + k(k_t; z_t) - (1-\delta)k_t = z_t f(k_t, n(k_t; z_t)), \text{ for all t}$$

given the initial capital stock k_0 and (Markov) transition process for $z_t \rightarrow z_{t+1}$